Descartes, Gödel and Kuhn: Epiphenomenalism Defines a Limit on Reductive Logic

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Abstract. René Descartes' enduring contribution to philosophy, natural science and mathematics includes the unresolved residue of Cartesian dualism, as well as a singular 'bottom-up' interpretation of reductive logic sustained within the modern structure of the reductive natural science paradigm. Application of strong reductive logic leads to the perplexing *reductive epiphenomenalism* proposition.

Kurt Gödel's two famous incompleteness theorems provide an argument through analogy, demonstrating that *reductive epiphenomenalism of consciousness* is a logical and demonstrably *true* 'bottom-up' reductive proposition; characterized by conceptual paradox that cannot be resolved from inside the modern reductive science paradigm using sustained singular 'bottom-up' reductive logic. The argument by analogy concludes reductive epiphenomenalism is an *undecidable reductive proposition* declaring strong reductive logic to be *fundamentally incomplete*.

Thomas Kuhn's historical conception of a scientific revolution and modern explorations of contextual paradigm adaptation do not include descriptions of a limit on reductive logic associated with *reductive incompleteness*. One analogous implication of reductive incompleteness is the potential for an *unresolvable* and *undecidable reductive proposition*, stated in the paradigm and strong logic of reductive science, to become a *resolvable* and *decidable reductive proposition* within a closely related meta-reductive paradigm, preserving strong reductive logic but employing slightly different assumptions and premises. This opens the door to exploring functional adaptation of the reductive paradigm with the creation of adjacent possible meta-reductive paradigms.

Adjacent possible meta-reductive paradigms responding to reductive incompleteness, may be able to more closely mimic Nature's inherent evolutionary logic, provide novel solutions to unresolved or anomalous reductive scientific problems, and clarify the relationship formal reductive incompleteness might have with the natural logic of evolving systems.

Keywords: Descartes · Reductive logic · Reductive science · Reductionism
Reductive epiphenomenalism of consciousness
Unresolvable logical and conceptual paradox · Gödel
Incompleteness theorems · Undecidable reductive propositions
Reductive incompleteness · Kuhn · Reductive paradigm
Meta-reductive paradigm

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1 René Descartes

René Descartes' enduring contribution to philosophy, natural science and mathematics includes the residue of Cartesian dualism associated with consciousness studies and the persistent application of a 'bottom-up' interpretation of reductive logic in modern science. The ongoing difficulty resolving the remaining brain/mind split may be directly related to Descartes' interpretation of reductive logic and the sustained application of his understanding within the modern structure of the reductive natural science paradigm.

René Descartes' initiated the consciousness debate with his famous pronouncement: *Cogito ergo sum* – "I think therefore I am." [21] Descartes' dictum declares sufficient, first-hand subjective knowledge of the existence of his own mind. Intangible and immaterial consciousness exists because he experiences it. Descartes' dualistic separation of fundamental substances, dividing *res extensa* (material substance that occupies space) from *res cogitans* (immaterial substance of the mind), ceded authority over immaterial spirit and mind to the Catholic Church and Inquisition, effectively excluding subjective consciousness and mind from the domain of scientific inquiry [80].

Descartes also suggested the material world must be approached with scientific skepticism, using objective observation and reductive analysis as the experimental and methodological tools for the nascent, historical natural sciences [3, 37]. Descartes' definition of *reduction* and his remarkably enduring initial formulation of reductive science appear first in his "*Discourse on the Method of Rightly Conducting the Reason, and Seeking Truth in the Sciences*". He composed four precepts, including, (1) *doubt*—scientific *doubt*; (2) *divide into parts*—*reduction*; (3) *ascend by little and little*—*synthesis*; and, (4) *enumeration*—declaring the essential importance of *mathematics* in science. Descartes envisioned a unified conception of a hierarchy of reductive natural sciences, bound together by the powerful and singular logic of reductive thought and the precise reasoning of mathematics [25]. A unified application of reductive logic metaphorically approaches phenomena with a 'downward' reductive focus, in order to decompose and understand the phenomena so that a subsequent 'bottom-up' synthesis can be achieved, often modeled with mathematical formalisms.

The underlying mechanisms or natural structures upon which any kind of unification of the sciences could be defined remains contentious [16]. However, in conformity with Descartes' solitary 'bottom-up' unifying interpretation of reductive logic, the modern natural sciences align themselves in a hierarchy from the most fundamental phenomena 'upward' toward more complex phenomena. Descartes' formulation of reductive science therefore survives in the effective modern reductive program, which proceeds in *three* steps: (1) *Reduction and Analysis*: Begin by taking apart a higher order phenomena into its disjoint elements and individually investigate these; (2) *Theoretical Formulation*: Using experimental evidence, imagination, and luck, formulate a model describing how the components relate and interact; (3) *Synthesis and Construction*: Using theory and experimental evidence, again compare the theoretical qualitative and quantitative success of the model with the experimental qualitative and quantitative behavior of the higher-level phenomena of interest, in order to demonstrate the scientific understanding of the phenomena is complete. Where possible then synthesize and construct the phenomena from its disjoint elements [70].

The application of reductive logic and the basic frame of the reductive science paradigm have not changed over four hundred years of vastly successful reductive exploration. With the exponential growth of scientific knowledge and increased understanding of cosmic evolution, biological evolution, self-organization and emergence in the hierarchy of complexity, it is worth reflecting on whether and under what circumstance, an adaptation of reductive logic and the structure of the reductive science paradigm, might better reflect the modern understanding of Nature.

2 Reduction

'*Reduction*' in modern science is associated with a reductionist claim that the logic of reductive thought and the epistemology of reductive science [79] mimic the ontology of Nature in specifiable ways [41]. The reductionist claim is justified by scientific evidence and the ongoing success of reductive science. The scientific reductive assertion states that the 'whole' can be *reduced* to the 'parts' constituting the 'whole', and the 'whole', including any emergent properties, can be fully accounted for by the 'parts', their 'interactions' and their 'relationships' [83]. The reductive understanding of '*fundamental'* is based on a belief that successful reduction of higher-order or more complex phenomena will ultimately arrive at an epistemological scientific description of an ontological substrate defining the most primitive components of the Universe [33]. In modern science, quantum physics is considered the most fundamental science describing the most fundamental and experimentally accessible entities, states and processes.

Nagel [57] composed an account of reduction as a kind of covering-law explanation, in which, one higher-level theory can be reduced to a second lower-level theory when it is possible to recognize that the theoretical terms of the first theory are related to or correspond to the theoretical terms of the second theory and it is possible to literally derive the first theory from the second. The reductive assumption that scientific disciplines and theories can correspond to one another in this way, allows reduction to serve as a framework for describing inter-level relationships and inter-level theories providing a route toward interdisciplinary integration. Nagel's account of covering theories suggests the inter-level relationship can be formally specified using reductive logic. The logic of the abstract relationship should not depend on either the content of the theories or the material structures the theories describe. Such clear and explicit logical and material correspondence between higher and lower theories is an abstract goal of reductive thought but is not that easy to achieve. This well-defined reductive goal has not, for instance, been accomplished in relation to the transition from quantum physics to relativity and Newtonian mechanics [50].

Reductive logic and the method of reductive science have been vastly successful, yet reductive epistemology does not quite capture Nature's ontological logic, as expressed in evolution or the self-organization of emergence in the hierarchy of complexity. Multiple lines of argument suggest reductive thought has limits and some authors offer partial remedial approaches to perceived shortcomings. In physics a broader view of emergence is suggested [51] or a more comprehensive narrative is recommended detailing the natural history of phenomena [82]. In evolutionary biology, beyond natural selection, an enhanced awareness of complex natural histories involving multiple intersecting causal factors is advocated [27]. In philosophy of mind [32] and the study of mind and consciousness [78], it is argued that there is something false, wrong or unfinished about the modern, reductive, materialist, neo-Darwinian scientific conception of Nature and consciousness. Further, it is difficult to articulate the boundary of the limitation in the absence of alternative conceptual frameworks [59].

Concern about the limits of reductive thought shape the mechanist perspective. The mechanist perspective offers an alternative to placing reduction at the conceptual center of natural science [22]. Mechanists criticize the conception that reduction should be assumed primarily to be a relationship between *theories*. For mechanists, scientists integrating their results are not simply building more elegant layered and corresponding theories; they are building theories about mechanisms. The mechanist perspective therefore tends to emphasize integrative pluralism in scientific research [54, 55]. Scientific achievements are collaborative and disjointed, adding incremental constraints to a developing representation describing how a mechanism works at one level and across levels [6, 22, 81]. Reductive logic and the mechanist perspective can work together and may ultimately belong in a novel paradigm placing entirely different concepts at the center of natural science.

Practical limits on reductive logic seem to be particularly relevant in relation to the often causally convoluted system dynamics in non-replicable or un-predictable settings associated with non-linear phenomena [69, 70, 84]. Some theorists go so far as postulating the existence of *non-reducible* hierarchically organized complex emergent phenomena [45, 46], in which case reductive logic is presumed to fail in part or entirely. Postulating non-reducible phenomena may be premature.

There may be unrecognized hard limits on the application of reductive logic in natural science. For instance, reductive logic is effective but may be limited *in principle* by a fundamental attribute of the logic that has not yet been recognized or successfully spelled-out. Understanding these unfamiliar limits and their implications needs to be addressed prior to advancing any concept or theory that rejects reductive logic in part or in whole.

3 Descartes, Reduction, Paradox and Epiphenomenalism

In *principle*, reduction works and reductive logic does not fail, despite *practical* difficulty conducting reduction in complicated contexts. Careful reductive analysis reveals even convoluted, non-linear, evolved, self-organized, hierarchically complex, emergent phenomena, such as consciousness, remain open to unyielding reduction [73]. However, among the anomalies associated with reductive logic, there are particular reductive conceptions suggesting the existence of hard limits.

Assuming Nature does not contain paradox, reductive theorists generally try to eliminate contradiction and paradox from scientific theory [85]. Despite the fact that reductive logic relentlessly works, particular well-composed reductive arguments using reductive logic nevertheless arrive at contradictory and paradoxical outcomes.

Careful dissection of reductive logic, seeking errors and omissions, as well as identifying partial or outright abandonment of reductive logic, often leads to clarification of paradox, contradiction or inconsistency; as well as providing deeper insights into the relationship between logic, science and natural domains [7]. Many scientific paradoxes and contradictions [8] can be resolved, when the source and structure of a conflict between observed phenomena, experimental results, theoretical constructions, mathematical models or narrative languages of description, can be determined.

Sometimes, however, no matter how hard scientists try, theorists are unable to extract or remove paradox from a particular reductive proposition. When a reductive proposition produces a paradox that cannot be resolved through careful examination of the logic and other identifiable sources of conflict, there may be something going on revealing a hard limit on reductive logic in natural science. For instance, in the hands of a careful and skeptical philosopher explicitly attempting to find a flaw in the strong reductive argument, William Seager's detailed logical, philosophical, scientific and mathematical analysis of the *reductive epiphenomenalism of consciousness proposition* and its *paradoxical* conclusion, reveals with some disappointment that the epiphenomenal argument is a very solid, step-by-step, logical and thus '*true*' reductive scientific proposal [73]. Seager carefully reviews all available historic and modern attempts to resolve the paradox of *reductive epiphenomenalism of consciousness* and finds no clear winner. The absurdly paradoxical conclusion of epiphenomenalism therefore must stand—*reductive epiphenomenalism* paradoxically insists that the complex phenomena and properties of subjective consciousness and causally impactful mind are only illusory epiphenomena of more fundamental quantum states [74]. It is this disturbing unresolved reductive paradox that may reveal the presence of a hard limit on reductive logic in science.

Thus, we see that René Descartes is responsible for two major modern philosophical and scientific conundrums. He initiated a four hundred year consciousness debate regarding his dualistic conception of material brain and immaterial mind, effectively pushing consciousness and mind into a bin of anomalies in relation to reductive natural science. Descartes is also responsible for the early formulation of unitary, 'bottom-up' reductive logic; a logic still applied in modern reductive science and the only available formal scientific logic used by natural science [23]. In the modern day, Descartes' conception of reductive logic arrives at a complex paradox associated with the reductive epiphenomenalism of consciousness proposition; which can be stated as a bizarre *self-referencing paradox* [8]—"I think *reductively*, therefore I am *not.*"

If the enigma of epiphenomenalism *can't be resolved*, the rigor of 'bottom-up' reductive logic *paradoxically erases consciousness* from the library of 'real' natural phenomena by translating consciousness into *illusory epiphenomena of quantum physics*. Descartes' complicated legacy challenges philosophers, scientists and mathematicians alike to find better solutions for both the residual Cartesian gap and the unresolved paradox of reductive logic and epiphenomenalism.

4 Kurt Gödel

'Reductive thinking', is a very special 'kind' of scientific and mathematical *'formal logic thinking'*. Kurt Gödel most succinctly defined a relationship shared by contradiction and paradox and formal logic, in his two famous incompleteness theorems published in 1931 [28–30]. A short summary of Gödel's important work provides an analogy and framework for confronting the contradiction and paradox at the center of *reductive epiphenomenalism of consciousness.*

There are four parts to Gödel's argument proving the two incompleteness theorems. First, there is Gödel's doctoral thesis, in which he learned about *formal completeness*. Then there is Gödel's exploration of *paradox*. This leads directly to the formulation and the precise, detailed, logical, proof of the *two incompleteness theorems*. Finally, there are the *implications of incompleteness* Gödel explores as a consequence of the incompleteness theorems. By responding to all four steps in the sequence of development of Gödel's thought, the analogy constructed links Gödel's work on incompleteness with reductive science and epiphenomenalism.

Gödel was interested in systems of abstract formal logic and mathematics. The components of an abstract formal system include, an 'alphabet', 'rules of grammar', 'axioms', 'rules of inference', definitions of 'grammatically well-formed statements', as well as derived and proven 'theorems'. The precise formal logic employed in a formal system is like a very finely tooled machine that leaves no space for fuzzy interpretation. The logic provides a mechanical procedure determining whether any given statement conforms to the system. To be useful, the logic of a formal system must be *consistent*. A formal system "is *consistent* if there is no statement such that the statement itself and its negation are both derivable in the system. Only consistent systems are of any interest in this context, for it is an elementary fact of logic that in an inconsistent formal system every statement is derivable, and consequently, such a system is trivially complete." [65].

Prior to beginning his work on *incompleteness*, Gödel began his academic career with a doctoral thesis, exploring *completeness* and the conditions in which an abstract formal system could be considered *closed, resolved, decidable* and *complete.* The mathematical ethos of Gödel's day was set by the agenda of David Hilbert [86] and a common belief that mathematics was on the verge of finalizing a formulation providing a complete understanding of modern mathematics [47]. Gödel began a journey that unraveled Hilbert's intention and the expectation of the mathematical community, when he focused on paradox and composed his detailed logical proof of two incompleteness theorems, which spell-out the conditions in which any formal system of sufficient complexity must be considered *open*, *unresolved*, *undecidable* and *incomplete* [64].

Gödel was interested in paradoxes of self-reference [11]. He began by drawing an analogy with two paradoxes of self-reference he specifically mentions; the 'Epimenides' or 'Liar's' paradox and antinomy [5], and 'Richards' paradox and antinomy [29, 38]. The 'Liar's' paradox creates a semantic analogue of Gödel's first incompleteness theorem, through a syntactical and abstract mathematical demonstration of an *undecidable proposition* within a formal system. 'Richard's' antinomy is a semantic antinomy that highlights the significance of logical *consistency* and the importance of differentiating clearly *mathematics* from *meta-mathematics*. This provides a semantic analogue for Gödel's second incompleteness theorem, through a syntactical, abstract mathematical demonstration of mathematical and meta-mathematical levels of argument.

The 'Liars Paradox' [13] is stated: "This statement is false." If "this statement is false" is true, then the sentence is false, but if the sentence states that it is false, and it is false, then it must be true, and one goes back and forth in the paradox. The paradox in part depends on the fact that English sentences can be constructed that cannot consistently or unambiguously be assigned a 'truth value', true or false, even though they are completely in accord with grammar and semantic rules.

The 'Liar's' Paradox is in part dependent upon an assumed binary decision that must be made between true and false—this is the assigned 'truth value'. From the vantage point of the paradox and the logic of binary decisions, the paradox cannot be resolved. From a meta-logical vantage point one can see that the ambiguity of English is a problem and binary decisions are not the only available kind of logical decision one might wish to consider in dealing with a statement created by a liar; nor are binary decisions the only way of thinking or dealing with a lie.

'Richard's' antinomy or paradox [58], starts with unambiguous real numbers and translates these precise mathematical objects into English statements. 'Richard's' paradox then results in an untenable contradiction between the level of unambiguous real numbers and the level of English statements about the numbers. The real number level and the English meta-level must be examined carefully to find the 'error'. The argument proceeds by demonstrating that a specific real number is unambiguous but the meta-level English statement describing the real number in natural language creates a statement defining the real number, for which there is no way to decide whether the meta-level English description is unambiguous or not. The paradox hinges on the realization that particular expressions of natural language describe real numbers unambiguously, while other expressions of natural language do not. While there is a way to demonstrate that real numbers are unambiguous, there is no way of determining unambiguously, which English statements unambiguously define a real number. The resolution of 'Richard's' paradox, then, is that there is no way to unambiguously determine exactly which English sentences are unambiguous when they define real numbers which also means there is no way to describe in a finite number of words whether an arbitrary English expression is a potential unambiguous definition of a real number [31].

The Halting problem is related to Richard's antinomy. Alan Turing, in studying computing, defined the Halting problem in 1936 in a response to Gödel's theorems. Turing proved that computational 'halting' is *undecidable* over all computing machines, meaning it is impossible to determine from the description of an arbitrary computer program and a given input, whether the program is going to finish running or run on for ever, thus one cannot determine from *inside* a computer program whether it is complete or forever incomplete [12]. Turing, aware of Gödel's incompleteness proof from 1931, was in search of significant examples that extended Gödel's very important result [4, 40, 47, 64].

Gödel drew an analogy from the Liar and Richard's paradox, since both are focused on *truth* and *falsehood*. Kurt Gödel constructed his analogy by shifting the attention of his readers toward a different paradox involving *truth* and *proof*. Gödel shifts attention from: "This sentence is *false*", to: "This statement is *not provable*" [14]. Gödel then proceeded to construct his 'logic bomb', which created a gulf between *truth* and *proof*, by shifting attention from the idea of *paradox* toward the mathematical concept of an *undecidable proposition. Undecidable propositions* are propositions that are "neither provable nor disprovable" [29, 30].

The *first* of Gödel's two theorems demonstrates the following: In abstract mathematical formal systems of sufficient complexity, which use a definable and consistent logic, there will inevitably be found *undecidable propositions* declaring the formal system to be *incomplete*. An *undecidable proposition* within a formal system is subtly different from a paradox or contradiction. Gödel's careful and detailed abstract logical argument creates a coding scheme that translates every statement, logical formula and proof, in Russell and Whiteheads' *Principia Mathematica* [43] into a mirror statement about natural numbers. He then takes the notion of 'truth' out of the Epimenides Paradox or the Liar Paradox, stated, "This statement is false" [14]; and replaces 'false' with an assertion about 'proof', in the form: "*This statement is not provable*". This statement about provability rather than true and false, is then also coded as an arithmetical, mirroring statement or counterpart, called a 'Gödel sentence' in abstract logic. He then coded the 'Gödel sentence', in a carefully detailed logical sequence, into the language of arithmetic [15]. The *informal* undecidable Gödel sentence is stated in the form, "This statement is not provable". The *formal* version of the Gödel sentence or statement, appears in Gödel's theorems in the form $[R(q); q]$, creating a self-referential statement that asserts its own unprovability and declares its own undecidability [38]. "By focusing on provability rather than on truth, Gödel's sentence avoids the paradox. If formal arithmetic is consistent, meaning that only true statements can be proven, then Gödel's statement *must* be true. If it were false then it *could* be proven, contrary to the consistency! Furthermore, it cannot be proven, because that would demonstrate just the opposite of what it asserts, its unprovability!" [39].

Thus, in Gödel's first theorem, *truth* becomes separate from *proof*. It is possible for a statement to be grammatically correct, logical and consistent within the framework of a formal system and in this sense for the statement to be *true*. The same '*true'* statement may also be a statement that because of its own assertion about itself, *cannot be proven to be true or proven to be false*. This differentiation effectively separates *truth* from *proof*. A 'Gödel sentence' becomes a *true statement* and an *undecidable proposition* in a formal system, a statement that subtly stops short of paradox and contradiction by declaring itself, through self-reference, to be *undecidable*—"This statement *cannot be decided* as to its *truth* or *falsehood*". A Gödel sentence is logically *true* but it *cannot be proven to be true or false*. As long as the undecidable Gödel statement is not forced into paradox and contradiction through decision, it remains part of a logical system and in effect *protects* the logical system from the dangers of paradox, contradiction and inconsistency. Gödel's first theorem goes on to prove that every version of the 'Gödel sentence' in every conceivable formalization of arithmetic must be '*true*' if the formal systems are sufficiently complex and consistent [64].

Consistency is carefully addressed in Gödel's second incompleteness theorem. As long as the Gödel statement located in a formal system is left *undecided* the statement can be logical and true but cannot be proven to be true or false within the formal system and its rules. If the statement is *decided* to be *true* or *false,* the statement then becomes a *contradiction* or *paradox,* threatening the logic and the consistency of the formal system. The idea of an *undecidable proposition* or 'Gödel sentence' forms a bridge between Gödel's *first* and *second* incompleteness theorems. The *first* theorem is focused on *undecidable propositions* in relation to the 'logic', the 'truth' and the 'provability' of statements within a formal system. The *second* incompleteness theorem is focused on *undecidable propositions* in relation to '*proof*' of '*consistency*' of statements within a formal system.

Gödel's *second* incompleteness theorem demonstrates that the logic of a formal system, when it is sufficiently strong enough that it contains *undecidable propositions*; is also too weak, or not strong enough, to 'prove' its own 'consistency'. Therefore, the *consistency* of a formal system cannot be determined from inside the system itself—*consistency* must be approached through *meta-consideration* derived outside the particular formal system and its logic [15]. "Gödel showed that if the consistency of the formal system could be demonstrated inside the system itself, then the informal argument just given could be formalized and the formalized version of the statement, "This statement is unprovable," would itself be proven, thereby contradicting itself and demonstrating the inconsistency of the system!" [39]. Consistency, therefore, *must* be determined from outside a formal system, in order to once again protect the system from illogic, contradiction, paradox and inconsistency. The necessity of meta-consideration, in relation to determining the consistency of a formal system of sufficient complexity, reveals another way in which abstract pure mathematical formal systems of sufficient complexity, *cannot be closed, resolved, decided or complete* structures.

Gödel goes on to prove two further significant implications of the incompleteness theorems. *First,* what must remain an *undecidable proposition* or 'Gödel sentence' in *one* abstract formal mathematical system, can often *be decided* and have a significantly different meaning and implication within a closely related formal system using a slightly different set of axioms and theorems [13, 56, 58, 64]. *Second*, Gödel's work additionally reveals that in an alternative formal system composed expressly to get around the presence of an *undecidable proposition* in *one* formal system, by adding axioms or theorems and by successfully making the 'Gödel sentence' provable or decidable: In such an alternative formal system, the system will inevitably run into its own *unprovable* 'Gödel sentence' or *undecidable* proposition [15, 47, 64].

5 Gödel and Reductive Epiphenomenalism of Consciousness

It is conceivable that reductive logic and reductive science create a sufficiently complex system that it is possible to construct or discover a logically *'true'* reductive theoretical proposition stating a linguistically '*unresolvable'* and formally '*undecidable'* self-referencing contradiction or paradox.

Is *reductive epiphenomenalism of consciousness* such an *unresolvable* reductive scientific proposition? More specifically, by analogy with Gödel's four-part proof of his two incompleteness theorems, is *reductive epiphenomenalism of consciousness* a *logical, true*, but also *unresolvable* and *undecidable reductive* proposition? If *reductive epiphenomenalism of consciousness* is a *true* but *undecidable* reductive proposition, as in the first of Gödel's theorems, it must be *left undecided* within the reductive formal system of logic, in order to *protect* the reductive logical system from *illogic, contradiction, paradox*, and *inconsistency*. Further, as in the second of Gödel's theorems, in order to avoid contradiction and paradox it may also be necessary that the *consistency* of reductive logic, just as in abstract formal systems, must be determined through meta-consideration, demonstrating again that the *reductive system of logic cannot be contained, closed, resolved, fully decided* or *complete*. By analogy, if r*eductive epiphenomenalism of consciousness* is an *unresolvable* and *undecidable* reductive statement, it effectively declares reductive logic and the reductive science paradigm to be a system employing a formal system of logic of sufficient complexity that it exhibits *formal logical reductive incompleteness*. It is worth a closer look at the epiphenomenalism proposition.

Reductive epiphenomenalism of consciousness is a reductive proposition employing strong reductive logic that reduces mind to brain, brain to material body, and material body to the most fundamental physical description possible in natural science; a description composed in the language of quantum physics. The *reductive epiphenomenalism of consciousness proposition* implies the conclusion that mind and all its manifestations, including subjective awareness and experience, including the qualia of perception [17–19], including the experience of freewill and willful and intentional action [36], including meaningful subjective emotional experience, including any sense of causal authority as an active agent composing aspects of our own environment in interactions and relationships in the world [35]; *all* these properties and all other defined properties of mind and consciousness are entirely transformed into an *illusion* and a *fantasy,* mere *epiphenomena* of a fundamental physical quantum description [73]. As a generalization, the *epiphenomenalism proposition* [66] reduces *all* evolved, emergent, hierarchically organized complexity, all higher-level causal interaction and relationships, whether associated with consciousness or not, *fully* and *totally* to a quantum description wherein all causal explanation ultimately resides in the quantum account.

The '*true'* logic of reductive epiphenomenalism finds experimental '*proof*' and support in the work of Libet [52]. Taking the true logic and experimental proof to heart, Harris [36] and Harari [35] try to comprehend the absurd meaning for human existence of the paradoxical illusion created by epiphenomenalism. Meanwhile, other corners of the philosophical community often sidestep epiphenomenalism through a variety of adaptations of reductive logic or by altering the relationship consciousness has with the rest of reality [74]. Concurrently, most of the scientific community try and ignore the theoretical conflict, contradiction and paradox posed by epiphenomenalism while they continue the study of consciousness 'as if' it is a valid phenomena rather than an illusion.

The vast enterprise of functional neuroscience, including interpersonal neuroscience [76, 77], often *accept 'as a given'* causally significant brain, mind and consciousness. *Assuming* causally effective consciousness, research then paradoxically proceeds with a reductive scientific approach to function, effectively trying to reduce mind and complex conscious phenomena to manifestations of brain [9]. In the case of interpersonal neuroscience, mind is something interpersonal, more than, not equivalent to, and not reducible to brain [78]. This entire enterprise, in one way or another, abandons, partially implements or alters the reductive logic of natural science. Functional neuroscience and interpersonal neuroscience sidestep the strong reductive logic of the epiphenomenal argument and its further implications, mostly on the basis that its implications are absurd and not because of any careful validation of logical failure or demonstrated experimental falsification.

Popper's criteria for a *scientific epistemology* [62] state that in order to be called "scientific", hypotheses have to be *verifiable* and *falsifiable*. Thus, hypotheses must be both *logically* and *experimentally* accessible and not merely descriptive in nature [61]. In light of Popper's criteria, there are a number of inter-related postulates, premises, assumptions and logical, experimental, mathematical and narrative considerations, that must be accounted for, in order to assess the *logical* and *scientific* merit of the epiphenomenalism proposition and in order to determine whether or not it is *possible* to unravel its' contradictions and paradoxes.

Start from the *postulate*; reductive science must avoid contradiction and paradox in scientific theory [85]. Accept the *premise*; ontologically neutral *reductive logic works* in natural science [16, 83]. Permit the *possibility*; *beyond* the known *practical limits* of reductive logic, the epistemology [79] of applied *reductive logic may have unrecognized, in principle, limits* [41]. Admit the *assumption*; *reductive epiphenomenalism of consciousness*, as a *hypothesis*, is a *logically true* reductive scientific proposition with some experimental support that has never been found to contain any kind of logical flaw [73]. Then, consider the following:

1. The outcome of *proving* reductive epiphenomenalism *true* is a *complex reductive paradox of self-reference* [8]—consciousness *is and is not*. The paradox can be stated in two parts: "A *conscious participatory* and *intentional* human *mind* composes the *logically true statement of reductive epiphenomenalism of consciousness*: When the *true* statement is *proven true*, the conscious mind and author of the statement are, by direct implication, obliterated from reality, as the statement *paradoxically* translates consciousness and mind into illusory quantum *epiphenomena*".
2. In a process of verification or falsification through observation and experimental evidence [34], *presume* a *necessary* weight of observational and experimental evidence accumulates that it is considered *sufficient* [10] to declare the epiphenomenal proposition *experimentally proven to be true*. The entire community of mindful, conscious, willful, participatory scientists conducting the scientific experimental exercise are then an *illusion, paradoxically erased* from the Universe and transformed into *quantum epiphenomena* [67].
3. Some philosophers and theorists suggest reductive logic should only be used judiciously, in particular limited contexts, in order to avoid contradiction, paradox and the 'excessive claims' of strong reductive logic, such as postulated by reductive epiphenomenalism [23]. However, this diluted approach sidesteps important points:
   a. Partially watering down or totally abandoning reductive logic is a bad idea—it's the only formal logic natural science has at its disposal. Formal reductive logic,

its paradoxes *and its limits* should therefore be spelled-out before any adaptation of the logic should be considered.
   b. Science intends to model Nature and Nature should be consulted in order to clarify how far reductive logic can be trusted in the task of approximating Nature. This exploration might prove strong reductive logic too weak to model Nature.
4. Reductive science might accumulate a large enough body of observation, theoretical and experimentally verifiable *contradictory* evidence that it effectively *proves* the epiphenomenal proposition *theoretically* and *experimentally false*—for example: fundamental physics may *enable* the existence of consciousness and mind, but quantum physics *may not limit* the causal power of hierarchically organized complex consciousness and emergent mind [26]. In this case, Nature gets the last word, because science is after all about attempting to understand and approximate Nature. If this does happen, reductive logic would remain logical and internally consistent, but it would be in deep trouble in relation to its scientific task. Faced with this 'minor catastrophe', formal reductive logic, at best, would become a weak mimic or a good approximation of a very limited aspect of natural evolutionary 'logic'—at worst, it might need to be abandoned entirely, to be replaced by what?
   a. Hypothetically, exploration of the relationship between, reductive logic (its defined and as yet unrecognized limits) *and* Nature (with its defined and as yet unrecognized evolutionary pattern), might discover, with surprise, a natural evolutionary pattern sharing similarities with the structure and application of reductive logic *and* its limits. A symmetric pattern might only become visible if strong reductive logic is sustained and reductive incompleteness is understood. Reductive logic might then *conform* to a repeating configuration in Nature and natural evolution!
5. Imagine a circumstance in which an annoyed, conscious, willful, reductive scientist, smugly succeeds in creating *a rigorously formal, logical, true and proven to be true, reductive argument* demonstrating that consciousness and mind exist as *more* than mere quantum epiphenomena—blatantly *contradicting* the formal logic and implication of the reductive epiphenomenalism of consciousness proposition. Such a contradictory argument, should it ever exist, would be a 'serious logical catastrophe' for reductive science. If *both contradictory* arguments are *true* and *proven true,* then reductive logic, through meta-level consideration, is an *inconsistent form of logic*—capable of arriving at *true* and *false* statements in relation to the *same* proposition and the same phenomena. Reductive logic in science is then a *trivial* logic capable of proving anything and incapable of differentiating true from false.

Thus, in all five circumstances focused on reductive logic and the scientific method, reductive logic is in trouble! The least troubling option above is the 'minor catastrophe' in which reductive logic must be supplemented with some alternative approach in order to more closely mimic Nature and evolution. William Seager demonstrated there are presently no clear philosophical winners capable of unraveling the paradox of epiphenomenalism [73]. The present discussion demonstrates there are presently no clear formal logic or scientific resolutions available either—*reductive epiphenomenalism of consciousness* creates contradiction and paradox that *cannot be resolved* using reductive logic. Therefore, the paradox of reductive epiphenomenalism *cannot be resolved* from *inside* the paradigm of modern reductive natural science.

Overall, to *avoid paradox, contradiction* or *logical inconsistency*, the *unresolvable* reductive epiphenomenalism of consciousness proposition must be left *undecided*. It must stand as an *undecidable reductive proposition* declaring reductive logic and reductive science to be *a sufficiently complex system of logic and thought capable of formal incompleteness*. *Truth* and *proof* in reductive natural science can be separated in reductive logic just as in formal logic and abstract formal systems. The undecidable epiphenomenalism proposition defines a *hard limit* on the application of reductive logic in the modern reductive science paradigm.

6 Thomas Kuhn: Epiphenomenalism Resolved and Decided

Thomas Kuhn [48, 49] envisioned periods of normal science in which scientists focus on problems considered acceptable and accessible, within the structure of a given theoretical frame or paradigm. The frame of normal reductive science naturally leaves some problems in categories defined as unsolved, unacceptable, or anomalous. Kuhn's initial statement defining the structure of scientific revolutions used 'theory' and 'paradigm' as interchangeable terms, leaving somewhat vague the potential scale of scientific transformations and revolutions. Modern work has extended Kuhn's exploration of scientific transformative events, studying lesser and greater scales of adaptation or revolution. When science enters into a period of upheaval on lesser scales, theories and methods can be modified and adapted to new contexts, but in particularly profound cases, on a much larger scale, an entire theory or scientific paradigm can be put aside in the face of novel ideas and successful alternative theoretical or paradigm structures [60].

The general framework of the reductive science paradigm and the 'bottom-up' application of reductive logic have remained in place for four centuries of successful scientific exploration. Descartes' historical 'bottom-up' conception of reductive science and the singular 'bottom-up' application of reductive logic in modern science has never faced a conceptual revolution of the kind or scale envisioned by Kuhn. However, a large library of 'unacceptable', unsolved or anomalous problems has accumulated within the reductive paradigm and a major paradigm shift or transformation of reductive science may be on the horizon.

The likely future structure of a potential paradigm shift and transformation of reductive science can be spelled-out through a further analogy with Kurt Gödel's work. Most important among Kurt Gödel's stated implications, is the general expectation that *an undecidable proposition in one formal system might be found to be resolvable and decidable, in a closely related formal system using slightly different axioms and assumptions* [64].

The history of enigmas and anomalies in mathematics and reductive scientific theory contains examples of statements and propositions posing conflict, contradiction and paradox that on careful examination are found to be *unresolvable* and *undecidable* in an existing frame of reference. By analogy, within a different mathematical or scientific frame of reference using slightly different premises, the unresolvable and undecidable propositions may prove to be *resolvable* and *decidable*. The ultimate resolution and decision of these unresolvable propositions in the past has depended on imagining or discovering an altered frame of reference creating a new meaning for the unresolved proposition, often transforming and opening whole domains of mathematics and science.

Two examples come to mind: *First*, Pythagoras and his conception of a 'closed universe' of whole numbers, which led to a confrontation with the unresolvable, undecidable and incomprehensible number $\sqrt{2}$. The resolution took the form of recognizing the incompleteness of whole numbers and the discovery of irrational numbers. This event opened up the vista of modern mathematics. *Second*, Isaac Newton's incredibly effective gravitational equation is very successful as an approximation but an incomprehensible mechanism it proposed, requiring *instantaneous action at a distance* formulated in the assumed deterministic fixed space and invariant time of Newtonian mechanics. This was considered to be a disturbing, incongruous, unresolvable and undecidable proposition. Newton's "great absurdity" and unsolvable puzzle was ultimately resolved and decided by Einstein in the form of special and general relativity theory [85].

The Pythagorean and Newtonian examples reveal an abstract pattern of mathematical and scientific transformation, not fully envisioned by Thomas Kuhn or any recent theorist, but implicit in Kurt Gödel's stated implications of his incompleteness theorems. The abstract pattern involves, recognition of potential paradox, subtle avoidance of paradox, the introduction of an undecidable proposition, generalization across multiple possible or similar formal system structures, as well as potential resolution of the undecidable statement within an altered system, frame of reference or paradigm structure. *Reductive epiphenomenalism of consciousness* exemplifies a troubling and unsolved modern paradox that can be subtly formulated as an *unresolvable* and *undecidable* proposition within the reductive paradigm. If, the reductive paradigm, reductive logic and its successes, can then be *preserved* and *encompassed as a special case* within an adjacent possible meta-reductive paradigm (MRP) using slightly different premises and assumptions, then, the unresolvable paradox and undecidable proposition of epiphenomenalism might become resolvable and decidable within the altered paradigm, with very different meaning and implication.

Such a successful adapted meta-reductive paradigm structure might entail *many* inter-locked, closely related but slightly different premises and assumptions, suggesting novel interpretations of natural phenomena and the complexity [71] explored by meta-reductive science [72]. In the present context, we are searching, at a minimum, for *one* adapted premise or assumption that offers an approach in which a novel adjacent possible meta-reductive paradigm can respond to the specific, unresolvable and undecidable, reductive epiphenomenalism proposition, by finding a path to its resolution, decision and novel meaning within a meta-reductive frame.

Imagine one slightly altered premise and assumption that could resolve the paradox and substantially alter the meaning of reductive epiphenomenalism. An imagined premise involves abandoning Descartes' and modern sciences' historical premise of singular 'bottom-up' reductive logic and replacing it, at a minimum, with an adapted premise; recognizing a need for *dual reductive description* involving 'bottom-up' causation and 'top-down' causal influences [26]. Implement the adapted premise in the context of evolved, hierarchically emergent [42] complex systems, or in the context of a complex adaptive system (CAS) model describing consciousness and the boundary of brain and emergent mind in a social context [53]. Such an adapted 'dual description' premise has been suggested and applied in the context of a CAS model of consciousness, in a psychiatric and psychotherapy context involving Dynamical Systems Therapy [75].

An adapted premise encompassing the reductive paradigm in a synthesis of dual reductive 'bottom-up' and 'top-down' descriptions in a meta-reductive paradigm (MRP), resolves the paradox of epiphenomenalism and decides in favor of a different meaning for reductive epiphenomenalism of consciousness. In a dual meta-reductive paradigm, the epiphenomenal proposition no longer stands as a singular, complete, rigorously logical 'bottom-up' reductive proposition. The 'bottom-up' brain can influence the mind and the 'top-down' mind can influence the brain [68, 76–78].

Deciding the meaning of the reductive epiphenomenal proposition takes the form of deciding in favor of a novel interpretation of the proposition based on the epistemology of a dual meta-reductive paradigm. Reductive logic must be applied twice and the interpretive meaning moves away from paradoxical epiphenomenal absurdity toward a narrative in which fundamental physics enables causally efficacious, hierarchically organized complexity [26]. In an encompassing complex dual MRP scientific narrative, reductive epiphenomenalism becomes an incomplete and partial statement composed by a historic and incomplete reductive natural science. Subjective conscious experience and the sense of personal agency, personal will power and intentionality can be reinterpreted in light of the dual causal description created by a dual MRP epistemology.

The adapted premise of a dual MRP responds to reductive incompleteness and responds to a reductionist blind spot [2]. In addition, the dual premise ultimately provides a more complete and resolved statement and narrative in which consciousness and emergent mind possess real emergent causal power and can do real emergent work [1]. Dual reductive accounting in a MRP significantly alters the basic understanding and theory of emergence in natural evolution and complexity [20]. While a dual MRP may not yet address all the philosophical obstacles to a modern science of consciousness [24], the multiple descriptions required in a MRP ultimately provide a better approximation of Nature's evolved, self-organized and emergent complexity.

An adapted premise demanding *dual* 'bottom-up' and 'top-down' multiple reductive descriptions might not be restricted to hierarchically organized complex models of CAS. The adapted premise might more generally make a demand for *dual* 'bottom-up' and 'top-down' multiple reductive descriptions throughout the entire hierarchy of sciences encompassed by the meta-reductive paradigm [26]. This adaptation of a basic premise in a meta-science might then have implication for resolving a number of other unresolved scientific problems. For instance, the lack of correspondence between quantum physics and relativity theory [50], and the incommensurate realities composed by quantum physics and relativity theory [63], might require a fundamental shift, from 'bottom-up' reductive interpretations, toward *dual* 'reductive thinking' and dual 'bottom-up' and 'top-down' reductive synthesis. It may be necessary to develop a generalized understanding of *dual* 'reductive thinking' integrated with 'emergent thinking', throughout all fundamental sciences and the entire hierarchy of sciences, up to and including the neurosciences studying brain and emergent mind [51].

The consequence of further adaptations of other premises and assumptions in a more complex meta-reductive paradigm is worth considering.

7 Conclusion

The discovery of one undecidable reductive proposition defines reductive logic and the reductive paradigm as sufficiently complex and susceptible to formal reductive incompleteness. The implications of reductive incompleteness associated with reductive epiphenomenalism of consciousness, opens a window on a panorama of novel adjacent possible meta-reductive paradigms, involving one or many altered and adapted premises and assumptions.

The existence of reductive incompleteness raises many further unanswered questions. What makes a theory or a paradigm sufficiently complex that it contains necessary incompleteness? Can incompleteness be generalized across all sciences? Does Nature instantiate natural forms of 'incompleteness' in the sequence of change, evolution, and the self-organization of the hierarchy of complexity?

It is possible for reductive science to compose, closed, resolved, decided and complete theoretical frameworks. By analogy, consistent with Gödel's work on completeness and his work on the two incompleteness theorems, any consistent reductive scientific theory of sufficient complexity will reveal the presence of undecidable reductive propositions and reductive incompleteness. When sufficiently complex, reductive scientific theory, the reductive paradigm and ultimately any meta-reductive paradigm created precisely to respond to identified incompleteness, will inevitably reveal, they too are open, contain unresolvable paradox, formulate undecidable propositions, and are fundamentally incomplete.

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