Chapter 13
Conclusions and Future Research Directions

Recent research advances have been made in different types of hyper-heuristics (HH), namely selection HH and generation HH, employing both constructive and perturbative low-level heuristics (*llh*). Among the four types of HH, selection HH (Chapters 2, 3) received more research attention than generation HH (Chapters 4, 5). This may be due to the research challenges in developing genetic programming and grammatical evolution, which are the main high-level techniques used in generation HH. These include the issue of bloating, which leads to the problem of readability and interpretability [14]. Among most of the generation HH, the newly generated *llh* have thus rarely been reused on new instances or problems. This presents challenges but interesting research directions for further investigations.

A large number of high-level methods have been investigated in HH. These include single-point and multiple-point meta-heuristics including local search and evolutionary algorithms, and various techniques including case-based reasoning [16, 37, 40], choice function [17, 52, 89, 132], fuzzy logic [6, 7], grammatical evolution [57, 146, 164], genetic programming [84, 96, 104, 82, 174, 193], Markov chains [92, 91], Monte Carlo [168, 34], rules [4], simple random method [17, 54], and hybridizations between them. Most of these have been studied in both selection and generation HH for examination timetabling problems (Chapter 10). The investigations of genetic programming have been mostly conducted in generation HH for vehicle routing problems (Chapter 7) but not in nurse rostering problems (Chapter 8). Investigations of these various techniques across different problem domains of diverse problem characteristics can lead to further research findings and strengthen fundamental discoveries on landscapes of high-level and low-level search spaces in HH (see Chapter 6).

A good range of *llh* have been employed; some are problem specific while others are commonly used across different applications. In the case of perturbative *llh*, these can be combined together with acceptance criteria. These research findings on different *llh* for different problem domains provide good ground for further in-depth

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investigations in terms of the generality and efficiency of HH. For example, different groups of *llh* with different execution speed and the number of changes to problem solutions in selection HH [119] have been investigated to gain insights into their contributions to the generality of HH performance. It is proposed that features of *llh* in relation to the generality in HH should be analysed by particular mechanisms to adaptively manage and select *llh* and to design general HH. The synergy between constructive and perturbative *llh* should also be examined to further improve the efficiency of HH.

A new formal definition for a general HH of different types is presented in Chapter 6, based on an existing definition in [151] for selection HH with constructive *llh*. Within the general HH, two optimization problems have been defined at two levels, respectively, each associated with an objective function, namely $f(s)$ for the low-level search space of problem solutions s, and $F(h)$ for the high-level search space of heuristics h. A mapping function M associates the search within the two spaces, i.e. M: $f(s) \rightarrow F(h)$. Some fundamental issues on landscape studies and analysis of the features of the search spaces have been discussed. Further investigations and understanding of the search spaces can facilitate the design of more effective HH. Other fundamental studies, such as runtime analysis of selection perturbative HH, have been conducted in [100]. It is shown that online reinforcement learning in HH does not outperform that of HH with a fixed distribution of *llh* operators. Such investigations into other types of HH may reveal further interesting findings; thus underpinning the fundamentals and theory of general HH across more problems.

A good range of applications has been studied in recent HH research, including vehicle routing in Chapter 7, nurse rostering in Chapter 8, packing problems in Chapter 9, examination timetabling in Chapter 10, as well as real-world combinatorial optimization problems [30]. This presents a nice and diverse range of representative applications. Compared to the other applications, more results have been obtained on generation HH for packing problems. At the time of writing this book, there is a lack of research on generation HH for nurse rostering, which, compared to the other applications, involves more types of constraints. For all the applications covered in this book, there exist well established benchmark datasets in the existing literature; thus comparison studies can be conducted, leading to interesting observations in both HH and meta-heuristic communities.

Although HH aims to increase the generality of search algorithms in solving different problems and instances, in the existing literature the majority of HH approaches have been tested on a single, and some on several specific problem domains, each evaluated separately against a particular objective function. The generality of the HH approaches has not been measured against certain standard or unified criteria across different problem domains. In a recent study, an initial attempt has been made to establish the measurement of four different levels of generality when assessing the generality of HH approaches [147], compared against specific evaluations for different problems. More such measurements in future HH developments will underpin research towards designing general algorithms across a range of different combinatorial optimization problems.

Since the inception of the field, there have been various advances in HH research. One such area is hybrid HH (Section 12.2). While there have been initial studies in this area, there is a need for further investigations such as the hybridization of more than two hyper-heuristics. HH have successfully been used for automated design (Section 12.3). The design decisions that have been automated using HH range from parameter tuning to creating new operators. An emerging area is the automated design of HH, which has contributed to reducing the man-hours involved in HH design [125]. The majority of HH research has focused on solving discrete optimization problems; however, more recently this been extended to continuous optimization problems as well (Section 12.5). Additional emerging areas include using HH to solve multi-objective optimization problems [108] and dynamic optimization problems [95].

In HH, domain specific knowledge can be considered by the *llh* for the problem under consideration, leaving the high-level search problem independent. That is, the general search is handled at the high level, isolated from the details of constraints and structure of solutions for the specific problem. In all the existing research, constraint handling has been conducted at the low-level solution space, by either discarding infeasible solutions constructed or generated, or by employing targeted operators that explore only feasible solutions. Investigations on effective constraint handling techniques, in conjunction with their effect on the connectivity of both search spaces, can enhance the performance HH for highly complex and constrained problems.

In HH approaches, both online and offline learning have been used to improve the efficiency of search upon *llh*. These include the offline learning of rules by using artificial neural networks [4] to construct nurse rostering solutions, and learning and storing constructive heuristics in a case-based reasoning system to construct timetables at different stages [37]. Online learning is usually conducted by adaptively adjusting the rewards or scores of *llh* based on the solutions generated. Examples include choice function [89] and reinforcement learning [113, 132]. There is, however, no extensive study on different types of learning in HH. Such investigations, employing for example machine learning techniques, could open new interesting research directions and further enhance the generality of HH approaches. For example, in [103], artificial neural networks have been trained offline to identify potentially high-quality nurse rostering solutions. During the problem solving on new instances, only those potential rosters of high quality are selected and evaluated, to reduce the large amount of computational time spent unnecessarily evaluating all roster solutions. Such a mechanism is highly effective in solving those complex and constrained problems, which is the case in HH, where a large amount of computational time is spent evaluating the generated solutions at the low level. Other existing research in machine learning, for example on fitness estimation in evolutionary algorithms [86], could also be explored within HH in future research.