Chapter 14
A Tale of Two Journeys

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Abstract Two decades ago we met: Charles, a young business man needing assistance with a debilitating fear of mathematics; Barbara, an experienced teacher of maths-avoidant adults, in the early stages of research for her doctorate. Both of us were embarking on big challenges. An initial six-month course, set up for our mutual benefit, enabled Charles to progress from viewing mathematics as "the most disgusting, unappealing building" to one "with form, balance and symmetry" and, on the other hand, the metaphors gathered from Charles became an illuminating part of Barbara's Ph.D. thesis (Miller-Reilly in Affective change in adult students in second chance mathematics courses: Three different teaching approaches. University of Auckland, Auckland, New Zealand, 2006). Recently Charles asked Barbara to teach him again, trying to meet the mathematical prerequisites for entry to a post-graduate business degree. The aim of this paper is for each of us to reflect on our respective journeys over two decades. Each of our narratives is presented in four sections: firstly, relevant background experiences before we met; secondly, two decades ago when the six-month course occurred; thirdly, some recent study; and finally, our reflections over two decades.

Keywords Adult · Mathematics · Education

14.1 Introduction

Two decades ago we met: Charles, a young business man needing assistance with a debilitating fear of mathematics; Barbara, an experienced teacher of maths-avoidant adults, in the early stages of research for her doctorate. Both of us were embarking on big challenges. A one-to-one course of seventeen sessions spread over six

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months, set up for our mutual benefit, enabled Charles to progress from viewing mathematics as "the most disgusting, unappealing building" to one "with form, balance and symmetry". On the other hand, the metaphors gathered from Charles about his personal conceptions of mathematics became an illuminating part of Barbara's Ph.D. thesis (Miller-Reilly, 2006). Recently Charles asked Barbara to teach him again, trying to meet the mathematical prerequisites for entry to a post-graduate business degree. The aim of this paper is for each of us to reflect on our respective journeys over two decades.

What could be the possible theme of this joint paper? Charles and Barbara met to discuss this issue. Charles' initial reaction: it could *not* be two decades since we first met! It felt "like yesterday"[1] because "it's real". The knowledge gained then is "applied everyday". Charles said he feels calm now when mathematics arises whereas he would have felt fear and panic. The six-month course had been profoundly beneficial with long-term benefits. It was like "facing a demon", he recalled, not a phobia because it was "not specific enough". It was "all encompassing" because mathematics "is in everything". It was not a "disability", this would be an "overstatement" because he has "ability". It took hard work, he faced "the struggle head on" and to address it was "massive" for him. Other people need this opportunity also, Charles emphasized. There would be "massive benefits" if these teaching methods ("professional skills and knowledge; innate ability to connect as a person; ability to understand the problem") were delivered by teachers with patience, enthusiasm and kindness. He remembered that Barbara did not say 'do more' or 'apply yourself more', that is, you just need to work harder—he had experienced this advice and believes it is a myth! He believes that adult numeracy is very different from adult literacy and that both need to be addressed—Barbara agreed. She recalled his incredible fear and how it seriously affected his career. Then also she remembered her amazement at his ability to express his fears as graphic metaphors and how this inspired her research which she hoped would inform further generations of teachers.

Four topics emerged from this meeting: our concern for all the other adults (and children) who have little mathematical confidence and competence and often suffer from the debilitating effects of this lack; he remembered the helpful characteristics of her teaching method and emphasized that others would need this type of teaching [described in Miller-Reilly (2008)]; he tried to describe the nature of the difficulties he had experienced which had motivated him to seek help two decades ago; and we talked about the long term benefits of our work together.

Each of our narratives is presented in four sections: firstly, our relevant background experiences before we met; secondly, two decades ago when the six-month course occurred; thirdly, some recent study; finally, our reflections over two decades. Overall conclusions complete the paper.

[1]Quotes in this section are from notes taken at our discussion.

14.2 Literature Review

For many adult students "a major life change, transition, or developmental task is probably involved in the decision" to return to study (Smith, 1990, p. 50). This is a rich source of their "readiness to learn", one of the six key assumptions underlying andragogy (Merriam & Brockett, 1997). Merriam (2005, p. 5) suggests that "our adult life can be seen as a structure consisting of periods of structured maintenance and stability, alternating with periods of change and transition" and that "all types of transitions hold a potential for learning and development" (ibid, p. 4). A common theme among adults learning mathematics is that it is a "means to achieve future change" (FitzSimons & Godden, 2000, p. 19). This paper investigates specific transitions in the life cycle: for Charles, tackling his debilitating fear of mathematics; for Barbara, undertaking the huge task of a Ph.D.

Drawing on their theoretical study of women's development and ways of knowing, Belenky, Clinchy, Goldberger, and Tarule (1986) developed the theory of connected teaching, which Renne (2001, p. 166) succinctly describes as a "shared conversation in which the teacher and students collaborate to jointly construct new understanding". Morrow and Morrow (1995, p. 19) discuss the aims of connected teaching in mathematics: students' "confirmation of self in the learning community"; "learning in a believing mode of communication and questioning"; "taking on challenges with support"; "gaining a sense of their own voice in mathematics"; and "becoming excited about possibilities of posing their own problems and inventing new knowledge". These ideas inform their SummerMath program for high school students. Jacobs and Becker (1997) suggest that connected teaching will help students who have not previously been successful in mathematics. Barbara's research of her teaching practice, using Charles' responses to aspects of her practice to illustrate each goal of connected teaching, confirms that it is a powerful and lasting experience for such students (Miller-Reilly, 2008). Buerk (1986) and Kalinowski and Buerk (1995) write about an undergraduate class/course called The Writing Seminar in Mathematics, informed by the theory of connected teaching, where students' development as mathematical thinkers is visible in their journals which are written throughout the course. In all her undergraduate mathematics classes, Buerk (1996, p. 29) tries "to create environments to help these students to connect with their own mathematical intuitions and ideas, and to gain confidence in their own abilities to do mathematics". She includes "extensive use of writing, of both cooperative and collaborative learning activities, of open-ended questions with individual student feedback, and of mathematical situations that yield surprise or doubt" (Buerk, 1996, p. 30).

Women's relative lack of doctoral degrees means that they are over represented in the lower ranks of academia, where they make large contributions to teaching and outreach/initiatives within and outside the department and university but without the material or status rewards that come with a research profile. Women more often participate in doctoral study later in life when they are likely to have adult children (Reyes and Stanic 1998 in Harding et al., 2010, p. 395). Mura (1987)

found that undergraduate women feel less confident than men in their ability to undertake Ph.D. study in mathematics. Becher, Hendel, and Kagan (1994 in Harding et al., 2010, p. 395) found that mature students enrolled for doctoral studies for personal development. Harding et al. (2010, p. 10) have used the personal narratives of seven women from several countries who completed Ph.D.s later in life, and found that their stories detail "the strong intellectual appeal of asking and answering a burning question and the personal and professional growth that can result from achieving the terminal degree". They conclude "no matter how late a Ph.D. is obtained, the qualification has a direct and beneficial influence on women's career paths" (ibid, p. 416).

Important mathematical skills and competencies in the workplace are "an ability to think in a mathematical way and to make decisions based on the interplay of a mathematical and situational sense of a problem", an aspect of 'mathematical literacy' (Noss & Hoyles, 1996, p. 4). Mathematical literacy is defined by the OECD's Programme of International Student Assessment (PISA) as follows:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2003, p. 24).

A large report about the mathematical skills needed in several sectors of the workplace suggests that education for adults in the workplace should "reflect mathematical literacy, rather than decontextualized skills" (Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002, p. 14.) Techno-mathematical literacies are an extension of mathematical literacy. In workplaces, as much as in the broader culture, this kind of necessary mathematical literacy is supplemented by the ubiquity of the computer: the new forms of computational technology that are used for doing mathematics are connected with new 'mathematical literacies'. The most complete description and examples of techno-mathematical literacies is to be found in Hoyles, Noss, Kent, and Bakker (2010).

14.3 Method

At the beginning of the two-decade time period for this paper, qualitative and quantitative data was gathered as part of Barbara's large study (Miller-Reilly, 2006) in which she investigated several teaching approaches used for adults returning to study mathematics. Additional data was gathered for the limited number of students who were interviewed, in particular, metaphors about mathematics were gathered using the Mathematics Metaphor Questionnaire (Buerk, 1996; Gibson, 1994). All the teaching sessions between Charles and Barbara were audiotaped and Charles responded to the questionnaires completed by participants in the large study.

Literature on metaphors, particularly in educational research, seems to fall into a number of categories—for this paper three categories are relevant. One category is

when metaphors are used to enable students to become aware their personal conceptions of mathematics (Buerk, 1994, 1996; Dooley, 1998; Gibson, 1994), and sensitive ensuing discussions create a supportive classroom environment. Another category is as qualitative data when exploring attitudes towards, and beliefs about, mathematics (Leder & Forgasz, 2006; Miller-Reilly, 2006, 2010). A third category is the use of metaphors to illuminate teachers' personal perspectives in both pre-service teacher education and in their teaching practice (Algar, 2009; Briscoe, 1991; Bullough, 1991; Chapman, 1997). In much of this literature the term metaphor is used in a broad sense, referring to similes or metaphors.

Buerk (1996) reports how she first "became conscious of mathematical metaphors" when she realised how often she "heard metaphors" as her "math-avoidant teaching colleagues responded to mathematical situations" in her earlier research. Gibson (1994) and Buerk (1996, p. 26) devised a "protocol for the collection of metaphors" in 1988, a protocol which Barbara adapted and used (Miller-Reilly, 2006)—the Mathematics Metaphor Questionaire. It allows students to write their metaphors about their conceptions of mathematics, which are often not easy to describe literally (Bowman, 1995). Both Gibson and Buerk use their protocol regularly in their classes which are informed by the theory of connected teaching; Gibson in her high school mathematics classes and Buerk in her undergraduate mathematics classes. "Collecting metaphors had a profound effect on me", wrote Gibson (1994, p. 9). The sense of powerlessness and frustration evident in so many of her students' metaphors motivated her to create a "positive environment in which students could risk small doses of frustration" and "meaningful lessons and activities that involved all students and relied more on cooperative learning" (Gibson, 1994, p. 9). Buerk (1994) suggests that metaphors give clues about students' learning strategies and their conceptions of mathematics. Writing their metaphors for mathematics is also another way that students "become aware of their own thoughts and the extent to which their teachers value this independent thinking" (Buerk, 1994, p. 46), which is particularly important for adults returning to study mathematics. In addition, gathering students' metaphors is valuable because the experience and ensuing discussion broadens mathematics to include language, imagery and reflection (Buerk, 1996).

In their review of research on affect and mathematics education Leder and Forgasz (2006, p. 409) list projective techniques as an approach "adopted by those who favour qualitative approaches to the measurement of beliefs and by those concerned that respondents to Likert items may not express the beliefs they actually hold but those they consider socially acceptable". Miller-Reilly (2006, 2010) found that sensitively introducing metaphors into interviews for her research allowed energized and emotional accounts of mathematics learning experiences, providing rich additional qualitative data for her study of teaching approaches used for adults returning to the study of mathematics.

Chapman (1997, p. 209) found that teachers unconsciously constructed personal metaphors and that these became the "basis of their conceptualization of problems and made sense of their teaching". Metaphors "frame the meaning one assigns to events", a way of understanding "how we think about things, make sense of reality,

… a perspective or way of looking at things" (Schön 1983 cited in Chapman, 1997, p. 209). Dooley (1998, p. 105) suggests that pre-service teachers can discover "unarticulated beliefs and assumptions" from an exploration of the "unique meanings that they have constructed for their metaphors and images" about teaching. Bullough and Stokes (1994, p. 197) furthered the development of pre-service teachers by gathering their personal teaching metaphors as a means of exploring their conceptions of teaching. Algar (2009) studied changes in teachers' beliefs over their career span by asking experienced teachers to select from six of the most common conceptual teaching metaphors (from the literature on teaching) one which indicated their current beliefs, as well as one that illustrated their beliefs early in their careers. Algar (2009, p. 743) found that while "very experienced teachers began teaching with teacher-centered conceptual metaphors", over the course of their careers about two-thirds "moved toward student-centered metaphors".

The stories of the journeys of Barbara and Charles over two decades seem to be examples of life stories or personal narratives, in particular for Charles, it is a mathematics life history (Coben, 2000). "We know or discover ourselves, and reveal ourselves to others, by the stories we tell" (Lieblich, Tuval-Mashiach, & Zilber, 1998, p. 7). "In many studies … the narrative is used to represent the character … of specific subgroups in society … minorities whose narratives express their unheard voices" (Lieblich et al., 1998, p. 5). Each of the four sections of our narratives, our experiences at four different stages of our journeys over two decades, will be "laminated". Just as lamination can provide more strength and stability in the resulting material, it is hoped that laminating our narratives will strengthen the tale of our two journeys.

The first stage of each of our journeys addresses each of our background experiences. Barbara writes about some of her experiences over the three decades since she had completed her tertiary education at bachelors and masters levels. Charles includes his mathematics autobiography, written two decades ago, describing his background mathematical experiences before we met.

14.4 Background—Barbara

My professional life and family commitments were interwoven. In common with many other women of my generation, having a career, although important to me, was secondary to embarking on family life. During my family's pre-school years I left my teaching position at the university and became involved in many community and parenting activities as well as teaching adults at night school. One class which stands out, and which influenced my later work, was when a maths-avoidant friend and I developed a new adult education course called "Maths Anxiety" which we taught as part of a community education program. When my youngest child

started school, I came back to paid work at the university (job-sharing). Many interesting challenges emerged: developing the mathematics and science program in a new centre within the university, the Student Learning Centre; initiating, with others, a day-long careers seminar for girls in the mathematical sciences which became an annual event; initiating, with others, a network of teachers from the tertiary, secondary and primary sectors (EQUALS Maths/Science Network) that worked toward making the classroom a more inclusive place to learn mathematics. Five years later, research and study leave in the U.S. allowed me to study programs for encouraging women in computing, mathematics, and science, and to present my first paper at an international conference, describing our equity initiatives in NZ. I incorporated many of the ideas and resources from EQUALS into my own teaching practice, that of teaching mathematics to intellectually-able maths-avoidant adult students in the fledgling Student Learning Centre at the University of Auckland. Evaluations from students indicated my teaching was successful. The support of colleagues and friends in the Auckland EQUALS group was important to me, as my mathematics teaching practice was atypical within the university. My concern about the under-representation of other groups (for example, Māori and Pacific Nations' students) within the university saw me join the Bi-cultural Staff Group where, for several years, we organized workshops for staff (faculty) within the university to share information about special initiatives taken by a number of university departments to support such groups. My academic research began at this time. A colleague and I started a quantitative study considering gender differences in achievement in the mathematics examinations in the last year of high school. The findings were presented for the first time at the 1989 NZ Association of Mathematics Teachers Conference. I found it satisfying being able to do research as well as teach. In the early 1990s I was asked to supervise teachers' research projects, which expanded my knowledge of the link between teaching and research. An invitation to attend the 1993 ICMI Conference on Gender and Mathematics Education in Sweden provided another turning point career-wise, and with the family leaving home, I decided to start doctoral studies, partly motivated by the limited promotion possibilities for me in my academic career without a Ph.D. I did not fully realize how huge a challenge this would be for me, partly due to many unforeseen circumstances in the family, and the size of the Ph.D. enterprise.

14.5 Background—Charles

During the initial stages of the 6-month course of study, at Barbara's request, I wrote my mathematics autobiography (Table 14.1).

The second section of our journeys describes parts of the six-month course, particularly at the start (two decades ago) and then when it was completed. These details, and Charles' quotes, were taken from the transcribed audio-tapes of the classes.

Table 14.1 My mathematics autobiography written in week 3 of our course

Age	Experiences
13 years	IQ test for streaming[b], no problems whatsoever with English and comprehension, knew I was hopeless at maths, and therefore ended up in a lower stream than that which really I should have been in, which pulled me back enormously. On all other subjects I was bored because I could do them. I wasn't being extended. I didn't realise that because I was poor at maths, though good at English, that I'd been held back—this affected my self-confidence
15 years	I was working to apply myself but failed School Certificate Maths. Homework was a complete nightmare. Despite listening attentively and taking the necessary notes, I could never[a] apply the concepts to a different set of numbers. It was totally frustrating! Despite assistance from Dad, I never really improved
16, 17 years	The same applied as per age 15, failed UE and Bursary maths and nobody seemed to understand. I failed every maths exam throughout my secondary school career. Therefore my association with and attitude towards maths was completely negative. Application was not the issue, understanding was!
20 years	After the completion of my 1st degree, majoring in English literature, I was unable to complete my 2nd degree (Commerce) because of the mathematical component
21 years	I had to leave a fabulous job, sharebroking, trading and providing advice for clients on equities, fixed interest. I was offered an opportunity to become a partner at 21, but I had this total fear of maths and recall breaking out into a sweat at the prospect of undertaking this work. I was surrounded by numbers, and I just thought, I can't do this, this is ridiculous. But, in reality, they could see skills that were good for them in me, which I can recognise now but …
33 years	An experience I had in the past month: sensitivity analysis, and a feasibility study on a large block of land. I recall thinking 'how can I do this?' When I'd complete any part of the exercise I'd review it and think 'how did I get this result? Is it correct? I'm not sure.' Then I'd review it again and again … In other words, complete insecurity in my own ability. In general my analysis was correct, however it took me forever to reach a result and the task seemed quite daunting. It took an unbelievable amount of energy. It's so tiring Continued failure with maths is frustrating, hurtful and demeaning. I wonder whether I have the maths version of dyslexia

[a]Underlining was that done at the time I wrote this
[b]Terminology for grouping by 'ability' varies by country: commonly called streams (in NZ), tracks (in US) or sets (in UK)

14.6 Two Decades Ago—Barbara

14.6.1 At the Beginning of the Six-Month Course

The year before I met Charles, I had embarked on doctoral studies, researching mathematics courses set up to teach adults coming back to the study of mathematics.

Comparison of two different bridging/access/second-chance courses was the topic. I had started gathering data from students using a questionnaire, which I had

developed and trialed. I was planning to interview some students from key groups in each course, identified through analysis of the questionnaire data, and to interview the teachers of these courses. Some open questions about how students had experienced each course were included in the questionnaire at the end of the course.

When Charles contacted me, wanting some mathematical help, I decided that researching my own teaching practice, which I had not examined in this way before, would be an interesting third comparison, another teaching approach with adults returning to the study of mathematics. I agreed to address his mathematical needs and he agreed to be part of my research project.

With Charles' permission, I audiotaped all our classes, and he completed the questionnaire I had developed for my doctorate. He also wrote his mathematics autobiography (see in the previous section of this paper) so that I became aware of his past mathematical experiences. Charles had completed a degree in English literature in his twenties. He was now 33 years old, a business man, and he felt at a turning point in his career because of his debilitating fear of mathematics.

My aim was to understand his mathematics avoidance and fear and let Charles experience mathematics as an enjoyable, creative, pattern-searching discipline, with connections to his context. I believed in taking plenty of time in the first few classes to acknowledge a student's feelings about mathematics so I asked Charles if he would complete the Mathematics Metaphor Questionnaire (Buerk, 1996; Gibson, 1994).

I was very fortunate that Charles had great ability in language and creativity as well as high motivation. His answers to the metaphor questionnaire were amazing: graphic, but profoundly negative, metaphors. For example, in answer to the question *What does using or doing maths feel like?* he said "Skiing on blue ice, with no edges, blindfolded". [Many more of his metaphors are listed in my other work (Ocean & Miller-Reilly, 1997; Miller-Reilly, 2006, 2008, 2010).] In all my experience of teaching maths-avoidant adults, I had never encountered this level of fear in a student. His earlier mathematical experiences were also very negative. I acknowledged them, took care not to blame him (Zaslavsky, 1994; Taylor & Shea, 1996), and promised that I would not give him any similar experiences. After the first few weeks I was so pleased when Charles said to me "you're the first person I have come across who can genuinely understand—it's a huge relief". I had already started gently and carefully doing some mathematics with him using mathematical games, manipulatives and explorative materials. I saw changes in his confidence and his mathematical knowledge as we worked together.

14.6.2 At the End of the Six-Month Course

At the start of the fifth month of the six-month course, I introduced algebra. We explored word and number patterns, using 'function machines' as a model (Langbort & Thompson, 1985). Charles commented that he had "gone a long way—especially on the fear side—I think I'm going to enjoy algebra". I started introducing a

spreadsheet for generating (linear) functions since spreadsheets are commonly used in the business world. We entered formulae into the spreadsheet, 'filling down' to generate a sequence of numbers, which led to the need to introduce the convention for the order of operations (using the mnemonic BEMA). His reaction was: "I am enjoying this—this is the best thing that has happened to me this year!" For our last session, our seventeenth session, he said the course had been "rewarding—very" and he had a "great feeling of accomplishment". He now "knows he can (do maths), given time—before it felt impossible". The change in his metaphors amazed me.[2] I was very pleased. We had both worked very hard! He wrote:

> Barbara approached my problem, which was very real, in a thoughtful, gentle and completely encouraging manner. She was able to empathise with me and fully understand what had been an on-going and seemingly never ending horrible experience.

I asked how it affected his day-to-day work and he replied:

> Oh just general confidence when it comes to using maths. When it comes to business matters, it's not a problem, but when it's something complex, some spreadsheet which I haven't done, it doesn't concern me if at first I don't see what's happening, because I know if I analyse it, slowly I will. So my attitude, that's what has changed. It's a confidence thing. You look at it more objectively, feel more at ease. It's a huge difference—enormous! I'm just so grateful.

I also was very pleased, and very grateful for his thoughtful clear statements, and told him so.

14.7 Two Decades Ago—Charles

14.7.1 At the Beginning of the Six-Month Course

During my first appointment with Barbara (two decades ago) I mentioned some of my strengths and discussed other reasons why I wanted to study mathematics. I said I "loved language, economics, creativity, colour, form, business", "especially colour and form and trading, they're instinctive".[3] I continued:

> On the one hand there's this business trading thing and on the other is this creative side. To really succeed on the business side, I think that I really need to understand maths. Now I could run with my creative side, and I may still do so, however if I do it now I'm doing so without a choice.

[2]"Black Holes and Beginning Teachers" (Ocean & Miller-Reilly, 1997), a paper jointly written by Jude Ocean and me, published in a journal for teachers, lists many of Charles' metaphors and alerts beginning teachers to what some of their students may be feeling about mathematics. It is now required reading for pre-service teachers in the University of Auckland teacher education program.

[3]All the quotes come from the transcripts of the audiotaped teaching sessions two decades ago.

The second time we met I again talked about the lack of choice of career options because my mathematics was "abysmally weak" and continued:

> In order to make a valid career choice I have to either say 'OK, it is, for want of a better description, that's a horrible black hole, or it's not'. But if it is that horrible big black hole, then I have to say 'it's there Charles, it's a reality, you've got to live with it, you've got to work with it'. I've reached an age (32–33) that I have to start making some pretty serious choices. OK, I do have other skills which are quite strong and are undeveloped. Can you see the importance of this? I mean it's actually quite a big life decision.

Later, during our second appointment, I again checked with Barbara if she realised how significant this chance was for my career—she replied that she understood. Barbara also emphasized that it would take a lot of work on my part—I understood that and was prepared to put in a lot of effort.

My metaphors were profoundly negative. I felt they very effectively captured the nature of my fear and negative experiences. I felt that mathematics was most like a hyena, "a scavenger-predator, rearing its head when I least want it; always succeeding in removing my self-confidence and sense of self. I hate it." If maths had been a way to communicate I thought it was "a forgotten dialect, making two-way communication impossible".

14.7.2 At the End of the Six-Month Course

At the end of the 6-month course my metaphors were very different. In answer to the question *If maths were a building what kind of building would it be?* at the start of this course I described mathematics as "the most disgusting, unappealing structure in history—maybe a prison, white, grey and ugly" but at the end "it would have a lot of form, it would have a symmetry, quite classical, pleasing to the eye—I would never have said that before. I can now see how a mathematician and an artist can be one and the same."

Barbara asked if I could describe what it was about the course that had contributed to these positive changes. I wrote this statement:

What Has Made The Changes

In the very first instance, actually facing up to the problem and seeking advice from a psychologist. Had the said psychologist told me in the very first session that I didn't have a problem with mathematics I would have thought him inept. However, by way of a process he was able to illustrate to me that in all likelihood I had the inherent capacity to cope with maths. Stage two was making contact with Barbara, who had taught students of a similar nature. What exactly has Barbara done which has been so enlightening? I need to list the points and ideas which have made the change as there have been so many.

1. Identifying the gaps.
2. Really starting with the basics. Letting me know it was fine not to understand some rudimentary concepts with certainty. Previously I had been greatly embarrassed by my lack of certainty.
3. Encouraging my memory and actually saying on many occasions: "you'll probably find you realise more than you first thought".
4. Approaching problems in bite-sized pieces.
5. Patterns. These were a major as they illustrated a concept (an alien one at that) in a visual fashion which I could relate to.
6. Building on knowledge and the basics as I learnt engendered great confidence.
7. Being completely encouraging and motivating. What an amazing teacher.
8. Letting me know it was OK if I didn't get it first hit.
9. Once the logic emerged from the patterns it was fun. Now I thoroughly enjoy maths and would like to spend more time discovering it. In some instances it's like reading a fabulous book, interesting and expanding your perceptions. At other times it's like feeling a cog turn in your brain for the very first time.

Four years later, when I asked Barbara if we could discuss some mathematical problems again, I wrote this statement:

> Prior to commencing tuition with Barbara I would have described my pure mathematics skills as severely limited. This limitation had, from my own perspective, been a significant impediment. Despite my abilities with basic and business mathematics I was acutely aware of my shortfall in what I referred to generally as "maths". This shortfall manifested itself as a lack of self-confidence when approaching most situations involving mathematics. This mindset was frustrated by an innate knowledge that this should not be the case, as my other skills were well developed. In a way fear and frustration reigned.
>
> I am delighted to say that I now approach mathematics with a degree of pleasure and relative confidence.
>
> Business mathematics seldom becomes extremely complex, other than for say a business analyst, as such some principles provide a guiding hand and the whole process is manageable. An appropriate example of the great value of this tuition is my ability to understand, use and enjoy algebra.
>
> The knowledge gained from expert tuition has enabled me to tackle such problems without fear and in a reasonably timely fashion. The method of teaching adopted by Barbara has opened up doors within my brain which were firmly closed and enabled me to progress with a greater sense of confidence.
>
> I would rate the knowledge gained and consequent confidence as the highlight of the last four years.

Now, in the third section of our journeys, we consider our recent tuition and discussions, two decades after the first course of study.

14.8 Two Decades After the First Course

14.8.1 Recent Tuition and Discussions—Charles

By changing my inadequate maths experience to an adequate maths experience two decades ago I was prepared to embark on a new journey, to try to meet the entry requirements to enter a leading business school in Australia for post-graduate business studies. However, the institution under-pitched the reality of their requirements. Even though Barbara and I worked on some of these requirements over several months, the additional algebraic knowledge that was required was not made clear until after I took the Harvard pre-entry mathematics test (for MBA). They then informed me that I required a score of at least 80% in this test. It was a massive disappointment to me when I found out about this requirement after I did not make this score. I had worked hard to try to enter this university for more tertiary study. However, on the positive side, without the work done two decades ago I would not have even attempted to get entry.

14.8.2 Recent Tuition and Discussions—Barbara

I also felt very disappointed for Charles. Is this another example of mathematics being used as a gatekeeper (FitzSimons & Godden, 2000), where an inappropriate level of mathematics knowledge is required for entry? I am not familiar with this course so do not know. Familiarity with several chapters of a finance textbook were part of the requirements for entry to this graduate course so we worked our way through these chapters. I became aware again of the fact that Charles, coming from part of the financial world, spoke a different language from me—the discourse of the financial world. I was coming from the mathematical world and was familiar with the mathematics register. I found that, as did Noss and Hoyles (1996, p. 9), "financial instruments which, from a mathematical point of view, seemed to be more or less the same phenomenon, have different names" depending on the context. Charles became the teacher at times to make this classification of financial instruments more understandable to me. I learned about this classification of financial instruments while bringing the attention of Charles to the relationships between their underlying structures, as Noss and Hoyles (1996) also did in their study of the mathematics of banking.

Charles often mentioned how valuable learning about the convention for the order of operations (BEMA) has been for him. The process of searching for patterns and discovering "principles" (Charles), which we did so frequently in the six-month course, has also been very useful and this knowledge has been used many times over the years.

Reflections over the previous two decades are the final aspects of our journeys.

14.9 Reflections of Our Journeys

14.9.1 Barbara's Reflections

The last two decades have been dominated by the journey to complete my doctorate then by my continuing teaching and research, and by my growing family—the birth of my doctorate as well as the births of my nine amazing grandchildren!

Teaching Charles, and collecting his responses to my teaching practice, was intertwined with the challenging journey, more than 10 years ago, to complete my doctorate. Charles' ability with language and his creativity, which produced such amazing metaphors, brought my Ph.D. experience to life. His responses were inspirational and hugely energizing for me both as a teacher and as a researcher. They provided the motivation for me to write about the importance of the utilization of metaphors in research (Miller-Reilly, 2010).

As a teacher, it was a powerful and satisfying experience for me to successfully help an adult with such a debilitating fear of mathematics. Reading about the theory of connected teaching I recognized most of my goals as a teacher. I also saw these goals realised in Charles' metaphors and in his statement "What has Made the Changes" (in a preceding section of this paper). I researched my teaching practice using the theory of connected teaching, using Charles' responses to illustrate each goal (Miller-Reilly, 2008). Charles' comments confirm Morrow and Morrow's (1995, p. 20) statement that "gaining a sense of their own voices in mathematics", as a result of connected teaching, is a "very powerful experience" for a student. Gibson (1994) and Buerk (1996) had similar experiences as teachers of their mathematics classes, described in a previous section of this paper.

My role metaphor would be a swimming teacher who gets into the water with the student, holding the student, keeping their head out of the water, as they slowly feel more confident in the water, knowing when to let go, knowing when to let them take risks with their new-found confidence to gain new skills and knowledge. This metaphor is similar to Algar's (2009) final (sixth) teaching metaphor which is student-centred, involving equitable distribution of power and the ethic of care.

In common with other women who do doctorates at a later stage of their lives, I agree that "a major benefit of doing a Ph.D. late in life is the personal growth experienced" (Harding et al., 2010, p. 416). Completing this degree also has given me added recognition in my academic environment and has positively influenced my retirement, enabling me to continue doing research in an honorary position within my Department.

14.9.2 Charles' Reflections

You presented pure mathematics from a considered position. In the first instance you acknowledged my fear and approached the subject gently, this created a sense

of comfort and a platform for learning, critical to the acquisition of knowledge. Then you proceeded to enlighten me in respect to the subject by first presenting it as a series of principles and patterns versus rules. I found this enlightening and entirely understandable. This foundation enabled us to embark on a series of subjects which I have been able to utilise on both a specific and general level.

Broadly my confidence when approaching subject matters with a general pure mathematics component has ensured I address the same with a high degree of calm, a stark contrast to my previous position. Adoption of the principles learnt and application of a step by step approach has been a significant guide.

The benefits of the study have enabled me, in part, to maximise returns via sound due diligence—invariably with a mathematical component, strategy, implementation and leadership. As a consequence I have relished completing some large commercial transactions—significantly increasing the probability of success via the methodologies and collaboration I employ throughout the process. I agree with Noss and Hoyles' (1996, p. 4) statement that "what matters (in a workplace situation) is an ability to think in a mathematical way and make decisions based on the interplay of a mathematical and situational sense of a problem".

On a specific basis I have applied algebra when developing spreadsheets, considering problems with a number of variables and building discounted cash flows plus simple valuations. The application of our study has also been relevant when reading and interpreting balance sheets; inherently this has provided me with the confidence to apply my more natural lateral thinking skills, to interpret high level information and look for connectivity based on principles. So the union of my learnt mathematical skills and more innate strategic skills has proven most beneficial. A simple example of the same was balancing the primary costs, administration, with a primary income stream and seeking to achieve parity between the two by the application of an airline model requiring 95% occupancy. Whilst simplified this assessment was core to the development of a strategy which, combined with a series of other components, led to a significant financial turnaround in the affairs of the entity in question.

Had I not embarked on the journey with Barbara life would have been different, better or worse I'm not sure but certainly the study and positive consequences have created comfort in a subject area that was previously faced with fear and dread. I think it would be reasonable to say that this comfort was a key contributor in some of my more recent career achievements for example:

(a) Achieved an overall recovery rate of NZ$0.95 in the NZ$1.00, including a New Zealand transaction record, for a series of property receivership transactions.
(b) Delivered a negotiated capital gain of NZ$7.0 m, within the confines of the Public Works Act.
(c) For the financial year ended 30 June 2015 achieved a return of 13.1% vs. the NZX Gross at 10.8% for a diversified managed funds portfolio.
(d) Set a four year New Zealand transaction record for an international property divestment programme inclusive of a NZ$50 m capital raise and capital gain of NZ$12 m.

My learning experience with Barbara was overwhelmingly positive with significant constructive spin-offs. I'm most grateful to have had this opportunity and sincerely hope that the lessons learnt will have application within further learning environments.

14.10 Discussion

Charles' experience supports Parker's (1997, cited in Safford-Ramus, 2004, p. 57) conclusion, that "overcoming mathematics anxiety during adulthood" is a "transition of major magnitude", an important "life event". Parker identified this process in a number of adults, interviewed in her research, who were mathematics anxiety success stories. The first five stages are clearly visible in Charles' story: his "perception of a need", his "commitment to address the problem" by taking "specific actions to become more comfortable with mathematics", his recognition of a turning point having been reached", which has resulted in a change in his "mathematical perspectives". It created comfort in a subject area that Charles previously faced with fear, with beneficial outcomes at work. Charles' concern that others have the same opportunities that he has found so valuable illustrates that he is in the final stage of the six-stage process identified by Parker (1997, cited in Safford-Ramus, 2004, p. 57).

Barbara felt a great personal achievement completing her doctorate, as it was the culmination of many of her previous experiences both in life and in her profession. It also resulted in further professional recognition, supporting Harding et al. (2010) conclusion that such doctorates result in personal and professional growth.

We each took on big challenges during these two decades and both feel very positive about the outcomes. Remarkably our lives have intersected in ways that have been markedly mutually beneficial, a level of reciprocity that had not been anticipated. The lamination of the narratives of our two journeys (Lieblich et al., 1998) has created our unusual and inspiring tale. We come from different subgroups in our society: an academically-able maths-avoidant adult (Charles); and a mature woman undertaking doctoral studies (Barbara). We were both undertaking different transitions (Merriam, 2005): for Charles, tackling his debilitating fear of mathematics; for Barbara, undertaking the challenging task of a Ph.D. Unexpectedly we each provided a crucial component of the other's journey.

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