# Hardening Soil Model - Influence of Plasticity Index on Unloading - Reloading Modulus

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**Abstract.** Numerical analyses based on Finite Element Method (FEM) to solve geotechnical problems are carried out using software implementing various soil models. Currently, Hardening Soil Model (HS) is often used in the geotechnical engineering practice. One of the parameters describing the model is the unloading-reloading ($E_{ur}$) modulus. This paper presents triaxial test results of cohesive soils aimed at determination of unloading-reloading ($E_{ur}$) modulus. Based on test results, empirical correlations for evaluation of $E_{ur}$ modulus were determined. The correlations determine $E_{ur}$ modulus on the basis of plasticity index and vertical effective stress.

**Keywords:** Hardening Soil Model · Triaxial tests
Unloading-Reloading modulus

## 1 Introduction

Hardening Soil Model (HS) was formulated in 1999 by Schanz et al. [4]. Its modification, which includes elastic behaviour of soil for the small strain (HSS) was presented by Benz [1]. Both models are among the most popular constitutive models used in the geotechnical practice.

Strain parameters of the model are defined in Fig. 1. The manual [3] and publication [2] listed in the bibliography below, proposed the relationship between $E_{ur}$ and $E_{50}$ as $E_{ur}/E_{50} = 3$. However, Obrzud [2] and Truty [5] noted that $E_{ur}/E_{50}$ ratio should be higher.

Below an empirical equation for $E_{ur}$ modulus calculation based on effective stress and plasticity index (PI) is proposed.


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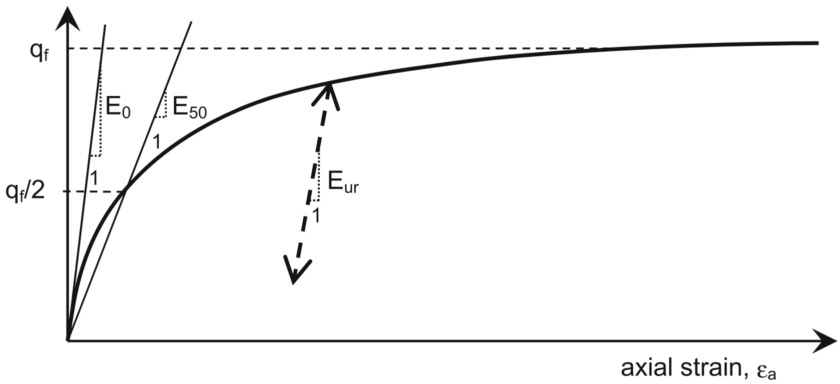


**Fig. 1.** Evaluation of $E_0$, $E_{ur}$ and $E_{50}$ based on stress-strain characteristics.

# 2 Test Methodology and Analyses

## 2.1 Triaxial Tests

The tests were carried out on undisturbed soil samples in GEOTEKO's laboratory. The triaxial tests included the following stages: back pressure saturation, isotropic consolidation and strain controlled drained shearing along standard stress path i.e. with constant cell pressure and increasing vertical stress.

## 2.2 Analyses

The analyses were performed using the results of triaxial tests carried out in terms of effective stress of values higher than *in situ* stress. Below, Fig. 2 shows a relationship

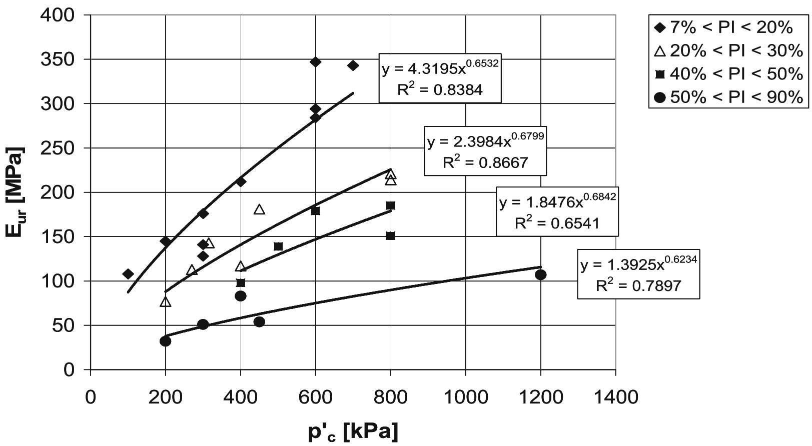


**Fig. 2.** Eur vs. mean effective stress at the end of consolidation stage.

between $E_{ur}$ modulus and mean effective stress at the end of consolidation stage ($p'_c$) taking into account plasticity index value.

The relationship between $E_{ur}$ and $p'_c$ can be described by the following general formula:

$$E_{ur} = a \cdot p_c'^{n} \quad (1)$$

Parameter (a) in Eq. (1) changes together with plasticity index variations. Figure 3 shows the relationship between parameter (a) and plasticity index.

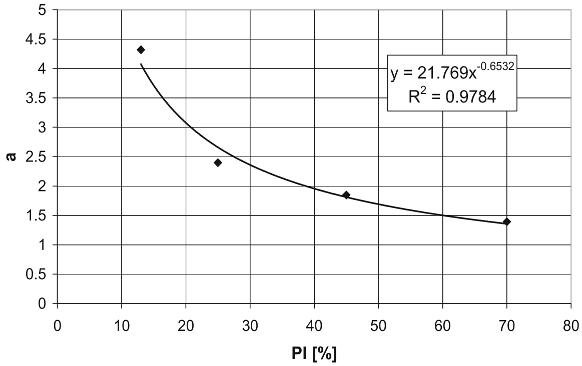


**Fig. 3.** "a" vs. $I_p$

Based on the above, $E_{ur}$ moduli values can be calculated from the following formula:

$$E_{ur} = 21.769 \cdot PI^{-0.65} \cdot p_c'^{0.65} \quad (2)$$

Using the proposed empirical formula, $E_{ur}$ moduli were determined and compared with the relevant values resulting from the laboratory tests (Fig. 4). In conclusion one may declare that a significant part of $E_{ur}$ value determined by empirical formula is within ±30% of $E_{ur}$ value determined in the laboratory tests. Thus it seems to form a better approximation than the ratio $E_{ur}/E_{50} = 3$ which was mentioned earlier.

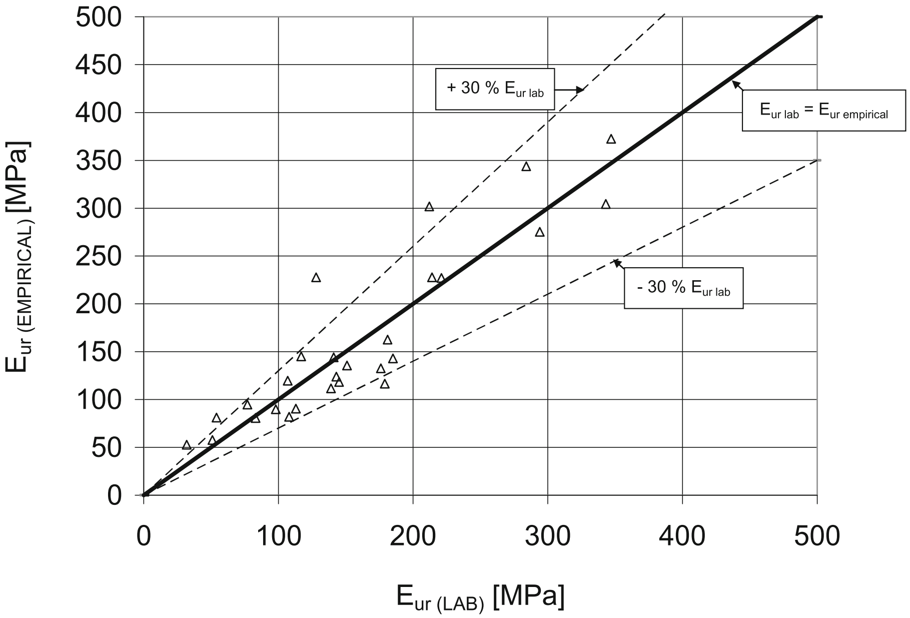


**Fig. 4.** Eur values from empirical formula and from the laboratory tests

## 3 Conclusions

The following conclusions can be drawn on the basis of the performed tests and analyses:

- as soil plasticity index increases, the influence of effective stress on $E_{ur}$ value becomes smaller.
- the best correlation between $E_{ur}$ modulus and effective stress was obtained for soils of plasticity index smaller than 30%.
- the proposed empirical relationship allows calculating $E_{ur}$ modulus on the basis of plasticity index (PI) and average effective stress ($p'_c$) values. A significant part of $E_{ur}$ value determined from the proposed empirical formula is included within the range of ±30% of $E_{ur}$ value determined in the laboratory tests.

The analyses presented in the paper were carried out within R&D activity of Geoteko Geotechnical Consultants Ltd. The a/m works resulted also in the proposal of other empirical formulas for estimation of geotechnical parameters. They will be presented in the next papers to be published.

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