Chapter 7
The *Uaratio* and Its Possible Use in Roman Urban Planning to Obtain Astronomical Orientations

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Abstract Several works have tried either to demonstrate or reject the notion that the orientation of the main axis of a Roman city was deliberate since its choice might add an extra sacred dimension to the entire urban space [González-García et al. (Mediterranean Archaeology and Archaeometry 14(3):107–119, 2014; Magli (Oxford Journal of Archaeology 21(6):63–71, 2008)]. There exist ancient texts that support the hypothesis of the existence of astronomical orientations, such as those of Frontinus (*De Agrimmensura*, 27) or Hyginus Gromaticus (*Constitutio*, I). In the case that these precepts were fulfilled: how to achieve it? Besides the astronomical hypothesis, some scholars have pointed to the use of a geometrical technique: the *uaratio* (Orfila et al. La orientación de las estructuras ortogonales de nueva planta en época romana. De la varatio y sus variaciones. 2014). By this, the short sides of a regular triangle that are in ratios of integer numbers (for example 1:2, 2:3) are laid along the cardinal axes. In this work we present a comparison of the orientation of 81 Roman towns in the Iberian Peninsula, measured in situ, with *uaratio* angles with aspect ratios up to 12:12. By this exercise we want to discern whether the orientations were astronomical, purely geometrical, or if geometry could have fostered astronomical aims by using selected and well-known angles to trace lines that fitted the desired astronomical purposes. It is then, an attempt to shed more light to the issue of the orientation of Roman towns by combining two hypotheses that, in contrast to what it might seem, could be complementary but not contrary.

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Introduction

Roman society is characterised, among other factors, by the incorporation of norms used in daily life in measuring units, in time reckoning and in modular division when constructing buildings. These norms also were applied during the division and careful planning of conquered or confiscated territory.

At that time, land division played a key role in the expansionist process and was essential in reorganizing the territory after a conquest, when new structures, such as towns, were erected. The foundation or re-foundation of towns involved important decisions about their spatial organization and orientation (Orfila Pons et al. 2017a). Furthermore, Romans accepted the idea of well-planned cities, and urban organization was applied with common standards and objectives. Importantly, it provided a reliable means of calculating the taxes that should be collected per each plot of land (Gilman Romano 2003).

Land division was carried out by a number of individuals: the *curator operis*, who coordinated the project, the *architectus* or *mensor* who defined the plan, and the *gromatici* or *agrimensores,* who laid out the divisions. They also measured irregular plots, understood the complexities of land law, and by the Late Empire even became judges in land disputes (Lewis 2001). Each plot of land, or *centuria*, was demarcated by *limites* or roads at right angles known as *decumani* and *cardine* that, ideally, should run east-west and north-south, respectively. However, previous studies have revealed that cardinal orientation was not a common trend (González-García and Magli 2015; González-García et al. 2014; Orfila et al. 2014).

Besides the practical component, a symbolic aspect should not be dismissed, since the creation or a new *territorium* contained an ideological factor (Castillo Pascual 1996). The foundation of new urban entities was accompanied by the performance of a ritual that might have influenced the selection of the spatial disposition of the main axes (Rykwert 1988). In this sense, by considering a combination of practicality, adaptation to the terrain conditions and symbolism, an unavoidable question arises about the motivations that were used in selecting the orientation of Roman urban grids.

Although some of the first publications on this topic (Le Gall 1975; Peterson 2007) refused to accept the existence of any intentionality behind the orientation of Roman towns, a number of later studies suggested the presence of well-defined patterns of orientation in different regions across the Roman Empire (Orfila et al. 2014; Richardson 2005). Furthermore, astronomy-based explanations for the orientation of many of the settlements in the studied samples were proposed by a number of authors (González-García and Costa Ferrer 2011; González-García et al. 2014, 2015). If this was so, how did Roman surveyors achieve these astronomical designs?

In this work we propose a geometrical method that Roman topographers might have applied to urban layout in order to set the direction of the streets according to some celestial events. It consists of the application of a geometrical technique used in Roman times and explained by the ancient writer Nypsius: the *uaratio*. Basically, this technique is based on the use of right-angled triangles where the length of its short sides are in ratios of integer numbers or where the lengths of the three sides are in ratios of integer numbers. These last are called Pythagorean triangles because their sides form a Pythagorean triple (like 3:4:5 for instance, that obey the Pythagoras Theorem).

The possible use of a standardized geometrical method to determine the orientation of Roman towns has been explored previously (Orfila 2012; Orfila and Moranta 2001; Orfila et al. 2014; Orfila Pons et al. 2017b). In order to test this hypothesis, we compared the orientations of 76 Roman settlements in the Iberian Peninsula, which were measured in situ, with a set of *uaratio* angles. This test was complemented by the results of practical experiments with reconstructions of instrumentation used by the *agrimensores*, such as the *groma, metae* and a *gnomon*, developed by the SOTOER group at Universidad de Granada (Costa and Orfila 2014; Orfila et al. 2014).

If the astronomical and the geometrical hypotheses converge, we might have found a relatively simple technique that worked as a standard procedure in the various processes of expansion (Adams 1982).

The *Uaratio*: A Proposal for Its Implementation

The *uaratio* was a technique used by Roman topographers, and was described by Marcus Iunius Nypsius in the second century CE (*Fluminis uaratio*; *Limitis Repositio*: La. 285.1–295.15, Bouma 1993). Its application allowed the identification of the *limites* of the land lots assigned to each community. It was also useful in determining the orientation of every orthogonal project when the land lots were not cardinally oriented (Orfila Pons et al. 2017b; Roth-Congés 1996). These interpretations are included in the *Corpus* of the *Gromatici Veteres*, which Chouquer (2004) interpreted and associated with the cadastre (land registry) that was made after the fire at the Roman *archivum* in CE 80, under Vespasian. In this, the location and limits of the different pieces of land was recorded.

On the basis that every architectural endeavour must start from a primordial line, we assume that this line was the meridian, as proposed by some scholars (see e.g. Orfila and Moranta 2001; Orfila et al. 2014; Orfila Pons et al. 2017b). The meridian line does not depend on the site location, and it is relatively easy to obtain by using a gnomon. This procedure is described by ancient authors such as Vitruvius (*De Architectura* I, 6.6) and Hyginus Gromaticus (*Constitutio* I, 52.4–22). As mentioned above, in most cases the urban axes were not cardinally oriented but were at different angles to the meridian line. Our proposal is that this may have originated from the use of the *uaratio*, as described below.

We follow Orfila et al. (2014) and consider two possible ways that this technique could have been applied. The first one consists of tracing the sides of a right-angled triangle, whose lengths present integer ratios, on the cardinal axes previously obtained with a gnomon, as shown in Fig. 7.1(3ab). The ratios are called *uariationes*. The hypotenuse of the resulting triangle defines the direction of one of the streets, and

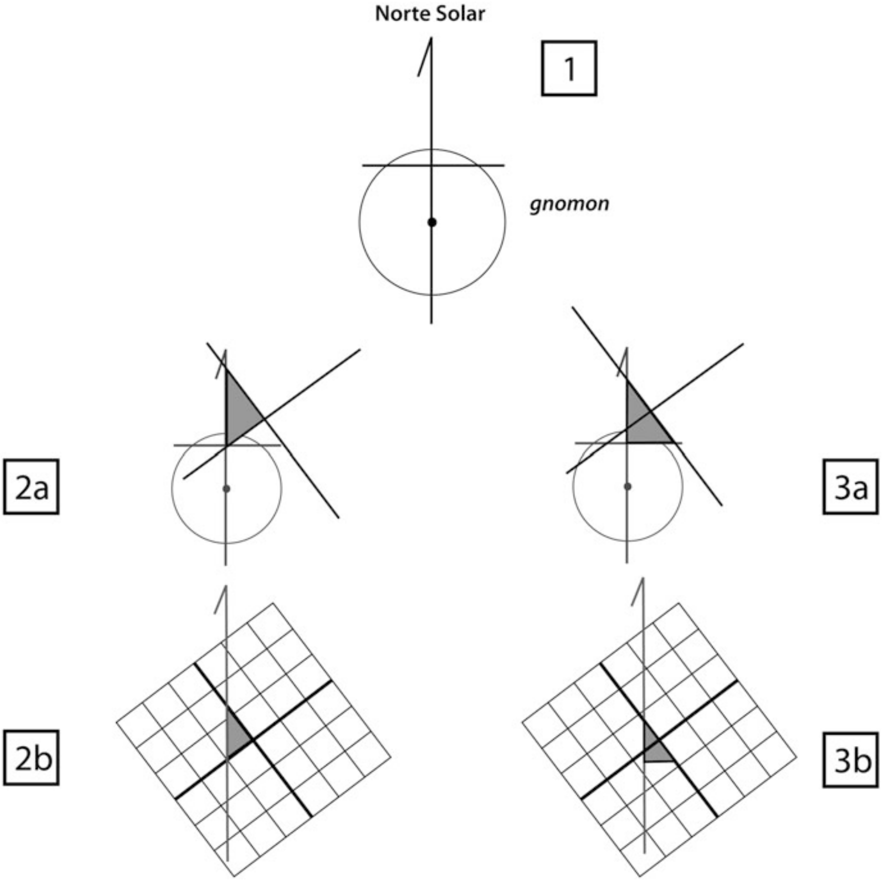

Fig. 7.1 The two hypothetical procedures for the application of the *uaratio* to obtain the direction of a new urban grid (adapted from Orfila et al. 2014)

that would depend on the ratio chosen for the triangle sides (1:2, 2:3, 3:5, etc). The perpendicular street could have been obtained by using a *groma*.[1]

Alternatively, this technique can be used by considering right-angled triangles whose sides are Pythagorean triples.[2] Our proposal is that the hypotenuse was drawn along the meridian line and the sides of the triangle, both with lengths that used integer numbers, would define the direction of the urban grid as seen in Fig. 7.1(2ab) (see Orfila et al. 2014). This idea was first proposed by Margarita Orfila and Luis Moranta (2001) for the lowest Pythagorean triples of 3:4:5 and 5:12:13. In addition,

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[1] This is an instrument based on a horizontal cross with arms in right angles, on a vertical support. From the end of each of the four arms hung a cord or plum-line tensioned by a bob. It was used for sighting and setting out straight lines and right angles (Lewis 2001).

[2] These have three positive integers (a, b, c) such that $a^2 + b^2 = c^2$.

the fact that the use of fractions to obtain irrational numbers was common in Antiquity (Neugebauer 1969; Peterson 1992) constitutes further motivation to explore their possible application in determining astronomical orientations of *centuriae* and urban grids.

Sample and Methodology

The first step was to test whether the *uaratio* was present in the city plans. To accomplish this, we compared the azimuth of the orientation of a number of Roman cities in the Iberian Peninsula with the distribution of *uaratio* angles.

The sample contains the azimuths of the main streets of 76 Roman towns in ancient *Hispania*, corresponding to 81 orientations (Fig. 7.2) (Rodríguez-Antón 2017). This is due to the presence of more than one urban grid in some sites, such as *Italica* or *Corduba*. The data were collected in situ during several fieldwork campaigns (Rodríguez-Antón 2017). The altitude of the horizon in the direction marked by each city grid was also measured in order to compute the astronomical declination of a hypothetical object that would rise or set at that specific point on the horizon.

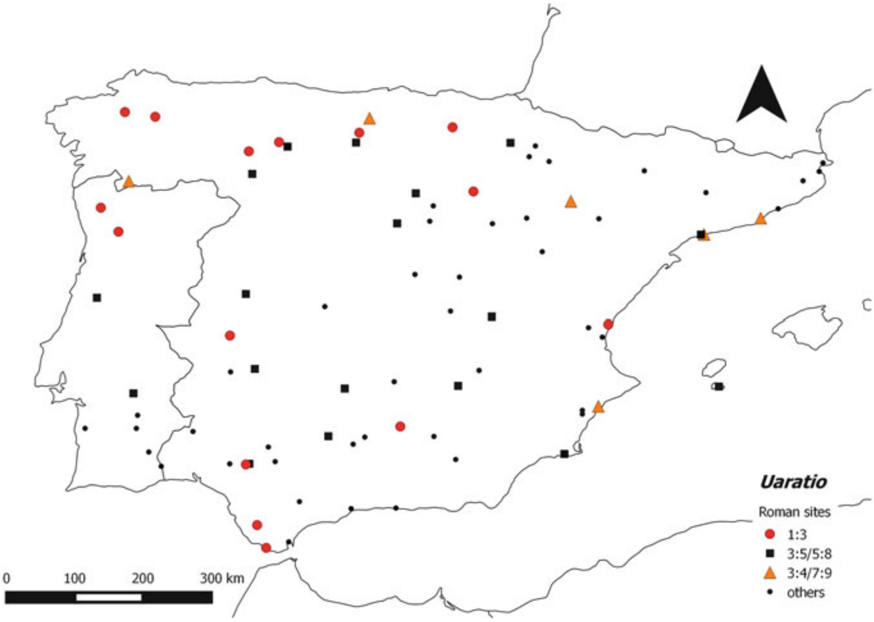

Fig. 7.2 Map showing the Roman cities studied, classified by the azimuth groups present in the sample that fit low *uariationes*

We used a professional compass to obtain the azimuths and a clinometer to measure the altitudes of the horizon. These instruments introduce nominal errors of ±¼° and ±½° respectively. However, azimuth errors were sometimes larger owing to external factors, such as the preservation state of the structures under consideration. The measured azimuths were corrected for magnetic declination, which was either estimated from the model WMM2012, or updated ones, available at: https://www.ngdc.noaa.gov/geomag-web/#declination. When altitudes could not be measured due to blocked horizons, we used reconstructions of the horizon from the Digital Terrain Model SRTM (Shuttle Radar Topography Mission) of NASA through the on-line panorama generator HeywhatsThat, available at: http://www.heywhatsthat.com/.

The comparison sample is a dataset of angles derived from the application of the *uaratio* as explained in the previous section. We considered *uariationes* ranging from 1:1 to 12:12. Since equivalent fractions correspond to equal angles, these were considered only once. That is, we did not include the angles resulting from, e.g., ratios 1:3, 2:6 and 3:9, but just those of 1:3 (Fig. 7.3). Figure 7.4a shows the computed distribution of the *uariationes* computed from 1:1 to 12:12 across the entire horizon.

A density distribution with an Epanechnikov kernel has been used to calculate the histograms in the present work. The bandwidth in each case depends on the sample size and the errors in the measurements, so that fine details can be appreciated while avoiding the over-smoothing of the distribution (González-García and Šprajc 2016). The value of the bandwidth is indicated in each particular case.

In the next section we will compare *uaratio* angles with azimuths (Fig. 7.4b), and angles bearing from north in the sample from *Hispania* with possible *uariationes* and angles from triangles in relation of Pythagorean triples (Fig. 7.5). Finally, we will compute the declinations of the orientations of the sample from *Hispania* to test whether the main values fit the use of simple *uariationes* (Fig. 7.6).

Testing the *Uaratio* on Roman Settlements

Due to the orthogonality of the street grids, the azimuth distribution, as well as the *uaratio* angles, are repeated four times along the 360° of the horizon (Fig. 7.4a). This allowed us to limit the analysis to a 90° wide sector in order to better appreciate the coincidence or divergences of both angle distributions (Fig. 7.4b). Due to the azimuth error, in general estimated in ±0.5°, and the sample size (81 measurements) the bandwidth in the histograms in Fig. 7.4 is 1°. We established a margin of ±2° to consider that a ratio of *uaratio* fitted a city orientation. For that we took into account the intrinsic error of the azimuths and the irregularities that could have derived during the tracing of the meridian line and the triangles on the ground.

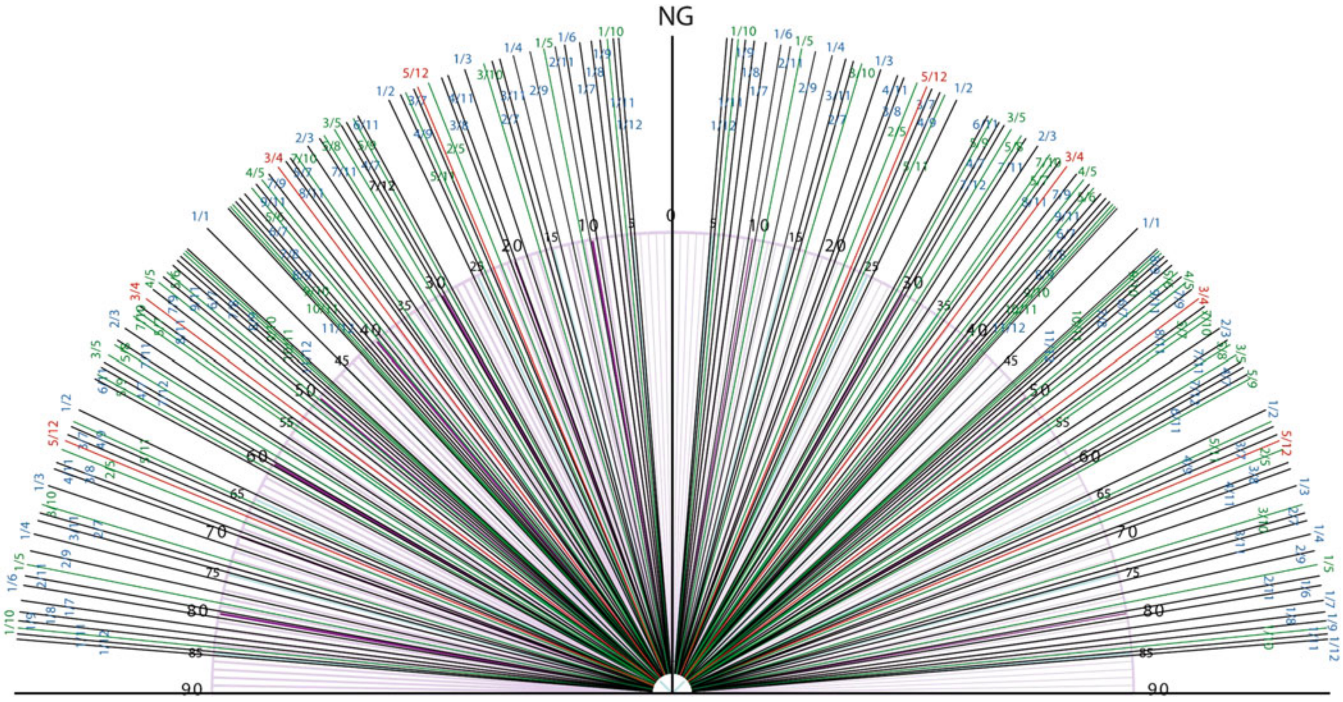

Fig. 7.3 Possible *uariationes* by using fractions from 1:1 to 12:12. 47 orientations per sector of 45°. Pythagorean uariationes in red (adapted from Orfila et al. 2014)

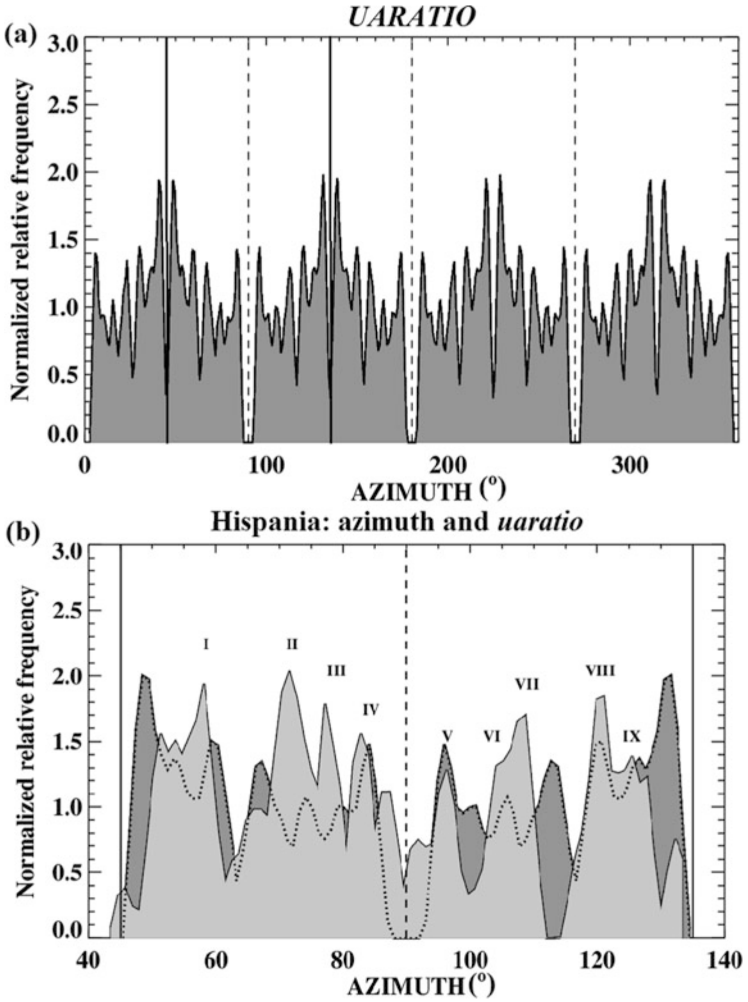

Fig. 7.4 (**a**) Distribution of *uaratio* angles computed from *uariationes* 1:1 to 12:12. Equivalent proportions are considered just once. Dashed vertical lines indicate the cardinal points and solid vertical lines the interval of 90° wide [45°, 135°] chosen for (**b**). (**b**) Distribution of *uaratio* angles from the interval indicated in (**a**) within vertical solid lines (dark grey) and the azimuths of the *decumani* of the sample of Roman towns in *Hispania* (light grey). Both distributions are the same in the three remaining symmetrical divisions. Roman numbers indicate the azimuth groups of the sample, whose equivalent values are shown in Table 7.1; read the text for further details

The election of the azimuth sector [45°, 135°] is because we define a decumanus—the street that should run east-west—within that range and within its symmetrical interval towards the west [225°, 315°]. At latitudes of the Iberian Peninsula, both intervals comprise the azimuth sectors within which the Sun and the Moon move during a full solar or lunar cycle, so it is adequate to compare the azimuths of the *decumani* with solar positions. It is interesting to note that the azimuth distribution in Fig. 7.4b presents well-defined orientation groups, and that

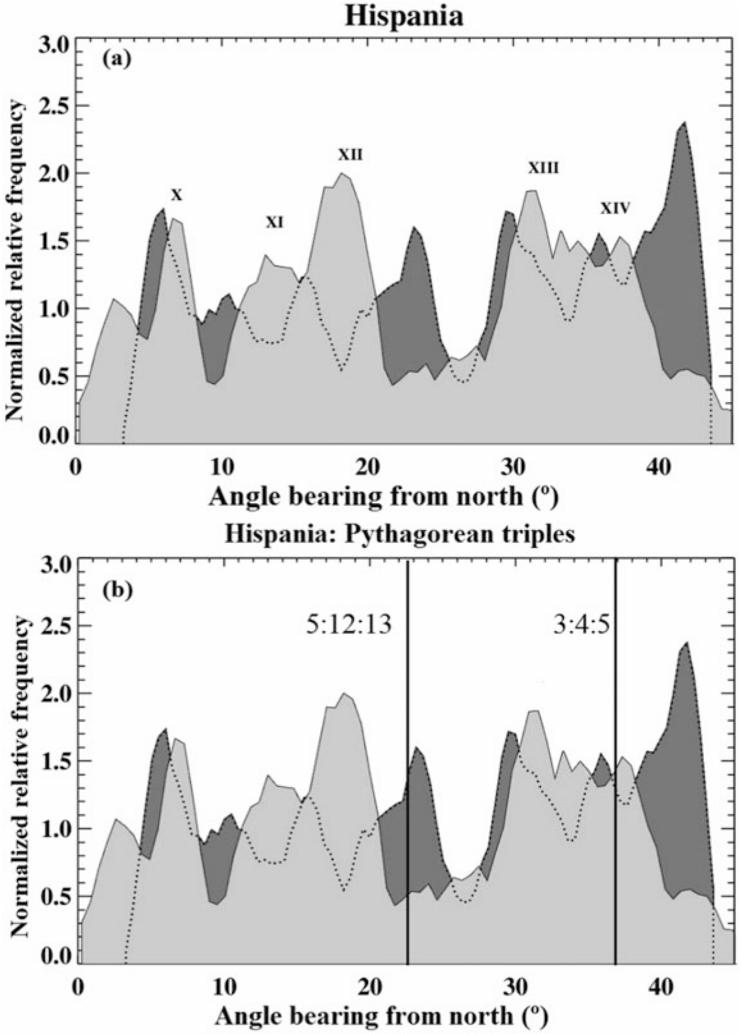

Fig. 7.5 Histograms with the orientation of the angles bearing from north of the cities in the sample in Hispania (light grey) and *uaratio* angles within an interval of [0°, 45°] (dark grey). Roman numbers in (**a**) are described in Table 7.1. Solid vertical lines in (**b**) indicate the equivalent angles by using Pythagorean triples to orientate towns

the main minima of the distributions of both datasets coincide. This means that there is a lack of cities oriented towards directions whose angles do not correspond to a *uaratio* proportion.

In order to further simplify the comparison, we took into account the symmetry of the *uaratio* angles within a 90° sector. That is, we considered symmetrical proportions together (e.g., 1:3 and 3:1 are considered to be equal). Since in this study

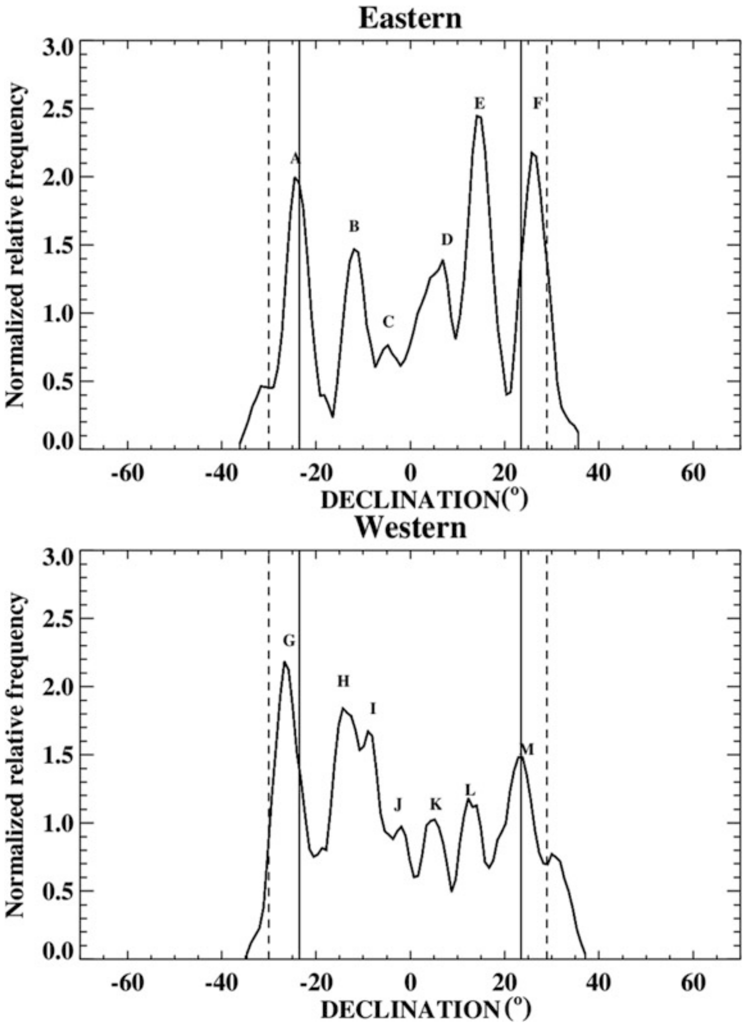

Fig. 7.6 Declination histograms of the *decumani* of the sample of Roman towns in Hispania, defined within azimuth sectors of [45°, 135°] towards the east and [225°, 315°] towards the west. Vertical solid lines represent the declination of the Sun at the solstices and dashed lines those of the Moon at its major lunastices

our main reference is the local meridian, we focused on the range [0°, 45°]. To take advantage of the mentioned symmetry, we converted the azimuths in the sector [45°, 90°] to the range [0°, 45°] by considering their value relating to 45° (called Φ) and using the formula $\alpha = 45° - \Phi$. Now α is the symmetric angle to Φ bearing from 45° (Fig. 7.5). Angles in the range of [0°, 45°] were left unmodified. We stress that with this choice in our analysis now concerns the application of particular proportions rather than the orientation of the town. The angles of the maxima of Fig. 7.5a and the

Table 7.1 Values for angles of the maxima in the histogram in Fig. 7.5a, *uariationes* that fit these angles, and in the last column equivalent peaks from Fig. 7.4b

		Uaratio		Equivalent maxima
Maxima	A (°)	(°)	X:Y	
X	7	63	1:9	IV V
		8.1	1:7	
		7.1	1:8	
XI	12	11.3	1:5	III VI
		12.5	2:9	
XII	18	18.4	1:3	II/VII
XIII	31.5	30.5	7:12	I VIII
		30.9	3:5	
		32.0	5:8	
		32.5	7:11	
XIV	37	36.0	8:11	IX
		36.9	3:4	
		38.5	4:5	
		37.9	7:9	

possible *uariationes* that fit each azimuth are shown in Table 7.1. What is observed in the histograms in Fig. 7.5 is that symmetrical peaks relating to the cardinal points in Fig. 7.4b merge and give those wider maxima in Fig. 7.5 (Table 7.1).

To compute the histogram we used a density distribution with an Epanechnikov kernel with a bandwidth of ¾°, since now there are more elements within a smaller interval and it is convenient in order to better appreciate fine details in the distributions. Once again, there is a coincidence of the main minima of both distributions. Furthermore, it is interesting to note that three of those maxima (XII, XIII and XIV, Table 7.1) fit angles that derive from relatively simple *uariationes* such as 1:3 or 3:4.

We next tested whether city azimuths matched the angles resulting from implementing right-angled triangles with sides in relation to Pythagorean triples 3:4:5 and 5:12:13. The reason for using these triples is that these are the lowest ones; that is, the ones that represent the smallest fractions. Additionally, these ratios were often used in Antiquity (Gros 1976; Orfila 2014; Roskams 2001). As can be shown in Fig. 7.5b, there is a quite appreciable group of azimuths that approximate to the equivalent angles of using 3:4:5, while not many match the angles resulting from 5:12:13.

Finally, we conducted a Kolmogorov-Smirnov test to broaden our understanding of the results and to check whether both datasets were drawn from the same parent population. That is, if the azimuths of these groups of Roman settlements could have been obtained by using the *uaratio*. If the resulting probability was low enough, the idea that both samples had the same origin could be excluded. A probability of 0.05 or smaller is required to dismiss this hypothesis with a confidence level of 95%. In this case the test yields a probability of 0.14, so we cannot discard the possibility that both samples come from the same parent population with sufficient confidence level.

A Possible Link Between *Uariationes* and Astronomical Positions

As seen above, the use of the *uaratio* to determine the orientation of new cities cannot be discarded. In case this was indeed a common practice, what could have motivated the election of specific angles?

Certain texts by ancient topographers such as Frontinus (*De Agrimensura*, 27) and Hyginus Gromaticus (*Consitutio* 1) contain practical recipes for land demarcation and surveys (Gilman Romano 2003). In particular, they indicate the necessity to follow the path of the Sun when orienting the decumanus of a town. The sacred character of this task and the performance of a ritual in the foundation of a new urban, or even military, space should not be underestimated either (Rykwert 1988), and give support to the astronomical hypothesis for the orientation of towns. Furthermore, a number of studies also suggest that the main streets of several Roman towns could have been laid out according to some celestial phenomena, both in the Iberian Peninsula (e.g. González-García and Costa Ferrer 2011; González-García et al. 2014, 2015) and in other regions of the Roman Empire (e.g. González-García and Magli 2015; Rodríguez-Antón et al. 2016a, b).

In this exercise we have selected the lowest *uariationes*, or fractions, from Table 7.1 whose equivalent angles fit maxima of the sample present in the diagram in Fig. 7.5 and calculated the resulting declination in the direction of the hypothetical *decumani* that bear azimuths equal to these *uaratio* angles. We considered a flat horizon ($h = 0°$) and latitudes of 40° and 42°, because these are approximately the mean values for the Iberian Peninsula and Rome respectively.

By this, our aim was to check whether astronomical orientations were obtained by the standardized application of specific *uariationes* when observations of astronomical rises and sets could not be observed directly. The idea of comparing the declinations for the latitude of Rome arises from the possibility that, in case the *uaratio* were actually used to point towards particular celestial objects, the *Urbs* might have been the reference places when the standards were stablished and later exported to different regions.

Assuming that the application of *uariationes* that involved low proportions would have been easier, and thus more common, we began this test by exploring the resulting declination values from the use of the simplest ratios that fitted some of the maxima of the azimuth in the sample. In particular, we calculated the declination of hypothetical *decumani* whose azimuths coincided with the corresponding angles of using the lowest *uariationes* in Table 7.1. From the peaks in Fig. 7.5, the orientation groups of the sample that derived from low *uaratio* proportions were XII (1:3), XIII (3:5) and XIV (3:4 or 4:5). Maxima X and XI coincide with angles that result from bigger proportions; maybe with the exception of 1:5 in XI.

It should be highlighted that because of the estimated error margin to differentiate an azimuth from a *uaratio* proportion (of ±2°), and the significant density of possible *uaratio* angles within each interval of 45° width, there is normally more than one fraction associated with each azimuth. In these cases we tend to assume that the lower the relation of the triangle sides, the more common the use of that fraction.

It must be stressed that not all the angles in Fig. 7.5 and Table 7.1 are real azimuths, but are angles bearing from north regardless of whether they were to the right or left of the meridian line. For this reason, and because our interest is to compare these values with those of the *decumani*, we have used the azimuths shown in Fig. 7.4b to compute the declinations, within [45°, 135°], which are equivalent to those shown in Fig. 7.5 and listed in Table 7.1.

For example, the value of maximum XII (Fig. 7.5a, Table 7.1), which is centred on 18° (α), approximates to that of the proportion 1:3. Due to the symmetry regarding cardinal points, this fraction comprises peaks II and VII in Fig. 7.4b, which derive from 3:1 and 1:3, respectively. In this way, the azimuths considered to relate to the declination were 72° ($90° - \alpha$) and 108° ($90° + \alpha$), that resulted in $\delta \approx \pm 13.5°$ at latitude 40° and $\delta \approx \pm 13.25°$ at latitude 42°. In the map shown in Fig. 7.2 there is a concentration of cities that bear these azimuths in northern *Hispania*, while the rest are spread in the south, with just one in the Levantine coast.

Following the same criteria, maximum XIII (Fig. 7.5a), which is centred on 31.5° (Table 7.1), is associated with simple proportions such as 3:5, as well as 5:8, 7:11 and 7:12, and comprises the symmetrical maxima I and VII in Fig. 7.4b. The resultant declinations are $\delta \approx \pm 23.5°$ at latitude 40° and $\delta \approx \pm 22.75°$ at latitude 42°. Sites with this orientation are spread throughout the entire Iberian territory.

The last maximum analysed is XIV (Fig. 7.5), whose angle can be associated with *uariationes* 3:4 and 4:5, which are relatively simple, as well as sides of the Pythagorean triple 3:4:5. The resultant declination does not fall within a solar range but in a lunar one. The declinations are now $\delta \approx \pm 27.25°$ at latitude 40° and $\delta \approx \pm 26.5°$ at latitude 42°.

As can be observed from the histograms in Fig. 7.6, all of the formerly mentioned declination values correspond to the most significant maxima present in the studied sample. The declinations of peak XII approximate maxima B, E, H and L in Fig. 7.6, whose values coincide with that of the Sun on last days of April and October. Peak XIII gives solstitial or almost solstitial declinations and, as already mentioned, the resultant values for peak XIV offer declinations out of the solar range. To represent the histograms in Fig. 7.6 we have used an Epanechnikov kernel with a bandwidth of 1.5°, following the same criteria that we used in Figs. 7.4 and 7.5. Interestingly, the *uariationes* that fit the values of maxima X and XI, which involve higher proportions than those of XII, XXX and XIV (Table 7.1), give the less significant declinations groups of Fig. 7.6: C, D, J and K.

Finally, the same procedure was followed with the triangles in relation to the lowest Pythagorean triples: 3:4:5 and 5:12:13 if they were used as proposed in Fig. 7.1(2ab); that is, if the hypotenuse was drawn along the meridian line. For the triple 3:4:5 the resultant declinations were $\delta \approx \pm 27.25°$ at latitude 40° and $\delta \approx \pm 26.5°$ at latitude 42°. In the case of using 5:12:13, the results were δ ≈ ±17° at latitude 40° and δ ≈ ±16.5° at 42°. As expected, values from 3:4:5 give the same declination as maximum XIV. The resulting declinations by using a triple 5:12:13 coincide with positions of the Sun on the first days of February and August and in mid-November and May.

Discussion

The fact that some azimuths in the sample give declinations for a flat horizon similar to the real ones in the Hispanic sample is not a surprise but something that one may expect. It is obvious as well that almost all the azimuths matched the *uaratio* angles, due to the high density of *uariationes* within an interval of 45° wide. However, the striking result is that the main declination groups of the sample (Fig. 7.6) coincide with significant dates when the Sun had azimuth values that resulted from the use of low *uariationes* (maxima XII, XIII and XIV). Furthermore, these declination groups are the same ones that contain the most significant astronomical relationships for the Roman context in *Hispania* (González-García et al. 2014).

In the first case, we observed that the fraction 1:3 (peak XII, Fig. 7.5) gives declinations at the studied latitudes that can be roughly associated with the position of the Sun on the day of the mythical foundation of Rome (21 April) or 1 May (as well as 1 November and 1 February for its negative value). In the case of 21 April, the connection with the Roman tradition is certainly obvious, but for the other dates, a reasonable explanation arises from pre-Roman traditions. In particular, these can be related to the so-called Celtic mid-season festivals (García Quintela and González-García 2017). This hypothesis makes sense for the group of cities in the Iberian northwest that bear azimuths from group XII (Fig. 7.5a), for being a region traditionally considered to have been inhabited by local societies with a Celtic substratum. In addition, similar results had been found in other regions with ancient Celtic traditions, such as *Galia* (García Quintela and González-García 2016; González-García and García Quintela 2014; González-García et al. 2016) or *Germania* (Espinosa Espinosa et al. 2016). We thus have found simple *uariationes* that relate to relevant astronomical orientations.

The second interesting result is that of peak XIII, which gives roughly solstitial declinations that are also present in the histograms in Fig. 7.6, and in the orientation patterns of Roman towns studied in other areas of the Empire (González-García and Magli 2015; González-García et al. 2018; Rodríguez-Antón et al. 2016b), that were explained by the symbolism of Augustus's propaganda, among other possibilities. In fact, 50% of towns that bear this azimuth have Augustan foundations or re-foundations: *Petavonium*, *Pisoraca*, *Valeria*, *Liberalitas Iulia* and the Imperial urban grids of *Corduba* and *Tarraco*. Once again, low proportions, such as the 3:5, give conspicuous astronomical patterns.

Finally, the use of the fraction 3:4 could have been applied to obtain orientations outside the solar range, like in the two maxima at δ ≈ ±27° present in the declination histogram (F, G in Fig. 7.6). One hypothesis connects this result with the Julio-

Claudian dynasty, because the goddess Venus was regarded as the ancestral mother of the *Gens Iulia*, which Cesar and Augustus belonged to. There are three cities from this period with an azimuth of 3:4:5: *Juliobriga*, *Cesaraugusta* and *Barcino,* all in northern *Hispania* (Fig. 7.2). Similar values were also obtained in some Roman towns in North Africa (Rodríguez-Antón et al. 2017). In this particular case, some were explained by positions of the Moon and Venus as well, having also *Uthina* and *Thuburbo Majus* Augustan foundations (see González-García et al. 2018). However, it is just an hypothesis and a deeper analysis of these values is needed.

It must be stressed that not all the cities grouped in a declination pattern have the same azimuth, since the horizon is not always flat. There are also sites oriented according to some of the listed *uaratio* angles, for example 3:5, that have different declination values from those predicted for the selected latitudes and flat horizons. One example is that of the Roman theatre at *Carthago Nova* (present-day Cartagena), where the *frons scenae* and *cavea* line follows a solstitial azimuth that could have been drawn by the application of proportion 3:5 elsewhere or transported from the forum of the city (González-García et al. 2015), but the structure itself does not point to the real sunrise at the summer solstice due to the local topography.

In particular, just 57% of the cities in the declination group E have an azimuth that could be obtained by using a 1:3 relation and 66% of those with a solstitial declination have an azimuth corresponding to a fraction 3:5.

In the case of the relation 3:4, it is present in six cities with almost solstitial orientations or close to the extreme declination of Venus as a morning star ($\delta \approx \pm 24°$), and in eight towns where the *decumani* point towards extremes of Venus as an evening star ($\delta \approx \pm 27°$).

As a matter of fact we find sites with astronomical declinations and with azimuths derived from low *uariationes,* but there are also cities with the same declination but different azimuth values. In this last case, orientations might have been obtained by direct observation of astronomical phenomena. One example would be the orientation of Via Appia, where it seems that the Romans chose the setting of the star Castor as the direction where the via should face instead of the use of a simple fraction (1/1) (Magli et al. 2014). In this scenario, it is difficult to discern in cases where:

1. Direct astronomical observations give similar azimuths.
2. The orientations considered as astronomical resulted from the application of specific *uariationes*.
3. Standardised relations between *uariationes* and astronomical positions did exist so those fractions were used when direct observations of the astronomical phenomena were not possible.

Conclusions

In this analysis we have explored whether a standardised geometrical technique was used by ancient Roman topographers to set specific directions of land plots and, in particular, of city grids. By this, we can enumerate a couple of main outcomes:

1. The coincidence of the main minima of both distributions, *uaratio* angles and azimuths of the sample, suggests that there is a lack of orientations towards directions that are not associated with a *uaratio* proportion. Furthermore, the result of the Kolmogorov-Smirnov test does not allow us to refute the null hypothesis that both datasets come from the same parent population. That is, it is not possible to discard the idea that the azimuths were obtained by the use of the *uaratio* with a sufficient level of confidence.
2. Secondly, the azimuth groups of the sample that can be obtained by the application of low *uariationes* (XII, XIII and XIV) and the Pythagorean triple 3:4:5 give declination values that coincide with the main declination groups of the sample as well as with interesting astronomical positions within a Roman context. That is, there is an agreement between simple *uariationes* and relevant astronomical orientations present in the sample in *Hispania*. Conversely, maxima X and XI fit the angles of higher *uariationes* such as 1:9 and 2:9, and currently the resulting declinations cannot be explained by the relevant positions of any celestial bodies.

In the light of this, although sometimes orientations seem to hint an astronomical use of the *uaratio*, to better understand these results, an analysis of a wider sample at different latitudes, or restricted to delimited areas, is needed. Besides this, it would be also interesting to examine more local or chronological conditions, in order to determine whether astronomical motivations may have been favoured over more practical ones.

To sum up, it could be said that currently we cannot refute the use of a geometrical technique such as the *uaratio* to explain the orientation of Roman rural and urban spaces. In fact, this procedure could have been something extremely useful to obtain the characteristic regular organization of Roman landscapes according to the sky, since only the determination of the meridian line would have been needed to repeat the same orientation across the different territories. Furthermore, if a connection between the *uariationes* and the positions of some celestial bodies did really exist, geometry and astronomy might have represented a combination of practicality and symbolism when direct observations of the celestial phenomena in question were not possible.

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