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Katherine Safford-Ramus
Jürgen Maaß
Evelyn Süss-Stepancik *Editors*

Contemporary Research in Adult and Lifelong Learning of Mathematics

International Perspectives

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Each volume in the series presents state-of-the art research on a particular topic in mathematics education and reflects the international debate as broadly as possible, while also incorporating insights into lesser-known areas of the discussion. Each volume is based on the discussions and presentations during the ICME-13 Congress and includes the best papers from one of the ICME-13 Topical Study Groups or Discussion Groups.

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Editors
Katherine Safford-Ramus
Mathematics Department
Saint Peter's University
Jersey City, NJ, USA

Evelyn Süss-Stepancik
Department 3: Faculties
Pädagogische Hochschule Niederösterreich
Baden, Austria

Jürgen Maaß
Institut für Didaktik der Mathematik
Johannes Kepler Universität Linz
Linz-Auhof, Austria

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Chapter 1
Introduction

Evelyn Süss-Stepancik

Abstract At ICME-13 in Hamburg, Germany, the participants of Topic Study Group 6 "Adult and Lifelong Learning" presented papers that spanned a wide range of topics. The areas that they covered fell into four groups. Those in the first part of this book give an overview of the concept "Numeracy", the second part addresses a student's focus in adult learning while the third looks at adult mathematics education from teacher's viewpoint. The final group consists of papers that touch on multiple aspects of teaching mathematics to adults.

Keywords Adult · Mathematics · Education

1.1 An Outline of Adult Learning, Lifelong Learning and Lifelong Mathematics Learning

Looking in the manifold literature on adult learning it becomes clear that this topic has been discussed scientifically in different aspects for about twenty-five years. First of all, one has to know what constitutes this particular group of learners. Usually adult learners are defined as a very diverse group with the typically age 25 and older. The heterogeneity of this group is large and covers learners who start, resume or continue their education beyond the normal age of schooling in their societies (ICME 13, 2015). So, one can imagine that adult learners are different from traditional college students. Most of them are highly motivated but have a lot of responsibilities and situations that can interfere their educational purpose (Merriam & Caffarella, 1999). Discussing the topic of adult learning, lifelong learning must also be considered. With the decreasing number of young population and the increasing number of elderly population the demand of lifelong learning

E. Süss-Stepancik (✉)
University College of Teacher Education—Lower Austria,
Baden bei Wien, Austria
e-mail: Evelyn.Stepancik@ph-noe.ac.at

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gets more pertinent. Although there are different definitions of lifelong learning, it can be assumed, that lifelong learning will help adult learners to continue developing on a personal level. Beyond that Agget and Neild (2014) see a wide range of benefits that flow from lifelong learning to society and the economy. But what about lifelong mathematics learning for adult learners? Safford-Ramus, Misra and Maguire state that:

> Adult learners practicing lifelong mathematical learning are supposed to be more productive, economically active, and individually satisfied. (Safford-Ramus, Misra, & Maguire, 2016, p. 1)

Despite this fact, adult learners still feel reluctant to lifelong mathematics learning. The reasons are many, but three fundamental ones are to be presented here. First the negative perception about mathematics that occurs often together with math anxiety must be called. Second, although adult learners do a lot of mathematics in their everyday life, they do not perceive it. Wedege pointed out, that:

> They do not connect the everyday activity and their own competences with mathematics. Most of them only associate mathematics with the school subject. (Wedge, 2010, p. 89)

Finally, it should be mentioned that mathematics instructions often emphasis on memorizing procedures and rarely focus on conceptual understanding which lead to a situation where learners started hating mathematics (Chisman, 2011).

This brief discussion reveals that adults' mathematics education is facing a number of challenges. For that reason, it becomes obvious to discuss different researches about adult mathematics education.

1.2 The Field of Adult Mathematics Education

The research in the field of adult mathematics education is a very broad one. Therefore, only those topics that were issued at the Congress in Hamburg 2016 will be highlighted in this part.

One of these topics concerns the obstacles to and the advantages of the adult learners. Actually there is an extensive scope of research that link beliefs and attitudes about mathematics. A large number deals with math histories and math biographies of adult learners. The core findings show, that students see a massive gap between the mathematics they encounter in their personal or work life and the math they could not do. The latter is what they called mathematics; the first is what they call "common sense" (Coben, 1997). Also related to this topic is the phenomenon "math anxiety". Some investigations are devoted to the teachers' behavior (e.g. Yuen, 2013); some analyze how adult learners can overcome math anxiety. In contrast to math anxiety self-efficacy is a predictor of success in the adult mathematics classroom. However, there are fewer studies about self-efficacy. These studies mirror self-efficacy from two angles. The first angle reveals the teachers' opportunities to enhance students' self-efficacy (e.g. Dweck, 2006). The second angles looks at own contributions adult learners can make to enhance their self-efficacy (e.g. Safford-Ramus, 2015).

From the very beginning, the publishing conduit for adult mathematics educators has had a strong pedagogy spirit. In this view mathematic education for adults always aims for empowerment. Benn (1998) for instance is convinced that a low level of numeracy in our society limit participation and critical citizenship. In this context Freire's pedagogy plays an important role. His approach to education aims to transform oppressive structures by engaging people drawing on what they already know. In Brazil teachers were accustomed to using his work in literacy program but struggled to transfer that experience to the teaching of mathematics (Dias, 2000). Another aspect in this area is the empowerment of parents who often return to study mathematics to be able to help their children learn math. Whether in Ireland, Denmark, in Hispanic communities or anywhere else in the world there are research projects and workshops on parents and grandparents as adult learners of math. Some of them not only lead the parents to a better conceptual understanding of mathematics but also allows them to experience the pedagogical changes being implemented in the children's classroom (Ginsburg, 2008).

As already stated above, mathematics makes an important contribution to citizenship but also for further credentialing. Some adults learn mathematics because the lack of basic mathematics education or because they were unsuccessful in school or because they need it for their career path. The common feature of many offers and curriculums for adult learners is that mathematic tasks are drawn from real life situations, everyday life and workplace tasks. Overall, it can be observed that adult learners bring a treasure of intrinsic and extrinsic motivation, and a desire to understand "why", not just "how", the procedures work (Safford-Ramus, Misra, & Maguire, 2016). Finally, it should be noted, that the rapid evolution of the technology also affects adult math education and first approaches to design multimedia tools for teaching math in adult basic education are made.

Another important group of adult learners are teachers. Two basic categories of teachers as adult learners can be distinguished. The first includes students that are becoming teacher while the second addresses practicing teachers who seek to upgrade their mathematical knowledge and/or their teaching methods. In addition to this distinction, however, they each have a different target audience. Some of them teach children, the others teach adults. Unfortunately, only a few countries and international projects are supporting initiatives to improve adult mathematics teachers' competencies.

Last, but not least, the disparate and competing conceptualization of numeracy should be mentioned here, because it often occurs in literature as well as in the Topic Study Group "Adult Learning". It is a matter of fact that the discourse on how numeracy is conceptualized and its relationship with mathematics and literacy is still going on. Following Maguire and O'Donoghue (2003) the concept of numeracy should be considered as a continuum starting with a limited concept of numeracy and culminating in a conceptualization of numeracy as a complex and multifaceted sophistic construct. Some others considered numeracy to be "not less than maths but more" (Johnston & Tout, 1995). Withnall (1995) has defined numeracy as a socially based activity requiring the ability to integrate mathematics and communications skills. However, Safford-Ramus, Misra and Maguire (2016) rightly pointed out, that there is a clear need for numeracy to remain a dynamic construct. Many Governments are reacting to the poor results in International surveys (e.g. International Adult Literacy Survey) and are developing a wide range of initiatives to improve adults' literacy and numeracy. Crucial to the success of these provisions, however, are three factors. These are, the policy environment within teachers must operate, the conceptualization of numeracy being employed and the appropriateness of the teacher training provided.

This briefest synopsis of the work that has been accomplished in the field of adult mathematics education intents to introduce readers to the field and open the door to look at the following contributions that were made by the speakers at the ICME-13 in the Topic Study Group 6 "Adult Learning".

1.3 The ICME-13 Scientific Discourse of Topic Study Group 6 "Adult Learning"

At the ICME-13 the participants of Topic Study Group 6 "Adult Learning" had a lot of thought-provoking discussions based on the topics outlined above. To group the essential themes the contributions were grouped into four sections. The first section resumes the conceptualization of numeracy, the second addresses a student focus in adult learning, the third a teacher focus and the last covers overarching themes. A large number of publications deal with the math histories or math biographies of adult learners.

1.3.1 Numeracy

The articles from David Kaye, John O'Donoghue, Diana Coben and Jürgen Maaß debate numeracy as an important issue of adult learning. Kaye underlines the need to recognize how the terms "numeracy", "mathematics" and "maths" are used and by whom. During his long research with reference to Adults Learning Mathematics and numeracy he has identified different uses and interpretations of concept numeracy. In addition to this Kaye reflects on the term "non-traditional-student" as a new research problem in mathematics education because the adult sector is often ignored. O'Donoghue illustrates the specific situation and developments in mathematics education in Ireland as well as the impact on adult learners of mathematics. Furthermore, O'Donoghue discusses the so-called "Irish mathematical education landscape" (IMEL) and the Adult numeracy education in Ireland. Finally, he makes a very strong case fore the inclusion of numeracy within a school mathematics framework. Coben from New Zealand treats numeracy and literacy as both social practices and technical skills. She presents a conceptual framework encompassing numeracy, reading, writing, speaking and listening practices, in real and virtual (digital) environments. The ultimate aim of this work is to develop a way of tracking changes how adults use numeracy in the workplace, at home and elsewhere. In contrast to these three articles with a rather national focus, Maaß gives an excellent political analysis and discusses numeracy—the confidence and skill to use mathematics in all aspects of life, at work and in practical everyday activities—at a global point. He raises the questions: What is reality? What is the reality of adults learning mathematics? How can adult teachers show the practical use of mathematics to their pupils?

1.3.2 Student Focus

The articles from Eun Young Cho and Rae Young Kim, Andrea Maffia and Maria Alessandra Mariotti, R. Ramanujam, and Zekiye Morkoyunlu are based on various studies where the subjects were adult learners. Eun and Rae emphasize the needs of mathematics education programs for adults in Korea. They interviewed seven adults. The reasons for the most adults to return to study mathematics are similar to the aspect (e.g. empowerment) discussed above. But there is also a small group that just for the pleasure of mathematics pursues their mathematics education. These findings may provide implications on educational policy and research in the field of adult learning mathematics in Korea.

Maffia and Mariotti are combining cognitive psychology with adult mathematics education. They were investigating adults' conceptions of multiplication. Therefore, they interviewed ten adults and made a qualitative analysis of the math histories and math biographies of these adults. The found out that the level of instruction and the duration of schooling are important variables in recalling and in choosing strategies for multiplications.

Ramanujam, an Indian adult math teacher and researcher present an experiential account of an adult mathematical classroom. He demonstrates the possibilities available to qualified and open-minded mathematics educators when they are released from the demands of following mandated curricula. In his class women that are working as vegetable sellers expected that mathematics lesson helps to increase their earning capacity. The staring point of the lessons was the typically daily routine of vegetable sellers and at the end they infer functional variations and optimize their physical efforts for packing the vegetables. From this experience Ramanujam requires a re-orientation of what we expect the learner to achieve but respects the learner's maturity and offers the learner a way of thinking that is enriching. His experience confronts the popular myth that adults and early school leavers need to be restricted to a remedial mathematics diet.

Zekiye Morkoyunlu analyses the impact of parental involvement in mathematics education for children as well for parents themselves. For this quantitative and qualitative research, a sample of students with their parents was selected from a middle school. One important result is, that students need support from their parents from primary school to middle school. Another result is, that the common learning of parents and students led them to engage with mathematics in real life. As a conclusion of this study parents remark the necessity to deepen their mathematic knowledge and to extend their how to support their children properly.

1.3.3 Teacher Focus

The articles from Sonja Beeli-Zimmermann, Neomar Lacerda da Silva and Maria Elzabete Souza Couto, Terry Maguire and Afoie M. Smith and Lena Lindenskov focus on the teacher perspective. Beeli-Zimmermann reports on her finding of a qualitative study describing the mathematical beliefs of eight adult education teachers in Switzerland. Even if adult education, particularly adult basic education, does not play an important role in Switzerland's educational field, the vocational education enjoys a very high status in Switzerland and the Swiss Federation of Adult Learning is running different projects to professionalise the training of adult basic education teachers. She collected her data in the summer 2012. It is interesting that she asked the participants to create an image of mathematics as a starting point for her data collection. Her findings identify three dualities and clarify the beneficial of this research for adult education teachers.

Lacerda da Silva and Souza Couto present their research results from a qualitative study where they analyzed how Freire's premises influence the teaching practice of adult mathematic teachers. As already explained above Freire's pedagogy belongs to the spirit of adult mathematics education (citizenship) and plays an important role in Brazil. The results indicate that his pedagogy can slowly be translated into mathematics lessons.

Maguire and Smith are illustrating the concept of "Math Eyes" and its popularity in adult education, school and community in Ireland and internationally. Their article offers substantial information and guidance for adult mathematics professionals in what can be done to encourage public engagement with mathematics, as well as in the related and important area of teacher professional development for adult numeracy tutors. They also describe connections across Math Eyes and "big ideas". Their findings of their study also show possibilities to develop Math Eyes and the look at familiar things trough the lens of mathematics.

Lindenskov a researcher in the field of adults learning mathematics in Denmark opens with a very interesting historical and philosophical analysis of the evolution of adult education in Denmark. She focuses on two settings in this field. She is pointing out that mathematics in labor market training is only directed towards industry needs and preparatory adult education is only directed towards the personal development and citizenship. These differences go hand in hand with a lot of others aspects like teacher qualification. To conclude her contribution, she raises some questions such as whether vocational mathematics education should also contribute to personal development beyond employers' needs, and the effect of placing a degree of compulsion on adult learners of mathematics.

1.3.4 At the Crossroads—Overarching Themes

The articles from Barbara Miller-Reilly and Charles O'Brien, Pradeep Kumar Misra, and Katherine Safford-Ramus draw attention to three different aspects in the research field of adult learning. Miller-Reilly and O'Brien, a pair of adult learners composed of an adult doctoral student and an adult mathematics student, reflect on their two decades journey interacting through their learning. Their literature review focused on adults returning to study mathematics and the relationship between women's way of knowing and adult education. Miller-Reilly was working on her doctorate when she taught O'Brien, a young man with completely negative association and attitude towards mathematics, in an initial six-month math course. His creative descriptions of his mathematics anxiety and overcoming it during this course provided inspiration for Miller-Reilly's doctoral study. Their mutually beneficial experiences resulted in personal and professional growth for both.

Kumar Misra opens his article with a literature review covering background information on life long learning. Afterwards he leads into the main theme, Open Educational Resources. While emphasizing the necessity of lifelong mathematics learning Misra is also outlining the different challenges to practice it. He affirms the importance of Open Educational Resources to overcome these challenges and presents details about the benefit and global initiatives to use Open Education Resources for lifelong mathematics learning.

Safford-Ramus presents an excellent summary of research and themes that surfaced in her literature review in the field of adult learning. Her paper highlights the voices of the students and teachers. She concludes with suggestions for future research and the necessity for an advanced degree specific to adult mathematics education as well as the incorporating of technology in adult learning mathematics.

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Part I
Adult Numeracy

Chapter 2
Defining Adult and Numeracy: An Academic and Political Investigation

David Kaye

Abstract I have for some time been working on the use of the term 'numeracy', particularly with reference to adult education research. I have identified many uses and interpretations of 'numeracy' in ALM proceedings and other publications. Over time I recognized the need to not only identify how 'numeracy' is used, but also 'mathematics' and 'maths'. The first part of this paper will summarize how these terms are used, when they are used and by whom, as presented at TSG 6 (ICME-13). The second part of this paper reflects on the experience of attending ICME-13 as an adult numeracy specialist. A new research problem is developed that explores how adults are considered in mathematics education research and the term 'non-traditional student' is considered. The content of the 'Scientific Program' of ICME-13 is explored to identify research topics of relevance to non-traditional students. Further investigations reveal the extent to which the adult sector is ignored. Some comments are made on the need to redress this situation.

Keywords Numeracy · Adult · Adult education · Non-traditional student Lifelong learning · ICME-13

2.1 Section 1—TSG 6 Introductory Paper

2.1.1 Introduction

The theme of the first part of this paper is apparently simple: defining 'numeracy'. I have been considering what is meant by adult numeracy for most of my 'teaching' life, from around 1990. I personally always felt much more comfortable calling myself a numeracy teacher, rather than a mathematics teacher, but I did not feel a need to define it. However, I discovered that many questioned what numeracy is. I set myself the task of looking for answers, mainly in the papers published in the

D. Kaye (✉)
Adults Learning Mathematics (ALM) and Learning Unlimited (LU), London, UK
e-mail: david.m.kaye@btopenworld.com

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proceedings of the annual international conferences of Adults Learning Mathematics (Kaye, 2003b). I was inspired by the variety of what I found.

I have continued to identify definitions of numeracy and what has been said about mathematics. As an adult numeracy practitioner and latterly a teacher trainer (educator), I have always seen a direct link between how numeracy (or mathematics) is described and the content of my teaching. This makes the task a political one, as curricula are the subject of political debate. This paper identifies some key issues in defining adult numeracy, identifies the political significance of discussing what numeracy is (and what it is not), and considers why other terms are sometimes thought to be more appropriate.

2.1.2 Key Definitions

I will begin with some examples of definitions that have been published over the last 20 years or so. I have not addressed the evolution of the terms numeracy or numerate; a very good summary of this is provided by O'Donoghue (2003). Though I speak of 'definitions', I include descriptions of courses or teaching strategies in which 'numeracy' forms part of the explanation. It should be noted that the extraction of these quotations, from longer papers, has been my decision, and for my purposes, and the original author's intentions may not be that implied by my analysis.

2.1.3 Using 'Numeracy'

There are many definitions of numeracy and what they have in common includes: context; relevance to real situations; used for solving problems; personal choice of methods; and favors personal empowerment. Mathematics, or mathematical, is always mentioned to identify the nature of the activity, but something else is always there as well, and may well be more important. As Tout (1997) said "we can say that numeracy is not less than mathematics but more". Here are a few examples.

> Indeed, the idea of numeracy has been used to emphasise the need for "the maths" to be learned (and often used) in context. What distinguishes the context of everyday numerate thinking and problem solving from that of academic mathematics is the different activities and practices which form the different contexts. Thus, the numeracies used in the work of builders, pharmacists and shop-managers are all different - because they are based in different practices and hence are specific to them. (Evans & Thorstad, 1995)

> The term mathematics triggers very strong negative feelings in many people: anxiety, panic, fear, or anger. They may unconsciously label the everyday maths they use as 'common sense' to avoid triggering those emotions. Part of the problem of definition is nomenclature: what I am referring to as 'mathematics' is often called 'numeracy' by academics and policy-makers, (but not by the general population) although they often mean a much narrower field of knowledge and skills: what used to be called 'arithmetic'. (Colwell, 1997)

> We believe that numeracy is about making meaning in mathematics and being critical about maths. This view of numeracy is very different from numeracy being just about numbers, and it is a big step forward from numeracy or everyday maths that meant doing some functional maths. It is about using mathematics in all its guises - space and shape, measurement, data and statistics, algebra, and of course, number - to make sense of the real world, and using maths critically and being critical of maths itself. (Tout, 1997)

> When we talk about numeracy what do we mean? In practice the term may signify any one of a number of things including, basic computational arithmetic, essential mathematics, social mathematics, survival skills for everyday life, quantitative literacy, mathematical literacy and an aspect of mathematical power. These descriptions span a spectrum of personal abilities from basic skills to high-level cognitive abilities such as problem solving and communication. (O'Donoghue, 2003)

> We also wanted to have research evidence of the numeracy knowledge of domestic activities which emphasises understanding by the participation of adults and to leave behind traditional methods that stand for conception of adult as persons with a simple knowledge, lazy, dumb and with concrete reasoning. Some research at job settings have showed how mathematical procedures and explanations to solve problems are highly context dependent, talking about quantities, measures and procedures in a mathematical way is a form of knowledge when is built by cooperative learning, through processes of social interchange. (de Agüero, 2008)

> *Inbalance* defines numeracy as a particular behaviour involving mathematical skills to manage a situation or solving a problem in a real life context. In this regard, numeracy involves actions such as identifying and locating, acting upon (order/sort, count, estimate, compute, measure, model), interpreting and communicating. Of course, numeracy is more than just "do basic arithmetic". … Behind this picture, *Inbalance* assumes that numeracy concerns four different domains: everyday life, work-related, societal or community and further learning. Numeracy occurs every day. (Díez-Palomar, 2012)

> … we needed to define the domain within which we laboured. We decided to adopt the following generic definition of what it means to be numerate: To be numerate means to be competent, confident, and comfortable with one's judgments on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context. (Quoted in Coben & Weeks, 2013)

Each of these quotations has been chosen to represent a key factor. Evans and Thorstad raise the issue of multiple numeracies, which differ in trades, professions and cultures. Tout introduces the idea that through numeracy we can be critical, not only in social and political analysis, but of mathematics itself. Colwell illustrates here an example of using mathematics, but in a way that others would use numeracy. Along the way she also recognises that other terms are also commonly used—such as 'everyday mathematics', 'arithmetic' or even 'common sense'. The brief statement from O'Donoghue presents us with the scope of numeracy, and also, by the way, provides us with other terms that have been used instead of numeracy, though each may be more limited, if used alone. The example from de Agüero places numeracy in an analysis of using mathematics in a very specific domestic context, with an emphasis on gender and class. Díez-Palomar is referring to a European project, Inbalance, which aimed to improve teaching adult numeracy by providing resources to support learning and assessment. Here we see a very rich definition of numeracy as a tool for European citizens to improve their lives. The final example from Diana Coben is one that has been quoted on many occasions since it was first published in 2000 (in *Adults Numeracy Development: Theory, research and practice* edited by I Gal) and is here re-used by Coben herself.

All of these examples see numeracy as—'using mathematics'. This of course raises a considerable dilemma for teachers, and those researching adult education, as we struggle to decide what does it actually mean to be 'using mathematics' in a classroom—particularly in an adult numeracy classroom?

2.1.4 Models of Numeracy

Some authors have sought to produce models of numeracy, as tools for further analysis, for example Maguire's and O'Donoghue's framework for *Numeracy Concept Sophistication.*

> The organising framework of numeracy concept sophistication views the development of the concept of numeracy as a continuum with three phases: Formative, Mathematical and Integrative. The phases represent an incrementally-increasing degree of sophistication in the conceptualisation of numeracy. The phases are seen as a continuum with gradual rather than sharp boundaries. (Maguire & O'Donoghue, 2003)

This model presents numeracy as a dynamic system, which allows for the many different ways in which numeracy is observed. However, in any one situation the sophistication can be lost, movement along the continuum is blocked and the numeracy experience is restricted by curriculum boundaries.

Gail FitzSimons provides a model that places numeracy within an analysis of the nature of mathematical knowledge

> Bemstein describes mathematics as being a *vertical discourse* due to its coherent, explicit, and systematically principled structure. It takes the form of a series of specialised, codified languages, with many sub–disciplines (e.g., algebra, geometry, trigonometry). In formal education, the discipline of mathematics is recontextualised for the purpose of enculturation.
>
> Following Bernstein, I argue that the construct of numeracy is an example of a *horizontal discourse.* This is due to the strong affinity between the burgeoning corpus of research reports on workplace and everyday activities involving the use and re/construction of mathematical knowledges.
>
> It is well recognised that the mathematics classroom is a community of practice distinct from that of professional mathematicians; also, from the workplace. In my opinion, then, numeracy is composed of mathematical knowledges and skills, however derived (i.e., formally & informally), in combination with reflective knowing in context – knowing which draws upon a lifetime of experience. (FitzSimons, 2007)

This situates the discussion about numeracy as one of epistemology and is of considerable importance in debates about what the content of an adult numeracy curriculum should be. Unfortunately more recent experience has shown that such perceptive analysis has not informed the thinking of policy makers in England, as we will see below.

The final model is taken from Tine Wedege and Lena Lindenskov who worked on a new adult numeracy curriculum for Denmark.

> We, ourselves, operate with three analytical terms: numeracy, mathematics knowledge and maths-containing knowledge. In the research project "People's mathematics knowledge in technologies undergoing change", we employ the term 'mathematics knowledge' instead of 'mathematical knowledge' or 'mathematical skills': using this new term we wish partly to signal the extensiveness of the concept, partly to avoid the schism between knowledge and skills. The term *math-containing knowledge* is needed to speak about knowledge in which mathematics understanding is integrated with other kinds of knowledge. We shall continue our considerations concerning the relations between numeracy, mathematics knowledge and math-containing knowledge. (Lindenskov & Wedege, 2001)

This indicates the complex investigation by these two researchers in producing a national curriculum and shows the importance of discussing and interpreting the content and values that are implied by various terms associated with adults' mathematics education.

2.1.5 Numeracy in England 2001 and 2011

> **Numeracy** – 'Mathematics equips pupils with a uniquely powerful set of tools to understand and change the world' (The National Curriculum, (QCA)). Changing the world may not be the immediate goal of adult learners, but being numerate - acquainted with the basic principles of mathematics is essential to functioning independently within the world. It is important that as well as developing skills in manipulating numbers, learners understand and make connections between different areas of mathematics so that they are able to apply skills to solving problems in a range of contexts. In the process, they may also begin to discover the joy and power of mathematics. (Basic Skills Agency, 2001, p. 10)

This is the definition from the Adult Numeracy Core Curriculum (ANCC), published in 2001. We see as usual the immediate reference to mathematics, but that itself is then qualified by; 'being numerate', 'basic principles of mathematics', 'skills in manipulating numbers' and 'make connections between different areas of mathematics'. The publication of the ANCC was part of a whole package of developments addressing adults' basic skills, brought in through policy changes which included increased government funding, which started with the publication of the Moser Report (DfEE, 1999). These developments are summarised by Coben et al. (2003) in her review of research of adult numeracy. As an interesting aside this report, though speaking of numeracy throughout actually uses the definition of 'quantitative literacy as a proxy for numeracy skills' as used in the British IALS survey of 1997. The three literacy definitions of that survey are worth quoting.

> Prose Literacy: understanding and using information from text e.g. understanding a newspaper article

> Document Literacy: Locating and using information from other formats e.g. reading a bus timetable
>
> Quantitative Literacy: Applying arithmetic operations to numbers embedded in print e.g. working out the price of a loan from an advert. (DfEE, 1999, p. 98)

This set of reforms set in train considerable growth in the provision of adult numeracy classes, training of specialist teachers and increased research into adults learning mathematics and numeracy. Many researchers, as shown above, focused on the learners' needs and the relevance of the numeracy to the learners' lives. In contrast policy making in England prioritised the skills economy and workforce development. I will say more about this below. These policy initiatives were informed by a completely different set of research than I have been presenting. These used the evidence from long cohort studies and the very large comparative international studies on adult basic skills. Full summaries of these studies over the last 25 years are given by Carpentieri, Litster, and Frumkin (2009) and Windisch (2015). For example:

> … the Leitch Report (2006) … argued that in order for the United Kingdom to remain an economically competitive nation, it would have to greatly improve its literacy and numeracy skills – and that it would have to improve the latter more than the former. … In this strategy, it was argued that while England was making good progress in improving the literacy and numeracy skills of the population, it would need to greatly increase progress in numeracy in particular in order to avoid losing economic ground to other nations. (Carpentieri et al., 2009)

and

> Identifying effective policy interventions for adults with low literacy and numeracy skills has become increasingly important. The PIAAC Survey of Adult Skills has revealed that a considerable number of adults in OECD countries possess only limited literacy and numeracy skills, and governments now recognise the need to up-skill low-skilled adults in order to maintain national prosperity, especially in the context of structural changes and projected population ageing. (Windisch, 2015)

These are very significant studies, which raise many different issues about the way adult numeracy and mathematics is provided for in England. I have introduced them for two reasons. Firstly, it would be misleading of me to suggest that the only research about adult numeracy had been focusing on the importance of seeing numeracy as an enlightening and critical set of skills and knowledge. Secondly it was important for me to show that it was numeracy that these reports were discussing. What was meant by 'numeracy' in these various reports remains part of this debate, but it was called numeracy.

This was not to remain the case in policy proposals for England. In 2011 the British Government published a report that specifically identified replacing 'numeracy' with 'maths' as part of its new policy review.

The report 'New Challenges, New Changes' stated (BIS, 2011, p. 10)

> There has been a large improvement since 2003 in Level 2 and above literacy, but no improvement in lower level literacy and the nation's numeracy skills have shown a small decline. So, despite considerable efforts over the last 10 years to improve the basic skills of

> adults, our new national survey shows that 24% of adults (8.1 million people) lack functional numeracy skills and 15% (5.1 million people) lack functional literacy skills. This is unacceptable.

This is followed by a table of 13 key points, the first four of which are:

- Re-establish the terms 'English' and 'Maths' for adults.
- Prioritise young adults who lack English and Maths skills, and those adults not in employment.
- Pilot in 2012 how providers can be funded on the basis of the distance a learner has travelled.
- Fund GCSE English and Maths qualifications from September 2012.

The first key point is the most significant; the change in government policy was being signalled by abandoning the terms literacy and numeracy; however, it seemed strange and somehow perverse to me to change 'numeracy' to 'Maths', rather than 'Mathematics'. Colleagues at the time thought I was raising a trivial point; arguing that it is a very common abbreviation. I must admit that it is what many people use in many contexts; but this context was different. This is a government policy paper written specifically to change the language used to discuss adult basic skills education. It is also unusual to use informal language, such as abbreviations, in a government report 'because everyone does'. What was the motivation for this particular change? This must also be part of our debate. It should also be noted that this same document specifically prioritises 'young adults'.

What is certain is that following this statement the use of 'numeracy' in relation to adult education has considerably reduced and also so has the reference to 'mathematics'.

Carpentieri et al. in their review of research use both numeracy and maths in this paragraph (my emphasis).

> It is commonly accepted that there is less stigma about poor **numeracy** skills than poor literacy skills. Successful learning is equated with learning skills that are applicable in life … . Over the life course, adults develop many of their **maths** skills through activities in their daily lives, but their beliefs about **maths** tend to be based on their school experiences. (Carpentieri et al., 2009, p. 53)

In another example from a recent OECD report (Kuczera, Field, & Windisch, 2016, p. 31) on one page, in a section headed 'The policy response in England' all three words are used (my emphasis).

> England has in recent years adopted a wide-ranging set of measures to address the literacy and **numeracy** weaknesses of young adults at 16-19 and beyond. …
>
> To increase completion rates, and improve basic skills among young people, the participation age in education has been raised from 16 to 18, and English and **mathematics** have become mandatory for those not meeting minimum requirements. …
>
> *New initiatives seek better preparation of further education (FE) teachers of **mathematics** and English*

> With a view to upskilling the FE workforce in the teaching of **maths** and English, a GBP 30m-package was put in place for 2014/2015. It includes bursaries of GBP 9 000 for English teachers, and of GBP 20 000 for **maths** teachers to attract good graduates into teaching, and programmes to enhance the skills of existing **maths** and English teachers so they can teach GCSE. Support will also be offered for the professional development of up to 2 000 teachers who want to teach **maths**.

2.1.6 Numeracy, Mathematics, Maths

The use of the three terms alongside each other is now common, but what differences there are in meaning is not clear. There is one theoretical approach that may help our investigation. Coben (2002) considered if a 'duality of discursive domains exists with respect to adult numeracy' (following the work of Catherine Kell in literacy). These are distinguished as follows

> … Domain One is … characterised by formalisation and standardisation of the curriculum, technologisation, unitisation and commodification of learning and learning materials. It is competency-based and outcomes-focussed with certification being the desired outcome, and explicit equivalence with educational levels in schools.
>
> By contrast, Domain Two numeracy is about informal and non-standard mathematics practices and processes in adults' lives, which may bear little relation to formal, taught mathematics. (p. 27)

These two domains reflect much of what has been said about numeracy above. Perhaps the authors of the 2011 BIS paper wished to clarify the 'confusion' between the two domains of numeracy and use 'maths' for Domain One and leave 'numeracy' for 'Domain Two. Unfortunately, this resulted in Domain Two numeracy almost disappearing from adult education courses, thus reducing the choices of adult numeracy practitioners and adult learners.

The British government since 2011 have followed policies that have generally reduced support for adult education courses, focused on the 16–19 year old cohort and given exceptional support for the GCSE mathematics qualification (standard mathematics qualification in schools at age 16); (see BIS, 2011). This has been questioned by many of the teaching profession and education researchers, for example in FE Week on-line

> A government move to continue insisting on widespread GCSE maths and English resits – through the heavily criticized condition of funding rule – has left sector bosses "extremely disappointed".
>
> All 16 to 18-year-old students with a near-pass (previously grade D, now grade three) GCSE in the subjects have since August 2015 had to continue studying and resit them through the rule, alongside FE courses, rather than a level two functional skills qualification. (Offord, 2017)

The OECD report (Kuczera et al., 2016) was quoted above as an example of the use of mathematics, numeracy and maths in a research report. However, the main purpose of this OECD report is to review the UK government's policy initiatives on adult skills, and within that adult numeracy and mathematics. There is a very strong emphasis on the skills economy.

> Given the evidence that mid-life changes of career trajectory are hard but not impossible, one option is to encourage a 'contextual' approach, … This means identifying weak basic skills in the context of other learning, or in employment programmes or working life, and pursuing interventions that, so far as possible, link basic skills to a practical context, occupational skills in particular. (Kuczera et al., 2016, p. 34)

These policy changes have largely been brought about by changes in government funding, enabling certain courses, and shrinking many others. This means that available courses and qualifications have developed to match available funding, rather than from research-based decisions on appropriate content and teaching strategies. Since mathematics is seen as an essential subject it has been very directly affected, and it is within these policy changes that it is repeatedly referred to as 'maths' in all official documents. This has meant that in England at present there has been a reduction in adult education, and particularly in the adult numeracy education which has been the main focus of this paper. For example, Wolf (2015) describes the change in priorities for adult education as a whole but taken with the BIS (2011) policy paper this can be seen as a serious political attack on adult numeracy.

> A situation where the majority of funding comes from payments for full-time, non-employed 16-18 year olds is bound to shift the nature and focus of colleges which had once been focused on day-release, adults and part-timers.
>
> This shift is accelerated when the adult skills budget shrinks in real terms and when more of it is intentionally directed towards non-college providers and non-college-based activities. The former and the latter have been marked features of the adult skills sector in England, and appear set to remain so. (Wolf, 2015, p. 10)

I have been pursuing the meaning and use of the terms numeracy, mathematics and maths as a way of monitoring and categorising the policy changes in this field particularly in England and UK. With the recent changes we (UK adult numeracy practitioners) have experienced it is now even more important to continue to analyse the use of these terms. I stress even more what I have consistently been asking about the use of numeracy, mathematics and maths: when and where each term is used; about what; who is using it; for what purpose (Kaye, 2013, p. 74).

Finally I wish to reinforce what I said about the importance of using numeracy because of its multiple meanings, rather than in spite of them. Numeracy is not this or that, but a dynamic concept that changes over time, place and people (van Groenestijn, 2002), a far more challenging and interesting concept than 'maths'. Numeracy enables questioning and inclusion, rather than enforcing and exclusion.

2.1.7 Widening the View

The examples of numeracy definitions and uses are drawn from diverse sources, from many places and across many decades. However, the recent political and educational changes in the UK, though providing a significant case study on the evolution of adult numeracy and mathematics education in government policy, did reduce the focus to the events in one country. In concluding the first part of this paper it is worth noting that the debate around teaching numeracy, both to adults and in schools, continues to be explored internationally. An example is the ZDM special issue on numeracy (Geiger et al., 2015).

Mike Askew (2015) provides a very comprehensive overview to this volume which demonstrates all the issues about context, the nature of mathematical learning and approaches and strategies for teaching numeracy are still being reviewed and in many cases justified against more traditional mathematics education policies. Askew concludes by contrasting the generally held views about mathematics with those of numeracy practice, which are similar to many of the views found in this investigation into adult numeracy.

> But whereas there is some consensus on the practices of mathematics: abstracting, looking for generalisations, proving and so forth, we have yet to reach consensus on the practices of numeracy. Numerate behaviour is, by nature of the contexts within which it takes place, much more contingent than mathematical practices (Askew, 2015).

2.2 Section 2—Reflection on the ICME-13 Conference

2.2.1 Adult Numeracy

I went to the conference, and particularly TSG 6, with a focus on investigating the meaning and significance of 'numeracy' (as described above). I was representing an organisation called Adults Learning Mathematics, the aims of which are:

> Encouraging research into adults learning mathematics at all levels and disseminating the results of this research ….
>
> Promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels. (ALM website; Homepage accessed March 2017)

TSG 6 described the scope of its content in its abstract as follows:

> The Study Group encompasses all mathematics and numeracy education undertaken by adults for the purposes of personal, social, political or economic development, and as a course of study in its own right, or in support of learning another subject, developing a skill or furthering an activity. (ICME-13 Website)

However, the structure of the conference discussion groups did not provide the opportunity for discussion about 'numeracy' that was expected when the introductory paper was prepared (as presented in the first part of this paper). It had been anticipated that some feedback from those working in different languages would provide a useful commentary on this particular use of English in the context of mathematics education research. Unfortunately, this did not happen, and there is a strong possibility that the whole premise of the paper, as presented, was irrelevant to many of the other TSG 6 participants.

In the Topical Survey associated with TSG 6 *The Troika of Adult Learners, Lifelong Learning and Mathematics* (Safford-Ramus, Misra, & Maguire, 2016) there is an extended discussion (pp 24–28) in which the concept of numeracy and adult mathematics education are identified very closely with each other. In the same publication there is a subtle distinction made between the term ‘numeracy’ and the concept within education that it represents, that may be described by many other words, depending on local contexts (p. 25).

Over the course of the ICME-13 conference I found the issue of what is teaching “adults” mathematics a far more significant issue to consider than my on-going investigations into the use of the term ‘numeracy’. I began to think more about **who** is being taught, and in what institutions or contexts, than how to describe **what** is being taught. Within school and university contexts the question of what is being learnt can be answered by reference to curricula, syllabi, and degree regulations. However, this is not necessarily the case when considering adult learners. In reflecting on this situation, I have realised that the need to define the scope, meaning and significance of the content of **adult numeracy** requires looking at ‘adult’ as much as at ‘numeracy’.

As stated in the aims of ALM above one of the motivating factors in this field of research is the lack of attention given to this sector, particularly when compared to mathematics education research for other sectors. Though there have been some improvements, the situation is still well described by Diana Coben in the summary of her review written in 2003.

> Adult numeracy is fast-developing but under-researched, under-theorised and underdeveloped. It is a deeply contested concept which may best be considered as mathematical activity situated in its cultural and historical context. Research and capacity-building are required in: theory; policy; teaching and learning; teacher education; communication between stakeholders; international comparative studies. (Coben, 2003)

2.2.2 Adults or Non-traditional Students

There were two sessions in particular (which I attended), which were pivotal in the evolution of this alternative research theme. The first was the Invited Lecture by Linda Furuto entitled *Pacific Ethnomathematics Navigating Ancient Wisdom and Modern Connections*. This was a session focused on looking at an adult context and made connections with a group of activities, educational approaches and research topics included under the heading of ‘ethnomathematics’. The second was one of the discussion groups *Research on Non-university Tertiary Mathematics* organised by Claire Wladis, and colleagues. I had not initially recognised this title as being relevant; the phrase 'non-university tertiary mathematics' was unfamiliar to me. The experience of these sessions suggested an alternative investigation rather than extending my discussion on defining numeracy and mathematics.

My focus now shifted to looking at mathematics education research of direct relevance to adults, but where the learners were not described as adults, or at all. As this research progressed I became aware that other researchers had considered this in other contexts and the term 'non-traditional student' was one that had attracted some attention.

For example, in a study of US college education Eckel and King (2007) use a United States Department of Education definition of the 'non-traditional' student.

> Three out of four American college students are considered *nontraditional* – that is they possess one or more of the following characteristics: they are age 25 or older, have delayed entry to higher education after completing high school, did not earn a traditional high school diploma, are married, attend part-time, work full-time or have children. (Eckel & King, 2007, p. 1042)

Against this background of an under-developed but growing field of research, and an existing focus on 'adult numeracy' (or 'adult mathematics') I saw the programme of ICME-13 as a suitable primary source to explore mathematics education research relevant to adults or non-traditional students. My original purpose of exploring the meaning and uses of 'numeracy' is still relevant as adult numeracy can be defined as a 'mathematical activity situated in its cultural and historical context' (Coben, 2003). This can serve very well as a basic guideline when looking for examples in the ICME-13 programme of research that has a primary focus on the non-traditional student engaged with mathematical activity situated in a social, cultural, political or historical context.

This clearly describes adult learning as something that happens after school age (locally defined), but otherwise can be of any nature.

In the Topical Survey associated with TSG6 the authors combine the idea of adult learners and lifelong learning making connections with the social and political aspects of people's lives and a very broad view of relevant mathematics education research.

> The above discussions [on lifelong mathematical learning] clearly reveal that lifelong mathematics learning is necessity of our times. Promotion of this learning among adult learners offers multiple benefits ranging from personal to social to economical to political. Efforts have been made in different parts of the world to realize this potential but success still eludes us.
>
> The reason is that mathematics education is facing a number of challenges and these are equally applicable to adults learning of mathematics. To know about these challenges, it becomes obvious that one must study different researches about adult mathematics education that are spread across the publications of several disciplines—adult learning, mathematics education, and educational theory—or lies hidden in doctoral dissertations. (Safford-Ramus et al., 2016, pp. 7/8)

During the conference I became aware that there were contributions that represented the ‘several disciplines’ referred to above, that made no, or very little, reference to ‘adults’. However, the focus of their themes did not specifically relate to children or school students. Therefore, I chose to take my research in a new direction, which looks at the content of the ICME-13 Scientific Program as a sample of mathematics education research.

2.2.3 ICME-13 Scientific Program

My new research task led to an investigation of all the lectures, groups and presentations at ICME-13 to identify those that I believed were ‘equally applicable to adults learning of mathematics.’ (Safford-Ramus et al., 2016) but were presented without any reference to adults or were labelled in such a way as to disguise the connection with adult learning.

I reviewed the whole of the ICME-13 conference content, using both the website and the printed programme distributed at the conference. I also made use of the Topical Surveys associated with some of the Topical Study Groups. In the appendix I present a summary of my investigation where I give the titles and presenters of the sessions I considered were significant and essential to researching adults’ mathematics education. I found that two of the plenaries, six of the invited lectures, one of the ICMI study and survey teams, one of the Affiliated Organisations, 13 of the Discussion Groups and 16 of the Topic Study Groups (besides TSG 6) were of interest to this investigation. Though these remain a minority of the presentations and sessions, for example the 16 TSGs is only 30% of all the topic study groups, this is still a good proportion when compared to a single topic group that carries reference to ‘adults’ in its title.

2.2.4 Plenaries

I was surprised and pleased to find that two of the plenary presentations, by Bill Barton and Günter Ziegler, were easily identifiable as being relevant to adult learning, though the direct connections were not made overt. Bill Barton gave considerable emphasis to the development of studies that are now referred to as ethno-mathematics, which in their essence are to do with the way people (adults) within their everyday lives, through work, culture and traditions engage with numeracy and mathematics.

Günter Ziegler presented a very wide-ranging view encompassing history, culture and technology, but at the core he was asking ‘what is mathematics?’ One of his slides presented

Mathematics is …

I. Tool box *for everyday life*

II. Part of culture, *6000 years old*

III. Basis for high-tech, *essential*

IV. *A* human activity. (Ziegler, 2016 slide 26)

He was also concerned with the popular perception of mathematics and how images of numeracy are produced and received. This again easily identifies this presentation as relevant to those seeking to enhance the awareness of adults learning mathematics and is at the core of discussing people returning to learn mathematics at a different time in life.

2.2.5 Invited Lectures

Amongst the Invited Lectures there were some that addressed topics which supported the view that a considerable amount of research was taking place of direct relevance to studying mathematics teaching and learning amongst adults, but not labelled as such. I have already made reference to Linda Furuto's Invited Lecture within the context of ethnomathematics, *Pacific Ethnomathematics: navigating ancient wisdom and modern connections*; but there is also Cynthia Nicol's *Connecting mathematics, community, culture and place: promise possibilities and problems* falling into the same category. Another Invited Lecture is Yukihiko Namikawa's *Mathematical literacy and curriculum based on it—with several realizations in Japan*. The abstract makes a suggested link to the school curriculum, but the guidance given on what is understood by mathematical literacy goes far beyond learning in schools, and so yet again shares much with what we have as a focus for adult learners and the lifelong learning sector. In fact, of course, mathematical literacy is often used instead of the term 'numeracy', but referring to a similar concept, thus providing a direct link with adult numeracy research.

2.2.6 Discussion Groups

The conversation I had with Claire Wladis, John Smith, and Irene Duranczyk in the Discussion Group: *Research on Non-university Tertiary Mathematics* was the start of this investigation. I discovered that the work they were doing in the United States, mainly in Community Colleges, was very similar to my experiences in the Further and Adult Education sector in the United Kingdom. In fact the more we discussed the more we realised we faced the same institutional pressures, the same groups of learners, the same barriers to learning and the same aspirations for change. There was another Discussion Group entitled *Current problems and* *challenges in non-university tertiary mathematics education (NTME)* (Jim Roznowski, Dr. Low-Ee Huei, Younes Karimi Fardinpour and Dr. Vilma Mesa). It also uses the phrase 'non-university tertiary mathematics' in its title, but since I did not attend the group led by Roznowski I cannot be sure how similar the approaches were. I can identify that one major link is the US institutions called two-year or community colleges, which are defined by who is being educated as much as what mathematics they are learning. We can note the Fulbright Commission states

> Two-year colleges in the US offer **associate's degrees**, which are an alternative to the more traditional four-year bachelor's degree programs. Known as **community, technical or junior colleges**, these two-year institutions offer study in a wide range of subjects to post-secondary students of all ages and academic levels. (Fulbright scholar website; accessed March 2017)

However, the Roznowski group also tried to identify some international connections, with two members of the organising group being from outside the USA. Unfortunately, I do not have enough information about what those contributions were to draw any further conclusions at this stage.

In addition, it is worth noting Dietmar Guderian's *Mathematics in contemporary art and design as a tool for math-education in school* Discussion Group. With 'school' in the title this looked an unlikely candidate to be included as a relevant example. However, the theme of this group is of more significance in learning out of school than in it. It is further interesting to note that in the abstract for this DG the author includes 'adults' in parenthesis.

> Aim Nr. I (Tuesday): Mathematics in contemporary art with special view on roots in countries of origin of refugees actually entering Europe. The idea is to give indigenous children (and adults) as well as immigrated children (and adults) by the way an idea of the cultural level in their formerly homelands and some proudness, too.

This suggests a realisation that adults are part of (or even central to) this topic, but perhaps felt the topic would be more attractive or popular by suggesting its relevance to schools, as that is the focus of the majority of participants. This perhaps introduces another aspect to this review of mathematics education research —the 'hidden adult'.

In general, the emphasis on social and cultural contexts would appear to have greater relevance to education and learning outside of the formal school structure or environment, than within it. See for example the Topical Survey on adults and lifelong learning (Safford-Ramus et al., 2016) and the discussion about adult numeracy and mathematics in the first part of this paper.

A fourth example from the Discussion Groups is *Transition from secondary to tertiary education* (Gregory D. Foley, Sergio Celis, Hala M. Alshawa, S. Nihan Er, Heba Bakr Khoshaim and Jane D. Tanner). Again, this appears to be rooted in the world of the traditional student. However, since its focus is on students' lack of mathematical preparedness on entering tertiary education, we are in fact dealing with young adults in the no-man's-land after leaving school and therefore are non-traditional or 'adult' students. Though it may be at a different level, the teaching approaches for this group would be very similar to those common to the adult sector of providing ‘adult classes’, particularly those that are seen as providing a ‘second chance’.

2.2.7 Topic Study Groups (TSG)

Finally, there are examples from amongst the Topic Study Groups (TSGs). It should be noted that in the case of the TSGs more substantial research evidence is available in the form of the ICME-13 Topical Surveys Series (2016b) than in the abstracts on the ICME-13 website. Firstly, TSG 35 which has a focus on ‘Ethnomathematics’ which I have already referred to when discussing how my ideas first evolved with reference to the lecture by Linda Furuto. In the Topical Survey associated with TSG 35 this statement is made in the introduction.

> Ethnomathematics researchers investigate ways in which different cultural groups comprehend, articulate, and apply ideas, procedures, and techniques identified as mathematical practice. (Rosa & Shirley, 2016)

This is precisely the way adult numeracy researchers look at the mathematics and numeracy practices of their adult learners. Of course, such cross-cultural approaches do not exclude the traditional school student, but the similarity with adult education research is significant.

In a similar way there are a number of other Topic Study Groups that to a greater or lesser extent share many themes with adult numeracy. The most obvious is TSG 3 *Mathematics education in and for work* (Geoff Wake and Diana Coben) which is taking a particular approach to adults’ mathematical education. For example, the papers for this TSG include: *The numeracy of vocational students: exploring the nature of the mathematics used in daily life and work* (Kees Hoogland and Birgit Pepin) and *Re-contextualising mathematics for the workplace* (John Keogh and Theresa Maguire). It should also be noted that the main presenters to this study group have contributed considerably to the discussion about adult numeracy (see the first part of this paper and associated references) and recognise that they are focused on the same cohort of learners.

Another significant Topic Study Group is TSG 28 *Affect, beliefs and identity in mathematics education* (Makku Hannula and Francesca Morselli). This again speaks of such broad human experiences that it cannot but connect to those of us focusing on adults’ education. However, looking in more detail at the Topical Survey associated with TSG 28 (Goldin et al., 2016) there is some suggestion that the focus is still associated with a traditional school or college student. This is disappointing and again can be considered another case of the ‘hidden adult’ as the relevance of affect and motivation are so significant to the adult learner. What is all the more striking about this commentary is that in the introduction, which lists topics that are not addressed in this slim volume is ‘mathematics anxiety’. This is a topic researched widely in adults mathematics education research; see for example Evans (2000, Chap. 4), Sheila Macrae in Coben (2003, p, 100), Chris Klinger in the proceedings of ALM 14 (Klinger, 2008) and in the Topical Survey on adults learning (Safford-Ramus et al., 2016, pp. 11/12). Yet the two references given are to neuroscience, one of which does not even refer to mathematics. This is another example where it is important to identify that the research discourses over-lap, but the lack of regard for adult (or non-traditional) students diminishes the scope and validity of the research. Perhaps this is more a case of the 'invisible adult', rather than the 'hidden'.

Another of the topic groups that was 'easy' to include was TSG 53 *Philosophy of Mathematics Education* (Paul Ernest and Ladislav Kvasz). This has a very wide remit including the philosophy and history of mathematics, the philosophy of education and critical mathematics. Built around a series of questions including:

What is mathematics?
How does mathematics relate to society?
Why teach mathematics?
What is the nature of learning (mathematics)?
What is the nature of mathematics teaching?
What is the status of mathematics education as knowledge field?

These apply equally to adult learning as any other education sector. They are also, of course, similar questions to the ones I am asking in the first part of this paper about adult numeracy and have published previously (Kaye, 2003b, 2013, 2015). In fact, I have previously suggested (Kaye, 2003a) a thought experiment in which questions, such as those above about mathematics are re-read, but with 'numeracy' replacing 'mathematics'. Whatever your re-action to this experiment is, I would venture that the questions take on a different meaning. However, my current purpose is to identify that the philosophy of mathematics is as significant (if not more so) to adults learning, as to the school sector.

The Topic Study Groups were the most difficult to investigate and evaluate for my purposes as they are a complex collection of individual papers and include the oral presentations and posters. I describe below two contrasting cases to give some indication of the sort of criteria I used to include a topic or not. I admit I did not follow a strict quantitative coding process for this paper, though this could be done for a more robust analysis.

We can consider TSG 37 *Mathematics curriculum development* as an example of a general mathematics education topic group that I chose **not** to include. It has some papers very specific to schools (*Paradigms of curriculum development in schools—a personal view* Z Usiskin), some papers less specific about the teaching sector (*The center of everything—insiders and outsiders working together developing mathematics curricula* J Lipka) and some that seem to be part of the 'adult' field (*What math for all? For and from life* A Rampal). I decided that as a whole it is better to exclude TSGs like this from my list, even though I have noted some individual papers are of interest, and if a more thorough analysis were to be carried out at the individual paper level some papers would be relevant.

In contrast we can consider TSG 34 *Social and political dimensions of mathematical education* as a mathematics education topic that I **do** include, as it is by definition placing mathematics education in the 'real world'. Like the example of TSG 37 it is helpful to look at a few of the papers (titles) that were presented at ICME-13. There is *Financial education and mathematics education: a critical approach* (Celso Ribeiro Campos and Aurelio Hess) which makes obvious connections with a very important topic for adult learners. There are some that appear to have as much, if not more relevance to adults around ethical questions such as *Truths and powers in mathematics education* (Alexandre Pais) and there are some that do refer directly to the school sector (*Enacting hybridity in a home-school mathematics activity* Laura Black et al.)

In the case of TSG 34 I concluded that the topic as a whole, with a focus on 'social and political dimensions' was so significant to adult mathematics education research that it was of great importance to include it. In contrast TSG 37 tended to have a more institutional feel to it, and therefore have a greater focus on the school sector, even though many papers may be about breaking out of those institutional and policy constraints.

To conclude this review of ICME-13 TSGs I will refer briefly to the one on Semiotics (TSG 54) which like a number of others had an associated Topical Survey published (Presmeg, Radford, Roth, & Kadunz, 2016). This poses a slightly different aspect on this analysis which is perhaps shared with others such as 'Mathematical Literacy' (TSG 23) and 'The role of history of mathematics in mathematics education' (TSG 25) (see Appendix). A topic with a broad philosophical and ideological approach must be relevant to any mathematics education research, including adults. This is certainly a position taken by those researching adults' learning (see Safford-Ramus et al., 2016) but this is not always recognised or expressed within the other topics. This omission may at times be to the detriment of the research topic in question, where once again we have at best the 'hidden adult', but too often the 'invisible adult'.

2.2.8 A Way Forward

I have taken some time to establish how much of the content of ICME-13 is 'relevant to' adults mathematical education. I have been cautious in identifying how much of the content of ICME-13 has connections with adult or non-traditional students. But I am surprised at how many of the contributions to ICME-13 are relevant to adults (and non-traditional students) and yet this sector is represented **in name** by one Topic Study Group only.

I have also been surprised (and disappointed) to discover some situations in which I have identified the adult is 'invisible', by which I mean the research focus is obviously relevant and significant to the adult learner, but there is no reference to the sector at all. I will give one further example of this. I return to TSG 34 and the Topical Survey associated with it. The authors of *Social and political dimensions of mathematics education—current thinking* (Jurdak, Vithal, De Freitas, Gates, & Kollosche, 2016) though applying a thorough social and political analysis of mathematics education refer mainly to the school sector, though such an analysis is at the heart of adults' mathematical education research. For example

> At a most basic level, equity and quality issues in mathematics education arise when individual students engage in the collective activity of learning mathematics at the level of the classroom. To deal with issues of equitable access and distribution of quality in the mathematics classroom, the teacher has access to many possible practices—such as differentiation of instruction and developing high expectations of achievement from students. However, teachers' practices to promote equity and quality mathematics education are constrained, among other things, by school policies regarding reward structure, teacher professional development, improved technology, or attention to social circumstances. Thus teachers' practices in this regard are shaped, to a large extent, by school policies. (Jurdak et al., 2016, p. 5)

This limited view is questionable for a number of reasons. The school students are objectified and do not play any role in the complex interplay of political, social and ethical forces that are discussed. They are merely a passive component of the school system. This contrasts with research into the social and political context of adult learners' education which identifies the adult student as an active participant with a social and political role in relation to the learning process, especially when issues of equity and equality are under consideration; see for example Yasukawa (2006) and Baker (2006) and also Benn (1997).

This demonstrates again the 'invisible adult'. Not only is adult learning missing from the scope of the research but that the analysis would be greatly enhanced by the inclusion of the adult or non-traditional student experience. By remaining in the bunker of school education the conceptual growth of the topic is limited.

My conceptualisation of the research problem is still in formation and this is an exploratory paper. The direction of future work will largely be determined by initial reactions to this paper. However, my motivation is emotional at this stage: a serious obfuscation has taken place. I have given an overview of the investigations and research programmes being pursued about learning mathematics which put a very strong emphasis on the social, political, cultural and personal aspects of the mathematics learners' lives from the ICME-13 conference. These I have always considered to be a core part of the research into adults learning mathematics (or lifelong learning, or adults' mathematics education). However, it is disturbing to find that such research does not only fail to recognise the body of work developed under the heading of 'adult', but makes claims that exclude the adult learner. The overt recognition of the adult learner and the associated body of research knowledge is essential, as is the inclusion of the adult or non-traditional student overtly in other research topics.

I will conclude this section with another short quote from the topical survey associated with TSG 34 *Social and political dimensions of mathematics education* (Jurdak et al., 2016).

> Mathematics education is a social institution which is inseparably linked to power. Mathematicians and scientists, education researchers, politicians, teachers, students and parents are interested in mathematics education for various reasons, for example for the recruitment of future specialists, for the education of the enlightened citizen, for the vitality of the state economy, for the pursuit of a meaningful and dignified purpose in life or for the allocation of beneficial opportunities in further education and work. (Jurdak et al., 2016, p. 10)

This could not be a clearer statement about the research and practice of adults learning mathematics. Yet the context in which the statement is presented makes no reference this sector of education. The next step is a programme to change this in the future.

2.2.9 Conclusion

My original purpose at ICME-13 was to continue to pursue my longstanding exploration of identifying 'numeracy' as a significant concept to understand and to investigate when the term 'numeracy' is used and when an alternative term is used for the same concept. This took place within the research field of adults' mathematics education. At ICME-13 I began a new intellectual journey based on a few casual conversations and a couple of sessions that I attended by chance.

My original aims are still valid and so I have preserved my Topic Study Group (TSG 6) paper as the first part of this extended paper. It also serves to present some examples of education research into the teaching and learning of adult numeracy for comparison with other contributions to ICME-13.

The second part of the paper examines the sessions that were presented as part of the ICME-13 Scientific Program. The results of that investigation are summarised in the appendix. The purpose was to identify those sessions that I consider are similar to or include adult education research, even if the papers themselves give no indication they are relevant to adult education. I moved my focus from exploring numeracy and mathematics (the content of the learning) to exploring what 'adult' means in mathematics education research (the learner and the teaching context). I found that in some research discourses the term 'non traditional student' is preferred to 'adult'. More importantly I found that major research themes throughout the scientific programme of the conference made no reference to adult education, though the themes are ones that are consistently at the centre of research into adults learning mathematics … and numeracy.

Appendix

Table of ICME-13 sessions of interest to adults' mathematics education

Type	Title	Presenter(s)		Category	Grade
Plenary	Mathematics, education and culture: a contemporary moral imperative	Bill	Barton	Socio-cultural	High
Plenary	"What is mathematics?" and why we should ask, where one should experience or learn that, and who can teach it	Günter M	Zigler	Popular perception	High
Invited lecture	Connecting mathematics, community culture and place: promise possibilities and problems	Cynthia	Nicol	Socio-cultural	High
Invited lecture	History of mathematics, mathematics education and the liberal arts	Michael	Fried	History of mathematics	Medium
Invited lecture	Mathematical literacy and curriculum based on it—with several realizations in Japan	Yukihiko	Namikawa	Mathematical literacy	Medium
Invited lecture	The role of storytelling in teaching mathematics	Ansie	Harding	Teaching techniques	Low
Invited lecture	Pacific ethnomathematics: navigating ancient wisdom and modern connections	Linda	Furuto	Socio-cultural	High
Invited lecture	ICMI 1966–2016, a double insiders' view of key issues from the last half century of the international commission	Bernard R Mogens	Hodgson Niss	Mathematics education	Low
Survey	Conceptualisation of the role of competencies, knowing and knowledge in mathematical education research	Mogens	Niss	Meta theories	High

(continued)

(continued)

Type	Title	Presenter(s)		Category	Grade
Affiliated org	HPM: International study group on the relations between history and pedagogy of mathematics	Luis Fulvia	Radford Furinghetti	History of Mathematics	High
Discussion group	Mathematical teacher noticing: expanding the terrains of this hidden skill of teaching	Ben Heng	Choy	Teaching techniques	Low
Discussion group	National and international investment strategies for mathematics education	Joan	Ferrini-Mundy	Education opportunities	Low
Discussion group	Transition from secondary to tertiary education	Gregory	Foley	Tertiary education	Medium
Discussion group	Mathematics in contemporary art and design as a tool for math-education in school	Dietmar	Guderian	Mathematics and creativity	High
Discussion group	Theoretical frameworks and ways of assessment of teachers' professional competencies	Johannes	Koenig	Teacher training	Medium
Discussion group	Applying contemporary philosophy in mathematics and statistics education: the perspective of inferentialism	Kate	Mackrell	Teacher training	Medium
Discussion group	How does mathematics education evolve in the digital era? Discussing a vision for mathematics education	Dragana	Martinovic	Teaching techniques	Low
Discussion group	Mathematics Houses and their impact on mathematics education	Ali Peter	Rejali Taylor	Education opportunities	Low
Discussion group	Current problems and challenges in non-university tertiary mathematics education (NTME)	James	Roznowski	Tertiary education	High

(continued)

(continued)

Type	Title	Presenter(s)		Category	Grade
Discussion group	Creativity, Aha! Moments and teaching -research	Hannes	Stoppel	Creativity	Low
Discussion group	Sharing experiences about the capacity and network projects initiated by ICMI	Alphonse	Uwerwabayeho	Education opportunities	High
Discussion group	White supremacy, anti-black racism, and mathematics education: local and global perspectives	Luz	Valoyes-Chavez	Socio-cultural	High
Discussion group	Research on non-university tertiary mathematics	Claire	Wladis	Tertiary education	High
Topic study groups					
TSG 2*	Mathematics education at tertiary level	Victor	Giraldo	Tertiary education	Low
		Chris	Rasmussen		
TSG 3	Mathematics education in and for work	Geoff	Wake	Work/ vocational	High
		Diana	Coben		
TSG 6*	Adult learning of mathematics—lifelong learning	Jürgen	Maaß	Adults and lifelong learning	
		Pradeep Kumar	Misra		
TSG 7	Popularisation of mathematics	Christian	Mercat	Popular perception	Medium
		Patrick	Vennebush		
TSG 23	Mathematical literacy	Ido	Gal	Mathematical literacy	Medium
		Hamsa	Venkat		
TSG 24	History of the teaching and learning of mathematics	Fulvia	Furinghetti	Mathematics education	Low
		Alexander	Karp		
TSG 25*	The role of history of mathematics in mathematics education	Costas	Tzanakis	History of mathematics	Medium
		Xiaoqin	Wang		
TSG 28*	Affect, beliefs and identity in mathematics education	Markku	Hannula	Meta theories	High
		Francesca	Morselli		
TSG 29	Mathematics and creativity	Dace	Kuma	Creativity	Medium
		Demetra	Pitta-Pantazi		
TSG 31	Language and communication in mathematics education	Judit	Moschkovich	Communication	Medium
		David	Wagner		

(continued)

(continued)

Type	Title	Presenter(s)		Category	Grade
TSG 32	Mathematics education in a multilingual and multicultural environment	Richard	Barwell	Socio-cultural	Medium
		Anjum	Halai		
TSG 33	Equity in mathematics education (including gender)	Bill	Atweh	Socio-cultural	Low
		Joanne Rossi	Becker		
TSG 34*	Social and political dimensions of mathematics education	Murad	Jurdak	Socio-cultural	Medium
		Renuka	Vithal		
TSG 35*	Role of ethnomathematics in mathematical education	Milton	Rosa	Socio-cultural	High
		Lawrence	Shirley		
TSG 51	Diversity of theories in mathematics education	Tommy	Dreyfus	Meta theories	Medium
		Anna	Sierpinska		
TSG 53*	Philosophy of mathematics education	Paul	Ernest	Meta theories	High
		Ladislav	Kvasz		
TSG 54*	Semiotics in mathematics education	Norma	Presmeg	Meta theories	Medium
		Luis	Radford		
TSG nn*	These Topic Study Groups had Topical Surveys published as part of the preparation for the conference				

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Chapter 3
Mathematics Education and Adult Learners in Ireland

John O'Donoghue

Abstract This paper attempts to develop an understanding of the concerns, issues and developments in mathematics education in Ireland in recent years as they impact on adult learners of mathematics and adult mathematics education generally. The paper offers three brief sketches as background including the Irish mathematics education landscape (IMEL), Adult education in Ireland, and Adult mathematics education/numeracy in Ireland. A small number of perspectives are offered and used later to clarify issues and priorities and generate insights regarding adult mathematics education in Ireland.

Keywords Adult mathematics education · Numeracy · Literacy Policy

3.1 Introduction

This paper may be seen as an argument for locating adult mathematics education (AME) in the wider field of mathematics education based on a careful examination of issues, concerns and developments in AME in Ireland. The author believes that such bottom-up and country by country work as this is necessary to understand the current state of adult mathematics education and policy influences and directions nationally, and by extension internationally, and could serve AME well if other researchers were to follow this example. Such understandings would offer a foundation for better implementation of policy initiatives and for mapping future directions.

This is a first attempt to develop a coherent perspective on adult mathematics education in Ireland against a backdrop of mainstream mathematics education in Ireland and major policy influences. The paper offers three brief sketches as

J. O'Donoghue (✉)
EPI*STEM: The National Centre for STEM Education,
University of Limerick, Limerick, Ireland
e-mail: John.ODonoghue@ul.ie

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background including the Irish mathematics education landscape (IMEL), Adult education in Ireland, and Adult mathematics education/numeracy in Ireland. While the author acknowledges the interdependence of literacy and numeracy, the emphasis in this paper is on the mathematical underpinnings of numeracy. A small number of perspectives are offered and used to clarify issues and priorities and generate insights regarding adult mathematics education in Ireland. But first a brief overview of the Irish education system is developed so that readers may have an appropriate reference point. A more detailed description of the Irish Education system is available in the country description written for the TIMSS 2015 Encyclopaedia (DES, 2016).

3.1.1 A Brief Overview of the Irish Education System

The Irish education system is highly centralised and consists of four sectors, primary, post-primary, higher education and further education and training. The Higher Education (HE) sector comprises 7 universities and 14 Institutes of Technology (IOTs). The Further Education and Training (FET) sector operates mainly in and through the Education and Training Boards (ETBs) and the voluntary sector and intersects the HE sector through the IOTs. The Minister for Education and Skills has overall responsibility for education in Ireland and discharges this responsibility through the relevant government department/ministry, the Department of Education and Skills (DES) and its agencies.

All citizens are entitled to free primary and post-primary education. There are a small number of fee-paying schools. Virtually all primary and post-primary schools are state-funded but ownership of the majority of schools is in the hands of church authorities and religious bodies that run and manage schools in accordance with agreed DES guidelines. A significant number of post-primary schools are in state-ownership and comprise all the vocational schools in the state. All state-funded schools must follow the relevant centrally devised curriculum viz. the Primary School Curriculum or Post-Primary School Curriculum.

Primary schools offer 2 pre-primary years and Grades 1–6 (8 years). Most pupils start school aged 4–5 and transfer to post-primary school aged 12–13. The period of compulsory education is from 6 to 16 years. Post-primary schools offer a Junior Cycle programme (3 years) followed by a Senior Cycle (2 years) where curriculum subjects are offered at Higher and Ordinary levels. Students may avail of a sixth year called Transition year at the end of Junior Cycle. The National Council for Curriculum and Assessment (NCCA) advises the DES on all matters related to the national curriculum and assessment. The State Examinations Commission (SEC) is responsible for the national Junior Certificate Examinations and the high stakes Leaving Certificate Examinations (school leaving certificate examinations).

3.2 The Irish Mathematics Education Landscape (IMEL)

The mathematics education landscape encompasses primary and post-primary school mathematics; mathematics in Institutes of Technology (IOTs) mainly Service mathematics; and mathematics in universities including service mathematics and mathematics teacher education programmes. There is an important sector known as Further Education and Training (FET) that provides education for young adults upwards who have exited mainstream education, through courses and programmes accredited by the Further Education and Training Awards Council (FETAC). Provision in the FET sector complements the provision in a relatively small Adult Education sector. The main focus of this paper is adult learners of mathematics and adult mathematics education. Adult learners of mathematics are directly involved in all sectors except primary and post-primary school mathematics but are nevertheless impacted by developments and practices in these other sectors as will be outlined later in the paper.

Recent and on-going reforms in school mathematics in Ireland have been driven by international developments in mathematics education and Ireland's relatively poor performance on large scale international studies such as Third International Mathematics and Science Study (TIMSS), International Adult Literacy Study (IALS) and Programme for International Assessment (PISA) over several cycles and their successors e.g. Programme for the International Assessment of Adult Competences (PIACC). According to the most recent findings Ireland is doing very well in PISA results, reading (ranked 3rd), but maintains its position above the OECD average in science (ranked 13th) and mathematics (ranked 13th) (Shiel, Kelleher, McKeown, & Denner, 2016). Thus it confirms Ireland's performance as middle of the road and significantly short of the performance needed to support the Irish government's ambitions in education and economic development. These studies have put a spotlight on concepts such as quantitative literacy, mathematical literacy and numeracy and their importance for national development ensuring their consideration in policy discussions in national settings. Other concepts such as basic mathematics, functional mathematics and functional numeracy also feature in this debate. Developments and recent reforms in mathematics education are outlined in Sect. 3.3.4.

3.2.1 Adult Education in Ireland

The adult education sector in Ireland encompasses aspects of Higher Education, Further Education and Training (FET), Adult Basic Education and Community Education. Adult education policy leans heavily on the government policy paper produced in 2000 that underlines lifelong learning as a fundamental principle of education (DES, 2000). This document defines adult education as 'the systematic learning undertaken by adults who return to learning having concluded initial education or training' (p. 28). Policy as enunciated here (p. 12):

> … seeks to ensure that there is a fit and complementarity between education and training provision, so as to ensure that learners can move progressively and incrementally within an over-arching co-ordinated learner-centred framework.

The document further describes the sector as including aspects of further and higher education, continuing education and training, community education, and systematic deliberate learning by adults in formal and informal contexts. The current shape of the adult education sector in Ireland owes much to this policy document, and developments in the years since its publication may be seen largely as a working through of its main tenets. Recent reforms affecting this sector e.g. the establishment of the Education and Training Boards (ETBIs), and the reform of the awards bodies, may be seen in this light. Every National Programme for Government and policy document in recent times has enunciated the importance of education and training for economic development and underlined the relationship between education and training especially for adult learners in terms of employment and employability (e.g. DES, 2016).

Currently, the country is served by 16 statutory Education and Training Boards (ETBs) who are responsible for the education and training needs of their regions. These boards are the largest providers of adult education in the country through their schools (vocational schools), Further education colleges and programmes e.g. Vocational Training and Opportunities Schemes (VTOS), Youthreach, Post Leaving Certificate (PLC) programmes, adult literacy/numeracy, Community education, the Back to Education Initiative. Qualifications in these sectors are accredited by the Further Education and Training Awards Council (FETAC) on a 10-point quality scale for education qualifications described in the National Qualifications Framework (NFQ) that is administered and developed by Quality and Qualifications Ireland (QQI). In this sector relevant mathematics/numeracy qualifications relate to levels 1–5 which can be viewed as an integrated adult mathematics framework for functional mathematics (NALA, 2013). Level 3, 4 correspond to lower secondary school and levels 5, 6 to upper/senior secondary school. FETAC levels 1 and 2 are entry level basic mathematics while levels 3, 4 and 5 can be described as functional mathematics. The sector and education generally is very well served by two NGOs both supported by the government, the National Adult Literacy Agency (NALA) and the National Adult Learning Organisation (AONTAS). NALA makes a very significant contribution to the National Adult Literacy Programme and to adult numeracy, matters of specific concern in this paper. Provision in this sector is greatly influenced by economic policy and considerations that drive upskilling, re-training and back-to-work initiatives.

3.2.2 Brief Sketch of Adult Mathematics Education/ Numeracy in Ireland

In this paper adult mathematics education is viewed as part of a continuum of lifelong and life-wide mathematics learning and spans all sectors of the education system, and is based on a broad conception of mathematics and mathematics education (Coben, O'Donoghue, & FitzSimons, 2000). The domain includes: school mathematics; specialist mathematics and service mathematics in HE; vocational mathematics in FET; mathematics for everyday living and adult numeracy in adult basic education; and workplace mathematics.

In Ireland as in many countries the adult status of traditional-aged students in mainstream Higher Education (HE) is scarcely acknowledged and little or no attempt is made explicitly to match their mathematics teaching/learning to their adult status. More and more adults or mature students are visible in the HE institutes and these numbers of mature students (>23 years old) are projected to increase under current government policy. Government policy is focussed on increasing access and retention in HE and this has led to access programmes for adults and the development of support structures such as Mathematics Learning Centres that sometimes combine these functions with research in adult mathematics education in order to pursue their brief. Mathematics learner support is well established throughout the HE sector in Ireland and makes a significant contribution to adult mathematics education in access, retention, service mathematics, adult numeracy, and adult mathematics education research.

There is widespread expectation that numeracy/adult numeracy relates to the workplace. But there is an inherent difficulty here since there is no *generic* workplace. The real world is full of different workplaces! All workplaces are different and place different demands on workers in terms of their numeracy (Keogh, 2013). This analysis points to a need to clarify terms and understandings in the numeracy discourse. In Ireland, workplace mathematics that falls within the ambit of specialist mathematics/applied mathematics is generally developed through in-house courses and/or award bearing programmes in HE institutes. The bigger numeracy demands of working adults are addressed in context by the workers and very often not treated as mathematics or numeracy. These adults must rely on their school mathematics, upskilling opportunities provided by various agencies such as NALA and the FET sector and government initiatives related to workforce skills needs. In a recent reorganisation of the Training and Further Education sectors both sectors have been amalgamated into a single Further Education and Training sector under the aegis of a statutory agency, SOLAS, the Further Education and Training Authority. This authority is specifically tasked to develop and implement a literacy and numeracy strategy for adults in its sector (SOLAS, 2014).

3.3 Selected Framing Perspectives

While the framing of the landscape activity is of its nature 'macro level' work, the author also pursues some 'micro level' issues for a more balanced view where appropriate. The framing perspectives offered include: government policy imperatives; mathematics education; the 'mathematics problem'; recent reforms and developments in mathematics education in Ireland; numeracy policy. But first an overview of government policy and the mathematics education landscape is presented.

3.3.1 Government Policy Imperatives

Mathematics is an integral part of the Irish Government's economic strategy and it is central to its future success. The Government in various reports has identified mathematics as underpinning the knowledge economy, and they see mathematics, science, technology and engineering as important pillars of economic policy (e.g. EGFSN, 2008, p. 1). This perspective informs and drives mathematics education at all levels in Ireland. It is given an urgent and compelling dimension when it is linked to national competiveness as is increasingly the case in Ireland. In their paper circulated by the National Competitiveness Council, McDonagh and Quinlan (2012, Sect. 1.1) capture the essence of this latter position:

> The qualifications necessary for a competitive technology and knowledge based economy in areas such as Applied Sciences, Financial Services, Computing, Engineering, Statistics and Technology are Mathematics based.

The Minister for Education launched a review of STEM Education in Ireland in November 2016. The report endorses the role of mathematics as an underpinning discipline for the study of STEM disciplines thus underlining the importance of mathematics in Irish Economic and education policies (STEM Education Review Group, 2016).

The convergence of economic, education and competiveness policies is a very powerful influence on mathematics and STEM education in Ireland and has a direct influence on adult mathematics education/numeracy (DES, 2016). This influence is felt through DES initiatives and policy in Primary and Post-primary education; Further Education and Training (FET) and Adult education; the Higher Education Authority (HEA) policy in relation to mature student access and participation in Higher Education (HE); and various government initiatives sponsored by the Departments of Jobs and Enterprise, Social Protection, including Labour Market Activation schemes, Training and Return-to-Education schemes.

3.3.2 Mathematics Education

Using mathematics education as a framing perspective for developments in numeracy including adults' numeracy raises a number of interesting questions. A small selection is identified here for treatment in later sections of the paper including: the role of school mathematics in numeracy development; and numeracy as an educational task. Given the mathematical nature of numeracy it would be surprising if mathematics education research had nothing to offer researchers and practitioners in this area, and similarly for literacy research. Historically, there is a link between literacy and numeracy and this is recognised in an early definition of numeracy (Crowther, 1959). However, the language and communication aspects of mathematics as a language have scarcely been exploited for numeracy. This is not to undervalue any research done along these lines but rather to advocate for more research in this area in the future. It would be very surprising indeed if research in mathematics education did not supply starting points, insights and frameworks for numeracy practitioners and researchers as they work with their adult learners.

3.3.3 The 'Mathematics Problem'

Many learners who enter Higher Education (HE) in Ireland are mathematically under-prepared for the demands of their mathematics courses in HE, and in their future careers (Gill, 2006; O'Donoghue, 1999). While Ireland acknowledges a 'mathematics problem' that shares common features with several developed countries e.g. UK, US, Australia, we must be alert to unique local issues that impact on Ireland alone. On the surface what looked like a localised problem at the school/HE interface turned out to be a deep system-wide problem. The so-called 'mathematics problem' spans all levels of education and general mathematical and scientific literacy in the wider adult population. It is a universal problem that impacts on science, engineering and technology education, and increasingly business and finance. Together these concerns shape and drive what might be termed the Science, Technology, Engineering and Mathematics (STEM) agenda in Ireland which despite its undoubted impact on education, is still largely uncoordinated across education sectors and invisible.

The Irish 'Mathematics Problem' is multi-dimensional and multi-faceted and has been characterised by Irish researchers (Liston & O'Donoghue, 2009) and the DES and others and has led to the introduction of supports in HE such as Mathematics Learning Centres. For the purposes of this paper the following dimensions have been identified to add clarity and focus to the discussion on mathematics: quality; numbers; performance and failure rates; transition issues; mathematics in HE; adult numeracy and workplace mathematics. Each dimension has a number of strands that integrate the various dimensions in numerous ways—some of these dimensions are flagged as the paper develops e.g. quality, performance, and adult numeracy and workplace mathematics.

3.3.4 Recent Reforms and Developments in Mathematics Education in Ireland

Efforts to address issues in mathematics education in Ireland happen in a climate of ongoing reform across various sectors of the system and impact our primary area of interest in this paper. These rolling reforms are numerous and wide ranging and include:

- New mathematics curriculum (primary),
- New mathematics curriculum (Project Maths) (post-primary), including CPD (post-primary: national rollout completed and fully examined) (2008–2015),
- New bonus points scheme for mathematics (post-primary/HE) (2012),
- Reforms in Mathematics teacher education (primary/post-primary),
- Out-of-field teachers of mathematics initiative/NCE-MSTL (post-primary/HE) (2012–ongoing),
- National literacy and numeracy policy (primary/post-primary/adult) (2011),
- New assessment regime in mathematics (primary/post-primary/adult),
- New-style mathematics exams (Leaving Cert/Project Maths),
- Mathematics textbooks re-examined (post-primary),
- Mathematics learning centres (HE), (from 2001),
- National Digital Learning Resources (NDLR) (Leaving Cert/HE),
- National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL)(all levels/adult numeracy) (2008),
- STEM education review published (2016).

These actions and other initiatives indicate a very vibrant mathematics education sector in recent years. While all of these initiatives may be viewed as successful in their own right, because these developments span sectors and involve multiple agencies they are unlikely to deliver optimum outcomes overall. This observation has special relevance to Ireland where numeracy policy is not well articulated across the various education sectors including adult education.

3.4 Numeracy Policy in Ireland

There are numerous actors and stakeholders engaged in the formulation of numeracy policy and provision including adult mathematics education. These include government departments and agencies, providers and voluntary agencies and span all sectors of the education system and informal education. The principal actors are the government departments; Department of Education and Skills, Department of Jobs, Enterprise and Innovation and Department of Social Protection. DES agencies include the Higher Education Authority (HEA), the Further Education and Training Authority (SOLAS), Education and Training Boards (ETBs), National Council for Curriculum and Assessment (NCCA); voluntary membership organisations include the National Adult Literacy Agency (NALA), AONTAS (National Adult Learning Organisation); the business, industry and employers voice, IBEC (Irish Business and Employers Confederation).

The education landscape in Ireland in recent years has been greatly influenced by the country's performance in studies such as IALS, PISA, PIAAC and TIMSS and these in turn have driven numeracy and literacy policy in Ireland. National policy in Ireland as enunciated by the relevant government ministry, Department for Education and Skills (DES), in their report *Literacy and Numeracy for Learning and Life 2011–2020* (DES, 2011), places the burden of delivery squarely on the school mathematics curriculum at primary and post-primary levels but not exclusively so, as there is a role for other disciplines and initial and continuing teacher education in the development of literacy and numeracy. The recently reformed school curricula in Ireland, especially mathematics, have been identified as an appropriate vehicle.

School mathematics was and is still seen as the main vehicle for attending to these and other major concerns in this area. In this context, it is important that school mathematics is seen as an integral part of a lifelong mathematical experience extending from elementary education through to higher education including numeracy. Cockcroft (1982) underlined the mathematical nature of numeracy and suggested a continuum between school mathematics and adult numeracy. However, this is not the case in many countries and there is gap in this lifelong mathematics continuum especially as we look at adult mathematics education/numeracy and workplace numeracy in Ireland.

In Ireland, issues related to numeracy, insofar as there has been a separate coherent national debate on numeracy issues, have been largely subsumed in the discourse surrounding school mathematics.

The term numeracy as it features in current debates was introduced into official use in the report on adult education (DES, 1998). To show that numeracy was on the agenda this report advised readers that literacy <u>and</u> numeracy were intended when there was a reference to literacy. However, the subsequent government White Paper on adult education fails to repeat this message (DES, 2000). No official working definition of numeracy was available for some years afterwards. Historically issues related to numeracy were viewed in the context of the '3Rs' (<u>R</u>eading, w<u>R</u>iting, a<u>R</u>ithmetic, historically the principal concerns of elementary education) or treated under literacy. The concern in this regard was largely for competency in basic mathematics for school leavers and adults. However, the perceived scope of numeracy has widened in recent years but it is still bracketed with literacy, and in the case of adult education, included as an element of literacy (e.g. https://www.nala.ie/literacy). Despite this anomaly, NALA has been active in adult numeracy development and research doing very good work in the domain mapping a numeracy strategy and addressing the numeracy needs of tutors (see NALA, 2004, 2013).

SOLAS, the Further Education and Training Authority, is specifically responsible for developing and implementing a literacy and numeracy strategy for adults in the FET sector (SOLAS, 2014). The SOLAS FET strategy 'proposes that literacy and numeracy support should be integrated or embedded in FET programmes, as appropriate' (SOLAS, 2014, p. 99). This strategy is encapsulated in a 12 point plan with a number of points giving prominence to numeracy concerns. These concerns are summarised here as follows: point 7—prioritise numeracy and increase all forms of provision; point 10—develop fit-for-purpose screening and assessment instruments for literacy and numeracy through research; point 11—identifies the need for and supports staff development for literacy and numeracy tutors; point 12—supports research into effective literacy and numeracy practice. It is interesting to note that the FET strategy for adult numeracy is not based on any formal recognised definition of numeracy but rather on definitions of literacy e.g. DES (2013). The expectation is that literacy and numeracy strategies in both sectors would complement each other allowing for significant synergies between them to be exploited.

3.5 Numeracy as an Educational Task

The author has addressed this issue in a previous paper (O'Donoghue, 2011, p. 5) and inter alia proposed key elements of a national numeracy strategy built on school mathematics:

- an educational characterisation of numeracy and age-related clusters of learning outcomes
- an appropriate mathematics curriculum capable of generating numeracy as an outcome
- a well-aligned assessment regime (weighted towards assessment for learning especially for young school children)
- teachers capable of delivering the curriculum with a numeracy focus.

These elements are necessary but not sufficient and give us a framework for action. A national strategy needs to be driven and sustained; top-down and bottom-up actions are needed by different actors and stakeholders.

However, numeracy concerns do not end with school mathematics. Numeracy issues span the various education sectors especially the school sector and further education and training sectors in Ireland and their equivalents elsewhere in the world. An aspiration expressed by the Minister for Education and Skills that both literacy and numeracy strategies currently in operation in Ireland for children and adults would be articulated, has not yet been realised (Speaking notes, January 2016). A learner centred approach is well established in community education but a more traditional approach to mathematics/numeracy is evident in the programmes in the FET sector working at levels 1–3 of the National Qualifications Framework (Mathematics). This framework may be accessed at the Quality and Qualifications Ireland (QQI) website.

The paper also raises some important questions that are rarely dealt with in an explicit manner. These questions relate to both children and adults alike and are included here for consideration as they still demand attention. The list is not exhaustive but it does address:

- The meaning of the construct 'numeracy',
- The role of mathematics curriculum in numeracy development,
- Numeracy as a mathematical competence,
- Numeracy and 'real world' issues.

3.5.1 The Numeracy Construct

The literature is replete with descriptions and definitions of numeracy (Kaye, 2009). Some conceptions of numeracy relate specifically to school mathematics others treat numeracy as an adult competency (OECD, 2009). In the first instance it is important to realise that numeracy is a theoretical construct used variously inter alia to describe some policy target of government policy makers; educational outcome for school children; a desirable personal competence for school leavers, adults and workers. In practice it is often the case that there is no clear discrimination in the use of the concept and uses coalesce in people's minds.

Further confusion is engendered by multiple definitions and the undifferentiated use of related concepts such as *numeracy* (Cockcroft, 1982); *quantitative literacy,* International Adult Literacy Study (IALS/OECD, 1995); *numeracy as a life skill*, Adult Literacy and Life Skills Survey (ALLS, 2003); *mathematical literacy*, Programme for International Assessment (PISA, 2000–2012); *numeracy*, Programme for the International Assessment of Adult Competences (PIAAC, 2013). It should be noted that the relevant construct in these studies is carefully defined together with carefully designed assessment frameworks and assessment instruments. The official Irish definition used in the national numeracy strategy is set out below in Sect. 3.5.3.

3.5.2 The Role of School Mathematics in Numeracy Development

Clearly there is a role for the mathematics curriculum in the development and promotion of numeracy. There is widespread consensus on this point nationally and internationally. But that role is not well developed, or clearly defined or understood.

The contention made here by the author is that all mainstream school mathematics curricula should be capable of delivering numeracy and much more for the vast majority of school students provided they stay in school long enough, for example, until the end of compulsory education. That said, what is needed is a mathematics curriculum capable of developing in students/learners the bundle of mathematical competencies and dispositions and more that are associated with numeracy. Further, numeracy is not an automatic by-product of schooling and the appropriate mathematical competences must be prioritised and assessed in age-relevant ways at school level.

The official Irish report setting out government policy subscribes to the view that numeracy is achieved by mastering a wide range of mathematical content, processes and skills and involves a range of mathematical competences e.g. problem solving, mathematical modelling, and the use of mathematical language and communication (DES, 2011). The learner is numerate when s/he displays a capacity, confidence and disposition to use mathematics to meet the demands of living in a modern society including further learning and training, work, community and civic life. Opportunities to develop and display these competencies must be provided within the curriculum context for age-related categories of learners e.g. primary/elementary school children, junior secondary and senior secondary school students.

Since mathematics and society will evolve, numeracy has an implied 'lifelong' characteristic. This means that learners, adult or otherwise, must be equipped with the skills to engage in lifelong learning. Numeracy will evolve as a competency and must always serve the needs of individuals in society as learners, workers and citizens. It must therefore be seen as time dependent and technology dependent.

3.5.3 Numeracy as a Mathematical Competence

The official definition of numeracy and its elaboration presented by the DES (2011, p. 8) in their major strategy document for Ireland is used to set the scene for this section:

> Numeracy is not limited to the ability to use numbers to add, subtract, multiply and divide. **Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings.** To have this ability a young person needs to be able to think and communicate quantitatively, to make sense of data, to have spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems.

It is interesting to note that while the Irish position acknowledges the interplay between literacy and numeracy in their policy document each is defined separately.

Clearly this definition of numeracy is unattainable without achieving many mathematical competencies e.g. problem- solving and communication skills. However, numeracy is not 'less than maths but more' (Johnson, Marr, & Tout, 1997, p. 167) or to interpret this perceptive insight in the current context, numeracy is not less mathematics but more mathematics and even more as it involves extra-mathematical competences. Using mathematics in context, problem solving and applying mathematics involves other skills such as reading, noticing, modelling, planning, monitoring, translating, verifying and communicating. These competences apply to children and adults representing a more immediate need for adults in the context of working and living in a modern society. While numeracy for children is defined in the national strategy document, there is no such characterisation for FET sector as a whole. In practice, this sector and adults benefit from the good work of the National Adult Literacy Agency (NALA) who has developed approaches and resources for numeracy practice based on a definition of literacy that includes numeracy but numeracy is defined in a glossary as '*A lifeskill that involves the competent use of everyday mathematical skills and the confidence to manage the mathematical demands of real-life situations*' (NALA, 2014, p. 21).

The need for policy makers to offer clear guidance in this area is important as educators in all sectors need clarity when they try to develop practice, resources and professional development and integrate and articulate provision across sectors. The PISA thinking as enunciated in their framework document is an example of clarity and adds weight to the above argument as it unambiguously defines their mathematical literacy construct in mathematical terms (OECD, 2009).

3.5.4 Numeracy and the 'Real World'

For the purposes of mathematics education and numeracy including adult numeracy, there is a lack of clarity on what is meant by the 'real world', or what constitutes the 'real world'. It is not fully or accurately captured in terms that are commonly used in this context such as 'everyday life', 'work',' active citizenship', or in phrases such as 'mathematics for life', 'using mathematics in context' and 'mathematics applied to the real world'. From the perspective of numeracy and adult numeracy development it is important that these descriptors are treated as problematic, since they demand more than a common colloquial meaning that is found in much discourse around numeracy.

Even a superficial analysis of issues surrounding the 'real world' descriptor leads to practical considerations around formulating educational outcomes for numeracy in the context of school mathematics and mathematics education generally. For example, what do we posit as the 'real world' for young children in elementary school? Or posit for students in the Secondary and Compulsory education sectors? Or posit for adults in Adult education? Is it the same in all circumstances? Such an approach is essential at national level when numeracy policy is defined and implemented for all learners.

PISA addresses this issue head-on and defines its understanding of the 'real world' as it is used in their studies (OECD, 2009). PISA characterises the 'real world' using four broad contexts identified as personal, occupational, public and scientific. These contexts give rise to mathematical problems in everyday life.

A separate, fifth category, was used in the first cycle of PISA (2000) assessment (OECD, 2003). This category was called 'intra-mathematical' in order to include purely mathematical items. This intra-mathematical context was later subsumed in the scientific category. Interestingly, the PISA description of the 'real world' includes mathematics itself which is often excluded from such discussions. Further, when the Realistic Mathematics Education is examined it sheds light on what can be expected from children of all ages as it makes available for use everything a child can be expected to experience and *imagine*. Generally, the 'real world' is interpreted in various ways to include the 'real physical world' excluding the world of ideas (e.g. mathematics); the 'world we live in and experience'; 'everyday life' usually qualified to mean the life of ordinary people who are not scientists, engineers or other mathematics-using professionals; 'the world of work' is usually taken at face value without serious discrimination between work situations and demands on individuals.

3.6 Issues Related to an 'Embedded' View of Numeracy

International studies such as IALS, ALLS, PISA, PIAAC and TIMSS have driven numeracy and literacy policy in many countries in recent years. One consequence of this influence is that the burden of delivery of numeracy is placed squarely on the school mathematics curriculum and mathematics teachers. More generally, school mathematics is seen by governments as having a major role in developing individuals, society and the economy. For example, mathematics is now acknowledged as an underpinning discipline for all the STEM disciplines (STEM Education Review Group, 2016). The goal of school mathematics is to provide a worthwhile, relevant, meaningful mathematical experience for all children. Increasingly for many, this means teaching children to be positive about mathematics; to learn mathematical concepts in a connected way; how to reason and be creative; how to communicate mathematics; developing tools for solving problems in mathematics and real contexts; and also incorporates facts, skills, routines and strategies. This is a very comprehensive mission for school mathematics and as experience suggests is a very difficult one to achieve. It implicates learners now as young students and later as adults who may have reaped the benefits of school mathematics or not. Additionally in this context, school mathematics is expected to deliver several specific contributions for the nation including:

1. Supplying the next generation of mathematicians,
2. Providing the necessary numbers and quality of candidates for the STEM pipeline,
3. Providing an abundant supply of candidates to become mathematics professionals, including mathematics teachers, to service education, business and industry,

4. Raising the standard of mathematics in the general population,
5. Solving the adult numeracy problem.

In Ireland the consensus among policy makers, educators and relevant stakeholders on the success or otherwise of the education system has been consistent for many years and it points to a lack of success in several areas related to literacy, numeracy, mathematics and science. The following extract from a major policy document (DES, 2011, p. 13) regarding numeracy describes the situation:

Numeracy

– Repeated assessments of mathematics at primary level have revealed weak performance in important areas of the mathematics curriculum such as problem solving and measures
– The proportion of students who are studying mathematics at Higher Level in post-primary schools is disappointing. The proportion taking Higher Level in the Leaving Certificate examination has been in the region of 16% for a number of years, despite an aspiration in the design of the Leaving Certificate mathematics syllabus to have 30% of the cohort taking the Higher-Level examination
– The performance of students in Irish schools in international assessments of mathematics has been disappointing and has declined in recent years. In the most recent PISA tests, the performance of Irish fifteen-year-olds was at “below average” standard, ranking 26th out of 34 OECD countries
– About one-fifth of Irish students in the PISA 2009 tests did not have sufficient mathematical skills to cope with every-day life and Ireland also had significantly fewer high performing students than other countries.

In addition to highlighting specific concerns in mathematics education/numeracy this extract clearly demonstrates an ‘embedded’ view of numeracy. On the positive side action has been taken on all fronts leading to major gains in literacy and numeracy as measured and reported in national and international studies. Since 2011 when the National Literacy and Numeracy Strategy was launched, Ireland has participated in several international studies including TIMSS (2015), PISA (2012, 2015), PIAAC (2012), PIRLS (2015), and a National Assessments of English Reading and Mathematics (NAERM) (2014) for grades 2 and 6. In a recent report (DES, 2017) the government reported significant gains in all areas of the national strategy surpassing the 10 year strategy targets by midterm based on a consideration of these studies and other factors (see Central Statistics Office, 2013; Clerkin, Perkins, & Cunningham, 2016). Interestingly, the review showed a need for a greater focus on numeracy in the second half of the strategy period.

When the demands on school mathematics are unpacked in this manner (items 1–3 above) it is easy to see the potential for obstacles to success to arise in practice. Since the concern in this paper is for adult mathematics education/numeracy items 4 and 5 are candidates for closer scrutiny. For example, how does one reconcile expected contributions 4 and 5 above? School mathematics is the primary vehicle for raising the standard of mathematics in the general population, and is also seen in many countries, including Ireland, as a major vehicle for developing numeracy in the wider population. However, this dual function is difficult to implement in practice. The situation is complicated by incomplete understanding of what these functions/goals might mean in terms of educational outcomes for students. For example, numeracy outcomes involve a range of competencies not always treated effectively in school mathematics such as reading, noticing, modelling, planning, monitoring, translating, verifying and communicating, and also rely on non-mathematical knowledge from real life and other disciplines. These competencies may be part of school mathematics but that is not always the case or they are paid insufficient attention. Such circumstances render numeracy outcomes difficult or impossible to achieve within the context of school mathematics.

Also this policy is directed at future populations and even if it were completely successful there will always be a legacy mathematics/numeracy problem from the current adult population and those who are not well-served contemporaneously by the system. This situation sets up what I term the permanent innumeracy pipeline (see Sect. 3.7.1).

The situation is further complicated if the idea of raising mathematics standards in the general population is examined closely. Is this goal synonymous with numeracy? The demand for higher standards in the general population surely must be interpreted as higher than what is commonly understood as numeracy if even if numeracy were to be accepted as a mathematical standard.

There are more issues related to teacher education/knowledge. In secondary schools it is fair to say that mathematics teachers will be expected to carry the burden of numeracy education as champions with assistance from other subject teachers especially STEM teachers. But this is not unproblematic as real world applications of mathematics involves crossing subject boundaries especially at secondary level, and mathematics teachers' ability to do this cannot be assumed as a given. Long-serving secondary mathematics teachers cannot be faulted for their lack of expertise in new areas of emphasis in mathematics and new approaches as these concerns were not part of their original initial teacher education and little provision was made for their continuing professional development during their professional life. While these issues have been addressed in a front-loaded manner for all serving mathematics teachers in the most recent curriculum reform initiatives in mathematics education, and the ongoing need for relevant CPD is recognised, a suitable permanent framework for CPD has yet to be implemented.

The situation is equally pressing for primary school or elementary teachers who are in the main generalist teachers. Ireland has taken steps to ensure success in their dual objectives (Item 4 and 5) by implementing a national strategy for literacy and numeracy that maintains a focus on numeracy outcomes and provides for numeracy training for primary and secondary teachers, and is greatly assisted by the implementation of a new mathematics curriculum at secondary level together with extensive CPD for existing mathematics teachers and upskilling of out-of-field teachers of mathematics.

Measures to improve access, participation and retention at second level and to reduce failure rates will contribute to improved standards in the general adult population. Similar moves at HE level will also yield considerable benefits in this regard. More can be done through targeted initiatives such as recruitment and support of mature students, outreach activities, and second chance education where the gatekeeper function of mathematics needs to be ameliorated.

A special case could be argued for more resources for adult mathematics education as it intersects the whole area of access, disadvantage, second chance education, up-skilling, and workplace mathematics. This neglected area of mathematics education is likely to exercise a drag effect on the national agenda for STEM education and the smart economy unless it is addressed coherently in the short and medium term.

3.7 Challenges for Adult Mathematics Education in Ireland

This overview of adult mathematics education in Ireland notwithstanding its brevity and local character does lead to a small number of insights that are likely to resonate with the wider community of researchers and practitioners in adult mathematics education, and challenge them as they seek to improve adult mathematics education.

3.7.1 The Innumeracy Pipeline

Why is there an adult numeracy (literacy) problem in a developed country like Ireland? (Ireland is not alone among developed nations with such problems). The reasons are complex and difficult to unravel and range beyond the scope of this paper. However, such discussion as is entered here is used to focus attention on the *innumeracy pipeline*.

Compulsory education for all Irish citizens covers the period from 6 to 16 years. If you exclude those who cannot engage fully with the education system for one reason or another, virtually the whole population has been in school for at least the compulsory years. The inescapable conclusion is that the education system is failing learners during their school years and contributing to the country's own numeracy problem in adulthood.

In particular, the quality of the student experience in mathematics education has been found wanting and acknowledged (Conway & Sloane, 2005) and this realization, and other wider policy considerations, (Sect. 3.3.1) has led to major initiatives in mathematics education outlined in Sect. 3.3.4.

The pool of adults with inadequate numeracy has held stubbornly at c.25% of the adult population for two decades. This estimate is based on the IALS (1997) data (DES, 2007) (25% at Level 1) and PIAAC (2012) (just over 25% at Level 1). These numbers are fed annually by an *innumeracy pipeline* that is simultaneously reduced by adult education and training, workplace training and upskilling, and learners' personal efforts. The innumeracy pipeline is replenished annually by a number of sources and the sources identified will persist until the reforms outlined in Sect. 3.3.4 begin to make an impact. They are:

- the number of students who drop-out of secondary school annually without a qualification;
- the number of students who achieve poor grades annually in two of the three levels of the School Leaving Certificate (Mathematics) viz. Ordinary Level mathematics; Foundation Mathematics at Leaving Certificate level.
- the number of students failing their School Leaving Certificate mathematics annually (c.5000 students).

Poor performance and failure rates limit the educational and employment prospects of young people (EGFSN, 2008). These numbers contribute annually to adult innumeracy in the general population and the workforce.

3.7.2 Disconnect Across Sectors

Very often the realization that mathematics is implicated in learners' numeracy leads to basic mathematics such as number work and calculations excluding all other aspects of mathematics. Cockcroft (1982) underlined the *mathematical* nature of numeracy and suggested a continuum between school mathematics and adult numeracy. Cockcroft's continuum can be extended to a 'lifelong continuum of mathematics' for individual learners that locates learners at any point in their mathematics journey and helps guide and integrate mathematics provision nationally. Here constructs such as the *numeracy construct sophistication continuum* (Maguire, 2003) and *big numeracy* (Kaye, 2009) offer useful models for guidance and coordination across sectors.

Such ideas are not fully exploited in numeracy work in Ireland because little continuity is evident in lifelong mathematics provision. Notably, a significant gap is evident in the continuum between school mathematics and adult mathematics education/numeracy and workplace numeracy in Ireland. Different conceptions of numeracy inform school mathematics, Further education and Training, and adult numeracy and workplace mathematics for adults. For adult learners, as that designation is widely understood, the situation in Ireland is very much centred on basic mathematics and literacy (Adult Basic Education/Community Education) and functional mathematics (in FET) while school mathematics promotes a wider mathematical interpretation of numeracy.

3.7.3 Mathematics Education Research

Potentially, significant benefits are available but largely unrealised for acknowledging the mathematical nature of numeracy. Understanding the mathematical nature of numeracy and its ramifications for adults' mathematics/numeracy is a mathematics education question that falls squarely under the remit of mathematics education researchers. Putting numeracy on mathematics educators' research agenda is an immediate benefit that could lead to more numeracy research, and it also makes available in a more systematic way the whole gamut of mathematics education research for application to the lifelong mathematics continuum encompassing numeracy for all learners including adults. Finally, such engagement creates a two way street between mathematics education research and research in adult mathematics education/numeracy that must lead to mutual benefits.

3.8 Conclusion

In this paper the author has attempted to develop a coherent perspective on adult mathematics education in Ireland against a backdrop of mainstream mathematics education in Ireland and major policy influences. Concerns arising in the Irish context are presented and discussed and it is anticipated some of these will appeal to a wider audience. The author is not aware of any previous attempt to address adult mathematics education in Ireland in this manner and despite inevitable shortcomings associated with first attempts offers the paper as a baseline for future work.

There is a natural evolution here as the principal motivation for engaging in this exercise was to locate adult mathematics education in the wider mathematics education field as opposed to some other discipline e.g. education, in order to establish identity and exploit the parent field for benefits of association. This perspective is exploited throughout to raise issues for consideration which are discussed using various mathematics education lenses. Consequently, throughout the paper when numeracy is discussed the emphasis is on the mathematical underpinnings of numeracy.

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Chapter 4
Thinking About Relations Between Adults Learning Mathematics and Reality

Jürgen Maaß

Abstract Mathematics education for adults (and youth) takes place in our world. Whatever is going on in our world, in the "reality", might have influence on our courses, our students and on us ourselves. My thinking about relations between adults learning mathematics and reality start with a simple question: What is reality? Simple questions often open doors to other questions and looking through one of the doors we see that social reality has a main influence on adults learning mathematics. Why? Hoping for a (better) job or fearing to lose a job is a strong extrinsic motivation to learn mathematics for adults even if they have bad experience with learning mathematics at school. This implies several needs for course construction, pedagogical behavior of teachers and the relations of the institution that offers the course, the teachers, and the learners. One very important aspect of the social reality of our area of research and teaching called "adults learning mathematics" is the difference between adults and younger students learning mathematics. Life and job experience causes many differences. An other important aspect is the didactical intention to teach mathematics with the aim to show its practical use in reality. But if you teach adults this is different from teaching mathematics with the same idea in school because adults know—sometimes much better than their teachers—the real job situation. The last but not the least aspect of the many relations I am thinking about is the impact of our work in reality. Researching, teaching and modelling might change the world. This implies ethical questions about our responsibility.

Keywords Adult · Mathematics · Education · Numeracy

J. Maaß (✉)
University of Linz, Linz, Austria
e-mail: juergen.maasz@jku.at

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4.1 What Is "Reality"?

Maybe you are a little bit astonished about this question? Everyone knows the answer; we all live in a common reality. Are you sure? Think about a soccer game. Fans of both teams see the same situation differently. Fans of team A have seen a foul and fans of team B have not. Both groups know without a doubt what they have seen and both groups are convinced that their observation is correct. Maybe your argument now is that fans are not objective. Belonging to a fan group is a synonym for not being objective. Yes, we agree. But if we found one example for two realities at the same time and the same place in the mind of many people we have opened the door to many more examples. The philosopher Rene Descartes has started his "Meditations" with one doubt and concluded that each observation might be wrong. We are never sure that we see the one and only one reality. His conclusion was: The only thing we know really without any doubt is that we exist if we think (have a look at https://en.wikipedia.org/wiki/Meditations_on_First_Philosophy to learn more about his book).

From my point of view the question "What is reality?" is not only a question that was interesting for a philosopher that died long time ago. Why? I ask you to read descriptions of what really happens in Syria now given by governments from Syria, Russia, USA, Iran, Saudi Arabia etc. and by people that are or were involved in the conflict (as refugees, members of UN and other helping organizations etc.). After reading statements about Syria for some hours I really do not know the reality in that region of the world. But I get the feeling that there are really different views on reality maybe driven by different ideologies, aims, experience etc. I do not want (and I can't) explain what is going on in Syria but I want to use this example for my argumentation: there are many reasons to think about "reality" in relation to adult mathematics education. One hint arising from this example is that a lot of refugees from Syria came to Austria (and other European countries). Our government is trying to organize some help to integrate these people by education, for example further education in mathematics. Thus, we have new courses with special needs and conditions. Teaching mathematics and other subjects in such courses are planned fundamental on a point of view of reality now and in future: What is the situation now and what are the general and special aims of the courses? Different views implicate different reactions, as you can see easily if you imagine extreme views like the following. If the government "knows" that these refugees all are agents of the devil that will destroy our country if they reach it, the government will not pay education courses for them. If the government thinks that these people are exactly those that should by attracted with some action like "green cards" the government will do a lot to welcome the refugees. The reality in European countries now is somewhere in the scale between these standpoints—but wherever it is: It is obvious that the governmental view on reality is very important for the frame and the content of education.

I will not explain this now but I think this example shows very clear how much our work as mathematics educators depends on what is going on in the real world around us. I come back to different aspects of the influence that reality (better: the different thinking about the question what reality is) on our work.

Excurse: I know that there are a lot of good reasons not to talk about politics (and reality and different views on it) while we are talking and writing about our science. Going back in history about 500 years we find very important reasons looking at the conflict of new arising sciences (connected with names like Copernicus, Galilei, Kepler, Bruno and many others) and the Catholic church in Europe. For example, M. Kokowski, a Polish professor working at the "Department for the History of Exact and Natural Sciences and Technology in the Institute for the History of Science of the Polish Academy of Sciences", argues that the main aspect of the Copernican revolution was not some new knowledge about sun, earth and movement of planets but the attempt to "see" reality independent from the authority of church, religion and the holy book using scientific methods and technology (telescope, physics, mathematics,…). This was recognized as a frontal attack against the authority of church. People discussed: If the church is wrong explaining what reality is in the sky maybe it is wrong in explaining other things, too? What about ruling and obeying, government and paying taxes? Is our king really a king of God's grace? A bloody conflict started. Do you remember how this conflict was calmed? Founding the Royal Academy of Sciences in England the members promised "not to meddle with politics" and the King promised to guarantee something called the freedom of science. In other words: If the scientists stop to interpret or transfer their results into the political sphere they are allowed to research and communicate about results within their group without (or with less) fear.

Additional remark: Do you think this is only an old story about the beginning of the period of history, when sciences started their successful development? I do not think so: I have in mind a lot of situations up to this day where politicians and religious leaders suppressed sciences or killed scientists because they had other views on reality. Some keywords are "German Physics" (Einstein's "Jewish" theory was forbidden by the fascist regime in Germany), the "Manhattan Project" (to build the first atomic bomb), the systematic killing of academic people by Red Khmer/ Khmer Rouge and many others. Today one of the battlefields of political conflicts including sciences is marked with the keyword "Neoliberalism", another one is known under that headline "intelligent design". In all these cases the important thing from my point of view is the fight for power to *decide* what is reality. Now I feel like a taboo breaker pointing out these hints to the history of sciences but I insist on the fact that I do not want to explain my opinion about things that happens in history or now. I just want to proof that there is a strong relation of what is defined (by power) as "the" reality and our work as teachers and researchers in the field of adults learning mathematics.

My second philosophical hint is related to a thesis written by Sir Karl Popper in his book "The Logic of Scientific Discovery" (published in German, Tübingen 1934. Popper rewrote his book in English and republished it in 1959 see: http://strangebeautiful.com/other-texts/popper-logic-scientific-discovery.pdf). He wrote that each observation of reality is done by more or less useful or correct theories in our brain. Ears and eyes and other senses transport data to the interpreting brain. This opens the gateway to our own knowledge about learning. Each student learns with his or her own brain by interpreting incoming information. The students are different individuals with different experiences and different brains. This is a simple explanation for different learning results. On the other hand, this is a good reason to improve the quality and efficiency of courses for adults by checking the knowledge and the competencies they bring with them at the beginning. It is a very good idea to start the course where the learners are, isn't it?

Before I come back to the philosophical aspects I should add that a lot of research has been done to show that not only mathematical competence and qualification is important but the emotional relation to mathematics (see for example the books published as proceedings of the MAVI—"Maths Views" conferences. Type "MAVI proceedings" in a search window to find them all! Next one will take place in Helsinki in Summer 2018—or Evans, Wedege, & Yasukawa, 2013), too. In many countries research has shown that many adults have a negative emotional relation to mathematics because they had a bad experience with their learning mathematics in school. Asking adults about their experience with their learning mathematics in school they often have in mind positive feeling when they remember how to recalculate a bill in a restaurant or a shop. Elementary geometry (how much color I need to paint the walls of my living room?) was also something they understood. Percentages are also seen as useful. But if we start to talk about equations, especially algebraic equations including letters (as constants and variables) instead of numbers I often was confronted with pure hate. "I did not understand this!" "I was not able to calculate this letters—they are not numbers!" "The teacher blamed me in front of the classroom because I could not solve such tasks!" Some adults told me that they still (at age 40 or 50) have bad dreams about failing mathematics tests. These adults know that higher level mathematics (starting with algebra) is completely useless. This knowledge about reality combined with negative emotions is a great barrier against learning mathematics as adult learner. A lot of education problems are arising from this blocking combination and a lot of research is done to find strategies to handle this. I think a good way is to build up personal good relation (trust) that is the fundament for a load-bearing capacity to go the way into the area of higher level mathematics together.

This is a very important example for educational problems caused by different knowing what mathematics (algebra) really "is". Many adults know that it is useless, something made by the devil to make them feel blamed and angry. Hopefully many teachers know how useful algebra is in reality.

The next step in my argumentation about philosophical theories and our relation to reality is a hint to **common** theories that help us to understand and to "see" reality. These theories are result of education and socialization in a society. Therefore, they depend on common convictions about the world and everything around. Some of these theories are fixed as rules for good behavior like "respect your parents", "do not steal anything", or a set of rules for visiting relatives or friends or staying in a hotel and so on. Many of these theories depend on scientific research taught in school or described in a magazine or a TV documentary. For example, knowledge about healthy food is often condensed and spread by media.

Habermas (1968) has written a general theory and a lot of interesting examples for common knowledge of societies, generated by communication and agreement. On one hand, it is fine if a society has a common knowledge base, common ideas about good rules for living and behavior etc. But on the other hand, some (or many) members of a society tend to believe that a common view on reality is the one and only one always true view on reality. Some of these people are tolerant and some are not: They try to force other people to share their view. Being intolerant and forcing people to join a group is not acceptable from my point of view. In addition to other reasons such people often are very much convinced to know the reality and the norms to change it along their wishes. I try to hit them by putting this argumentation in the center: There is no reason to think that anyone knows the objective and correct truth about reality. Going back to the example "conflict in Syria" I mentioned on the first page I think it is easy to see that all conflict parties have different views on the reality in Syria, on the history, on the activities of the other parties and so on. If there would be only one true view it would seem to be easy to solve the conflict by convincing all parties to share this one true view. Do you think this is possible? It is very difficult to argue about social and religious conflicts that are part of an ongoing civil war. I will not try this. I go back to my intention of "thinking about reality".

I select an example about physics to point on limits and the origin of common knowledge about reality. What happens if you take a stone, lift it to your eyes to watch it and open your hand afterwards? I think your answer is like this: The stone will fall down to earth. Looking at such an answer we find the term "fall down" and we remember that these words reminds us to the theory our brain uses to interpret the movement of the stone. In physics lessons at school we have learned something about gravitation. The enormous mass of the earth attracts everything with a specific acceleration (about 9.81 m/s^2). Now I offer you another basic theory about the reason for this acceleration. This reason is very much different from that we learned in school. I call it "the love of Mother Earth". If you like to decorate my new theory to interpret the moving of the stone (and everything else) in direction of the heart of mother earth you maybe like to know that Mother Earth loves everything on her surface so much that she attracts it as far as possible each moment. When you open the hand Mother Earth at once starts to attract the stone (with the acceleration about 9.81 m/s^2).

Are you amused about this theory? I am! But I think you should keep in mind that this theory is absolutely identical with our well-established theory about gravitation. Only the name for the reason of the movement and some decoration around is different. Imagine we all would have learned the Mother Earth theory for the reason of the movement of the stone and other objects at school. In this case, we would be astonished or amused about the new theory and naming "gravitation". O.K.? This is the important point. Our knowledge, the theories we use to interpret what we see and the naming for this in a product of our education or learning from consuming information given by media. In other words: Our understanding of what we think is "reality" depends on social consensus about basic theories and our individual understanding of these theories. Nothing is fixed, everything is flowing. The old Greek philosopher Heraklit (Heraclitus) said this about 2500 years ago in his language: *panta rhei*.

Sidestep: When I talked about Mother Earth in Hamburg at the end of my presentation the co—chair of our TSG 6, Pradeep Kumar Misra, told me that in India many people really believe that there is a Mother Earth. He explained that his father every morning before he stands up touches the earth with his hands and to excuse for walking on her. I did not know this when I was looking for an example to explain the relation of common education and common point of view.

4.1.1 Framing the Personal View on Reality

Looking at another important aspect of the Mother Earth example gives a chance to understand the importance of the theories we use to understand what we see as **frames** of understanding. Research in mathematics education (for example done by the Bauersfeld group in Bielefeld, Germany) offers us a lot a very interesting examples for different understandings of mathematics caused by different framings. I remember one nice example from a personal talk with Jörg Voigt. He filmed a teaching sequence about calculating triangles with Pythagoras. The students learned how to solve tasks like "We have a rectangular triangle where c is the hypotenuse with given sides a = 5 cm and b = 7 cm. Please find out the length of side c." After several tasks from this type the learners used to do like this: $c = SQRT(a^2 + b^2)$. The frame was clear to everyone: Use Pythagoras to calculate the length of c. Then somethings unexpected happened: The teacher asked to find out something about money: If a = 400 € and b = 300 € is given. What percent c of a is b? Many of the learners did not check the change of the frame and found out c = 500 €. They used Pythagoras instead of a division 300/400 * 100 = 75%.

Maybe you think that this is an extreme example. But I think you will agree if I write that framing is a very important strategy that our brain uses to understand the world. Converted to a good learning strategy this is called "networking". If a learner can connect a new chapter with something he or she knows it is often much easier to understand the new. But sometimes old knowledge is like an obstacle. If you are very sure that it is impossible to drive a car it will be much more difficult to learn it. This is a well-known particular problem with adult learners. It is hard to erase an erroneous frame constructed earlier in their education. They return to the old one when they are in a situation.

Final remark: Bringing together these two philosophers we have an idea why the opening question is an important question: "What is reality"? If there is no one and only one exact and correct answer for all situations at all times we need a lot of communication to come to a common view of what we think what reality really is. From this point of view (going back to Aristoteles) it is an interpersonal construction. The opposite view is known as the world of ideas created by Platoon (and similar also known from many religions).

In mathematics different ways to answers these questions are main part of the philosophy of mathematics. I give a short hint to the two starting points in old Greek philosophy: Plato gave us the world of ideas that exist independent from mankind and any human activity as such. Our activity as researcher is to explore this world: we find something new for us like an expedition to an unknown part of the earth finds a new part of the world. For example, the north pole exists without any people visiting it. The opposite position is given by Aristotele: He argued that we construct the world doing research. Cantor, Hilbert and the Bourbaki team based their work on Plato, Brouwer and Lorenzen referred to Aristotle. The German philosopher Oskar Becker has described the different points of view with a metaphor: Plato said that mathematicians discover new parts of mathematics and Aristotle said that they construct it.

In our scientific discipline (mathematics education) a constructive approach is used by many or even most of the researchers. Up to this moment nobody found an objective optimal teaching strategy that works independent from specific learning situation, learners, teachers, history and so on.

4.2 The Influence of Social Reality on Adults Learning Mathematics

Let us have a look at the adult people that are learning mathematics. The first question is: Why are they learning mathematics? Many of them have bad experience with learning mathematics at school. Therefore we cannot believe that they come back to mathematics because they love to learn it. There are some courses for groups of adults that like to solve logical and mathematical riddles or to use statistics just for fun or something like a hobby. But most of the adults that visit our courses are *forced* to come by economic reasons. One big group is looking for a job. Learning mathematics for them is part of a general and formal qualification like getting a school degree or getting access to high school courses. A second big group is working in a company and like to get a better job or stay in the company because the skills needed for their jobs are changing.

I give two examples. A specialized bookkeeper wants to become a better paid controller in his or her company. So he or she needs to know more about economics in business. This knowledge is a kind of applied mathematics, too. The second example tells about a carpenter that is cutting wood for furniture with a CNC machine. The company is going to buy a new and faster CNC machine and he or she has to learn how to program it. This knowledge includes a lot of mathematics, especially geometry and logical algorithms.

Teaching situations with adults heading to formal qualification is more similar to school than teaching adults that belong to the second group. These adults are strictly concentrated on job needs. This has several important consequences. Three of them are drafted in the next sections.

4.3 Adults Learners of Mathematics and Their Perceptions to Reality: What Is the Difference Between Adults and Younger Students Learning Mathematics?

The answer seems to be simple. The most important difference *is the age* of the learners. Why this age difference is so important? *The first point* is that the authority relation between teachers and learners varies from school where teachers are adults and students are (elder) children but not adults. The age difference at school leads to a type of natural authority and sometime to generation conflicts. These both aspects are not typical for courses with adults. Teachers in such courses generate their authority from their knowledge about mathematics and their power to judge about the success of the learners. In other words: Authority here is more social and not natural constituted.

A second important point is money. Adults (or their companies) use money to pay for the courses. There is no compulsory school attendance. The learners lose job chances or money if they do not attend the course, but their parents are not forced by law to send them to the course. In other words: The extrinsic motivation is different, intrinsic motivation is seldom. Participants that pay for the course are much more looking for "effective" teaching than students at school. What is "effective" teaching from their point of view? If they want get a formal qualification they urge teaching for the test, a one to one training to solve the test-questions correct. If they want to improve their job qualification they insist on aspects they know and they will need in the job (this includes that they sometimes do not know it correctly, but they have an idea what they want and—more important—what they do not want: proofs, mathematical argumentation, information about relations to other mathematical aspects, and other things that are important from a didactical point of view).

The adult learner's knowledge about "effective" teaching often includes their school experience about teaching methods. If you try to introduce "new" methods like group work maybe you get advice that this is not an effective method. I remember participants telling me: "You know how to solve this equation. Please show us how to do. We see and learn. There is no need to try it in subgroups. This would be a loss of time". We know from our colleague's research about the efficiency of different methods that there are no ideal methods, but a mix of methods depending on different situations is more useful than using only one method (like traditional teaching similar to university lectures). If we want to apply this knowledge in courses for adults, we often must show the better efficiency to the learners step by step.

Other differences between adult learners and younger students depend on age difference, too. Becoming adult in most cases means to collect experience, to mature, to build up a personality etc. In other words: In most cases adults know much better then children in school who they are, what they want and why they want to learn which aspect of mathematics. In many courses, they are paying customers. They pay for a special service. They want to get something for their money in short time. If they have the feeling that the course is not worth to be visited they leave it. And sometimes they go to the management and demand to get back their money. Having this in mind many teachers use different teaching methods and different communication strategy.

These hints should show the influence of the perception of reality of adult learners on course conception, course content, used methods and the way teachers act in such courses.

4.4 Teaching Mathematics to Show Its Practical Use in Reality

One of the consequences of testing with PISA test is that many countries shifted their curricula from pure mathematics to more applied mathematics and teaching real world problems at school. According to this we have many debates about this type of teaching. One of the questions discussed is how near to reality mathematics teaching should go. It is a well-known fact that complexity is rising if we go near reality. Maybe some of you remember Hegel's saying about the rising complexity on the way from something abstract to something concrete and real. One of the main reasons to use abstract mathematics is to reduce complexity and find solutions leaving out concrete details. To point it out more intensively we can say that the main advantage of using mathematics is that mathematics is an abstract way to look at complexity of reality. One the other hand it is well known that the motivation of learners is rising if they see why they should learn mathematics if they see how mathematics is helping them so understand and change nature, technology and economy better.

Adults learning mathematics are less patient with pure mathematics. They demand—with the hint on the fact that they pay for learning, and they have no time for not needed things—consequent applied mathematics that is close connected to real world problems. Now I come back to the philosophic spotlights at the beginning by asking: What are real world problems? Understanding that there is not one and only one real world for all of us we get an idea that "real world problems" for different groups of participants and even more for each individual member of such group might be something slightly or completely different.

I try to explain this with an example, a group of handcrafters that produces furniture. This group should learn how to calculate cost of a thing like a cabinet or a dresser or a chair. One question they should learn to answer exactly is "How much wood do you need to build this piece of furniture?" What could happen? A teacher that is not familiar with furniture production maybe starts like this: "Let us start with a cuboid (2 m high, 1 m width and 1/2 m depth). How much wood is needed to build it if we take it 1/2 cm, 1 cm or 2 cm thick?" At school this could be a good first step to start modelling and calculating (if students start in this way). The practitioners in this course will stop the teacher telling him that a cabinet build of wood that is only 1/2 cm thick will not be stable, it will break down soon. Their next hint might be that the wood in front of the cabinet is never the same as at the rear. Many cabinets have no wood but chipboard at the rear. Discussions like this reduce the authority and the reputation of the teacher because it is clear that he or she is not familiar with the practice he or she should qualify for.

Maybe a task like this is a starting point for a debate of several participants that exchange arguments about the quality and the costs of the furniture their company is producing. One might say that his company uses only wood and nothing else for high quality furniture and some other might argue that this is too expensive. The next expert might tell about his company using only chipboard and veneer to make very good furniture. This is a simple example for different views on different real worlds that are important for the participants but—in a way—dangerous for their teacher. If the teacher tries to explain each learner something useful for his individual job situation he will have problems with all the other participants of the course that might be not interested because the teacher is not talking about their job situation. This leads us back to a main advantage of mathematics offering very useful abstract tools for a lot of different applications.

What is that? We use to solve real-world problems with an abstract tool? Abstracted means abstracted from real world! Studying mathematics at a university we learn—reading Bourbaki and other books based on this way of thinking mathematics—that mathematics is something very abstract, a building based on logic, set theory and some fundamental ideas, the only part of human knowledge were everyone (if we forget Wittgenstein here) can see that a mathematical theorem is proofed and absolutely true for ever. Starting with some basic axioms theorem by theorem is proved by deduction to build up the mathematical building or to recover it step by step. Formal axiomatic mathematics (what we have learned at university) is based on Plato's philosophy and not part of the real world but situated somewhere in a world of ideas. If we accept that (and do not argue like Aristoteles) we need to solve some very difficult philosophical problem: What is the relation from the world of ideas to our world? Why is it possible to use mathematics to solve real-world problems? If we argue like Aristoteles we have no such problem but we are part of a very small minority of mathematicians.

I like to invite you to think about some other aspects of this question. One of the simplest mathematical tools we have learned is addition. $1 + 1 = 2$. Everyone knows that, everyone is able to proof it by taking objects and counting them. Do you agree? Not? This is not a proof at university level: Here you need Peano axioms and some algebraic theory. If you start to count fingers, apples, stones or other real-world objects you do not work as mathematician. If you want to prove an abstract formula about numbers you must remove real world objects like fingers from your mind. This process is called "abstraction". After you (or someone else) has given the proof it is allowed to apply the formula. But sometimes there are surprises. If you put together one bottle of color with a second one of different color you will not have two bottles of one color but a bigger one with new color. 1 + 1 = 2 is not a good description of this mixing process. Let us have a look at a second example: If you add two persons, John and Mary, you have 1 + 1 = 2 persons. But if they marry and have three children we have the equation 1 + 1 = 5 after some years.

Thinking about such well known examples we have to keep in mind that we have changes the focus from "truth of mathematics" to "is this way of applying mathematics reasonable?" In other words: We are talking about modelling real-world problems. One aspect of this activity is to check the quality of chosen mathematical tools for this situation. Are we happy with the results we calculated? Do we need better data, better tools, a better description of the problem? Is it necessary to reformulate the aim of work? If you like to read more about these questions please have a look at Maasz (2008) and Maasz et.al. (2018)

4.5 Responsibility of Adult Learners and Learning Providers to Use Reality in Mathematics Learning

Imagine the furniture selling company produces a cabinet with not closing doors or any other important mistakes. The customer will not buy it and very quick the question arises: Who is responsible for this unedifying situation? If the door is too tall or too short it might be an error in geometry (measuring)—an error in using mathematics causes trouble. If students make mistakes in using mathematics at school they have trouble, too. But the consequences are restricted to the learning situation. They are blamed and maybe they do not pass a test. Mistakes in applying mathematics outside school in job situations have "real" consequences. The company loses money and maybe the worker is terminated.

Therefore, I think it is a very good idea to include the question "Who is responsible for consequences of applying mathematics?" in school teaching and in courses for adults. We do not need a university study in philosophy concentrated on ethics to know that whatever we do or do not has ethical aspects; we and our students are always responsible for our activities. Is there any doubt about the thesis that mathematical activities have ethical aspects, too? If you are not sure, please think about some of the following examples.

Understanding nature often leads to formulas as we know from physics: Newton or Einstein have written their results of research in mechanics and relativity theory with mathematical symbols. Their formulas were used in many technical applications from railways to atomic bombs. If politicians like to introduce new or higher taxes they present a mathematical model that shows without any doubt that there is no alternative. The country needs the money to avoid bankruptcy. A seed company argues that people on earth need more food. Therefore, the company needs genetic engineering (based on mathematics) to develop better seeds. If we look at a newspaper, we find mathematical arguments for and against anything. This makes is easy to find examples taken from reality for learning situations.

I do not like to blame a special newspaper to give you an example for a misuse of mathematical modelling of real world problems. I construct one for this paper. Imagine there is a student direct from university that is employed in a company that is doing market research. His boss likes to have a proof that he studies statistical methods well and ask him to do a little project: Please ask people "What is the distance from earth to moon?" The student went out to a shopping mall and asked about 1000 people. Looking at the answers he found out that it is a good idea to filter the answers. Some people said "I have the moon always in my mind—the distance is zero!" Most of the people estimated about 400 km and some told him very big distances about 400,000 km. Remembering what he has learned about statistics is sorted out the crazy data (0 and about 400,000), applied his knowledge about statistics to the rest of the answers and came to this result. The average estimated distance is 410 km, more than 60% of the answers are in the range from 400 to 420 km. He used a computer to plot the results, made some nice-looking graphics and presented it for the boss. The boss was happy, and ordered him to do another series of interviews. A year later another new employee should do a little project to proof his qualification. The boss had in mind the distance question and asked to repeat this research. The research was finished with the result: this year people estimate that the distance is about 390 km. The boss was astonished. The moon is approaching! He asked both employees "did you work correct?" and decided to publish an important news for newspapers. Our company found out that the moon is approaching. People watch out, build air-raid shelter to be protected!

What happened? The boss did not take care for the difference of "people estimate" and "in reality it is". Do you think this will never happen in reality? Please have a look at your newspaper!

I like to mention two other parts of reality we are changing while we do our work as researchers or teachers. In both types of activity, we are responsible for the results of our work. If we start a research project, we plan to have a close look at or to change the way of learning under special conditions. A typical research method is to compare similar situations including one important difference: For example, one group of learning is using a new method or technology and the other group (control group) is going on traditional way. What happens if there really is a big difference? Are we responsible for a disadvantage of the slower or less learning group? In some universities, we find ethical codes and an advisory committee.

The other important change we initiate is a change in life of learners caused by the teaching. Some of the learners learn a lot, start a new career and change their view on mathematics and the world, some reach a positive final test and some not.

4.6 Potential Gains by Promoting the Use of Reality on Adult Learning Mathematics

The main gain is trust: If adults see that is course is oriented on their view of reality they will go to this course and stay there until they have finished it. If a course for bookkeeping starts with some basic lectures about Babylonian and old Roman numbers to proof how useful our decimal system is the participants will protest and leave this course. If a course for plumbers starts with calculating the volume of tubes that are not used in the reality of the plumbers' jobs they will soon criticize this gap between course and job reality. If a baker should learn how to calculate his purchasing of ingredients for the next weeks he will not believe that this is a good course if flour is not mentioned.

The second potential gain is a good reputation of a course and if several courses run well maybe a good reputation of the institution that offers these courses. This will attract more customers, more adults that wish to learn mathematics here.

The third advantage I like to mention is the improved situation for the teachers. Orientation on reality helps to sort the content. Teachers always have the problem that there is much more teach then possible within the short time. If they use the reality of the (job-)situation of the adult learners as filter they can select easier and they get a positive feedback from the learners.

Finally, I like to express my hope that I could motivate you to think about the many and close relations between adults learning mathematics and reality.I would like to thank Katherine Safford for her very helpful and constructive feedback to my first draft of this text.

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Chapter 5
Scoping the Development of a Measure of Adults' Numeracy (and Literacy) Practices

Diana Coben and Anne Alkema

Abstract We describe our research scoping the development of a measure of adults' numeracy and literacy practices, focusing in particular on numeracy. Our ultimate aim is to develop a way of tracking changes in how adults use numeracy and literacy in the workplace, community and at home, to inform educational efforts. This is particularly important for numeracy because there is often a gap between the numeracy adults use in their daily lives and their performance on proficiency tests designed to measure their progress and assess their suitability for work or further training. We treat numeracy and literacy as both social practices and technical skills, against prevailing polarized positions in the academic and policy literature, and present a conceptual framework encompassing numeracy, reading, writing, speaking and listening practices, in real and virtual (digital) environments.

Keywords Adult · Numeracy · Literacy · Practices · Measure

5.1 Introduction

Adults' numeracy and literacy practices matter. Employers, educators and adult learners themselves want people to be able to handle the numeracy and literacy demands of their lives efficiently and effectively. However, numeracy is typically assessed through standardized proficiency tests that are not designed to capture such activity directly. Accordingly, adults' performance on standardized numeracy proficiency tests may not reflect their capabilities in their daily lives.

Given that adults often need to pass proficiency tests to enter employment, progress within the workplace or engage in further education and training, a measure of adults' numeracy and literacy practices—of what they actually do—might help to

D. Coben (✉)
University of Waikato, Hamilton, New Zealand
e-mail: diana.coben@waikato.ac.nz

A. Alkema
Heathrose Research, Wellington, New Zealand

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narrow that gap and help adult learners to build on their strengths as well as remedy any weaknesses, while building on practical and familiar activities. There is thus a strong pedagogical argument for measuring adults' numeracy and literacy practices.

A wider societal and economic benefit may also accrue from greater knowledge and understanding of what the Organization for Economic Cooperation and Development (OECD) calls 'skills use'. Accordingly, the OECD has incorporated skills use questions into its series of international surveys of adult skills[1] since the mid-2000s and especially in the latest such survey, the Program for the International Assessment of Adult Competencies (PIAAC). It has also established an Expert Group on skills use.

In 2016 New Zealand's PIAAC results showed that adults' numeracy proficiency is (on average) higher than the OECD average but with no improvement since the previous international survey of adult skills in 2006 and significant gender and ethnic variation (MOE & MBIE, 2016). In future research we plan to analyse New Zealand's PIAAC results with respect to skills use.

Meanwhile, in New Zealand we have identified the need for a measure capable of tracking and reporting changes in adults' numeracy and literacy skills use (their practices): in the workplace; in the community; in the home; and elsewhere; over time-periods compatible with patterns of attendance in educational programs. With regard to numeracy, such practices include, for example, recording work-related data, budgeting and checking the cost of shopping, checking safety gauges on equipment, mixing petrol and oil in the correct ratio to fuel a lawn mower, etc.; they are strongly embedded in the context in which the activity takes place. Emotion (both positive and negative) is likely to be in play with regard to adults' numeracy practices, as Evans (2000) shows in his exploration of the inter-relationship between adults' mathematical thinking, emotions and numerate practices. Accordingly, we envisage that a practices measure will need to take account of emotion.

The proposed measure is intended to provide evidence to: inform policy, provision and professional practice; support research on the effectiveness of numeracy and literacy provision; support improved teaching geared to learners' numeracy and literacy practices; and support improved outcomes for adult learners by allowing teaching to be more finely attuned to the type of engagement that research shows is effective in building proficiency over the long term.

In the project outlined here[2] we have scoped the development of such a practices measure—the essential first step. In the process we synthesized relevant literature, reviewed existing instruments and developed an original conceptual framework for a proposed measure of adults' numeracy and literacy practices. Here we give a necessarily abbreviated overview of the evidence-base for our work, present our conceptual framework and outline factors to be taken into consideration in further work.

[1]Successive OECD international surveys of adult skills have been undertaken roughly ten years apart from the mid-1990s (see: OECD Directorate for Education, 2009).

[2]The project was entitled 'Scoping the Development of a Measure of Adults' Literacy and Numeracy Practices' and funded by the New Zealand Ministry of Education, February–June 2015.

Our focus here is on the numeracy aspects of our study. Since it is necessary to consider both numeracy and literacy in this paper, we have chosen to reverse the normal order ('literacy and numeracy') to 'numeracy and literacy' throughout this paper to emphasize this point. Terminology around numeracy is complex (Coben et al., 2003, p. 9) and it is also not always clear in the literature, whether, for example, 'quantitative literacy' is included as a specific form of literacy, as was the case in the OECD's International Adult Literacy Survey (IALS) in the 1990s (Murray, Kirsch, & Jenkins, 1998). Also, as Coben (2006, p. 103) points out, numeracy is sometimes casually included in literacy—and 'literacies' (Street, 1995)—with scant regard to its particularities. She warns that adult numeracy education must be taken seriously on its own terms, with an equal, rather than a subservient relationship to literacy and language studies, recognizing both the mathematical aspects of adults' social and individual practices and the power of mathematics as the discipline on which numeracy is built.

5.2 Seeing the 'Social Practice' in Numeracy and Literacy

The academic literature on numeracy and literacy is highly weighted to the social practice perspective. Meanwhile, in the policy literature, numeracy and literacy are viewed primarily as technical, rather than socially situated skills, in what Street (1984) terms the 'autonomous model', in keeping with the emphasis in many countries on human capital outcomes (Keeley, 2007). Numeracy is particularly vulnerable in this respect since a reductive "'limited proficiency' vision of numeracy, […] with the emphasis on equipping the workforce with the minimum skills required for industry and commerce, has proved remarkably persistent" (Coben et al., 2003, p. 9).

We are unusual in treating numeracy and literacy as *both* social practices *and* technical skills, against the background of what is at best a polarized debate, at worst an absence of debate, between protagonists of the different positions. We agree with Schuller (2001), who argues that human and social capital have complementary roles as intended outcomes of policy. He notes that social capital "requires attention to be paid to the relationships which shape the realization of human capital's potential, for the individual and collectively" and contends that the use of social capital opens up avenues of thought, conceptualization and empirical work which allow certain contemporary paradoxes to be explored. He identifies these paradoxes as: the dominance of individual choice; a strong policy consensus on the importance of lifelong learning; stronger than ever demands for accountability and evaluation in the public sphere; and measurement methodologies in some senses technically more sophisticated (Schuller, 2001, p. 90). Our project contributes to the development of a "technically more sophisticated" measurement methodology in Schuller's terms.

Against this background, we outline the case for numeracy and literacy to be characterized as social practices potentially productive of social capital, where the focus is on what people do with their knowledge, understandings and skills in a range of social and cultural contexts (Scottish Government, 2011, p. 4). In this perspective the application of these skills may also be characterized as 'situated practice' (Balatti, Black, & Falk, 2006; Hutchings, Yates, Isaacs, Whatman, & Bright, 2012; Reder, 2008), acknowledging that practice always takes place in a particular situation.

Social practice theories of adult numeracy and literacy take a number of forms (Perry, 2012) and draw on a range of disciplines. For example, the 'new literacy studies' (NLS) developed by Street and others (see Hull & Schultz, 2001, for a review of these) draws mainly on sociology, socio-linguistics and anthropology. As the name suggests, the 'new literacy studies' is stronger on literacy (and language) than numeracy, as Street's (2003) review of NLS attests. Other social practices approaches draw on psychology, including situated cognition (Lave, 1988) and cultural historical activity theory (CHAT), the latter drawing on the work of Leont'ev (1978) and Vygotsky (1978, 1986). Reder's (1994) practice-engagement theory draws on Vygotsky and contends that literacy skills and reading practices develop best within specific practice contexts. Practice-engagement theory specifies the relationships between "expressed literacy choices/preferences and perceived social meanings" in a detailed, practice-specific way, emphasizing "the patterns of individuals' access to and participation in various roles within as well as across cultural groups" (Reder, 1994, p. 59). It acknowledges the possibility of continued development or decline of numeracy and literacy skills in relation to the affordances of any given situation and individuals' use of numeracy and literacy. The adage: 'use it or lose it' comes to mind, for which there is sound evidence, especially for numeracy (Bynner & Parsons, 1998). In fact, data from major UK longitudinal studies show that "Poor numeracy imposes difficulties for functioning in all areas of life and represents a particular problem in the modern world for women" (Parsons & Bynner, 2005, p. 36).

For each of these theoretical perspectives, there is a corresponding methodology and unit of analysis. These will be explored in the next stage of our work as they will determine the focus of any forthcoming measure of adults' numeracy and literacy practices. For example, with regard to unit of analysis, for Vygotsky this is individual activity, for Leont'ev and 'Third Generation' CHAT researchers, it is the activity system (Engeström, 2001). Working in a situated cognition perspective, for Lave and Wenger the unit of analysis is practice, community of practice, and participation. In an NLS perspective, Street distinguishes between 'literacy events' and 'literacy practices' as units of analysis. Literacy practices are the "broader cultural conception of particular ways of thinking about and doing reading and writing in cultural contexts" (Street, 2000, p. 11), while "literacy events" refers to discrete situations in which people engage with reading or writing, as described by Heath (1982). Similarly, Barton and Hamilton (1998) describe "literacy events" as activities in which literacy has a role, including texts and talk around texts. Purcell-Gates, Degener, Jacobson, and Soler (2000, p. 3) define literacy events more narrowly as "the reading and writing of specific texts for socially-situated purposes and intents". While literacy practices are unobservable, the associated literacy events are observable.

Does the NLS distinction between 'literacy events' and 'literacy practices' work for numeracy? Can numeracy practices be conceived as the 'broader cultural conception of particular ways of thinking about and doing numeracy in cultural contexts', to adapt Street's formulation? Can Heath's definition of literacy events be similarly adapted to refer to discrete situations in which people engage in numeracy practices? And if literacy practices are unobservable and the associated literacy events are observable, is the same true for numeracy? Activities such as adding up a grocery bill or calculating savings in a sale involve the use of particular numeracy strategies in context—these might or might not be observable, depending on, for example, whether the person uses a calculator, counts on their fingers or does the calculation mentally. With silent reading, the reader's strategy would be unobservable. We shall explore these issues in further ethnographic study as part of future work to develop a proposed measure of adults' numeracy and literacy practices.

In this future work, we shall consider adults' purposes for improving their numeracy and literacy given that adults undertake numeracy practices in a variety of contexts, with a range of complexity, types and genres, each of which has a particular function and purpose. Adults seeking to improve their numeracy and literacy skills are likely to have a particular goal in mind (Stewart, 2011; Waite, Evans, & Kersh, 2014). It might be to improve their skills for work, for example, to read and understand safety manuals, complete job sheets, or use digital technology: in other words, to develop their human capital. Alternatively, they might want to improve their skills for life at home and in the community, for example, to read more, help children with their homework, use information technology to connect to the wider world, or add up the cost of shopping as they go round the supermarket: in other words, to develop their social capital. Intrinsic motivation is also a factor for some. A strong message from learner-participants in the 'Making Numeracy Teaching Meaningful to Adult Learners' study in England was that they wanted to prove to themselves that they could understand mathematics; they were not studying it primarily to help them in their everyday lives (Swain, Baker, Holder, Newmarch, & Coben, 2005).

5.3 Identifying and Codifying Adults' Numeracy and Literacy Practices

Purcell-Gates, Perry, and Briseño (2011) have codified adults' literacy practices in their Cultural Practices of Literacy Study,[3] focusing particularly on everyday activities, including reading and responding to formal documents, reading the newspaper, letters, DVD covers, and writing notes and messages. Gyarmati et al. (2014b) found that when hotel workers developed their workplace numeracy and

[3]http://www.cpls.educ.ubc.ca.

literacy skills they were able to transfer them into the wider family and community lives. Black, Yasukawa, and Brown (2013) investigated the numeracy and literacy practices of production workers and found they needed to understand written job specifications, speak at team meetings, collate and understand productivity data, and understand their work contracts and working conditions. The researchers noted that much of the numeracy and literacy was so embedded into job roles that it was invisible to the workers, especially where it related to computer technology. Numeracy is also particularly prone to becoming invisible to those engaged in it (Coben, 2000; Keogh, Maguire, & O'Donoghue, 2012; Noss & Hoyles, 1996). For example, as noted above, numeracy operations involving mathematics may be done mentally and hence be invisible to the observer, or they may be deeply embedded in activities which the person concerned does not regard as having anything to do with numeracy (Coben, 2006, p. 103).

International surveys also produce data on adults' numeracy and literacy practices. From his analysis of the OECD's Adult Literacy and Life Skills (ALL) survey questions, Earle (2011, p. 1) categorized three types of job practices involving literacy, two of which also involve numeracy:

- Financial literacy and numeracy—working with bills, invoices and prices
- Intensive literacy—reading and writing letters, emails, reports and manuals
- Practical literacy and numeracy—reading diagrams and directions, writing directions, measuring and estimating size and weight, and using numbers to keep track of things.

Meanwhile, Coben, Miller-Reilly, Satherley, and Earle (2016) have explored what secondary analysis of PIAAC numeracy data reveal about adults' numeracy practices. For example, simple univariate tabulations can provide a 'big picture' view, while multivariate analysis can provide measures of the contributions of different factors to associations with numeracy practice. Frequent numeracy activity at work may be associated with any of: higher levels of education; numeracy-related fields of study; specific groups of occupations; and/or higher measured numeracy skill. Multivariate analysis can show which factors are most strongly associated with frequent numeracy practice and how much increase in frequency of numeracy practice is associated with one unit of measured numeracy skill, whilst holding other factors constant. The authors note that PIAAC data cannot tell us to what extent frequent numeracy practice *causes* high numeracy skill and that numeracy practice, opportunity or requirement to undertake numeracy practice, and numeracy skill may be mutually reinforcing. Even where data show a strong association between two factors whilst controlling for other factors, we cannot infer that a change in one factor will result in a change in the other. Also, PIAAC does not measure the *intensity* of numeracy activity. For example, finance analysts doing nothing but calculating costs and budgets would report the same way as someone who worked on costs and budgets for 10 min every day. Issues relating to continuous and categorical variables also need to be considered, e.g., PIAAC frequency options are five separate categories, although for some analytical purposes it may be legitimate to derive a continuous frequency variable from the discrete categories.

Both the ALL and PIAAC surveys measure the *frequency* of practices but not their *complexity,* nor, as we have seen, their *intensity*. The importance of assessment in relation to a structured range of complexity of workplace demand is highlighted in work on numeracy for nursing (Coben & Weeks, 2014). In the case of nursing, Hutton has developed a unified nursing numeracy taxonomy mapped to the 42 UK Nursing and Midwifery Council's Essential Skills Clusters (Young, Weeks, & Hutton, 2013). Although specific to nursing, this taxonomy could nonetheless be a useful model for future work to develop a measure of adults' numeracy and literacy practices. We envisage that intensity needs to be incorporated into a richer notion of frequency of use and that the degree of complexity of practice will also be important in forthcoming work on this project.

5.4 Relationships Among the Nature of Adults' Numeracy and Literacy Practices, Their Proficiency and Program Participation

The release of PIAAC Round One data confirms the connection between proficiency and practice and shows how one reinforces the other:

> For example, adults with already-high levels of skills are more likely to gain access to jobs that require still higher levels of skills. In turn, holding a job that requires regular use of literacy, numeracy and problem-solving skills helps to develop and maintain these skills. (OECD, 2013, p. 212)

The PIAAC survey also highlights the connection between levels of proficiency and frequency of engagement with practices. While those with higher levels of proficiency have greater engagement with practices, the survey found that

> adults who practice their literacy skills nearly every day tend to score higher *(sic),* regardless of their level of education. This suggests that there might be practice effects independent of education effects that influence proficiency. (OECD, 2013, p. 212)

Sticht finds that the PIAAC Round One results confirm what he calls the "Triple Helix" concept for literacy development. By this he means:

> the three-way interaction of education, literacy skill, and engagement in literacy practices [i.e.,] that education produces some literacy skill, that leads to more practice in reading, which helps in the pursuit of more education, leading to more skill, leading to more engagement in reading, and so forth. (Sticht, 2013)

An evaluation of the implementation of England's *Skills for Life* strategy in workplaces had similar findings. Waite et al. (2014) report that a year after completing a 30-h workplace literacy program, those who had used their literacy skills in their work showed improvements. Those who had been on the program were more likely than those who had not to undertake further education; they also read more. Similar results were found in UPSKILL (Gyarmati et al., 2014b) in Canada. Here the researchers found that those who had participated in a workplace learning program were more likely than those who did not to show improvements on three or more of the project's four behavioral indicators. One indicator included two measures of numeracy and literacy practices and these showed improvements. This study also found improvements in literacy skills that were higher for those assessed 12 months or more after the program than for those assessed at six months or less and between 6–12 months. More modest gains were also made in numeracy.

While there is an expectation that adults' numeracy and literacy proficiency will improve as a result of attending a learning program, the findings from the U.S. Longitudinal Study of Adult Learning (LSAL)[4] show no relationship between change in proficiency and program participation "over the relatively short time intervals typical of program participation and of program accountability and improvement cycles" (Reder, 2011, p. 4). LSAL's Director, Reder (2008) notes that this is at odds with the small reported differences recorded in pre- and post-program tests, but that such proficiency gain can also be made by non-participants. However, adult numeracy and literacy programs do "have demonstrable impact on measures of literacy and numeracy practices" over relatively short time-periods (Reder, 2012, p. 5). The LSAL study thus found a direct relationship between participation in programs and engagement with numeracy and literacy practices, endorsing Reder's Practice-Engagement Theory:

> Adults at similar proficiency levels at one point in time wind up many years later at different proficiency levels depending in part on their earlier levels of engagement in literacy practices. Those with higher levels of engagement at an initial point in time have higher levels of proficiency at a later point in time even with initial levels of proficiency controlled. (Reder, 2009b, p. 47)

LSAL focused on frequency of use, and:

> With many statistical controls in place, LSAL found strong relationships between participation in adult education programs and increased engagement with literacy (e.g., reading books) and numeracy (e.g., using math at home) practices. (Reder, 2009a, 2011, p. 3)

Of particular interest here is the direction of causality demonstrated by LSAL:

> The sequence of observed changes makes it clear that program participation influences practices rather than vice versa, [programs] have demonstrable impact on measures of literacy and numeracy practices. (Reder, 2008, pp. 3–4)

Similarly, an Australian study investigating whether adult numeracy and literacy courses produced social capital outcomes found that 80% of the learners improved the structure of their networks and the way they communicated, as well as contributing to positive socio-economic benefits in the areas of education and learning, employment and the quality of working life, and access to goods and services (Balatti et al., 2006). Thus adults' increased numeracy and literacy proficiency may be expressed in the increased efficacy of their numeracy and literacy practices. This is borne out in recent research on adults' outcomes from participation in a numeracy

[4]LSAL http://www.lsal.pdx.edu/ followed a random sample of around 1000 Portland Oregon area high school dropouts from 1998–2007.

and literacy program, which found beneficial personal benefits in family and community life and in the labor market, as well as an increased sense of self, enhanced understanding of others and greater independence. The authors note that most had also developed aspirations for further learning and were aware of new opportunities and possibilities opening up for them (ACE PDSG Learner Outcomes Working Group, 2013, p. 2).

These studies bear out Sticht's view that programs act as a catalyst for engagement with practices that then in turn improve proficiency in his "Triple Helix". It would seem that programs may act as a circuit-breaker that jump-start adults into engaging in activities that use, and subsequently further develop, their numeracy and literacy skills. This is crucial if there is to be an improvement in the numeracy and literacy levels of those with low skills. As New Zealand ALL survey data show, those with the lowest numeracy and literacy skills (ALL Level 1) have less opportunity than those with higher skills to perform tasks that involve reading or mathematics on a regular basis (Dixon & Tuya, 2010). PIAAC, ALL and LSAL thus bear out the prediction of practice engagement theory (Reder, 1994; Sheehan-Holt & Smith, 2000), which holds that engagement in numeracy and literacy practice leads to growth in proficiency.

The extent to which numeracy and literacy practices are connected to and build from participation in programs is contingent on a range of factors. Firstly, as Reder (2008) found, using authentic contexts in learning programs increases the likelihood that there will be improvements in practices. Purcell-Gates, Degener, Jacobson, and Sole (2002) also found a positive relationship between the use of real life, authentic activities and changed literacy behaviors. Vaughan (2008) adds the idea that learning must be meaningful in order for it to be practiced in a valued way. Secondly, adults need to use their learning in different contexts, particularly 'transferring' learning from education into the workplace (Eraut, 2004). However, as Evans (1999) and others (Black et al., 2013) have found, this is not necessarily straightforward, particularly for numeracy. This is important because societal and economic changes consequent upon globalization and technical innovation are leading to more complex, changing working environments in which workers are expected to make connections between different contexts and ways of doing things. Cameron et al. (2011) found the likelihood of transfer was increased when certain conditions were met in learning programs and in workplaces. For example, when workers were given the opportunity to practice their communication skills they were more likely to transfer what had been learnt. This is in keeping with Vaughan's (2008) wider view that workplace affordances are essential to the success of learning programs and that workplaces need to operate as learning environments in their own right.

Learning from workplace numeracy and literacy programs also carries over to practices in everyday life:

> These included learners being able to assist with homework and read books with children for the first time, fill in forms, read local newspapers, or calculate value when shopping. Skills improved through use as learners practiced these in new contexts. Transfer was

> enhanced when tutors encouraged learners to bring examples of outside LLN tasks into class, so that they could be supported with these. (Cameron et al., 2011, p. x)

Other research also shows teaching and learning programs bringing about changes in practices. For example, a New Zealand evaluation report lists practices, including calculating and measuring, that changed as a result of workers' participation in workplace learning programs:

> I *don't have to use my fingers. I can work out how many there are on a* pallet [when multiplying rows of products]
>
> *I'm now working out the volume of concrete. The engineers used to come out, now they just double-check it*. (Department of Labour, 2010, pp. 56–57)

As yet we have no way of measuring these changes. Proficiency gain is relatively easy to measure through traditional reading, writing and numeracy proficiency tests. However, research, including LSAL, indicates that there is likely to be little if any improvement in skill levels in the short term from programs (Reder, 2009b; Waite et al., 2014).[5] By contrast, learners are likely to show improvements in their numeracy and literacy practices in both the short and longer-term.

It is for these reasons that Reder (2013) argues that measures of engagement with numeracy and literacy practices would be a better way of showing continuous improvement during and after engagement with learning programs. He does not suggest that proficiency measures be dropped, rather that practice measures be developed to complement them. The challenge is to find a way of measuring changes in adults' numeracy and literacy practices.

There is currently no such measure mandated for use in New Zealand tertiary education and consequently no data available on adult learners' numeracy and literacy practices. Changes in proficiency are measured by the Literacy and Numeracy for Adults Assessment Tool.[6] Analysis of Assessment Tool data (Lane, 2013a, b, 2014) shows that while in general there are gains for adults in programs that extend over longer time periods, there is little correlation between time on-program and proficiency gain in the short term, as in LSAL, and current data are insufficient to determine an expected proficiency gain for learners who participate in adult numeracy and literacy programs of various durations.[7]

We reviewed a range of practice measures available in English to see if anything could be used or adapted for the New Zealand context. These included measures developed for research and survey purposes such as UPSKILL in Canada (Gyarmati et al., 2014a), LSAL (USA) and PIAAC (international) and measures developed for pedagogical purposes such as *Mapping the Learning Journey* (Republic of Ireland)

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[5]Note, the UPSKILL project did find gains in proficiency.

[6]https://www.literacyandnumeracyforadults.com/resources/356174.

[7]Publicly-funded programs in New Zealand include 100-h intensive numeracy and literacy programs for native or fluent English speakers (200 h for speakers of English as an additional language) and 40-h workplace programs.

(Merrifield & McSkeane, 2005). Most measure frequency but not complexity of use and they vary considerably. None covers the whole terrain which our proposed 'practices' framework will cover.

5.5 Rationale for the Development a Measure of Adults' Numeracy and Literacy Practices in New Zealand

Against this background we consider that a measure of adults' numeracy and literacy practices geared to the New Zealand context is necessary for several reasons, summarized below:

- The development of adults' numeracy and literacy proficiency over time is strongly associated with their engagement in numeracy and literacy practices. There is robust evidence that the development of adults' numeracy and literacy proficiency over time is strongly associated with their engagement in numeracy and literacy practice.
- An effective measure is needed to capture learners' progress over the relatively short time periods typical of numeracy and literacy programs. Proficiency measures are effective over a longer time period than is typically available in numeracy and literacy programs; an effective measure is needed to capture learners' progress over shorter time periods.
- Practices-based measures reach the parts proficiency measures do not reach. They have the potential to capture information on learners' engagement in numeracy and literacy practices that proficiency measures are not designed to capture.
- A numeracy and literacy practices measure is intrinsically sensitive to cultural and linguistic diversity. The conceptual framework developed in this scoping project and the proposed measure will be based in adults' numeracy and literacy practices, which inevitably reflect the cultural and linguistic context of the learner in ways which it is difficult for a generic proficiency tool to do. The proposed framework and measure thus offer a way of complementing the existing proficiency data in a culturally and possibly also linguistically sensitive way.
- Educational programs that increase learners' engagement in numeracy and literacy practices show improved outcomes for learners in terms of increased numeracy and literacy proficiency. Educational programs produce increased levels of practice engagement amongst learners and these have long-term consequences in terms of increased proficiency that are valued by policy makers.
- There is evidence that adults' participation in numeracy and literacy learning programs is strongly associated with changes in their numeracy and literacy practices; these changes may be significant for adults' employability and other life chances but they are not currently acknowledged in the New Zealand adult numeracy and literacy infrastructure.

- Adults' numeracy and literacy proficiencies are directly relevant to their prospects, wellbeing and quality of life. Large-scale adult numeracy and literacy assessments, including LSAL, exhibit strong relationships among proficiency, employment and earnings and other positive life outcomes.

5.6 Conceptual Framework

So what does it take to measure numeracy and literacy practices? How can frequency (incorporating intensity) and complexity be measured? This is how we propose this can be done.

Our original conceptual framework encompasses numeracy, reading, writing, speaking and listening practices, in real and virtual (digital) environments. Adults engage in these in a range of contexts and for a range of purposes, through the affordances that occur in individual and group situations.

- Contexts are economic, social and cultural. As such they may be formal/informal; familiar/routine/unfamiliar. Practices occur within contexts that are situated in, for example, work, home, and social and community settings. Contexts are viewed here in relation to their affordances, i.e., the opportunities available to people that allow them to engage with numeracy and literacy practices at work, at home and in social and community settings.
- Purposes are the expected or intended outcomes of a practice. They are determined by both the adult (the 'practicer') and the context and its affordances in relation to the practice. Purposes may be multiple, complex and layered, conscious or unconscious and may include, for example, the purposes of: informing/being informed; enjoying; persuading; participating, interpreting; inquiring; consulting; calculating; measuring; checking; etc.

These factors mediate the type of practice that is engaged in, such as:

- Speaking [practice] during formal proceedings at a civic event [context] to consult [primary purpose] people about changes to land use;
- Speaking [practice] at a health and safety committee meeting at work [context], to inform [primary purpose] staff about hazard identification at a work site;
- Speaking [practice] informally with friends over Skype [context] to tell them [primary purpose] about a recent gig;
- Speaking [practice] on the phone [context] to make an appointment [primary purpose] with the hairdresser.

Individuals' capacity to engage with numeracy and literacy practices is influenced and shaped by three factors, their:

- Personal attributes, such as motivation, persistence, self-efficacy, confidence;
- Ability to engage in practices, i.e., the individual's skills and knowledge to engage in a practice in a given context;

- Agency, i.e., the control an individual has over the situation and the extent to which they have the freedom to act independently, make choices, and take action in a given situation;

Shaped in this way, individuals' capacity is made manifest in the activities they engage with that involve the practices of numeracy, reading, writing, speaking and listening in given contexts, such as: reading a technical work manual; texting a friend to arrange to meet; speaking at a community meeting; listening to evacuation instructions in a building; logging work hours; calculating the discount in a sale.

The combination of the context and its purpose determine the complexity of activity and the practices required to undertake the activity. The combination of personal attributes, capability and agency influence the extent to which an individual is able to engage with the practice required to undertake the activity.

Figure 5.1 shows how such measurement might be conceptualized in terms of varying degrees of frequency and complexity. For example, an individual might engage often with simple contexts, tasks and events on most days, such as finding the time of a bus on a given route, reading hazard signs or sending a text message. These practices would be in the lower right quadrant. The same individual might also engage with complex tasks on a daily basis, for example, as a trainee mechanic working with car manuals, or as a process worker collating data in spreadsheets and displaying production data in graph form. These practices would be in the upper

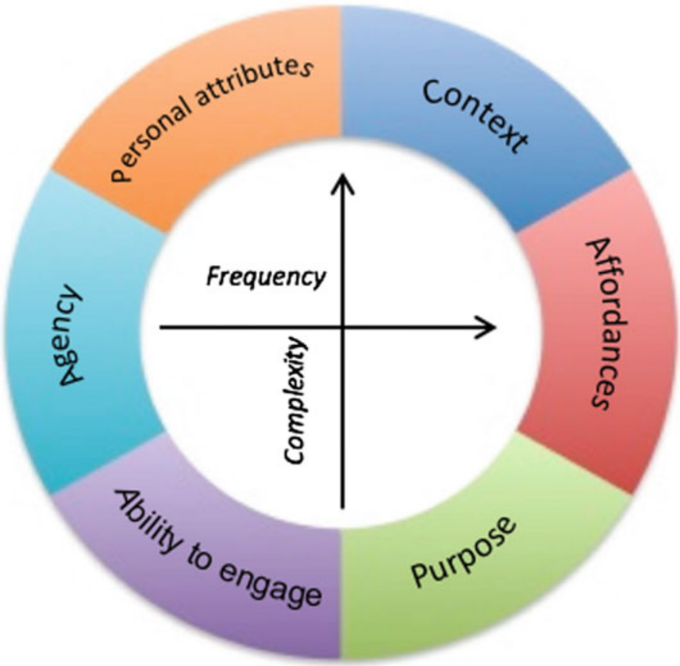

Fig. 5.1 Conceptualizing the measurement of adults' numeracy and literacy practices

right quadrant. Note that positioning on the quadrant refers to the practice and not the person. The outer rim should be considered as 'revolving' around the central quadrant. It represents factors applying to the individual and the context of their numeracy and literacy practices.

5.7 Looking Ahead

Our study indicates that a range of factors need to be taken into account when designing a measure of adults' numeracy and literacy practices.

The purpose, nature, scope, scale, validity, authenticity, relevance, cultural and linguistic sensitivity, practicability, stability over time and ethical quality of a practice measure and the need for it to be as simple as possible while covering both frequency and complexity of use will all need to be borne in mind.

It will be important to identify an appropriate theoretical and methodological framework and unit of analysis to inform further work, including the identification and selection of a sample of numeracy and literacy practices used in adults' work, everyday family and community lives, and those associated with learning programs, for example, those connected with particular trades and workplace roles.

We shall also need to consider the need for parity between numeracy and literacy and the degree of alignment between a practices measure and numeracy and literacy proficiency, as codified in the New Zealand numeracy and literacy infrastructure.

Given the wide range of contexts and settings in which numeracy and literacy are practiced and the contingencies on which these practices are based, a practice measure will need to: be feasible and practicable in use; lead to valid, authentic and fair measurements; be culturally and ethically sound, and accommodate emotional aspects of adults' numeracy and literacy practices.

Consideration will also need to be given to the ways in which the results of a measure of adults' numeracy and literacy practices will be used by various stakeholders, including providers of numeracy and literacy services, government and funding bodies, employers and adult learners themselves. Further work will also take account of the deliberations of the OECD Expert Group on skills use.

On the basis of our scoping project we know that developing a measure of adults' numeracy and literacy practices that is robust, evidence-based, culturally-sensitive, ethical, practicable and cost-effective will be a complex and challenging task. We are more than ever convinced that such a measure is needed.

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Part II
Student Focus

Chapter 6
Adults' Conception of Multiplication: Effects of Schooling on Multiplicative Conceptual Field

Andrea Maffia and Maria Alessandra Mariotti

Abstract School mathematics is often related to a needing for memorization of a lot of information, times-tables are a paradigmatic example. A large amount of research on arithmetical facts has been implemented within cognitive psychology but rarely it is related to mathematics education research and quantitative methods are always used. In this paper it is presented a pilot study: ten interviews with adults are qualitatively analyzed using constructs from both research fields, showing that times-tables organization in memory could depend on school instruction. Level of instruction of subjects is indicated as an important variable in recalling and in choosing computation strategies, namely in the construction of the concept of multiplication.

Keywords Multiplication · Long-term memory · Conceptual field
Schooling

6.1 Introduction

Students face many difficulties during mathematics learning and such difficulties are often justified in terms of the relationship between a student's abilities and the subject. An example of widespread belief is that a lot of memory is needed to learn mathematics (Zan & Di Martino, 2009). Maybe because of this kind of myth, so called rote-learning has been defined in the past as one of the worst enemies of mathematical learning (Fremont, 1967).

A. Maffia (✉)
Department of Education and Humanities, University of Modena and Reggio Emilia, Viale Allegri 9, 42121 Reggio Emilia, Italy
e-mail: andrea.maffia@unimore.it

M. A. Mariotti
Department of Mathematics and Information Science, University of Siena, Via Roma 56, 53100 Siena, Italy
e-mail: mariotti21@unisi.it

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Few researchers in mathematics education focused their attention on the theme of memorization, and mainly with a negative meaning. The lack of qualitative research on long term memory in the field of mathematics education has been noticed in the past by Byers and Erlwanger (1985); they highlighted a gap between the developments of research on memory and the theories and practices in mathematics education. More recently, Karsenty (2002) revealed her surprise in observing that "of the immense existing body of school research, only a minute portion is dedicated to questions of knowledge maintenance after school" (p. 117). Such ascertainment seems to still be true. However, the longevity of mathematical knowledge acquired in school seems to be relevant if we consider such knowledge to be relevant for all citizens. In this sense, research about long term memories of mathematical concepts appears as a suitable tool for critiquing teaching/learning practices.

There are researchers from the cognitive psychology field who dedicated attention to knowledge retention after schooling; mathematics is involved as the topic of some of these investigations. For instance, Bahrick and Hall (1991) discovered that, among their sample, there was a loss of information about high school algebra and geometry just for those who did not attend calculus classes in college. More attention has been dedicated to memory for basic contents: times-tables are, maybe, the most studied mathematical knowledge by researchers in psychology.

Most of the classical works on long term memory for arithmetical facts are based on written questionnaires or structured interviews and quantitative analytic methods are generally used. Many researchers collect their data among university students (usually attending Psychology classes) analysing their answer in term of speediness and correctness (e.g. Bahrick & Hall, 1991; García-Orza, Damas-López, & Matas, 2009; Geary, Widaman, & Little, 1986; LeFevre, Lei, Smith-Chant, & Mullins, 2001; Lemaire & Fayol, 1995). These studies provide strong evidence about the most common errors, and try to infer a model for multiplication facts' storage and retrieval drawing on this kind of information. In other works, a model for multiplication fact retrieval is built according to evidence from brain-damaged patients (e.g. Dagenbach & McCloskey, 1992; McCloskey, Harley, & Sokol1, 1991).

Only a minor part of literature is dedicated to investigating reconstructive strategies that are used when direct recalling is not possible. The few examples that we found use again just quantitative methods (Butterworth, Marchesini, & Girelli, 2003; Siegler, 1988), otherwise they concern just children (for a large review see Sherin & Fuson, 2005).

This paper reports a pilot study on the usage of qualitative methods to analyse the problem of mathematical knowledge retention in adults. Memories about multiplication between natural numbers are related to the level of schooling. Answers to open questions are analysed on the basis of a theoretical framework which draws constructs both from the cognitive psychology and mathematics education fields. The topic of multiplication is chosen because the wide literature on this specific topic that has already been produced through quantitative methods can constitute a "solid ground" for comparison.

6.2 Theoretical Framework

This paper focuses on *long term memory* (LTM), defined as the information acquired and still accessible after more than few minutes. We want to stress that we are talking about LTM which has to be distinguished from working memory which has recently received a wider attention from the community of researchers in mathematics education. In particular, for our purpose, it is useful to distinguish between the so called *episodic memory* and *semantic memory*, according to the definitions given by Tulving (1972). Episodic memory is the information associated to a particular moment or place, the rest of information constitutes the semantic memory. So, different representations of a mathematical concept, problems and procedures belong to the semantic memory which is the object of this study.

According to Vergnaud (1983), the study of knowledge related to a concept is realized through the analysis of the *conceptual field*, which is "a set of problems and situations for the treatment of which concepts, procedures and representations of different but narrowly interconnected types are necessary" (ibid., p. 127). A concept is characterized

> as a triplet of sets C = (S, I, *S*), where S is the set of situations which make the concept meaningful, I is a set of invariants (objects, properties, and relationships) that can be recognized and used by subjects to analyze and master these situations, and *S* is a set of symbolic representations that can be used to point to and represent these invariants and therefore to represent the situations and the process to deal with them. (Vergnaud, 1988, p. 85)

In particular, as the interviews' topic is times-tables, Vergnaud's classification of *multiplicative structures* (Vergnaud, 1983) is used. Multiplicative situations are distinguished in:

- *Isomorphisms of measures*: the structure of these problems is a direct proportionality between two measure-spaces. For instance, the case of calculating the total expense for a number of products when the cost of one product is known; it can be considered a direct proportionality between the number of products and the total cost. Also equal-group problems belongs to this category: when we want to know the total amount of objects that are equally distributed among several groups, the proportionality is between the number of groups and the total amount of objects. Also rate problems, like calculating the length of the travelled path, at a constant speed, depending on time, are isomorphisms of measures. "Isomorphism of measure involves only two variables and it is properly modelled by the linear function" (Vergnaud, 1983, p. 133).
- *Product of measures*: these problems involve three variables and a bilinear function's model. They are problems about the Cartesian product between two measure-spaces. For instance, calculating areas or volumes, counting of possible pairs of elements taken from two sets. An example of product of measure is the counting of all possible outfits that can be composed with a particular set of trousers and a set of shirts. Such problems consist of double proportions: e.g. both dimensions of a rectangle are proportional to its area. In this kind of

situations, the units of the product are expressed as products of elementary units: 1 m × 1 m = 1 m^2, 1 boy-dancer × 1 girl-dancer = 1 couple of dancers or 1 shirt × 1 trouser = 1 outfit.
- *Multiple proportions*: this is the case of a measure-space which is proportional to two other independent measure-spaces. An example is given by rate problems when the rate depends on two variables. For instance, the production of milk by a farm depends on the number of cows and on time. In these situations, the proportionality constant is a ratio between different magnitudes: amount of milk per day and per cow. Another example could be the following: "A family of 4 persons wants to spend 13 days at a resort. The cost per person is $35 per day. What will be the expense?" (Vergnaud, 1983, p. 139).

The presented interviews concern *arithmetical facts*; this term designate a couple made of a particular arithmetical operation and its result (e.g. 4 × 7, 28). The aim of this study is to describe adults' conceptual field for multiplication to answer the following questions:

- Which situations do adults associate to recalled multiplicative facts?
- Which invariants can be observed in adults' recalling or reconstruction of multiplicative facts?
- Do these situations and invariants differ among adults with different level of schooling?

To explore the conceptual field of basic multiplications, different questions have been predisposed to analyse the different components.

6.3 Methodology

In this work we decided to implement a qualitative analysis; semi-structured interviews are used. This method does not allow one to isolate all the possible variables but, as showed by Karsenty (2002), it can make possible the linkage between learned mathematical concepts even after many years.

The interview is composed of four question sets. Each one concerns a multiplicative arithmetical fact. An example of a question set is the following:

1. Can you tell me how much is 7 × 4?
2. Can you describe how you calculated it?
3. Did other solutions come up to your mind?
4. Can you imagine a different way to get the result?
5. Can you describe a situation in which this calculation can be useful?

The last question has been alternated with another one:

5. If you should explain to a child how to do this calculation, how would you do so?

Questions 2, 3 and 4 ask for the process that the interviewee realizes to obtain the multiplication result. So they are aimed at investigating the properties and relationships among the numbers recalled by memory; in other words, these questions investigate about the invariants. The goal of question 5 is to put in evidence situations associated with the particular arithmetical fact and, in general, to the multiplication between natural numbers. In particular, the second version of the question is introduced to ask for the most elementary and basilar situations. We conjectured that asking for an explanation to a child, we could reveal those situations and invariants which are considered "immature" by the interviewee but that are still connected in memory with the concept of multiplication. We decided to include such a question to observe also those situations that would not emerge from the most educated subject who could consider it inappropriate for a person of his/her own age to refer to such situations.

The chosen multiplications were 7×4, 8×6, 13×5 and 12×4. We decided to use both facts with numbers fewer and greater than 10 to investigate the main operational invariants associated to the multiplication even when direct memorization is unhelpful. We conjectured that the impossibility to recall directly the result can trigger different reconstruction strategies.

The sample was composed of five men and five women, aged between 35 and 42 years old (average age 37.5). This age range was chosen because the Italian national curriculum for mathematics remained unchanged during the period in which the interviewees attended primary school. We also decided to interview people who are no longer involved in any educational program (as university classes or similar).

The interview's location was chosen by the interviewee and usually it was his/her own house. So the interviewee had access to any kind of tool (paper and pencil, calculator, …) but in all cases just mental calculation was used. Interviews lasted generally about 10 min and they were all videotaped.

Interviewer and interviewees' utterances were fully transcribed and then answers to question 2, 3 and 4 were analysed looking for the different recalling/reconstruction strategies according to multiplication properties involved. Answers to the two versions of question 5 allowed us to investigate situations: a categorization is realized according to Vergnaud's (1983) framework as presented in the previous section. Answers to the first question of the questionnaire were analysed also in terms of response time and correctness. In this way we could have additional information about the fact that recalling or reconstruction is used, according to response time and typology of error. Furthermore, most of the psychological literature focuses on this kind of data and so we can compare our results with others.

One of the main research hypotheses was that memorized information and its organization depends on duration and typology of schooling. So, information about schooling level of interviewee is collected. Subjects are divided in three categories: high mathematical education (mathematical, science or engineering degree), medium mathematical education (other degrees or no degree), and low mathematical education (people who did not attend high school). In the sample there are three people with low mathematical education (LME), four people with a medium education (MME) and three people with a high education in mathematics (HME).

Other possible influences on multiplication facts recalling could come from a current job or from having children currently attending the first years of primary school. For this reason, we decided to collect information also about these factors but, as we will discuss in the closing section, they do not seem to be as influential as the level of schooling.

6.4 Results

6.4.1 Invariants

Errors appears just in LME interviews and they are mainly *interference errors*, namely it is recalled the result of a different arithmetical fact (Campbell, 1987). For example, 64 (the result of 8×8) is indicated as result of 8×6 by Interviewee 3. This kind of error is often associated to direct recalling from memory (Campbell, 1987). Indeed, adults usually prefer direct recall instead of reconstructive strategies used typically by children (Siegler, 1988); the association of a multiplication with its result relies often on semantic memory, this constitutes a first invariant. There are few cases in which the reconstruction of a fact relies on phonological aspects (as rhymes or doggerels). In the following, an example is transcribed. Translation from Italian to English was done by the authors and the original version of the interviews transcripts can be found in (Mariotti & Maffia, 2015).

Researcher: Can you say me how much is seven times four?
Interviewee 1: Twenty-eight.
Researcher: Can you describe how did you calculate it?
Interviewee 1: (smiling) Remembering the doggerel.
Researcher: Which one?
Interviewee 1: (singing) Seven, fourteen, twenty-one, twenty-eight.
Researcher: Ok. Did other answers come to your mind?
Interviewee 1: No (she smiles again).

Most of the reconstructive strategies are based on mathematical properties: Table 6.1 summarizes the properties used by the interviewees to reconstruct those results that they could not recall by heart. Such properties are applied mainly (but not only) for multiplicative facts that involve a number greater than ten.

There is no evidence of a direct linkage between the used arithmetical properties and level of schooling: all the interviewed people used mainly commutative and distributive properties, so such properties can be considered as invariants. In particular, the distributive property is often applied for calculating 13×5.

Table 6.1 Used multiplication properties

Multiplication properties	LME			MME				HME		
	Int1	Int2	Int3	Int4	Int5	Int6	Int7	Int8	Int9	Int10
Commutative property	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Associative property			✓		✓		✓	✓		✓
Distributive property	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Researcher: Can you tell me how much is thirteen times five?
Interviewee 1: Oh gosh! (some seconds of silence) sixty… (silence again) fifty-nine?
Researcher: Can you tell me how did you get this result?
Interviewee 1: Thirteen times five: I did fifty and then three, six, nine (silence) and it isn't what I said! (she smiles)
Researcher: And so, how much should it be?
Interviewee 1: (silence for some seconds) Fifteen, so… sixty… five! (nodding).

In this last excerpt, the interviewee recalls rapidly $10 \times 5 = 50$, but then, instead of $3 \times 5 = 15$, she recalls the result of 3×3. Hence, she gives 59 as answer. This kind of error is well documented in literature and it is defined as *operand distance effect* (McCloskey et al., 1991). It consists in giving the result corresponding to another multiplication with one operand in common. Interviewee 1 shows the same error while calculating 12×4: she answers "fifty" (that is $10 \times 4 + 2 \times 5$).

The commutative property was frequently used when answering to 8×6. In Italian, as in English, 6×8 rhymes with 48; such a rhyme seems to help in recalling:

Researcher: Can you say me how much is 8×6?
Interviewee 9: I thought the other way round. Six times eight, forty-eight. It comes easier from memory. […] To me, it comes the doggerel "Six times eight, forty-eight". So you can't do it wrong.

As can be noticed in the last two excerpts, commutative and distributive properties are used as *theorems in action* (Vergnaud, 1988), meaning that they are not explicitly stated in a general form but they are applied to contingent problems. There are no differences between the different levels of schooling; we can observe this behavior by all the subjects and the arithmetical properties are never explicitly mentioned or stated, even by those with a higher education in mathematics.

A difference can be noticed only in the use of the associative property. One of the LME subjects often used this property to multiply by four (doubling two times) even when it is not very useful (as in the case of 13×5 in which both the factors are odd numbers):

Researcher: Can you say me how much is seven times four?
Interviewee 3: Twenty-eight.
Researcher: Can you describe me how did you calculate it?

Interviewee 3: Seven, seven and seven and seven.
. […]
Researcher: What if I ask you how much is eight times six?
Interviewee 3: … Thirty-eight
Researcher: Can you tell me how did you do?
Interviewee 3: It is always the same strategy as before. Usually I do eight times two and it makes sixteen, I double it and it makes thirty-two then… then I add… then I have to add sixteen.
. […]
Researcher: Can you say me how much is thirteen times five?
Interviewee 3: Ehm… Thirty-two… Sixty-five.
Researcher: Can you say me how did you reach the result?
Interviewee 3: First of all thirteen times two, twenty-six. I doubled and added thirteen.

It seems that the usage of the associative property (in particular doubling) is for him a stereotyped action activated in the specific context of multiplications; it can be interpreted as *pragmatic schema* (in the sense of Nunes, Schliemann, & Carraher, 1993) meaning a behavioral schema that follows an *if-then* rule. In the case of the interviewee 3, doubling appears as the behavioral schema associated to multiplication facts reconstruction.

Finally, most of the subjects could express the relation between multiplication and addition. When asked to state a different way to perform a calculation (question 4), interviewees often referred to repeated addition as a way to get the same result in a different way; but this strategy was never used to get the very first answer. There are three exceptions: interviewees 2, 3 and 9 never mention repeated addition. Two of them are in the LME category and they often proposed direct recalling as the only strategy to get the result. Interviewee 9 belongs to HME and he proposed many alternative paths to the result, usually applying the distributive property.

6.4.2 Situations

Situations are classified according to Vergnaud's categorization of multiplicative structures (Vergnaud, 1983) as mentioned in the previous sections. The obtained classification is summarized in Table 6.2.

Most of the presented problems are *isomorphisms of measures*: the most frequent example is to find the total cost of a certain amount of objects/services when the unitary cost is known.

Researcher: Can you tell me a situation in which this calculation can be useful?
Interviewee 4: I have no idea!
Researcher: You can even invent it if you want.

Table 6.2 Classification of problems

Multiplicative problem category	LME			MME				HME		
	Int1	Int2	Int3	Int4	Int5	Int6	Int7	Int8	Int9	Int10
Isomorphism of measures	✓	✓	✓	✓	✓	✓	✓		✓	✓
Product of measures				✓				✓	✓	
Multiple proportions							✓			

Interviewee 4: Let's say that I do a monthly subscription at the swimming pool for four weeks and it costs seventy euros each month, and I want to know how much I do pay in total. I do this calculation.

Other examples consisted of calculating the ingredients for a recipe according to the number of people who have to eat or counting objects distributed in equal groups:

Researcher: Can you imagine a situation in which this calculation can be useful or necessary?
Interviewee 2: A practical one?
Researcher: Yes, any situation.
Interviewee 2: If I have four shelves and I have a certain amount of books on each shelf and I don't want to count them one by one. I can regroup them. But I wouldn't regroup them by seven, I would make groups of ten.

There are some examples of the other two categories, just from the MME and HME groups. As can be seen in Table 6.2, there is just one case of problem of *multiple proportion*:

Researcher: If you should propose a word-problem to a child. And in this problem the child has to calculate thirteen times five. How would you create this problem?
Interviewee 7: I don't know. I would image thirteen children and five…five days of vacation. They go to a scout camp and for each single day they have to buy food, and so they have to calculate the multiplication.

Some of the proposed *product-of-measures* situations are strictly related to the field of mathematics:

Researcher: Listen: if you would imagine a situation in which this calculation could be useful. Which one can it be?
Interviewee 9: The first thing that comes to my mind…find the surface of a rectangle. One side times the other one.

In the case of interviewee 6, who works as cashier in a supermarket, the workplace provides a situation in which objects are arranged as an array. Also this kind of problem is classified as a product of measures:

Researcher: Can you imagine a situation in which this calculation can be useful or necessary?

Interviewee 6: Well! If I think of my job… to count beer bottles, it comes to my mind when people have them in the shopping cart. As far as we are close to summer! […] Generally, they are in parallel lines with four bottles in each line and so… actually they are never in seven lines, but I could do this.

This is the only case in which the situation is taken from job's events, but most of the interviewees invented or recalled situations drawing on everyday life episodes. Such episodes can be related to hobbies or interests:

Researcher: Can you tell me a situation in which this calculation could be useful?

Interviewee 7: For example…to measure the pesticide for your own garden […] Dosage is never written for what you need. In this case you don't have just to calculate a multiplication, it is a proportion.

Karsenty (2002), investigating adult's memories about functions, notices that "they did not reproduce the learned material, but rather reconstructed it and in fact formed their own personal versions" (p. 141). This observation applies also to our case. There are relations between the life experiences associated to a mathematical concept; the ability to develop such relations seems to depend on the schooling level. In particular, people with MME and HME associate their personal life experiences to their conception of multiplication while the LME interviewed people recalls just situations in the school context (as classical school exercises):

Researcher: If you should imagine a situation in which this calculation is useful. A situation in which it can be useful to calculate seven times four: which one can it be?

Interviewee 3: Ehm… Seven times four…(long silence) Which kind of situation can it be?

Researcher: Any kind of situation, in real life. A context in which it can be useful to calculate it.

Interviewee 3: At school… I don't know… In mathematics classes.

Researcher: Because… for example: what can be asked in a mathematics class?

Interviewee 3: Times-table! Seven times four: the times-table of seven and the times-table of four.

Researcher: Do other examples come to your mind?

Interviewee 3: No.

On the contrary, the quantity and quality of problems associated to multiplication are wider in people with HME.

6.5 Discussion and Conclusion

In the previous sections, we showed examples of invariants and situations that are associated to multiplication by the ten interviewees. The analysis puts in evidence some differences between people with different levels of schooling.

A first result is the fact that mistakes appear just in the LME interviews; it has no easy explanation. Typically, times-table memorization takes place during primary school (usually in grade 2 or 3 in Italy). In our country, multiplication between naturals is revised in lower secondary school (grade 6–8) which was attended by all the interviewed people. So they were all exposed in the same way to this content when in school. Maybe, the better performances by those who had a HME can be explained in terms of *marginal knowledge* (Bahrick & Phelps, 1995). It is the set of information acquired in the past, even information that is no longer possible to recall directly. Bahrick and Phelps (ibid.) notice that the re-learning of marginal knowledge lasts longer than the newly acquired access to an equally difficult material which was not familiar before (regardless of the fact that the person recognizes it or not). Subjects who continued their mathematical education, in high school and further, had the opportunity to recall knowledge on arithmetical facts which was directly recallable or part of marginal knowledge. In both cases, the result of this recalling is the consolidation in terms of faster and efficient access to memorized information. This result aligns with that by Bahrick and Hall (1991) who observed that people who attended Calculus classes in college had better memories for high school algebra than those who did not attend such classes.

Interviewees with HME showed better abilities also on the meta-cognitive dimension, choosing the most convenient property to recall the particular required arithmetical fact. In the attempt to give a first answer to our research questions, we can notice that LME interviews show stereotyped *pragmatic schemas* (Nunes et al., 1993) while MME and HME subjects often changed their approach to calculation according to the required multiplicative fact. According to the definition by Schank and Abelson (1977), we can say that LME developed rigid *scripts* for multiplicative situations. This means that their semantic memory contains a small amount of possible actions admissible in these situations. This is evidenced by the fact that, for some multiplications, they declare that they cannot see any possible way for reconstructing the result if not direct recalling. On the contrary, HME subjects usually answered to question 4 (can you imagine a different way to get the result?) with sentences like "there are many" or "there are infinite ways", even if they also needed time to elaborate an example then.

Concerning the proposed problems, we noticed that almost all the interviewees propose at least a problem in which multiplication is considered as a *unary operation*: one factor is an operator which acts on the other factor (Vergnaud, 1983, 1988). This is coherent with the primitive model of multiplication as repeated sum (Fischbein, Deri, Nello, & Marino, 1985) that is often (but not always) recalled during the interviews.

Many of MME and HME interviewees draw problems from their everyday life. This can be interpreted as an activation of memories about past experiences. When a fact from semantic memory has been recalled in the past, a subsequent recalling of the same fact can cause an activation of memories about the moment or the place in which the past recalling took place. This interpretation implies a strong connection between semantic memory and episodic one. Such a connection cannot be generalized according to these data; it should be tested, especially in terms of the time gap between one recalling and the other.

On the contrary, most of the people in the LME group were not able to associate the mathematics they learnt inside the school's walls to everyday life contexts. An interpretation of this phenomenon can be that one of the effects of mathematical education is an improvement in transfer ability. This interpretation is compatible with the results by Nunes et al. (1993), who observed how workers with a low mathematical education could not transfer the knowledge acquired in school to the solution of problems at the workplace.

We could also suppose an opposite causality: the inability to easily recall multiplication facts and the difficulty in transferring them in new situations might be one of the root causes of pupils' disaffection to mathematics and eventually of dropping out of school. If this is the case, we can speculate that a rehabilitation of abilities in simple calculation can be thought as a possible intervention to stem the school drop-out rate.

Other factors than schooling should be taken into consideration. For example, we showed that a workplace situation was reported by an interviewee; but this is a unique case. Considering the same subject, she works as a cashier in a supermarket but she does not show a better flexibility in using arithmetical properties, nor does she have a faster recalling ability. The lack of differences could be ascribed to the fact that nowadays, in Italy, automatic cash registers are widely diffused and so people do not need to perform mental calculation even in buying and selling situations. Another interpretation can be taken from Nunes et al. (1993). They argue that workers elaborate specific pragmatic schemas to deal with money and that such schemas are not usually transferred to other kinds of calculations.

A second factor which could be considered is the fact that having children attending primary school could be the cause of a recent recalling of multiplicative facts. Indeed, it is a common practice for Italian parents to ask to their children to repeat the multiplication tables many times, with the aim of helping them in memorizing. However, three of the interviewed subjects are parents (interviewees 1, 5 and 8) but they present completely different behaviors that appear to be more similar to those of people sharing the same level of schooling.

This study is just a pilot one with a very small sample, so we must be careful in generalizing any result. Anyway, in this paper we have shown that it is possible to observe some differences in adults' invariants and situations associated to multiplication. According to our analysis of the collected data, the level of schooling appears to be the most related factor (in one direction of causality, or the opposite) to the differences in the conceptual field of multiplication developed by adults, even after many years from the end of studies.

In conclusion, we can get few answers from this analysis, and none of these answers is widely generalizable. Furthermore, our results can be influenced by the particular multiplications we used in the interviews. So, to strengthen our discussion, more interviewees are needed and a variation in the asked multiplications could be useful. Finally, we have to stress that all the interviewed people comes from the same country: this fact assures that they received a similar mathematical education but it constitutes also a limit to the possibility of generalizing results. Indeed, memory about multiplication fact can be strongly influenced by the spoken language (LeFevre et al., 2001).

We consider as the most important results of our investigation, the fact that many new research routes open up. In particular further qualitative studies on students and adults' memories about arithmetical facts and their reconstruction could be implemented to investigate at least three different research problems:

1. Adults' memories about schooling could shed new light on the causes of scripts development, highlighting those educational practices which results, in the long run, in rigid pragmatic schemas or in more flexible mental calculation.
2. Furthermore, few data are available to conjecture about the connection between semantic and episodic memories for mathematical situations. As far as realistic mathematics is currently promoted in many countries, it would be important to study if such real-life situations are recallable from long-term memory after many years.
3. Finally, we observed a relation between the reached level of schooling and difficulties in arithmetic facts recalling/reconstruction; however, we do not have enough information to suppose a unique direction of causality. This could be investigated more deeply through the analysis of the autobiographic memories of adults.

Again, an integration of theoretical frameworks from cognitive psychology and mathematics education may provide tools to study these research problems.

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Chapter 7
Toward Mathematics Education for Adults in South Korea

Eun Young Cho and Rae Young Kim

Abstract This study investigates requirements of programs for adults to address certain unmet demands for mathematical learning in South Korea. In doing so, we first explore historical contexts of lifelong education in South Korea. Then, by interviewing seven individuals about their understandings of mathematics, we found that the participants recognized the significance of mathematics in their lives even after a formal high school education because of its usefulness and intrinsic value. The participants indicated three different reasons for studying mathematics: their children's education, professional needs, and joy of studying mathematics itself. They as adults were eager to study mathematics after seeing it in a new way because they came to strongly believe that they could apply mathematics to their lives. Unfortunately, little effort and attention have been given to adult learning of mathematics in South Korea. The findings thus provide meaningful implications on educational policy and research for the development of mathematics education programs for adults in South Korea.

Keywords Lifelong education · Adult learner · Reconceptualization
Mathematics education

7.1 Introduction

Brianna had been always the class president in secondary schools and her grades were high as well. However, mathematics was difficult for her and her grades in mathematics-related subjects were bad. Brianna had frequently asked to herself,

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E. Y. Cho
Department of Mathematics Education, The Graduate School of Ewha Womans University, Seoul, Korea
e-mail: ceyceyoung@gmail.com

R. Y. Kim (✉)
Department of Mathematics Education, Ewha Womans University, Seoul, Korea
e-mail: kimrae@ewha.ac.kr

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"Why do I study mathematics? It has nothing to do with my everyday life. I won't major in mathematics. Why should I do it?" Solving mathematics problems, she felt like being surrounded by only numbers. Recalling her mathematics classrooms in secondary schools, Brianna described that as soon as a mathematics teacher entered her classroom, the teacher explained the contents in monotone without greeting, and then he or she called upon a few students to write their solutions of the given problems on the board. Brianna was too nervous at such a moment. It was hard for her to memorize the process of solving the problem in front of classmates.

Twenty years passed. Her daughter entered a gifted program of mathematics at 4th grade. She experienced many mathematical activities using various manipulatives. Brianna was interested in the mathematics activities. She thought that the mathematics she learned from schools was very different from the mathematics her daughter learned. When her daughter became a 7th grader, Brianna made up mind to teach her child mathematics by herself because she did not want her child to simply repeat routines and procedures at an academy. She started taking some courses for being a mathematics instructor. The courses were provided by the human resource development center of her district.

Studying mathematics as a grown-up was different from what she did in secondary schools. Brianna was a doctoral student specializing in chemistry before being a housewife when her children were in the 8th grade and 2nd grade respectively. She studied mathematics a lot in her school days, but her grades in mathematics were not as good as those in other subjects. She had some negative memories of mathematics. However, after being a grown-up, she had opportunities to think about why she should solve mathematics problems and how they were related to. Her understanding of mathematics has been changed as well. 'The reason for learning mathematics is to develop logical thinking, and mathematics is a necessary tool in modern society.' Although Brianna started to study mathematics for her child, she eventually found herself enjoying mathematics without any external pressures such as college entrance examination or expectations from family and society. Nevertheless, she complained that the mathematics curriculum for adults were almost the same with school mathematics which she had learned. She wanted to find some appropriate mathematics programs for adult learners like her.

This comes from the interview with Brianna about her experiences in mathematics. Even though Brianna's story is not representative, it allows us to see what demands for mathematics for adults exist. Unfortunately, only a few programs are available for adults to learn mathematics in South Korea. Even the existing programs do not effectively meet the demands for lifelong learning of mathematics. In this chapter, we will first explore social and historical background of lifelong education to understand the status of lifelong education in South Korea. Then, by analyzing the interviews with seven adults, we will describe the changes in their perceptions of mathematics across time and their demands for learning mathematics as adults. Drawing upon the results, we will discuss the meaning of lifelong education for mathematics and suggest a possible agenda to develop mathematics education programs for adults to meet the demands in South Korea.

7.2 Lifelong Education in South Korea

Lifelong education was first introduced in the late 1960s by UNESCO as a significant educational experience of each individual's life. Later on, the concept of lifelong education has been developed in two ways. In the globalization era of the 1990s, many international policy organizations including the Organisation for Economic Co-operation and Development (OECD) have used lifelong learning as a concept to maintain economic health. It can be accomplished by developing and strengthening skills of workers in accordance with a neoliberal economic trend (Hager, 2011). On the other hand, lifelong education begun awakening to the absurdity of the traditional education centered by teachers and at schools. It started to focus on learners and emphasizes autonomy of learner. Lifelong education is thus used to denote both concepts. In many countries, the value of lifelong education is put forward as individualism and economic utility (Kwak, 2005, p. 87). Considering that education persists in society, the two perspectives are probably complementary.

In this sense, lifelong education in South Korea is a good example to show how the two concepts of lifelong education could be applied in a real context.

7.2.1 Brief History of Lifelong Education in South Korea

The history of lifelong education in South Korea shows well how education has responded to social need. Informal education in Korea had been implemented in the form of literacy education with Korean alphabets during the period of Japanese occupation (1910–1945). Although there were a variety of learning activities through communities even before the occupation, the term *informal education* first appeared in the period of the US military government (1945–1948). The policies for lifelong learning appeared in this time as well. Thus, we generally refer to this period as the beginning of lifelong education in South Korea (Kwak & Choi, 2005).

The type of lifelong education varies depending on what the society needs. In the 1950s and 1960s, the education for rural enlightenment and literacy was the mainstream. This trend continued in the 1970s with the *Saemaul Undong*, i.e., new community-building movement, initiated by the government. *Saemaul* education is composed of moral education, job training, and technical training to empower individuals and communities. It aimed to cultivate leadership of local community leaders who have expertise and abilities to organize and lead this movement effectively (Roh & Roh, 2010). It began to modernize rural areas, expanding to the whole country later. It helped overcome the trauma and rebuild economy after the Korean War.

In the 1970s, education for the workforce was important. A decade later in the 1980s, social education, which denotes all types of education excluding formal school education and family education, had been administratively well-developed, and people began to distinguish between social education and lifelong learning. In the 1990s, a wide variety of lifelong education associations and national research institutions published a number of papers. A major change in policy about lifelong education was made at this time. The 5.31 Education Reform Bill in 1995 produced supportive systems for long-distance learning such as credit bank systems, a bachelor's degree by self-education, part-time enrollment and home schooling. Lifelong education was institutionalized in this manner (Kwak & Choi, 2005). In the 21st century, lifelong education has begun to be systematically implemented at a national level by the Lifelong Education Act amended in 1999. Lifelong education spread not only in the field of education but also in almost all areas including administrative, cultural, health, welfare, and religion.

The Lifelong Education Act was revised again in 2007. Overall, it strengthened the responsibilities of national and local governments, which let them manage lifelong education institutions efficiently and provide supportive programs (Yang, 2017, p. 95). The revision of law and systems seems to have affected the participation rate of the people in the lifelong education: 29.8% in 2007 and 35.7% in 2016 (Jeon, 2017, p. 103).

Despite the development of lifelong education over time, research and practical programs on lifelong learning of mathematics are still quite lacking. There are some studies on mathematics for the elderly to improve the quality of their lives by expanding their thinking power and logic (Ko, 2007, 2009, 2010) and to develop instructional materials for parents to help the children study mathematics (Lee, 2011; Lee, Kim, Cha, & Lee, 2013; Park & Kang, 2008; Park & Shin, 2009). However, more systematic research on lifelong learning programs for mathematics is in need.

7.2.2 Toward Mathematics Education for Adults

Mathematics has developed along with a human history. Numbers are products of human analysis and synthesis, arising from the process of cognitive development to other concepts such as abstraction. Ancient mathematics had developed as a tool to make human life efficient, including inventions such as tax law and calendars with development of civilization, and also as a tool to understand the nature. Now, mathematics as advanced technology is still crucial for our lives because it helps us to communicate with people and make reasonable decisions from the analysis of huge amount of information and data. Despite the fact that mathematics is integral to human beings, however, people often ask why studying mathematics is so important.

Safford-Ramus, Misra, and Maguire (2016) suggest three reasons why adults are reluctant to re/learn mathematics. The first is the negative perception of mathematics that many of them have. A negative perception of mathematics may hinder one's learning of mathematics. Secondly, adults may not perceive their mathematical activities in daily lives. Rather, many of people usually regard mathematics as a subject to be taught in school. Finally, there is a popular belief that people cannot ask "why" about principles in mathematics (Safford-Ramus et al., 2016, p. 7). It seems to be related to the way of learning mathematics in schools.

Considering the negative perception of mathematics, Cho (2015) argued that this was closely related to how to teach mathematics in schools, predominantly focusing on numbers and formulas. For this reason, people may lose an opportunity to learn mathematics connected to realistic contexts where they may use mathematics to solve problems. Furthermore, the realistic contexts are also related to the value of mathematics that adults have. Coben (2002) and Harris et al. (2015) found that adults have *use value* in mathematics related to their everyday lives. *Use value* means as the mathematical usefulness that people feel in their lives, and *exchange value* means the social utility of the mathematical knowledge such as professional distinction. The use and exchange values of mathematics tend not to be proportional (Coben, 2002). Williams (2012) deals with the 'enjoyment of mathematics' as *use value* because it is related with the process of consumption. But he also commented that these values are often hard to identify (Williams, 2012, p. 59).

To help adults to recognize they do mathematics in their daily lives, it is required to identify how school mathematics is related with our work as well. It was dealt with at TSG 03 of ICME 2013 as the topic, mathematics education in and for work (Wake, Coben, Alpers, Weeks, & Frejd, 2017, p. 387), which is one of the most important parts of mathematics education for adults. TSG 03 suggests how mathematics education meets the mathematical requirements of our lives as one of the focal topics (Wake et al., 2017, p. 388). It is one of fundamental topics to stimulate mathematics learning of adults. Moreover, it is related with our study which deals with how adult learners feel about mathematics and what they want to learn.

Furthermore, the results of the Programme for the International Assessment of Adult Competition (PIAAC) released in 2013 have many implications for mathematics education in Korea. According to it, the numeracy of the 16–24 age group in South Korea is much higher than the overall OECD average. By contrast, the numeracy of the 16–65 age group is much lower than the average (OECD, 2013, pp. 264–265). Song and Kwon (2016) critically approached that PIAAC might not be suitable for measuring cognitive and practical knowledge of Koreans. We should not only pay attention to this study which approached the validity of the contents of PIAAC critically but also need to identify which educational environment in South Korea caused these results. We assume that the follow-up studies of PIAAC would lead policies and programs regarding mathematics for adults.

7.3 Methodology

In order to reveal how adults think about mathematics, we interviewed seven adults as a case and analyzed the data using constant comparative method (Corbin & Strauss, 1990). The participants were initially selected for our other study aiming to understand the relationship between their experiences of learning mathematics as students and their guidance for their children's learning of mathematics. However, we discovered from the analysis of the data that all the participants had experienced the changes of their views on mathematics and had desired to have another opportunity to learn mathematics to meet their own needs. We thus decided to analyze the data from the view of lifelong education. Despite the limitation of the data, it is meaningful to see what they think about mathematics as a case because presently there is little evidence about the demands for lifelong learning of mathematics in South Korea.

We interviewed seven adults who have different socio-economic backgrounds (see Table 7.1). we restricted that all participants were in their 40s raising their children. This is because the average age of the first marriage in Korea has already been over 30 years and the adults in their 40s at the time of the study learned under the same national mathematics curriculum. There was the only one male participant because it was hard to find males who took care of their children in full time.

Each interview lasting about 90 min was recorded and transcribed. The semi-structured interview consisted of six main focal questions:

- What do you think about education you had through K–12 years?
- How did you participate in general class? Please, tell us some episodes about it.
- How did you participate in mathematics class? How did you study mathematics?
- What do you think about mathematics education that your child has in school?
- Have you ever been under stress due to your children's mathematics learning?
- Who was the most impressive mathematics teacher of your child? Why and how?

  (Cho, 2015, p. 25–27)

We also asked "how do adults understand mathematics? Is this different from what they learned during their school lives? What made them pursue further learning of mathematics?" as sub questions.

Using the constant comparative method (Corbin & Strauss, 1990), we analyzed their responses. At first we focused on both aspects: what mathematics they had studied in their primary and secondary education, what kinds of mathematics they experienced as parents. While analyzing them, we found some characteristics of data on the perspective of lifelong education. So we analyzed them again.

At this time, we focused on three aspects: what mathematics they had studied in their primary and secondary education, what they think about their schooling, and what kind of mathematics they discovered that they needed on the job. We made open codes and axial codes such as the perception of mathematics, the reason to study mathematics after iterative analyses to find specific selective codes. We were finally able to narrow down to the needed selective codes such as mathematical values and continued on to the final analysis.

Regarding mathematical values, our final selective codes are extrinsic value and intrinsic value. Prior studies (Coben, 2002; Harris et al., 2015; Williams, 2012) divided mathematical values into use and exchange values. Williams (2012) means potential academic or professional achievement as exchange value. And the use

Table 7.1 Participants' demographic information

Participant	Annie	Brianna	Carolyn	Dorothy	Elizabeth	Felix	Gale
Age	41	42	49	40	48	45	40
Sex	F	F	F	F	F	M	F
Highest degree in education	Bachelor's Degree	Master's Degree	Bachelor's Degree	Bachelor's Degree	Bachelor's Degree	Bachelor's Degree	Diploma
Occupation	Employee	Housewife	Housewife	Official	Dancer	Designer	Housewife

Note F Female, *M* Male

value is related with competence and comprehension that people attain when using mathematics (Williams, 2012, p. 59). We could find the exchange value and use value through the motives of our participants to study mathematics. Differently with Williams (2012), in our study, the ‘enjoyment of mathematics’ that participants felt could not be involved in use value. So we set extrinsic and intrinsic value as the final codes. The first one covers the use and exchange value. In other words, this can be divided into two sub-categories: use value and exchange value. The second means the enjoyment of mathematics itself that participants valued.

7.4 Results

7.4.1 Adult Perceptions of Mathematics

All the seven interviewees said that mathematics they learned in schools was not connected with realistic contexts. They mainly memorized formulas and solved problems without any application into realistic situations. They seemed to have similar images of mathematics regardless of their achievement in mathematics.

Of the seven participants, three of them graduated with good grades, while the remaining four had poor grades in mathematics. All but Dorothy said that the definition of mathematics at school changed drastically after they were grown up. Dorothy said that she still considered computational skills the most important. Even though she said so, she wanted her children to learn mathematics with manipulatives through exploratory activities. “I would like (mathematics) to be a subject that can be touched and touched…”

Annie said that she was a “sincere student” in schools and learned in a way that she did not have any special questions, but “studied, solved, solved, and solved.” But now Annie thinks that the reason for learning mathematics is to improve thinking ability.

> Annie: I did not know it at that time, but do they … do people study this number … such as differential or integral, these rules. The world has come to know as a rule of all things. Mathematics is not only mathematical, but also, for thinking.

Brianna said that the mathematics she learned during her school days was a process of struggling with numbers without knowing its meaning behind them. Now she thinks that the real reason for learning mathematics is in improving the power of logic, and that it should be easier with approaching in the context of the real life with lowering the overall difficulty of school mathematics.

> Brianna: It seemed that the study of mathematics was so far away from us that we were surrounded by numbers and only to find the solution with it, in order to find the solution in the number. When I came here, it was not mathematics that I had to do with numbers. In the end, I really think that the reason for doing mathematics is for thinking.

Carolyn and Gale thought that the mathematical problems that they had learned during their school days were not necessary in their current everyday lives. What Carolyn thinks of as mathematics are logical algorithms to think mathematically. Carolyn claims that “to think mathematically” is necessary even when she does home management ‘as a housewife’ such as ‘organizing some paperwork’ or ‘arranging clothes.’ Gale thinks that to be good at mathematics, one has to think a lot, so one can improve one’s thinking ability by solving math problems, but the mathematical problems one solves in school are not useful for real life.

Elizabeth and Felix felt the usefulness of mathematics as they studied their major in college. The dancer Elizabeth felt that the choreography in dance was similar to the mathematical formula.

Elizabeth: As soon as you apply the formula, this step is like this, the answer always comes out here. The rotation is finished in a few turns. If I had known some more math then I would have been able to get accessible to this faster and more easily.

Felix was apologetic for missing the interest in mathematics after he found that it was a useful tool in his field: “Bézier curves … It is mathematical tool used in computers made by a French scholar called Bézier … Mathematics is a powerful and wonderful discipline.” In Felix’s field, Bézier curves are tools to design various kinds of letter font. French engineer Pierre Bézier designed automobile bodies of Renault by using them. The reason he lost interest was that mathematics problems in his elementary school were too simple. At the time of middle and high school, mathematics was full of simple calculations.

The participants in this study showed a tendency to think of mathematics as a subject to develop their thinking ability relating to their majors or jobs. The participants’ definition of mathematics has changed significantly. They defined school mathematics as simple formulae, computational skill, and solving problems without real-life context whereas the current definition of mathematics is critical thinking, creativity, and algorithmic capabilities. It is consistent with the visions of the 2015 revised Korean mathematics education curriculum. The results show that the participants did not initially understand the value of mathematics, and only realized it after completing their formal education.

7.4.2 Expectations for Learning of Mathematics

After discovering the value of mathematics learning, the participants felt the necessity to reintroduce mathematics into their lives. The motivations to relearn mathematics have been shown in various ways, such as teaching their children mathematics, getting the logical power and professional works, and the enjoyment of mathematics learning itself. To achieve these goals, they do their own learning by themselves.

The mathematical needs of Annie, Brianna, Dorothy, and Gale are to help their children learn mathematics. To guide and provide solutions for her child’s studies, Gale solves the workbook that her children are solving: "I try to solve the problems ahead. Otherwise, I cannot teach them. I check the answer sheet when I do not know." Annie tries to find the right math book for her child ahead, and she wants to guide her children to "develop mathematical thinking skills, a certain mathematical mind". Annie is eager to be a parent who has more mathematical knowledge: "I want my children to keep their eye on it and do something more. Not just to help them to study the work books."

Dorothy, who wants her child to study mathematics with manipulatives, attempts to do hands-on activities by herself: "There is a mathematics book designed to make manipulatives by oneself. I have a lot of interests in it (laughs). I also try to buy one after another." Brianna takes a mathematics instructor course and teaches her children: "Because I have to teach my children, I also have to look at books and know them. But what I'm doing is so much fun. How fun a magic square is!" Brianna, who resumed her mathematics learning to teach her children, told several times that she got to know the fun of mathematics. Therefore, she would like to keep studying mathematics through the math programs offered in the non-formal education area.

A common finding among the participants who study mathematics to guide their children is that mathematics they want to teach is different from what they learned during their school days, but rather close to what matches the values of mathematics they have found after becoming adults. Carolyn, who does not currently perform any learning activities for her own goals for mathematics learning, also emphasizes developing creativity when guiding her child's mathematics study: "I think it's good to approach mathematics with funny, playful thinking skills to develop creativity, so I let my children do hands-on activities."

Like Brianna, who is currently studying mathematics to enjoy mathematics as well as education for children, Felix also watches documentaries on mathematics or looks at related books: "The Poincaré conjecture … an unusual mathematician. I watched the documentary with fun. That's good." He kept his own mathematic activity after recognizing that the tools he used at working was related with mathematics.

The choreographer Elizabeth had never thought that mathematics would be related with dance when young. One day when she choreographed a work, she felt that the step and beat matched together like a mathematical formula. She said that choreographing was similar with mathematics and she should have studied mathematics hard in her school days. She did not do any activity regarding mathematics.

Participants showed three different expectations- their children's education, professional needs, and joy of studying mathematics itself. The first one is use value which is valued by using at teaching their children and the second one is exchange value which is related with professional achievement at some studies (Coben, 2002; Harris et al., 2015; Williams, 2012). And we called these expectations as extrinsic value. Furthermore, Williams (2012) analyzed the joy of studying mathematics as use value because people felt a joy of mathematics 'in the process of consumption (p. 59). On the other hand, the joy of studying mathematics that Brianna felt is related with not consumption but the joy itself. So we analyzed it as intrinsic value.

Participants would like the opportunity to study mathematics directly associated with their own lives, but unfortunately that is extremely rare. They want to study mathematics to help their children study or to develop their ability to think mathematically. In particular, according to Cho (2015), parents want their children to have a positive attitude toward mathematics but they have difficulties in doing so due to lack of proper training. The results show that most of the participants have studied mathematics individually depending on what kind of needs they have because they could not find any program to meet their needs in formal education.

After all, we found that adults' attitudes towards mathematics have changed significantly even after finishing formal education. With this change, many of them are willing to study mathematics now. However, adult mathematics education programs that reflect their needs are hardly found in South Korea. While they had formal educational systems to help them in learning mathematics throughout their school years, there is minimal institutional support for learning mathematics on their jobs. It is thus conclusive that mathematics education programs for adults are necessary.

7.5 Conclusion

Our findings state that participants recognized the usefulness of mathematics as experiencing life outside of school. The participants all seemed to regard mathematics simply as formulae and numbers in K–12 education. Moreover, they had some negative experiences with mathematics in schools. Nevertheless, they would still like to have a chance to learn mathematics related to real-life applications, which may be geared towards their jobs, home-management, and for nurturing and guiding children. By relearning mathematics, they hope to be able to think and act more logically and systematically in their lives.

In international assessments such as PISA and TIMSS, a fair amount of Korean elementary and middle school students responded that they had great misgivings about mathematics (Choi, Park, & Hwang, 2014). However, the results from this study showed that the participants who had negative impressions of mathematics during their school days has changed to be positive, especially by recognizing the value of mathematics after becoming adults. Simply put, it could happen in current students. A mathematics education program for adults has not yet been developed in South Korea to meet these needs of adults.

In addition, the results of this study not only indicate the need for adult mathematics education but also designate that schools should focus on creativity and solving problems in real life that the 2015 revised Korean mathematics education curriculum emphasizes. The participants who re-conceptualize the value of mathematics as creativity—and logical power to solve problem—want to know how to use mathematics in a real-life context and to learn mathematics with hands-on activity. They were students who had mathematics anxiety and did not feel the necessity to study mathematics in their past. The mathematics of schools that reflect the adults' demands seems to be helpful to students who are not interested in mathematics.

Through this study, we found that mathematical anxiety decreased significantly after growing up rather than during their formal education. They seemed to recognize the necessity of mathematics and they were motivated to study mathematics as well. Moreover, they figured out more clearly why they wanted to study mathematics again. This feature is consistent with the characteristics of andragogy proposed by Knowles (1977): self-directedness, the distinct experience of adults, the readiness to learn, priority given to problem solving. Although it cannot be generalized only by a small number of cases in this study, it is meaningful to confirm that Knowles's conceptual framework might be valid in South Korea. This study presented the necessity of follow-up studies about the direction of development of programs for adults.

Korea is among the world's fastest aging countries in the world due to extremely low birth rates (Statistics Korea, 2015). There are national investments to improve birth promotion, the labor market, and overall quality of life, but require more research to be executed efficiently. Considering Korea's situation as well as rapidly changing societies globally, adult learners call for an educational program in order to gain skill sets required by society. In other words, programs and policies should be implemented to allow adult learners to access advancing mathematics according to their personal needs.

Furthermore, we propose amending of Lifelong Education Act to link mathematics education for adult learners with school mathematics education. Currently, the lifelong education is defined by "all types of systematic educational activities other than formal school curricula" by General Provisions of the Lifelong Education Act in Korea. The amendment to remove "other than formal school curricula" in it seems not only to help people turn their perspective into that educations happen in the whole of their lives but also to be a starting point to generate various programs in a rigid system to associate school education and lifelong education.

7.6 Discussion

In this session, we will experimentally present an agenda for mathematics education within the framework of lifelong education for adults in Korea. The results from this study show that Korean adults have three mathematical needs such as their children's education, professional requirements, and the pleasure of mathematics itself. The first is partially overlapped by the agenda of parental education currently held by the Ministry of Education, and the second is nested with the agenda of vocational competency development by the government.

Parents' requirements in mathematics education seem to be studied more appropriately within the agenda of adults' mathematics education. Research on parental learning is mainly focused on programs for interactions with their children and curricula of various subjects, especially, for K–6 grades (Kim, Rho, Ryu, Yoo, & Lee, 2008; Lee, 2016). The participants of this study wanted their children to think critically, creatively, algorithmically, and logically through studying mathematics. Naturally, they wanted to have various experiences with mathematics. To meet this need, parental education for mathematics can be considered as a part of lifelong education in Korea.

Second, we could grasp the professional requirements of adults in mathematics in this agenda. Currently, there are some mathematics programs to get qualified in occupations such engineers and financial specialists. It is hard to find mathematics courses to assist on their career activities once they get qualified. Mathematics programs for adults may need to be specified to support their profession. For instance, the dancer Elizabeth recognizes that the steps and rotations in her choreography are related to mathematical formulas. Designer Gale came to reflect the software he uses to design works on mathematical principles. When it comes to learning for adults, self-reflection through their own experiences and learning with them are prominent characteristics. Taken together, we need to study what kinds of mathematics are necessary in each job and how they could assist to work more efficiently.

The third is that one studies mathematics because of its intrinsic interests in mathematics. It could be implied in the first and second agenda shown above. In other words, when rediscovering mathematical elements in one's job, some participants became interested in mathematics and wanted to study mathematics. Besides some of those who start studying mathematics to guide their children also found out the enjoyability of mathematics. Overlapping research on it would be efficient in proceeding to follow-up studies for the first and second programs. Only then would the follow-up studies would be possible.

In general, when developing a lifelong learning program, we would follow ADDIE model: analysis, design, development, implementation, and evaluation stages (Caffarella & Daffron, 2013). There are studies on mathematics for the elderly and some vocational educational program such as 'mathematics instructor'. However, to find the data-driven program is still hard and research on adults' mathematics is its beginning stages. For the agenda for adults' mathematics education, to focus on the stage of requirements gathering and analysis seems more important.

In the stages of design and implementation, we might design the programs in the format of distance learning. In 2016, adult learners participated in non-formal education by types of participation in the format of distance learning, other types of learning including private tutoring, taking courses, study groups and professional seminars and workshops in order (Korean Educational Development Institute, 2016). Although it is difficult to manage the quality of a distance learning program, we can take advantages of distance learning by getting information about learning styles and preferences of adult learners more easily than other types of learning. Therefore, it would be useful for the feasibility study of adult mathematics education programs.

Currently, lifelong education programs in South Korea fall into six lifelong education areas listed in the Lifelong Education Act: supplementation of education, basic literacy and competence, cultural arts, civic participation, liberal arts, and vocational skills training. According to the 2016 Lifelong Education Statistics, there were a few mathematics education programs for adults in supplementation of education, basic literacy and competence, and vocational skills training (Korean Educational Development Institute, 2016). The controversy as to whether these six areas are appropriate criteria for monitoring the provision of programs continues and it seems necessary to redefine the criteria for dividing the content types of programs logically (Han, Yang, & Lee, 2011). In this process, the need for adult mathematics curriculum should be fully considered.

Furthermore, in order for the agenda of adult mathematics education to be linked organically with school education, it appears essential to amend the lifelong education ordinance as well. In South Korea, lifelong education is defined as all kinds of educational activities but such activities in formal schools:

> The term "lifelong education" means all types of systematic educational activities other than formal school curricula, including supplementary education to upgrade educational attainment, literacy education for adults, occupational education for ability enhancement, humanities and liberal education, culture and art education, and citizen's participation education (Lifelong Education Act of 2016).

Mathematical competence such as problem solving and reasoning, emphasized in the formal school mathematics curriculum, should be developed throughout the lifespan. Education is intertwined with social change and in the 21st century that changes rapidly, education is on-going through life. We need to expand our views on education from only focusing on school education to the ongoing process of learning in entire life. With the revision of the Lifelong Education Act, we will reach integrated view on education.

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Chapter 8
Mathematical Explorations in the Adult Classroom

Ramaswamy Ramanujam

Abstract We present an experiential account of an adult mathematics classroom and try to make a case that themes like optimization and processes like multiple representations, formal communication, argumentation etc. deserve a place in the curriculum for adults learning mathematics. This requires a re-orientation of what we expect the learner to achieve but respects the learner's maturity and offers the learner a way of thinking that is enriching.

Keywords Mathematization · Classroom processes · Argumentation
Goal setting by learners

8.1 Background and Rationale

What are the goals of adult mathematical education? In the words of Wheeler (1982) it is "more useful to know how to mathematize than to know a lot of mathematics". Presumably Wheeler had schools and children learning mathematics in mind when saying this, but would it be far off the mark to articulate the main goal of adult mathematical education as *the mathematization of the adult learner's thought processes*? How does such an articulation resonate with social and developmental goals of adult education that see education as an instrument of social justice? As a relative outsider to this research area of *Adults Learning Mathematics*, I wish to offer an experiential account and attempt to derive some (admittedly polemical) conclusions on curriculum and pedagogy in the adult mathematics classroom.

R. Ramanujam (✉)
Homi Bhabha National Institute, Mumbai, India
e-mail: jam@imsc.res.in

R. Ramanujam
Institute of Mathematical Sciences, Chennai, India

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The broad argument that we wish to present is the following:

- In an adults' class, the adult learner's consent in goal setting is indispensable, and the educational enterprise cannot quite take off without such consent.
- This implies a mathematics curriculum built on the learners' own lives, especially in their participation in economic activity and aesthetic activity, and takes their conceptual maturity seriously indeed.
- Topics from 'higher mathematics' such as optimization, probability, interpolation and extrapolation, transformation and symmetry are likely to be successful in engaging the mature adult learner. Simple familiarity with these notions may yet be better than struggles to achieve proficiency in "elementary school" level mathematics.
- Processes such as estimation, approximation and argumentation form an essential part of pedagogy in the adult classroom (perhaps even more so than in school).

As mentioned above, this argument is itself not built on a well-constructed theoretical framework, but only on (admittedly inadequate) personal experience. The presentation is based mainly on the conviction that there are some underlying principles worthy of discussion, but we will try and present a theoretical framework that may, in retrospect, offer a basis for such an argument.

8.1.1 Learners' Consent in Goal Setting

However we articulate the goals, the adult learner's consent in goal setting is indispensable. The educational enterprise cannot quite take off without such consent, especially since the adult learner has the option to walk out of the class at any moment, unlike children caught in compulsory school. In adult education, there is a clear contract: the learner entering the classroom is aware that she lacks some skills that formal education provides, that these skills are worth acquiring, and that classroom experience will (hopefully) lead to acquiring such skills. In turn, the teacher considers the learners to be lacking in such skills at entry, and confidently anticipates that classroom participation will help the learners in acquiring the skills.

Note that *numeracy* curricula fit this story very well. Invariably, the numeracy curriculum deals with number concepts (arithmetic, fractions, decimals, percentages, perhaps also ratio and proportion), measurement (shapes, space) and data handling. This is broadly the realm of mathematics in the elementary school, and it is often assumed that this is the *right* mathematical content for the adult learner. The learner's consent is then easily obtained: any school dropout is likely to remember this as what mathematics had seemed to resemble in school, and encounters with the world of numbers in the learner's daily life would broadly resonate with such content. This is a happy equilibrium of content selection but begs the question of the learner's engagement with the goals of mathematics education.

In such a scenario, goals such as enriching the inner world of the learner, or of teaching a way of thinking, are largely unaddressed. Algebra, geometry, proof, trigonometry etc. are likely to be outside the learner's realm and hence no learner is likely to question their exclusion. Not infrequently, the teacher of the adult education class carries largely unhappy memories of school classrooms when it comes to algebra, geometry and trigonometry, and is not unhappy with the exclusion of such content. In any case, goals of enriching the inner world are rather hard to communicate to the adult neo-literate learner, especially one who is perhaps skeptical on what such education might offer anyway. Once again, this leads to a happy equilibrium of content exclusion, but again steers adult mathematics education in a direction away from mathematization.

On the other hand, many adult learners do see mathematics as being immensely useful in life. Indeed, anyone involved in commerce of some variety has understood the importance of calculations and often sees the adult education class as an opportunity to learn something immediately useful in life. This is in marked contrast to school classrooms where a large number of children perceive mathematics as being entirely removed from life outside school, and as very likely being entirely useless in their lives ahead.

Given such reality, what is likely to be an adult learner's demand on mathematics education, if she were to articulate such a demand? I do not have access to field research on this, but in my experience, *usefulness* is certainly a criterion employed by adult learners. But I wish to emphasize that their preconception of useful mathematics is arithmetical, principally only because of their memory of school.

However, such use or usefulness relates only to what Polya (1969) refers to as the *narrow aims* of mathematics education. For him, the *higher aim* is to inculcate a way of thinking that mathematics is especially good at. There are many ways of thinking, and the kind of thinking one learns in mathematics is an ability to handle abstractions, with an emphasis on formal problem solving, use of heuristics, estimation and approximation, etc. Is it possible to give prominence to such processes in the adult mathematics classroom, when adult learners have very low levels of literacy and low levels of achievement in mathematics?

8.1.2 Life Based Curricula

It is in the lived reality that adult learners find the usefulness of mathematics, and indeed, lack of contextualization is often cited as the cause of alienating children in school mathematics classrooms. A natural corollary would be to base a curriculum on the *mathematical needs* of learners, emerging from their lives. Setting aside considerations of implementation, let us consider the conceptual space of such needs. Such needs can then be perceived as professional (broadly relating to economic activity) or personal (for leading a life of dignity and creative engagement).

While arithmetic does play an important role in commercial life, arithmetical needs can be largely met by experiential learning, as demonstrated easily in any bustling marketplace in India [similar to the experience reported in Nunes, Schliemann, and Carraher (1993)]. In fact, simple technological tools suffice to meet these needs in much of the world. As we will argue later, the professional mathematical needs of adult learners are more in the realm of domains such as probability, resource optimization etc. Not surprisingly, these topics are considered important by business schools.

I am not sure whether research exists on personal mathematical needs of adults. Certainly some amount of mathematical learning is needed to read a newspaper; for instance, statistical inference is needed to make sense of data. Similarly, being part of democratic decision making involves some mathematical ability in assessment of social or community situations. But beyond and above all these—social needs, mathematics may well have an intrinsic role in appreciating order and method, pattern and regularity, in the aesthetic experience of life. I would like to suggest that understanding notions like symmetry and transformation is a part of such mathematical learning. But whatever be its nature, it is very likely that the relevant themes belong to 'higher' mathematics rather than the mathematics of the elementary school that dominates adult education curricula.

8.2 The Context

In what follows, I will share my personal experience when I was pulled in willy-nilly to address the question of whether 'useful' mathematics could yet address the goals of mathematizing the learner's thought. This led to a sequence of educational experiences that suggest a tentatively positive answer to this question. The learners I worked with were women from the poorest sections of Indian society like in Nunes et al. (1993), daily wage earners, and yet they enthusiastically participated in the exercises described below. The book Rampal, Saraswathi, and Ramanujam (1998) describes the broader numeracy context in which the classes discussed below were held.

These classes took place in the broad context of Total Literacy Campaigns (TLCs) in India in the early 1990s (Rao, 1993). In 1989–1990, a voluntary movement in Ernakulam, Kerala, asked the National Literacy Mission of the Government of India for help to conduct a mass literacy program, whereby approximately 1000 volunteers would teach close to a 100,000 learners. This was followed the next year by a similar program in Pondicherry that took up a similar task for about 60,000 learners. Subsequently, a number of districts took up TLCs. During 1990–1991, a voluntary group called Tamil Nadu Science Forum was inspired by Ernakulam and Pondicherry to take up such a literacy exercise in Chennai (or Madras, as the city was then called). Without any governmental support, this effort ran classes for nearly 1200 learners in six slums of South Madras, with about 100 volunteers running the classes. All the experiences I report are from some of these classes that I taught as a volunteer.

8.3 A Framework

What follows is an experiential account, but before I present it, it is worthwhile attempting to frame the discussion in a theoretical framework of education research. In the sense that we are inferring systematic implications from specific experience, the study is *interpretative* (Carr & Kemmis, 1986) and since there is no attempt to quantify effectiveness in any manner, the approach is *qualitative* (Ernest, 1998). I am guided by Ernest's definition of a theoretical framework, which he describes as "*the set of epistemological (and ontological) assumptions that determine a way of viewing the world and, hence, that underpin the choice of research methods*" (p. 35).

We will consider three distinct educational experiences below, each one illustrating a theme in adult mathematics education. The first one focuses on mathematical demands made by the professional or economic life of adults, the second points to mathematical opportunities that arise from critical reflection on everyday experiences, and the third shows a capability for mathematical abstraction in learners that could be rooted in culture. All of these, narrated in episodic manner, yet lay emphasis on the conscious participation of the learner not only in mathematical activity directed by the teacher but also in goal setting, methodology and in setting limits to exploration as well. Such emphasis can naturally be placed within the theory of *Contextualizations and Intentional analysis* (Ryve, 2006).

In *Philosophical Investigations*, Wittgenstein (1953) introduces the notion of language games, where he discusses the rich behavioral aspects of language and communication. In particular, he draws our attention to the way in which these language games can be learned before we have mastered the individual concepts used in the game. The classical example is a child's answer to the question, "How old are you?", where the child holds up three fingers (correctly), without any understanding of the number three, nor the denotation of a year by a finger, nor indeed the far more complex notion of a year. Reliable strategization is an essential component of all game playing, and a player may reason locally within a game, develop strategies and fine tune them effectively, without being able to generalize them into statements of universal validity though the correctness of the strategies may well depend on the truth of such statements. What is critical is an acceptance of the rules of the game, sustained interest in playing and autonomy in joining the game or quitting it. In groups, each player may enter the game with differing objectives and capabilities, but communication and a shared interest in the game is sufficient to sustain the process. Such games can be dynamic and lead to new games; Wittgenstein remarks, intriguingly: "*And this multiplicity is not something fixed, given once for all; but new types of language, new language games, as we may say, come into existence and others become obsolete and get forgotten. (We can get a 'rough picture' of this from the changes in mathematics.)*"

The theory of contextualization (Halldén, 1999), operating within constructivist theories of learning, seeks to account for the situational character of a learner's construction of knowledge. Hallden distinguishes conceptual context, situational context and cultural context; these help to highlight the varying resources that learners bring to the classroom and their cultural situatedness. This is especially important if we need to engage not only with the learning processes in the classroom but also the judgments that learners bring in and their autonomy in decision making (for instance, on how far to explore a concept). Such contextualization emphasizes the communicative nature of the classroom in which every activity is a dynamic game in the sense of Wittgenstein above. Apart from the flexibility in varying content across context, it serves to place limits on validity of content (to a class of contexts) and to highlight the criticality of learners' intentions (especially as learners are autonomous adults).

Intentional analysis is a philosophical standpoint that is based on the assumption that all acts can be seen to be intentional. von Wright (1971) remarks that "behaviour gets its intentional character from being seen by the agent himself or by an outside observer in a wider perspective, from being set in a context of aims and cognitions." While Ryve (2006) and Nilsson and Ryve (2010) discuss contextualization and intentional analysis principally to show how learners bring situated resources to contexts by way of intentions and how this can be used in teaching by varying contexts, my purpose here is to highlight the role of contexts in bringing flexibility into the curriculum (by decoupling us from theorems of universal validity) and that of intentions in ensuring learner autonomy in the adult classroom.

Another important strand of related work is that of Critical Mathematics (Freire, 1994) and Reading and Writing the World with Mathematics (Gutstein, 2016), a framework developed extensively by Frankenstein (1995) and Skovsmose (2004). In this viewpoint, liberatory education has no option but to offer quality mathematics education to the marginalized; when learners learn to use mathematics to study reality, they come to view mathematics as useful, recognize some of its limitations and prepare to shape society. For Freire, it is necessary to *write* the world to *read* the world. Acting in the world using mathematical means empowers adult learners in ways that not only improve their standing in the world, but eases their relationship with knowledge systems, and thus power structures in society. I suggest in the sequel that themes from (what is typically regarded to be) higher mathematics (such as optimization heuristics, usually taught in business schools) are actually accessible to adult mathematics learners with little formal mathematical background, and that this adds to social empowerment as envisioned in critical pedagogy. We will also discuss the role of community knowledge (Civil, 2007) and using it as resource material for mathematics education; this again resonates with the emancipatory goals of adult education.

Lastly, there is a need to bring what is termed *ethnomathematics*, the mathematics of traditional knowledge systems, into classrooms, not only to build bridges between such knowledge and modern mathematics in the universities, but also for a deeper understanding of knowledge in itself. This is needed not only to acknowledge and authenticate such traditional (or folk) knowledge in the minds of learners, but also to actively explore a new mathematical world otherwise inaccessible to learners. The project of mathematically making sense of what the learners know by way of culture (Barton, 1996), opens the door for new open-ended mathematical learning (Gerdes, 1988). As we will see, this drags mathematics on the street into the classroom and makes it an object of critical inquiry, and thus contributes to contextualization as well as critical pedagogy.

8.4 A Demand

In India it is not yet uncommon to see itinerant vegetable sellers who carry vegetables in bags slung over shoulders and on an overhead basket. In 1991 this was a lot more common sight, and Velamma was one such vegetable seller. One day when she came to my place and saw piles of books (primers used in adult education classes), she asked if I was running a bookshop. Knowing her to be non-literate, this was my opportunity to make a big pitch for her to join literacy classes that I was involved in. Initially Velamma was very reluctant, complaining that she simply did not have the time in the evenings for "such things". She seemed to get further demoralized when she realized that the curriculum involved *three books*. At this point she hesitated, and asked: "*If I finish all three books will I be able to earn 10 rupees more daily?*"

All my preparation in education was tested in an instant. Whatever fancy formulations I might have had on goals of education, or even goals of adult education, none included an underwritten guarantee to increase earning capacity by any amount at all. (One US dollar equated to roughly 35 Indian Rupees those days.) Given that her daily earnings varied between Rs. 50 and 100, she was asking for a big value addition due to education. And yet, why not, indeed?

I was, of course, not sure how I could promise such a thing, but thought this was an attempt worth making. I explained to Velamma that I could not guarantee results but that we would try very hard to increase her earning capacity. This was perhaps foolhardy, but earnestly meant nonetheless. (Interestingly, Velamma's daughter was in school at that time, in Grade 7; but Velamma did not for a moment wonder whether spending full time in school for 10 years increased her daughter's earning capacity.)

Thus began a rather interesting journey. We started classes that were initially one-on-one, meeting roughly 3 times a week. Very soon, Velamma brought a few other friends, all of them itinerant vegetable sellers like her. Soon we were joined by some flower sellers. In five to six months, there were 20–30 learners, appearing in class at different frequencies. We followed the official primers for literacy exercises (and only a few of them at that) and entirely ignored the mathematics content (which was principally teaching arithmetical operations). What we did was entirely different, and had as its sole aim the goal of increasing daily earnings. The content was not fixed beforehand, nor was learning assessed systematically. In my opinion, what sustained the exercise was what I have referred to above as the *contract*, an articulated goal shared by learner and teacher alike. While there was no guarantee of fulfillment of contract, a sense of making progress was sufficient for the adult learner to commit to further exploration (and indeed, invite others to join in the experience).

To explain what we did, it is important to get an idea of the daily routine of someone like Velamma. Typically, she would start her day at dawn in the wholesale vegetable market, where she would buy a mix of vegetables for, say Rs. 200. These would be packed into her big basket and bags. Then she would choose between one of five or six localities where she operated, take a bus to that place, and spend much of the day alternately walking the streets and settling down at some fixed places (like one near a temple). Note that the basket rested on her head and one bag hung from each shoulder as she walked the streets under the scorching sun. At the end of the day, if she had unsold vegetables, she would sell the perishable ones to one of the bigger shops (for a loss) and carry over the rest to the next day.

Thus Velamma's choice points consisted of: what vegetable mix to buy at what rates, how much of each, and where to ply her wares. These decisions were subject to drastic constraints: weight (she had to carry them), volume (only three containers), perishability and expected sales. There was also a matter of packing her basket and bags, since each time she made a sale, she had to potentially unpack everything, and this might not always be possible. Her pricing was variable, and hence she could also choose to raise/lower prices as she wished. There were further considerations: a low-margin item such as tomato might sell well anywhere, whereas a high-margin cauliflower might sell better in an upscale neighborhood; but then she might not be allowed inside those houses at all.

One can easily set this up as a multi-dimensional combinatorial optimization problem and *prove* that it is computationally hard! At least, this was how I approached the problem initially and realized that my mathematical training (from some of the top academic institutions of my country) would not help directly. (Today, I have better *game theoretic* formulations of Velamma's problem, but computational intractability is unavoidable.) But then it was clear that what we needed at that time was not a *universal* solution, one that would yield the best answer to every such potential situation, but a solution that was *adequate*, simple and could well be sub-optimal.

What we did in class was to tabulate (more generally, *represent*) the data. We met in the evening and wrote down what Velamma had bought on that day, at what rates, and what she sold where, at what rates. Initially I was eager to record everything, and we noted attributes like perishability, weight, volume etc. Soon I realized that this was futile for several reasons: for one, the data became too vast to process; for another, these attributes were constant and the learners found it silly to write these down every day. This last point is important to the story here: Velamma had a pretty good idea of weights, volumes and perishability of the goods she was dealing with, and saw no utility in writing them down. The particular combination used on a particular day was relevant to decision making, but she could not be brought to see this relevance, whereas price variability was important to her. She was not interested in minimizing her physical discomforts, but only in increasing her earnings, keeping discomfort to her own manageable levels. Dropping these variables turned out to be wise as we progressed, because the learners were automatically correcting for these anyway. For instance, if we came up with a combination based on pricing that was difficult to handle physically, rather than discard it and seek another, they would modify the solution redistributing the mix appropriately. Once again, this might yield suboptimal outcomes but the simplicity achieved was well worth the ‘loss’. Moreover, the learners’ participation in not only broad goal setting, but also in methodology (to some extent) made the entire exercise more meaningful. They took pleasure in repeatedly pointing out that their teacher “sir” was inexperienced and impractical, a well-deserved admonition administered in an indulgent and affectionate tone, and very helpful in taking the classes forward.

It was lucky that the initial classes were taken up with reading and the valiant struggles to master shapes of letters, so Velamma was distracted from “mathematics”. Thus grew a set of tables, and graphs (‘pictures’ to the learners), and soon patterns began to appear. By the end of a month, we could infer functional variations and formulate many heuristic “rules of thumb” on product mix to buy from wholesale market. This led to a “**rule book**” (a filled and annotated notebook) that suggested different combinations at different prices. Icons were used to represent vegetables, units were left implicit so that 2 meant 2 kg whereas 500 meant 500 g, and so on. Initially we named rules by people (mainly due to my training that privileges X’s theorem, Y’s definition etc.), but this led only to meaningless disputes and everyone was happy to drop the convention. The result was an organic naming policy (though I could not have thought of such phrasing those days); to an outsider names like “Thursday rule” or “Brown tuber rule” would make no sense, but they carried meaning to us by way of reference to a discussion that took place in class. *Local definitions* are indeed a part of sound mathematical practice.

At this point I should emphasize the fact that Velamma was a remarkably intelligent woman. She entered into the ‘game’ with great gusto, made fun of my obvious inability to predict prices, and contributed to rule making in a big way. The entire exercise acquired significant depth when more learners joined in it. We could now compare different buying and selling patterns, make up more heuristic rules. In my opinion, the essential component in their learning was this process, of *systematic reflection on their own practices, aided by symbolic rule making activity.* They could also assess the effectiveness of rules themselves, though I had to urge patience on them, not give up on a rule on one failure, but amortize over time. Once the notion of amortization (definitely a higher mathematics notion) was intuitively understood by the learners, progress was rapid.

During the same period, we also went through many *packing exercises* so that the learners could minimize physical effort. The effort required for unpacking when a buyer asked for an item that lay deep inside needed to be taken into account. Discussion on this turned out to be very useful.

Thus, the classes comprised a form of problem centered learning, involving many sophisticated mathematical notions like optimization, functional variation, expectation, amortization, estimation, approximation etc. However the engagement of the learners with the notions themselves was highly informal, lacking in rigour and with *no universal validity*. Nonetheless, the engagement was sincere and committed, and more importantly, perceived as successful, its validity empirically verified within the restricted domain of practice. I suggest that the engagement was also mathematical.

In the end, the class never had a formal valediction either, but simply petered out, meeting over longer intervals. The techniques were surely limited and met their maxima; when the learners felt they were not getting more out of the rule book, they simply stopped participating. By then some minimal literacy goals were achieved as well, and my earnest attempts at science education perhaps hastened their departure. At the risk of belabouring the point, I would like to suggest that choosing the point of exit should be the prerogative of a discerning learner, a choice that our schools and universities are loathe to accept, but one that occurs naturally in the arena of adult education.

As a postscript, I must add that the rule book did help Velamma and her friends to earn more, though I was never sure by how much. A few years later, Velamma told me that she had thrown away the rule book as she remembered it all anyway, and made up better rules now on her own. I was interested to find out what the new rules were and how they were better, but she did not indulge me with explanations.

8.5 An Exploration

What follows is an account of an adult education class in one of the slums of what was then Madras city. The tenements did not have piped water, there were pumps every 100 m (or so) from which water was collected. In addition, tankers would arrive periodically bringing water, and residents stored them in containers.

Many learners were often late to class, and the standard reason was the water truck. Someone pointed out that the same story seemed to be told irrespective of which day of the week it was. The class met on Mondays, Wednesdays and Fridays, shifting to Tuesdays, Thursdays and Saturdays some weeks (for a variety of reasons). And yet, the water truck reason seemed to be uniform, though the trucks did not come every day.

This observation led to a very interesting discussion. Water trucks came *every third day* at that time. Assuming that the trucks came on a Monday, learners realized that within three weeks they would have come on all days of the week. Speculatively, we asked if the arrival would span all days of the week if the trucks came on alternate days. This was indeed verified to be true. A natural supplementary was to ask whether the observation held for trucks coming every fourth day. At this point, learners had difficulties, so we drew a diagram: the days of the week on a circle, and lines taking us from Monday to Friday to Tuesday etc. The result was a single closed trajectory that visited *all the day-vertices exactly once* (Fig. 8.1).

A logical next question was on trucks coming every fifth day (and then, on coming every sixth day), but learners found the question ill-motivated and most of them simply refused to "waste time" on these considerations. But then someone

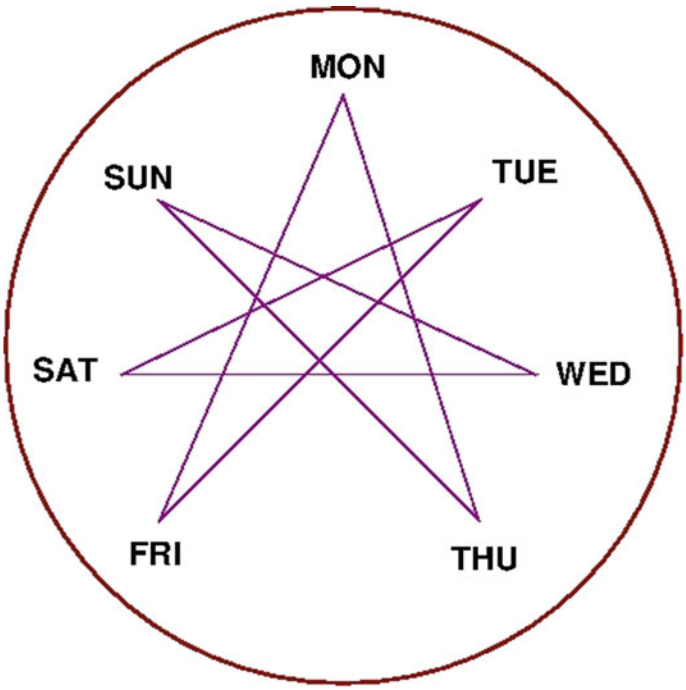

Fig. 8.1 Arrival every third day

pointed out that we could still see what kind of picture obtained, whether it was similar to the picture we already had for 'every fourth day'. This suggestion met with an enthusiastic response, and the curves were drawn. The conclusion that a "full visit" cycle obtained for "whatever" frequency of truck visit seemed heartening to the learners. I tried to spoil the party with the suggestion that trucks arriving every seventh day would always arrive on the same day of week and thus the statement was true only for frequency varying from 1 to 6. But this was indignantly dismissed as "obvious and meaningless", since only frequencies from 2 to 6 were "interesting".

The next exercise was to consider a different arrival event but once every three hours on the clock. My clumsy attempt at story-making met with derision and one of the learners said it was only about drawing pictures, so there was no need for stories! This led to a flurry of drawings, notebooks soon filled with circles and linear trajectories visiting vertices on them (Fig. 8.2).

The fact that a frequency of 5 led to a full visit on 12 vertices but that frequencies of 2, 3 and 4 led only to partial visits led to the conjecture that this was about division: if the frequency divided the total, only a partial visit would obtain, but if it did not divide, a full visit was sure. This conjecture was confirmed by 6 and 7 (hailed and celebrated at high decibel levels) but alas, falsified by 8 and 9. Most learners simply gave up and went home at this point (Figs. 8.3 and 8.4).

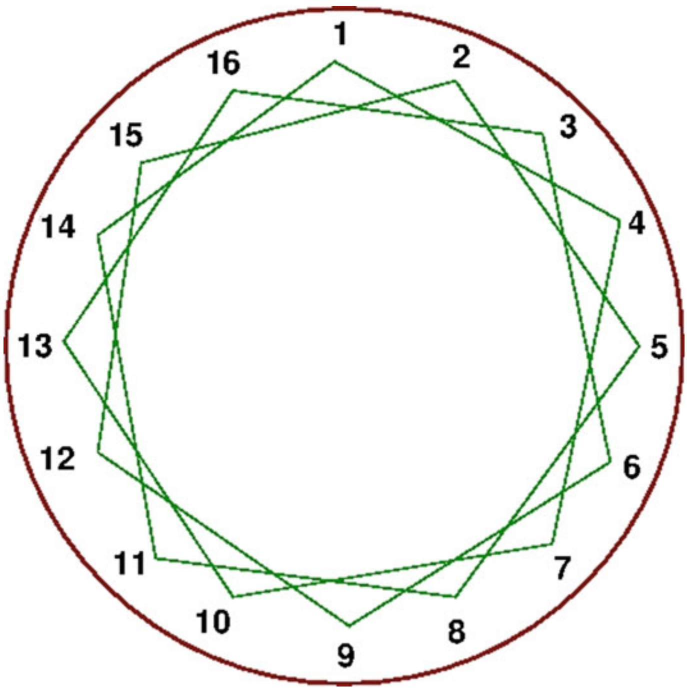

Fig. 8.2 Visiting every 3rd vertex, 16 vertices

But then a few persisted, and in a few days' time, not only did we have a rather large collection of drawings (some of them very beautiful), but we also had one of the bright learners identifying the pattern: full visits obtain exactly when the two numbers (frequency and total) were relatively prime (though not stated in this language). I tried to formalize the statement as a theorem, for any k and n, but most learners saw no point in that, seeing it as some mumbo jumbo. The few who were indeed curious that the statement would be true for any k and n, could not see how I could be sure for say 1500 points on a circle, with the curve visiting every 137th successor. I did try to explain that this was possible and that in some sense, that this was what mathematics was all about. But I did not succeed in the effort.

However, one thing was clear. *Modular arithmetic* was well motivated among the learners, they participated in many exercises involving them. There definitely was argumentation and inference, though it never graduated to proof and universally quantified statements. On the other hand, limited to experimental verification in the small, they could play around with notions like orbits and their composition, from the realm of abstract algebra, without ever learning such vocabulary.

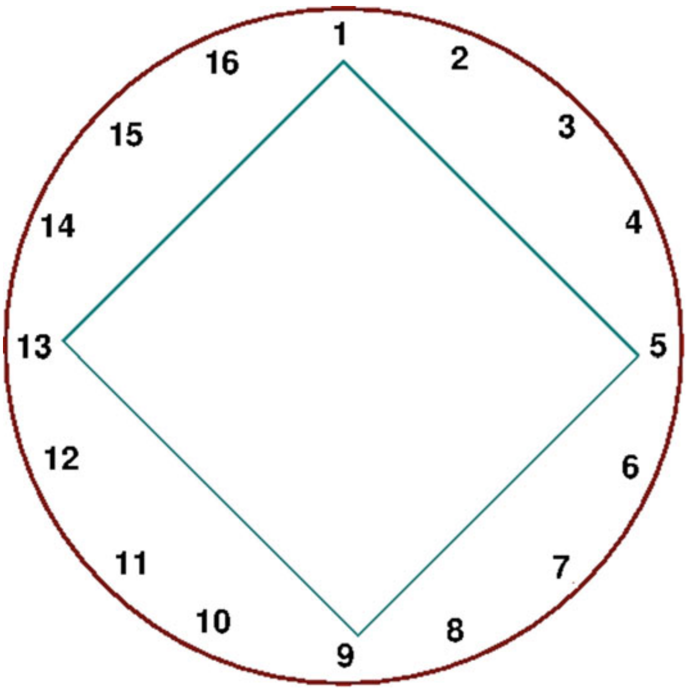

Fig. 8.3 Every 4th vertex, 16 vertices

8.6 Curves and Symmetries

In Southern India, especially in Tamil Nadu, there is a traditional practice of drawing **kolam**s, which are intricate patterns of curves enmeshed in one another (Ascher, 2002). These are drawn on the ground, typically on a 1 m by 1 m patch in front of a house, cleared and cleaned every day for just this purpose, often at dawn and at dusk. Coarsely ground rice flour is typically used for the kolam, and in a few hours it becomes insect fodder.

Kolams serve an obvious aesthetic purpose, but the ritual itself is believed to be rooted in myth and magic from ancient times, when closed curves were believed to trap evil spirits thus preventing them from entering the home. Pragmatically, kolams are supposed to help feed ants and insects so that they do not enter the house. There seem to be interesting connections between kolams and Vanuatu sand drawings.

Kolams are distinct from rangolis that are traditional in Northern India. Kolams usually are abstract, made of dots and curves in one kind (pulli kolams), line segments and curves in another kind (kambi kolams). There is a particular class of *chittu* kolams that are especially intricate: they contain curves that twist and turn compulsively, making for such complex traversals that it is hard to predict the global pattern by small local observations. See Naranan (2007) for an interesting account of mathematics in kolams.

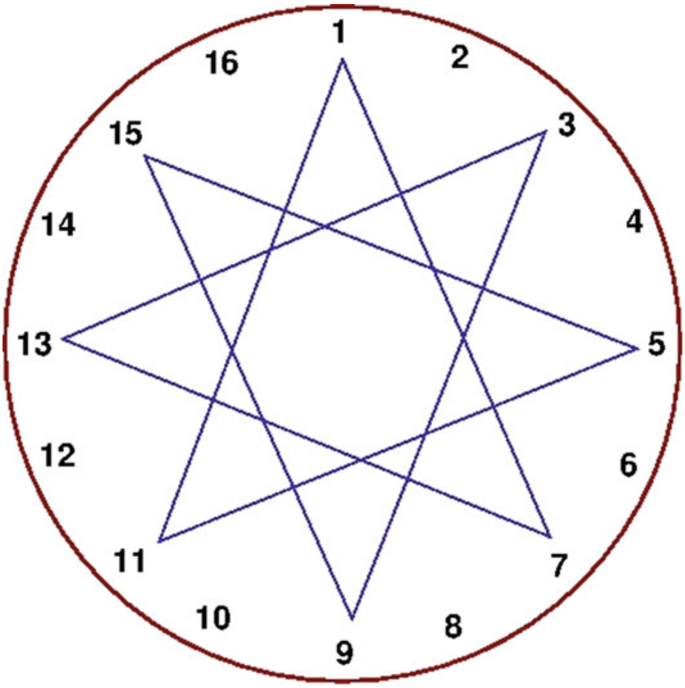

Fig. 8.4 Every 6th vertex, 16 vertices

The first three are examples of *kambi kolams* (drawn with lines or arcs connecting dots) and the latter are *pulli kolams* (which are curves that traverse around dots). There is no clear definition of what figure constitutes a kolam, but there is a clear grammar of construction, and practitioners agree on what makes for pleasing kolams. Open curves are definitely not kolams. The pulli kolams can get remarkably intricate. Note that these are not abstract paintings meant for galleries but for drawing in front of houses, with a lifetime of a few hours.

Admittedly, everyday kolams drawn in front of houses would not be intricate, but it would not be surprising to find one such as in Fig. 8.5d. It is to be emphasized that the people who draw the kolams (women, always) are not necessarily 'educated' in school, and have certainly never learnt to draw kolams in any art class.

Figures 8.6 and 8.7 are photographs taken by a friend in January 2015 in Chennai: one of a freshly drawn kolam on the floor and another of a woman drawing a kolam.

The adult education classes we have been discussing all had women who could draw kolams, with a few rare exceptions. Some of the learners were experts, proficient at the kind of *chittu* kolams (of very high complexity) mentioned above. The majority had a repertoire of about 40–50 simple kolams, of which they would use 5–10 on an everyday basis, the remainder reserved for special occasions. Philosophers might discuss symbolic meaning in kolams, but these women rarely

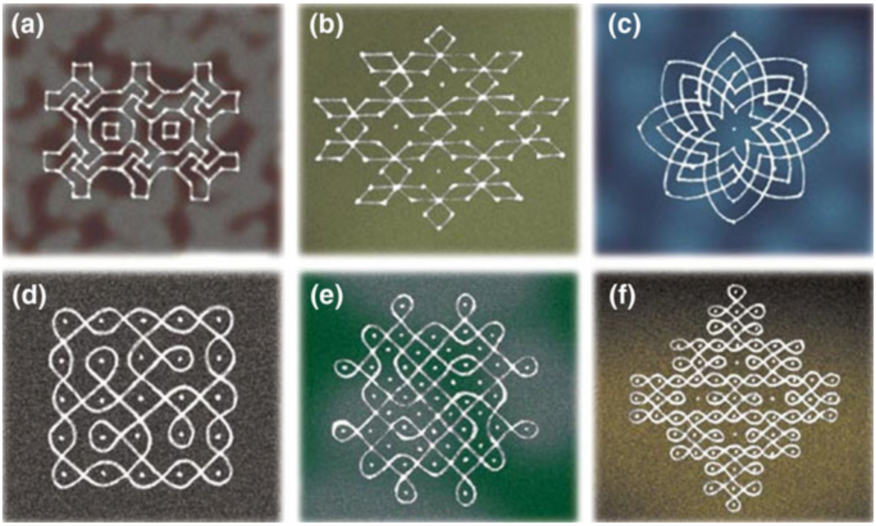

Fig. 8.5 Some kambi kolams and pulli kolams (I thank Prof. Naranan for the images)

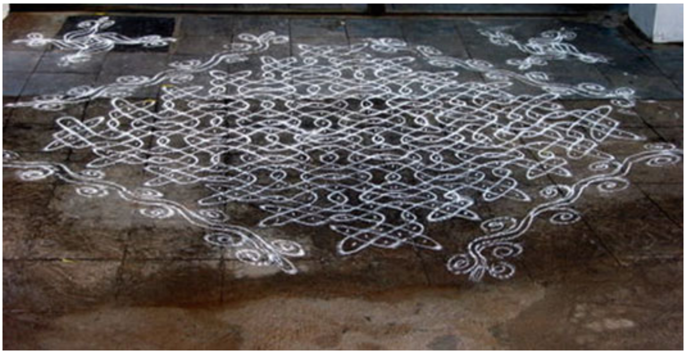

Fig. 8.6 A freshly drawn kolam

pondered meaning in kolams. They mostly saw it as structured drawings and enjoyed the activity for its own sake, thus showing facility in a form of abstract symbolic manipulation without the need for meaning attached to the symbols.

However, none of them saw this as *mathematical knowledge*, and laughed derisively when I described it as such. Indeed, while the class did enjoy kolam drawing sessions, they saw this as frivolous activity, and I could not easily engage them in exercises involving kolams. But when we did manage to take kolams as objects of interest and study them, it was a richly rewarding experience. I am not going to describe the trajectory of our engagement, but only list the areas of mathematical activity we engaged in.

Fig. 8.7 A kolam in progress

8.6.1 Curves

That kolams had curves going around points was obvious to our learners, but listing the kind of curves used turned out to be interesting. Eventually we brought them all into three classes: open curves, simple loops, and loops with one or more twists in them. We used strings, pieces of twine and rope to mimic what happens in kolams. That all open curves were in some sense “equivalent” to line segments took a great deal of discussion to agree on. That the length of the curve is not material was clear to some right away and was never clear to some at any time. Some could not see why we should bother with such ‘names’ for curves. Some were enthusiastically classifying a whole bunch of kolams given in a book. One learner was trying to reconstruct many known kolams with ropes and was finding it very difficult; her conviction that this should be possible, however difficult, was striking.

Perhaps the most interesting exercise that this investigation led to was the consideration of *pulli* kolams with exactly one closed curve in them. We can consider pulli kolams to be rectangular arrays of points, and the kolam is a single curve that turns around, encompasses every point and returns eventually to loop back to make a single traversal. The learners could construct such $m \times n$ kolams for many (small) values of m and n. They could also see, by trial and error, that this was difficult, even seemingly impossible for some values as well. Discerning the pattern that admits a single closed curve, proved to be challenging, and rather difficult for the class. In the class that also went through the water truck exercise discussed earlier, the learners were given a clue that this was somehow related to the clock figures we had drawn, curves that visited every vertex when skipping to the next k-th neighbour. Immediately one of the learners guessed that in rectangular $m \times n$ kolams, a single closed curve kolam obtains when the two numbers have no common factors. This was immediately verified by all, but why this should happen, or how the two situations were somehow “the same” was too hard to explain.

Some learners continued to be skeptical of the relevance of such exercises; some others not only enjoyed the activity, but also relished the challenge of reasoning involved.

8.6.2 Symmetries

The use of ropes to ‘name’ curves made it easy to appreciate *rotational symmetry*, and *mirror symmetry* seemed to occur naturally to learners. We did explore other symmetries in kolams but I found that rotational and mirror symmetries were the only ones that learners took to in an enthusiastic vein. Drawing known kolams and asking what symmetry it had, and convincing another that it had such symmetry, was a rather successful activity.

In my later classes, we could go beyond this easily, and ask how to *break* a symmetry in a kolam, or *introduce* a symmetry into a kolam. This is not a matter of snipping a curve, since we cannot have open curves in kolams. This led to a great deal of enjoyable experimentation. Quickly we were led to the essential question: what operations could be allowed on kolams (so that the result is acceptable as a kolam)? We had to name these operations too, but learners were more interested in trying them out than in their compositional properties.

We came up with many particular constructions but no general ‘method’ for transformations of this kind. What is worthy of note here is that the notion of transformation that preserves a property, or achieves a property, was intuitively grasped by the learners and applied in specific contexts. One or two learners developed a mental toolkit of such transformations that they could make a call on and use in context. But my attempts to economize, to do with as few operations as possible, met with bewilderment initially and pointed sarcasm later on.

8.6.3 Communication

Perhaps the most enjoyable activity was this: one learner would draw a kolam in her notebook, and another would be at the board. The former would need to give a sequence of instructions to the latter, so that the kolam may be reproduced identically on the board. ‘Curl right’, ‘twist left’, ‘skip two and curve around’ etc. became common usage. Describing the initial grid of points led to interesting numerical patterns and terms: 1-to-7-diamond would be a description of rows of points, 1-3-5-7-5-3-1. A 5 by 3 rectangle was merely 5-3 (orally, “five three”). While flips and rotations were common, learners never took to terminology like ‘rotation by 90 degrees’ etc.

It is fair to say that a **semi-formal language** of communication was consciously used by the group. Its syntax had ambiguities but the semantics was fairly precise and could be used reliably. In all the mathematical exercises carried out in class, this was the best use of stylized jargon, where the learners appreciated the use of jargon as such. Since attempts to make the language more formal and unambiguous were resisted, it is reasonable to conclude that the learners exhibited maturity in deciding the level of formalism they needed and were comfortable with. That learners appreciated the use of stylized jargon and decided the level of formalization needed can be construed as mathematical learning in itself.

The language developed was limited to the classroom, and not deployed by the learners in their communication with each other otherwise. This suggests that they understood it as *formal* communication as well.

8.7 Curricular Concerns

These experiences have been reported in an anecdotal fashion, mainly in the spirit of a "proof of concept" or "existence proof" to argue that it is indeed *possible* to have nontrivial mathematical explorations in an adult education classroom. Topics that are typically considered to be "higher" mathematics may still be accessible to adult learners without formal schooling.

8.7.1 A Quick Review

Are there any curricular/pedagogic lessons in an experiential account such as this? The story of Velamma and her colleagues points to a very important curricular demand that a large part of humanity has on mathematics. Much of the human population is involved in self-employed economic activity, albeit part of larger industrial or market processes. Their need to analyze their own practices and attempt an increase in productivity and earning capacity is not only to be expected but felt by themselves as well. It is not clear that the compulsory school, even when successful, at its distance from such economic activity, empowers these people. School education prepares its students well to be part of an industrial economy, and perhaps to be good at professions that have their own guilds and practices. The autonomy of decision making that Velamma and her colleagues enjoy comes with a strong mathematical need, one that is best addressed by adult education.

What kind of mathematics addresses such a need? Broadly speaking, this is the realm of *commercial* mathematics, profit and loss, ratio and proportion, data handling etc. included in school curricula, but also probability, expectations, optimization, risk assessment, etc. that are typically taught in *business schools*. Unfortunately, the latter aim at *proficiency,* and mathematical methods that value complete answers with universal applicability, and the preparation for such mathematics takes years of learning. Perhaps it is possible to revisit these curricular domains for engaging with such notions only to achieve *acquaintance*, a familiarity that suffices for creating and using *heuristics*, and seeking help when needed.

The story of the water truck points to mathematical opportunities inherent in reasoning about daily lives. Paolo Freire talked of reading the "world" and a political objective of adult education has always been to empower learners with ways to understand the world. Typically texts in adult classes are loaded with situations that get the learner to reason about her life experiences. Can the mathematics classroom attempt the same as well? It is then important to have a flexible curriculum that is alive to such opportunities and attempts to find puzzles and problems to solve from learner's lives. According to Polya, mathematics is best taught through problem solving, and this holds for adult learners as well.

The need for culturally rooted pedagogy is generally emphasized and this is of course one of the points illustrated by the kolam lessons. But there is surely more: it points to the possibility of engaging with rather abstract mathematical objects in the adult classroom, when they have aesthetically appealing concrete representations. It is easy to forget that stylized diagrams can be as symbolic a notation as the standard algebraic one. Operating on diagrams as objects opens up an avenue for mathematical exploration that seems largely unexplored. The exercise also motivates the need for stylized jargon, and in solving their own communication needs, learners can create mathematics.

On the other hand, these stories offer another hypothesis worth examining. These learners did explore, did infer 'theorems' but were interested in them in the specific setting they were posed in and not interested in their generalizations. Statements of universal validity, dear to mathematicians, seemed to be considered rather irrelevant. This perhaps also means that there are many mathematical journeys possible for adult learners if we approach the content in a different way, not in its generality, but in whatever specificity it can be embedded in. (Note that I am <u>not</u> conflating specificity with concreteness; the content may well be abstract.)

All this suggests that much more is possible in the adult mathematics classroom than what we typically attempt. However, the word *possible* comes with many (perhaps obvious) caveats.

8.7.2 No Scaling

As it happened, during the mass total literacy campaigns (TLCs) that took off in Tamil Nadu soon after the classroom explorations reported here, such attempts could in general not be taken up. The numeracy curricula in these campaigns did make feeble attempts to move beyond the arithmetical operations and elementary skills, and to relate to life experiences of adult learners, and generate discussions in the classroom (Rampal et al., 1998). There were many kolam "competitions" with learners enthusiastically participating in them. But the concept of mathematical explorations could not be communicated with any success to the largely school educated volunteer teachers who conducted literacy classes. Indeed, they might perhaps have found themselves less capable than the adult learners in discussing symmetries etc. as above.

In general, this has been a recurrent theme. Taking up open ended explorations of the kind discussed here has been difficult with school teachers of mathematics as well, though achieving far greater success than with teachers of adult education classes. As a rule, teachers prefer to take curricula as given and discuss curriculum transaction, seeking remedies for ‘hard spots’ (typically the arithmetic of fractions, introduction to algebra etc.). The ‘shifting of focus from content to process’ advocated by the National Focus Group on teaching of mathematics (NFG, 2006) has been hard to achieve.

8.7.3 Processes

Yet, the mathematical needs of the adult neoliterate learner can perhaps be better met by such concepts from ‘higher mathematics’ than those typically referred to as ‘skills for life’ but which rarely go beyond computational skills. The processes involved in learning mathematics, the processes that need emphasis, are no different for the adult learner than for a school child: according to NFG (2006) these are: formal problem solving, use of heuristics, estimation and approximation, optimization, use of patterns, visualization, representation, reasoning and proof, making connections, mathematical communication. Indeed, according to NFG (2006), “giving importance to these processes constitutes the difference between doing mathematics and swallowing mathematics, between mathematization of thinking and memorizing formulas, between trivial mathematics and important mathematics, between working towards the narrow aims and addressing the higher aims.” If this were so, it would be a crime to deny adult learners opportunities for ‘important mathematics’.

At this juncture, it is also worthwhile to stress on the difference between ‘doing mathematics’ and ‘swallowing mathematics’ referred to above. Elsewhere (Ramanujam, 2010) I have argued that doing mathematics involves “processes such as selecting between or devising new representations, looking for invariances, observing extreme cases and typical ones to come up with conjectures, looking actively for counterexamples, estimating quantities, approximating terms, simplifying or generalizing problems to make them easier to address, building on answers to generate new questions for exploration, and so on.” These are listed as ways in which “mathematics in school classrooms misses elements that are vital to mathematicians’ practice.” The question is: are such processes not relevant for the adult learner as well? Should the curriculum for the adult learner not be *ambitious*? [For an account of the reform process in school mathematics in India, situated in the larger context of mathematics education, see Ramanujam (2012).]

8.7.4 Some Principles

Given the significant difficulty among adult neoliterate learners of achieving even those competencies found at the elementary school, it seems utopian to speak of ambitious goals for mathematics in adult classrooms. For, is it not the case that mathematics learning is sequential, one concept building on another? Without proper foundations in school level mathematics, how can one attempt topics from higher mathematics?

I suggest that we need only a slight re-orientation of our expectations from adult mathematical learning to make this attempt at all, to reconcile the difficulty with the ambition. We need to move away from the stand that if a topic is introduced to the learner, then we demand proficiency in it. We could settle instead for *acquaintance* with the topic, whereby the learner is exposed to the mathematical core of the concept in experiential ways, sees applications illustrated and gets sufficiently familiar with it to be able to recognize the need for the concept and its use later on, and perhaps be able to look up or seek help as needed. This is typically what is expected of skilled workers in factories, we could fashion a learning environment on such principles as well.

Then the curricular areas of study for the adult mathematics learner would not be structured as Arithmetic, Geometry etc. but as Perceiving patterns, Dealing with uncertainty, Optimization, Processes, Symmetry, Form and content, Aesthetics, Modes of reasoning, and so on. Problem solving in groups would be the medium for engaging with such content, rather than as a skill in isolation. Such a structure would emphasize process rather than content, appeal to the maturity of the adult learner, and be more purposive.

We suggest that there are some central principles that can be gleaned from this discussion, which are worth debating, worth exploring:

1. There is a demand on adult mathematics education that comes from learners' productive lives that needs to be addressed seriously; this typically involves mathematics taught at the university, but whose underlying notions may be accessible to adult learners, in a manner that suffices for their needs.
2. Learners' lives offer rich opportunities for valuable mathematical exploration but a curriculum that prioritizes content transaction would typically be unable to seize such opportunities. A process oriented curriculum would be more alive to such opportunity.
3. Learners can engage not only with abstractions at a primary level, but can also *reify* them, create locally effective mathematical language for dealing with them, mainly thanks to the maturity that adults bring to the classroom. A curriculum modeled on school mathematics misses out on such potential in the classroom.
4. Treating mathematical content as given and only meant to be "delivered" in class deprives learners not only of the joy that creating mathematics offers, but also the training in analytical and abstract thought that it gives. This is a problem central to all mathematics education, but no less so for adult education.

Working out such a curriculum and implementing it would not be easy. Finding teachers who are in tune with such a philosophy and equipping them with the ability to take on such tasks would not be easy. But there is no reason to believe that the task is impossible either. On the other hand, the benefits to society in fostering clear and structured thinking in learners would be immense.

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Chapter 9
Parents' Training in Mathematics: A Societal Awareness Study

Zekiye Morkoyunlu, Alper Cihan Konyalıoğlu and Solmaz Damla Gedik

Abstract The aim of this study is to find out how the individually studying middle school students' mathematics achievement differs from the students getting support from their parents. The study is designed as a mixed method study. The study is quantitative in terms of being quasi-experimental and it is also qualitative in terms of the instruments; such as, mathematics seminars for parents, parents' written reflections, parent journals and interviews with parents. The quantitative data was analysed descriptively. The qualitative data was also analysed through descriptive and content analysis. Based on the findings, parents found the mathematics seminars both enjoyable and useful for both children and themselves. The findings indicate mostly positive effects of parent support during the 5th grade level mathematics; such as, productivity, being more careful, having interaction, doing with understanding. There is a 15 point difference between the average scores of the experiment and the control group students in favour of experiment group.

Keywords Mathematics education · Parental guidance · Middle school

9.1 Introduction

Mathematics is not a subject that is only taught by teachers at school. In Turkey it is common to support mathematics education through special courses in which children are trained for exams. However, the support provided by these special courses should not be considered sufficient for teaching children mathematics. Especially during children's early years of experience in mathematics, it should be understood

Z. Morkoyunlu (✉)
Ahi Evran University, Merkez/Kırşehir, Turkey
e-mail: zekiye.morkoyunlu@ahievran.edu.tr

A. C. Konyalıoğlu
Atatürk University, Erzurum, Turkey

S. D. Gedik
Hacı Bektaş Veli University, Merkez/Nevşehir, Turkey

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that a child can observe a great deal by experiencing different situations outside school or at home. In this way, children can understand mathematics in a more meaningful way. Thus, environmental interaction should be a significant factor, as social constructivism proposes, as well as teacher–student interaction. Parents are the most important component of this direct interaction, given that parents have the most important role in this informal education environment where children interact with other closely and constantly. Parental involvement in mathematics education is seen as significant, based on published literature, in terms of children's cognitive (Şişman, 2002; Yenilmez, Özer, & Yıldız, 2006) and affective (Yenilmez & ve Özabacı, 2003) development, helping to decrease the level of mathematical anxiety in children (Alkan, 2011), making positive impacts on children's attitudes towards mathematics and children's mathematical achievements (Alkan, 2011), being a crucial factor in children's internalization of mathematics (Al-Mahdi, 2010), and preventing the disconnection between the school and the home (Civil, 1998). The research revealing these results is generally about the correlational studies of "Parents in Education" and about the relationship between the "demographic features of parents" and "the students' achievements". Experimental studies emphasizing the theme "Teacher at home: The Parents" were not seen during the literature review. In other words, no study focusing on the parents' mission as an instructor or a guide was seen in published literature. Also, it is currently the case that parents need mathematics support to help their children.

An individual is a subject that cannot be thought of outside their social environment, based on social constructivism (Ernest, 1996). Individuals learn through interactions between themselves as well as by themselves. Thus, there is no figure of speech describing an individual's mind isolated from their environment. There is a figure of speech called "individuals chatting" based on meaningful verbal and nonverbal interactions and dialogues (Ernest, 1996). According to Ernest (1996), the mind, thought of as a part of a wider structure, is thought of as "constructing meaning socially". In other words, social constructivism is the aspect of constructivism emphasizing language and social interaction (Ernest, 1996). As Yıldırım (2010) stated, parents were stated as the primary source in the social support system. For children's learning, based on social constructivism, the role of the parents is obvious in this social support system.

However, parents' awareness has an important role. On this topic, some research reveals that parents have awareness about helping their children individually in terms of instruction. For example, Hoover-Dempsey and Sandler (1997) conducted a study on why parents get involved in their children's education. In this study, the decisions by the parents on their involvement points to three essential structures. These are: firstly, the parents' beliefs about what is important in their children's education; what is necessary in their children's education and what parents can allow; secondly, parents' self-awareness in terms of making an impact on their children's educational outcomes; and thirdly, the general wishes and advantages for parents (Hoover-Dempsey & Sandler, 1997). In the study, it was stated that children's academic achievements increased in line with their parents' self-awareness.

9.2 Method

9.2.1 Purpose of the Study

The purpose of this study is to find out how the students' success differs between students getting support from their parents and students who don't get support from their parents.

9.2.2 Research Problem of the Study

This looks at how the mathematical achievement of 5 fifth-grade students—studying with the support of their parents—differed from the mathematical achievements of 8 5th grade-students—studying on their own—and the differences in their attitudes towards mathematics in a middle school, randomly chosen in a rural city in Turkey.

9.2.3 Sample of the Study

The study sample consisted of 5, fifth-grade students and their parents, and 8 fifth grade students by themselves. These students were selected from a middle school, selected randomly, in a small developing city in Turkey. The students were also selected based on their grades and their mathematics teachers' opinion.

9.2.4 Instruments

The study used all of the following: a pre-test and post-test mathematics attitude scale for students; a mathematics attitude scale for parents; mathematics seminars for parents lasting 4 week-totally 8 h, activities for parents for each seminar; the written reflections of the parents; researcher observations; focus group discussions with parents; semi-structured interviews with parents, and semi-structured interviews with students who studied with their parents.

9.2.5 Data Analysis

In the study, the methods of descriptive statistics and content analysis were used to analyze the written reflections and journals. The achievement tests were analyzed using an SPSS 15.0 statistics program. The study is considered to be a mixed method study with the emphasis on qualitative analysis.

9.3 Findings

9.3.1 Findings of the Analysis of the Parents' Daily Journal Questions

See Tables 9.1, 9.2, 9.3, 9.4, 9.5, and 9.6.

Table 9.1 Themes emerged from the answers for 1st questions

Study weeks	Question 1: How does studying with your child affect your child's interest, awareness and curiosity, and how does it affect you?
1. Week	All the parents answered the question positively. The emerging themes were "happy", "understanding well", "good impact", "enjoyable", "productive", "being more careful", "student–teacher relationship
2. Week	All the parents answered the question positively. The themes that emerged were: "happy", "listening", "understanding", "seeing maths as easy", "seeing maths as creative", "great work", "studying together in other classes", "motivation to solve maths problems", "being more careful", "tiredness", "monitoring child", "revision of child's work"
3. Week	All the parents answered this question. The themes that emerged were: "being successful", "good impact on both the child and the parent", "being able to help in other cases", "getting bored", "decreasing concentration", "positive study", "being comfortable"
4. Week	Four of the six parents answered this question. The emerging themes were: "understanding well", "conducting arguments with the parent", "enjoying studying mathematics", "enjoying studying other courses", "studying together as two students", "producing solutions immediately", "thinking flexibly", "solving easier"

Table 9.2 Themes emerged from the answers for 2nd question

Study weeks	Question 2: Did you observe that studying together helps your child to understand the problem and explain it mathematically? Please explain by giving an example
1. Week	Five of the parents answered the question positively. One of them misunderstood the question. The themes that emerged were: "knowing and learning before parent", "making the process understandable", "parent explanation acted as a vehicle for clarity", "being more careful", "decreasing the mistakes", "interaction", "reading to each other", "explaining to each other"
2. Week	All the parents answered the question positively. The themes that emerged were: "explaining before parents", "demonstrating different method", "performing various mathematical knowledge", "rarely using different method", "understanding his own thoughts well", "explaining the given and required information", "producing clarity of the child's understanding"
3. Week	All the parents answered this question. The themes that emerged were: "comprehending coefficients", "solving problems faster", "having no difficulty in understanding", "understanding the age problems", "being more careful", "stating well when understanding", "interaction between the child and the parent", "wanting mother's help"
4. Week	Four of the six parents answered this question. The themes that emerged were: "understanding mathematical concepts better", "understanding solutions better", "increasing productivity in line with the performance", "understanding problems", "needing help rarely", "reaching the results himself", "figuring out the date problems together"

Table 9.3 Themes emerged from the answers for 3rd question

Study weeks	Question 3: Did you observe that your child produced solutions by thinking flexibly during studying together? Please give an example. How did studying together affect this situation? (Flexibility: Thinking flexibly is to think through different ways to produce solutions)
1. Week	All the parents answered the question positively. The themes that emerged were: "concept learning", "producing solutions", "having different ideas", "finding easy and practical ways", "explaining to the parent", "finding the right solution"
2. Week	All the parents answered the question positively. The themes that emerged were: "usefulness at school", "home-created study culture", "sharing different methods", "doing with just one method", "understanding better", "doing problems with different methods", "enthusiasm when working together"
3. Week	All the parents answered this question. The themes that emerged were: "quick answers", "quick results", "focusing on the problem", "producing solutions", "producing a solution immediately", "doing easier", "thinking more flexibly", "thinking of different methods", "studying together is better than studying individually", "producing more solutions during mutual study"
4. Week	Four of the six parents answered this question. The themes that emerged were: "increasing productivity in line with the performance", "producing solutions immediately", "thinking flexibly"

Table 9.4 Themes emerged from the answers for 4th question

Study weeks	Question 4: Did studying together have any impact on your child's understanding of mathematical concepts and their ability to state these concepts? Please explain it by giving an example
1. Week	All the parents answered the question positively. The themes that emerged were: "good concept learning", "asking questions", "increased self-confidence", "understanding with explanation", "reading with understanding", "thinking about the questions", "working with understanding", "sufficient background", "understanding easily"
2. Week	Five of the six parents answered this question. The themes that emerged were: "satisfaction of studying", "solving hour problems easier", "understanding mathematical concepts while problem solving", "stating well", "explaining"
3. Week	All the parents answered this question. The themes that emerged were: "comprehending quicker", "getting faster", "understanding what was read", "understanding the problem well", "good comprehension of the problem", "stating the problem well", "comprehension of multidimensional (complex) problems", "comprehension of the question", "knowing most of the mathematical concepts", "stating the concepts often"
4. Week	Four of the six parents answered this question. The themes that emerged were: "learning multiples together", "learning age problems together", "mutual study is effective", "making payment in a market", "knowing what to do by using deck (tens) or dozen (twelves')"

Table 9.5 Themes emerged from the answers for 5th question

Study weeks	Question 5: Did you observe that your child began to relate mathematics to real life as a result of studying together? Please explain by giving an example
1. Week	Five of the six parents answered the question. The themes that emerged were: "using mathematics in daily calculations", "using mathematics in sharing activities", "not relating", "using mathematics in money examples"
2. Week	Five of the six parents answered this question. The themes that emerged were: "advantages of mutual study", "love of mathematics", "love of other courses", "giving quick responses", "relating partially", "knowing the importance of mathematics in real life", "using mathematics to manage money"
3. Week	All the parents answered this question. The themes that emerged were: "mutual study helped to understand age problems", "happy", "love of mathematics", "calculations while shopping", "understanding instalment payments", "no relations", "using mathematics in careful spending"
4. Week	Four of the six parents answered this question. The themes that emerged were: "calculating and explaining the instalment prices", "making calculations in a restaurant, market or a grocery store", "using mathematics in careful spending"

Table 9.6 Themes emerged from the answers for 6th and 7th questions

Study weeks	Question 6: What are the problems that you encountered while you were studying with your child?
1. Week	Five of the six parents answered this question. The themes that emerged were: "not having any problem", "unnecessary hurrying", "reading incompletely", "understanding incompletely", "being impatient", "problem with time management"
2. Week	Five of the parents answered this question. The themes that emerged were: "a few problems", "mostly positive", "differences between the mother's explanation and the son's understanding", "reading incompletely", "understanding incompletely", "being impatient", "problem with time management"
Study weeks	Question 7: What kind of changes did you observe in your child's awareness and curiosity while working together?
3. Week	All the parents answered this question. The themes that emerged were: "being happier than before", "responding quicker", "keenly focusing on the problem", "getting bored when studying individually", "love of mathematics", "curiosity in mathematics", "understanding with clarity", "producing solutions", "no change in curiosity", "liking mathematics more", "liking studying together", "having enthusiasm to solve problems", "studying together"
4. Week	Four of the six parents answered this question. The themes that emerged are: "having curiosity", "being keen", "being aware of the problem solution", "being more curious", "mutual study makes his problem solving easier"

9.3.2 *Findings of the Parents' Interview*

See Table 9.7.

Table 9.7 The analysis of the parents' interview questions

Questions	Themes emerging from the data
1. Were you able to have discussions with your child during the study? How did communication change over the study?	1. All the parents answered this question. All the answers were positive. The themes that emerged were: "many discussions", "wanting to study a lot", "satisfaction in working together", "arguing with mother", "correcting the mother", "learning better from father", "explaining to the father", "showing ways to the mother", "better child performance", "mother interested in the study", "mathematics study culture at home", "learning when studying together", "parental guidance helped on unknown subject", "increasing self-confidence", "being more detailed", "being more careful", "not much change in communication", "parental guidance when needed"

(continued)

Table 9.7 (continued)

Questions	Themes emerging from the data
2. Did you feel able to guide your child easily during this study?	2. All the parents answered this question. The answers were positive. The themes that emerged were: “existing habits of studying together”, “began to guide the child more willingly”, “wider perspective in learning and teaching”, “not asking many questions”, “child loves math”, “parents have knowledge”, “parents feel more comfortable”, “double-checked homework, first the mother checks; second the father checks”, “being able to help the child”, “learning at the course (seminar)”, “parent’s feeling like a teacher”, “happy child because of mother’s help”, “existing differences from the beginning”
3. Has your willingness to encourage your child to study increased?	3. All the parents answered this question. Two of the parents gave negative answers. The themes that emerged were: “already liked to study together”, “increasing”, “beginning to study easier”, “being able to explain easily”, “having much more knowledge”, “child has much more self-confidence”, “not wanting much parental involvement”, “not many differences”
4. Did you observe any improvements in your child’s ability to evaluate herself/himself?	4. All the parents answered this question. The answers were positive. The themes that emerged were: “commenting to each other”, “checking the homework”, “understanding the previous mistakes”, “knowing where information was lacking”

9.3.3 Findings of the Parents’ Focus Group Discussions

See Table 9.8.

Table 9.8 Findings of the analysis of the parents' focus group discussion

Questions	Themes emerging from the data
1. How did you feel during the seminars? (For instance, return to the school life etc.)	1. The themes that emerged were: "feeling like a student", "feeling responsible", "feeling like a teacher at home", "transferring learning to the children", "good effect"
2. What are the advantages of the seminars in terms of ideas and experiences?	2. The themes that emerged were: "having different perspectives", "learning happens when sharing"
3. Were you able to share ideas among yourselves? Could you give examples?	3. The themes that emerged were: "sharing during the seminar", "doing comparisons"
4. Did you look at the 5th grade mathematics curriculum after the seminars?	4. The theme that emerged is: "not coming to mind"

9.3.4 Findings of the Students' Interviews

See Table 9.9.

Table 9.9 Findings of the analysis of the students' interview questions

Questions	Themes emerging from the data
1. How have your habits of doing mathematics homework changed during studying with the guidance of your parents?	1. Three of the five students answered this question. The themes that emerged from the study were: "asking parents", "learning by asking", "questions getting easier"
2. Did you feel more confident when studying with parental guidance?	2. Three of the five students answered this question by saying "Yes"
3. How has your perspective towards mathematics changed during your discussions, and sharing work with your parent?	3. Three of the five students answered this question. Themes emerging from the study were: "liking and enjoying math more"
4. Did your mathematical achievement increase because of parental guidance?	4. Three of the five students answered this question by saying "Yes"

9.3.5 Findings of Achievements Tests

See Tables 9.10, 9.11 and 9.12.

The academic achievement of the students in the mathematics class showed a significant difference at t (13) = 2.38, 2.86, p = .036, depending on whether or not they received parental support. The academic achievement ($\bar{X}$ = 83) of the mathematics course of the experiment group is more favourable than the academic achievement ($\bar{X}$ = 67.62) of the control group mathematics course. This finding can be interpreted as a result that there is a significant relationship between parental support and non-parental support in 5th grade students.

Table 9.10 Statistical results of the experiment and control group students' achievement

Groups	N	$\bar{X}$	S	SD	t	*p*
EG	5	83.00	5.38516	11	2.387	.036
CG	8	67.62	13.56400	9.877	2.865	.017

EG Experiment group
CG Control group

Table 9.11 Students' performance table with parental guidance

	Student	Pre-test	Post-test	Unit Eva. results	Written Ex.	Av.
1.	Su. A.	56	60	94	86.8	74
2.	Se. A.	76	84	90.2	93.2	86
3.	E. İ.	68	88	91	95	86
4.	İ. O.	76	72	88.6	92	82
5.	G. G.	80	–	86.25	95	87
					Av (1–5) = 83	

Table 9.12 Students' performance table without parental guidance

	Student	Pre-test	Post-test	Unit Eva. results	Writ. Ex.	Av.
1.	Ş. G.	60	72	33	89.0	64
2.	S. Y.	84	76	70	92.8	81
3.	Ş. A.	64	60	57	86.0	67
4.	V. K.	64	36	26	80.2	52
5.	S. Ç.	56	48	16	75.4	49
6.	M. D.	88	60	53	93.6	74
7.	O. T.	80	96	83	95.4	89
8.	E. A.	60	64	51	85.4	65
					Av (1–8) = 68	

The first student was able to increase her performance from pre-test to post-test. She and her mother already had the habit of studying together. They stated that the study seminars made them more systematic and knowledgeable. A school environment had been created at home. The mother felt she adopted the role of teacher, and the daughter adopted the role of the student.

The second student was also able to increase her performance from pre-test to post-test. She and her mother already had the habit of studying together. They stated that the study seminars made them more systematic and knowledgeable.

Like the first two students, the third student was able to increase his performance from pre-test to post-test. His performance on the written exams was also better from the beginning to the end of the term. He and his mother already had the habit of studying together. They also had a mathematics study culture composed of mother, son, father and brother. His homework had been double-checked by his mother and his father. His success can be related to this study habit.

The fourth student was not able to increase his performance from pre-test to post-test. His lack of attention and making mistakes are probably the reasons for this result. He and his mother already had the habit of studying together. The mother stated that this program made the mother (her) more knowledgeable, and this helped them to support their child's mathematics learning at home.

The fifth student did not take the post-test. His written exam results were successful. He has a problem studying. His father stated that his son's problem was getting bored immediately when he began to study. His father said that his son generally does his homework on the last day before submission.

These students were selected based on their similar success before the study to the students getting parental guidance in the subject of mathematics. Two of the eight students were able to increase their performance from pre-test to post-test. The performance decreased for all the others. Their unit exam results were lower than the students getting parental guidance.

9.4 Conclusions and Suggestions

Parents are the first teachers of their children. This situation is obvious during the primary years of schooling. However, during the middle school years, students are still in need of this support. They still need to be guided by their parents, especially the mother. Most of them may not be able to evaluate themselves in terms of their performance. At this point, parental guidance helps to evaluate the children's performances. In this study, those with an existing habit of studying together became more systematic; sometimes the role of parent turned into the role of teacher at home, and the students got more comfortable when supported by their parents. Both the parents and students thought that their academic achievement and their general success got better as a result of this study.

Both for the pre-test and post-test, students getting support from their parents made improvements from the beginning of the study to the end of the study. The problem-solving skills of the students getting support from their parents began to improve. Students getting support from their parents got used to studying with their parents and sharing their experiences with them. This made them engage with mathematics in real life. Even though the result of the study shows the positive impact of parental involvement in students' mathematical education, the parents needed more knowledge and guidance about mathematics in order to help their children.

Moreover, the parents involved in the study wanted to continue to learn mathematics both for their children (especially to help them with their exams) and for themselves. It's important to get parents actively involved in children's mathematical education. Parents should be provided seminars or courses about mathematics—both mathematical subjects and how to teach maths—in order to help their children. Parents should be guided to help their children in mathematics education at home. All these can be achieved with the collaboration of schools and education faculties.

The limitation of the study is the sample of the study involving five students as the experiment group and their parents. That's why the results could be seen as suggestive to other related studies. Based on the findings of the study, when this type of studies conducted for a long time period the results could be positive in terms of both the students and the parents on the mathematical achievement of students and the attitudes towards mathematics.

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Part III
Teacher Focus

Chapter 10
Mathematics in Youth and Adult Education: A Practice Under Construction

Neomar Lacerda da Silva and Maria Elizabete Souza Couto

Abstract This study aims to analyse how Freire's premises are present in the teaching practice of mathematics teachers working in Youth and Adult Education sector. To generate the data, we used the analysis of the Pedagogical Proposal of the Youth and Adult Education (Vitória da Conquista, 2007) in the city of Vitoria da Conquista—Bahia, Brazil, observing the teaching practice of a teacher and the focus group, in a qualitative research study. In the proposal the premises are understood as: dialogicity, problematization, meaning and world comprehension. The practice (class) was organized with a theme, starting with orality, mobilization of knowledge of the students, understanding and approach in the study of mathematical objects, which reinforces the perceptions of the teacher in the focus group, having in the premises the elements for the construction of mathematical knowledge.

Keywords Freirean assumptions · Mathematics teaching · Youth and Adult Education Sector · Teaching practice · Critical approach

10.1 Introduction

In this work we present some results of a Master's Degree research (Silva, 2014) which aimed to analyse and discuss the pedagogical practice of teachers of mathematics in Youth and Adults Education sector to understand the contribution of the studies of the Brazilian educator Paulo Freire. In order to do so, we analyse how the Freirean premises present in the Pedagogical Proposal for Youth and Adults (Vitória da Conquista, 2007) influence the organization of mathematics classes. The

N. L. da Silva (✉)
Teacher of the Bahia Education Department, Ilhéus-BA, Brazil
e-mail: nlsmat@hotmail.com

M. E. S. Couto (✉)
State University of Santa Cruz, Ilhéus-BA, Brazil
e-mail: melizabetesc@gmail.com

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research uses a qualitative approach by emphasizing the description and the study of personal perceptions (Bogdan & Biklen, 2006) of teachers and students.

Youth and Adults Education, as a teaching sector that supplies a demand of the students with specific characteristics, needs a definite positioning of public policies. Debates around the organization of the Youth and Adult Education sector as a specific field of public responsibility of the State (Arroyo, 2007) provided a growing broadening of the constitutional rights, even though the fight for recognition being translated in terms of constitutional norms is not sufficient, but also in terms of political actions in the institutional field. Thus the creation of public policies that require a guarantee to the right of education and/or the creation and education of educational proposals uses strategies that contemplate linguistic, social, economic, educational and other components.

In this way, in the discussions about mathematics teaching, its place and significance with regards to skills and abilities required of individuals in the present society, result in much pressure on this area of teaching, so that student's preparation develops abilities that go beyond specific mathematical knowledge. This is a process whereby he becomes conscious of his own place in the world in such a way that he may reflect on the role played by mathematics.

The study may contribute in the academic-scientific perspective with debates, proposals and projects. It is also possible to present alternatives for the creation of public policies for teacher training so that they take these suppositions as essential elements.

Thus we have the following questioning: how do the Freirean suppositions orient the teaching practice of educators for Youth and Adult Mathematics Education sector? This article is organized in 5 sections: Methodological considerations; the teaching of Youth and Adults Mathematics Education sector and the thoughts of Paulo Freire; the Pedagogical Proposal for Youth and Adults sector and the Freirean suppositions; the teaching practice in Youth and Adults Mathematics Education sector in a Freirean perspective and, finally, a discussion.

10.2 Methodological Considerations

The study took place in three schools that cater to Youth and Adult Education sector and was developed with four teachers who taught mathematics. After finishing the analysis of documents of the pedagogical proposal in order to identify which Freirean premises are present in the document, we analysed through observation of mathematics classes of research participant teachers the influence of such premises in their practice in Youth and Adults Education sector as well in their speech with the focus group. However, in this study, we present situations of the lessons of the teacher Iza (not her real name).

We observed four hours of lessons in the class that attended the II module (corresponding to the 7th year of elementary school). There were twelve students in this class, with ages between seventeen and twenty-five years. The students sat in a semicircle and were participative and inquisitive. Many questions emerged during

Iza's lesson and were used as a basis for new questions. Thus, a dynamic classroom where everyone participated was established.

We had four meetings with the focal group with the educators who were research subjects, with the purpose of creating debate about the influence of the Freirean suppositions on mathematics teaching. The meetings were organized according to a pre-structured schedule that contributed with the organization of the speeches and discussions about the comprehension the they possessed on Freirean studies in Youth and Adult Education. The focal group was composed in a rich process of empirical, material production (Pimenta, Ghedin, & Franco, 2006) because, other than verbal information, it also made it possible to observe the reactions of the individual members of the group before inquiries and cognitive conflicts caused by diverging opinions that came up.

In the focal group meetings, the teacher was invited to freely give their opinions about the Pedagogical Proposal for Youth and Adults (Vitória da Conquista, 2007), focusing on: the Freirean theoretical idea; the objectives; the orientating principles of work and the organization of mathematics classes according to the Freirean suppositions. The discussions happened in an organized manner with regards to the diverging opinions.

To analyse the research material we used the studies and contributions about the "*Educação Libertadora*" *(Liberating Education*) of the Brazilian educator Freire (2000, 2005, 2013), as well as the Skovsmose studies (2012), for expressing concerns about the socio-political-cultural character in the teaching of mathematics. In discussions regarding the specifics of Mathematics Education in the Youth and Adult Education sector, we used the studies of Fonseca (2007).

10.3 Mathematics in Youth and Adult Education Sector and Freirean Thinking

Youth and Adult Education sector in Brazil, according to Resolution CNE/CEB No. 1 of July 5, 2000, which establishes the Curricular Guidelines for this type of education in Brazil (Brasil, 2000), highlights the importance of analysing historical and social debt related to a part of the Brazilian population who had been denied the right to an education. This resolution allowed this population re-entry into the educational system, offering social, economic and educational aspects, and seeking a permanent, diversified and universal education.

Accordingly, teaching in this type of education must be guided by the principles of equity, difference and proportion, in a specific pedagogical model, in order to fulfil the specific distribution of curricular components, the identification and recognition of the otherness of the young and adults in their formative process and proportionality, with adequate allocation of the curricular components to the specific needs of these subjects. Thus, with the guidelines of the National Curriculum Guidelines for Youth and Adult Education sector (Brasil, 2000), it can be assumed that, even considering teaching for this sector as part of the general educational system, its own content and methodologies are needed.

The originality of the thought of the Brazilian educator Paulo Freire, with a reflection situated in his time and historical-cultural reality, made him known internationally. His works are deepened in a liberating educational proposal, concerned with the pedagogical bond developed between educators and learners. In a Freirean dimension, the subjects who attend classes at the Youth and Adult Education sector have specific characteristics that are related to a social and historical exclusion arising from policies that do not prioritize the rights denied to this audience, and understanding them can contribute to the adoption of a consistent pedagogical posture and responsible interaction with these students. These characteristics make up the political character of the Youth and Adult Education sector (Soares, Giovannetti, & Gomes, 2007) and situates this teaching field as committed to the lower classes education overcoming the various forms of exclusion and discrimination still existing in society.

The search for training for the labor market, as well as an education directed to the full exercise of citizenship is the primary motivation taken by young students and adults who to return to school. According to Fonseca (2007), students of the Youth and Adult Education sector return to classrooms because they perceive themselves pressured by the demands of the labour market that values institutional knowledge, but also by the desire to learn, because they did not have that opportunity before, or simply by the need to acquire their rights. It is therefore necessary that the Youth and Adult Education sector considers the need, the desire and the right expressed in the speeches of students as a base for their educational activities in this field.

With regard to the teaching of mathematics in Youth and Adult Education sector, considering the nature and trajectory of the public to be served, our characterization of Mathematics Education, is in line with what is espoused by Fonseca (2007, pp. 11–12), "not as a form of provision of basic and vocational education, but as a pedagogical action which has a specific audience, also defined by their age, but mainly an identity defined by socio-cultural exclusion traits".

Young and adult learners interact and relate continuously through everyday situations. These situations demand explanations, discussions, critical analysis and decisions about the problems inherent to the society in which we live. Even certain common situations, brought by students in the daily life of mathematics classes, allow for particularly fertile moments of meaning-making. That is, "the nature of mathematical knowledge can provide experiences of meaning that can not only be experienced, but also appreciated by the learner" (Fonseca, 2007, p. 25).

Comprehending that the Youth and Adult Education sector is a right of the citizen, a possibility of personal accomplishment, and a necessity of the society, prompts the educators in the field of ethics and citizenship to fight for a consistent and concerned training. Beyond this it encourages "them to make an effort to recognize and analyze the characteristics and demands which belongs to the public they attend, and align this attendance with the constant negotiation with these characteristics and demands" (Fonseca, 2007, p. 64). Such actions comprehend the political consciousness of the educator, as Freire (2005, p. 28) alerts us, "the ethical, political and professional responsibility of the educator obligates to ask himself the question, to capacitate himself, to qualify himself even before he begins his educational activities".

In this sense, understanding mathematics as a political as well as a cultural expression specifically in mathematics education for youth and adults, and the possibility of a school in a socio-political and cultural perspective, the pedagogical thought of Paulo Freire envisions an education committed to overcoming oppression and exclusion—conditions that acknowledge the cultural environment of students and their role as questioners of reality in order to attain knowledge and thus real changes.

In his work, *Pedagogia do Oprimido (Pedagogy of the Oppressed)*, published in Brazil in 1974, Freire (2005, p. 39) elaborates his experience with "liberating education" and proposes to overcome the horizontal educator-educating condition in the pedagogical relationship by inferring that " no one educates anyone, no one educates himself: men are educated in communion, mediated by the world". This problematizing education contrasts with the "*banking concept of education*" and presents strategic premises: dialogue, meaning and world comprehension, as a way to overcome the impasses of an oppressive education from the teaching ethics which can counter the concept of humanization to the dominant notion of social exclusion.

When the educator sees himself as the formative subject of the educational process and his students as "objects" that must be formed and, who passively receive knowledge belonging to the subject that knows and are transferred to them, there is an emptying of the dialectical relation of learning of the person. In this sense, Freire (2005) warns that education becomes an act of depositing knowledge, in which the learners are the depositaries and the educators the depositors. This characterizes the banking concept of education, where pedagogical practice is reduced to the act of depositing, transferring and transmitting values and knowledge:

> Instead of communicating, the educator makes "communications" and deposits that the learners, mere incidents, patiently receive, memorize and repeat. This is the "banking" concept of education, in which the only leeway available to students is to receive the deposits, to store them, and to file them away. (Freire, 2005, p. 66)

In the banking concept of education, knowledge becomes a gift of those who think they are wise to those who do not know. In opposition, Freire's liberating education, which privileges the exercise of critical understanding of reality and enables not only the reading of the word (the reading of the text), but also the reading of the context (the world comprehension), in which the educator committed to change can not only tell learners about their world view (trying to impose it), but must engage in dialogue. In *Educação Libertadora (Liberating Education*) (Freire, 2005) there is a search for the critical understanding of reality, in a reading of the world that enables the liberation of the subjects from their oppressive condition.

Thus, in a more directed bias towards the formation of educators, in the work *Pedagogia da Autonomia (Pedagogy of Freedom: ethics, democracy, and civic courage)*, the last work published in his life, and 22 years after the *Pedagogia do Oprimido (Pedagogy of the Oppressed)*, Freire (2013) brings to the reflection on the essential knowledge to the educational practice of critical educators, progressive educators, knowledge derived from educational practice.

Freire (2013, p. 25) believes that “there is no teaching without instruction”, since it is in this relationship that the teaching and learning process occurs. The critical-transformative educator knows that teaching requires: methodical rigor; research; respect for the learner’s knowledge; criticality; aesthetics and ethics, as well as the embodiment of the word by example; risk; acceptance of the new and rejection of any form of discrimination; critical reflection on the practice, recognition and assumption of cultural identity.

Teacher training in the Freirean perspective considers that “teaching is not transferring knowledge, but creating possibilities for its own production or its construction” (Freire, 2013, p. 47), which implies that teaching requires: unfinished consciousness; the recognition of being conditioned; respect for the being of the student; common sense; humility, tolerance and struggle to defend the rights of educators; apprehension of reality; joy and hope; the conviction that change is possible; and curiosity.

In the awareness that “teaching is a human specificity” (Freire, 2013, p. 89) and, because it is human, teaching demands: security, professional competence and generosity; commitment; understanding that education is a form of intervention in the world; freedom and authority; conscious decision-making; knowing how to listen; recognition that education is ideological; willingness to dialogue and to love learners.

In his works, Paulo Freire has always been concerned with the pedagogical relationship developed in the educational system. Such concern has its core in the relationship established between educator and educating. For Freire (2005), the act of teaching does not exist without learning because it was from the human condition that everyone is able to learn that over time men and women have been developing ways and methods of educating. In this way, it is the act of learning that justifies the relationship established between educator and educating.

The organization of the mathematics classes in the Education of Youths and Adults sector in the city of Vitoria da Conquista—Bahia—Brazil complies with the Pedagogical Proposal for Youth and Adults (Vitória da Conquista, 2007), whose theoretical basis is the studies of Paulo Freire. An analysis of Freirean thinking makes it possible to identify epistemological categories that can constitute educational premises guiding pedagogical actions (Gadotti, 1996), such as: dialogicity, meaning, problematization and world comprehension.

According to Freire (2000) dialogicity is an expression of the horizontal relationship between teachers and students—a respectful meeting, supportive and committed among those who believe that the world can be transformed when pronounced in the dimensions of action and reflection. In a Freirean pedagogical concept, the educator is a questioner and this role characterizes his pedagogy as posing a question to dialogue with intentions in the pursuit of reflection that leads to action.

Dialogicity is one of the primary premise of Freirean thought. It is a dialogue born in the practice of freedom, committed to life and that improves in its context as a constitutive and indispensable element in the educational perspective of Paulo

Freire. Therefore, it is not a dialogue reduced to simple conversation, but, according to Freire (2005), it is necessary to understand it as a human phenomenon that is constituted of action and reflection. In dialogicity, the dimensions of action and reflection are always present. In pronouncing the world in the dimensions of action and reflection the pronounced reality becomes problematized, demanding from the people new actions and reflections. In this way, humanly we exist and modify the world, reflecting on our limitations and projecting action to transform the reality that conditions us (Freire, 2005).

Meanings are a search engine for the programmatic content of education, in an appreciation of the social knowledge and strategies for overcoming the mediated reality, giving it new meanings (Freire, 2005). The issues that will be worked out by the mathematical content are significant, and they are not in isolated individuals, nor in the separate reality of the people, but in the world-human relations. From the context of the students it is a requirement to action, moving in search of reflection, asking—systematically and methodically for something they want to discover, building and re-doing during action and reflection.

The problematization may be handled as a mechanism of problematization of themes and reframing of reality in order to overcome the memorization and mechanical repetition of knowledge in a collective reflection/action (Freire, 2005). The questioning occurs when the educational process settles the dialogue about the questions. They are fundamentally essential in the knowledge construction process. The educator, in his pedagogical action, in view of problem-based education, questions his students, hears their arguments, reflects these in new questions. The room becomes a dialogue in search of knowledge. A search that is constant and effective for dealing with the questions about the problems.

World comprehension is the ability to question the reality and increase their knowledge in order to act consciously on the local reality of the subjects (Freire, 2013). In the pedagogical situation it corresponds to the expression of the students' utterances about the reality to be problematized. These are comments, complaints, opinions and doubts that involve the experiential reality and that can be problematized. From world comprehension the subjects, open to the different objects of knowledge present in the reality that surround them, transcend their already elaborated perceptions and reach new levels of perception of reality, thus increasing knowledge.

10.4 The Pedagogical Proposal in Youth and Adult Education Sector in the Freirean Assumptions: The Case of Vitoria da Conquista—Bahia—Brazil

The theoretical basis of the Pedagogical Proposal of the Youth and Adult Education sector in Vitoria da Conquista—Bahia—Brazil, in force since 2007, is supported by a Freirean conception of education, according to which the action-reflection in the educational process perceives the student as a subject that appropriates the capacity to find the contradictions of reality and to put itself in a critical posture in order to enter and seek solutions for community problems.

The Freirean idea of education is essentially problem-based, following from a liberal educational practice, with the rupture of the relationship of control of the educator over the student. Such educational practice breaks with the "banking" concept of education (Freire, 2005) and leads to a wide scope, integrated learning, organized on the horizontal aspect of the educating relationship, in the value of culture and orality, with strong ideological content. This political-educational proposal "establishes an articulation between a social cultural reality, the school, the teacher and the student, strengthening the role of the school, the participation of the community, the commitment of the teacher and the capacity of the student, which are subject of the process of the development and appropriation of knowledge process as a whole" (Vitória da Conquista, 2007, p. 12).

The vision of a liberating education is also expressed in one of the objectives of the proposal, which pursues "contributing to prioritizing identity, creation of citizenship, integral formation, critical consciousness and notions of liberty and democracy" (Vitória da Conquista, 2007, p. 18), and which is in agreement with the *Diretrizes Curriculares Nacionais (National Curriculum Guidelines)* for the Youth and Adult Education sector (Brasil, 2000).

This way, the pedagogical proposal orients that the educator studies constantly, in such a way that he understands the context and the challenges in which education is found, while creating a constant dialogue with theoretical references that contribute to the expansion of his professional evolution and of his studies as regards reality.

The document highlights, furthermore, the interdisciplinary work between the various areas of knowledge and, particularly in the mathematics teaching, clarifies that its role "is to stimulate curiosity, the inquisitive spirit, the capacity to generalize, predict and abstract, contributing in an effective manner to the growth of citizenship" (Vitória da Conquista, 2007, p. 28).

In this way, the proposal with the objective of an education that contributes to the development of identity, building citizenship, comprehensive training, critical consciousness, notions of freedom and democracy provides for a problem-based education and dialogue (Vitória da Conquista, 2007). Establishing dialogic moments in the mathematics classroom requires an attitude of the educator to allow the students to speak and, for that which the students do not have ready answers, he problem-solves with them together, is curious to know the perspectives of the students and continually reconsiders their knowledge and their practice (Alrø & Skovsmose, 2010).

With this, the youth and adult students have much knowledge to share; their life experiences, in work and in and in family relationships, constitute world readings that can be contemplated by the educational process. Reflection about the place of mathematics in Youth and Adult Education sector considers that personal, social or professional life situations demand recognition with regards to quantitative analysis and logical or interpretive matters of a mathematical instrument and, because of this, can be used as a form of organizing pedagogical work in mathematics classes (Fonseca, 2007).

In the proposal, the dialogue is a mediating element between educators and students and the everyday situations in which world reading is a possibility, in such a way that, "in search for knowledge, there is no absolute no one absolutely ignorant, but people that, collectively, seek to know more. One should believe in the other, believe in his own potential, and it's this belief that allows the dialogue" (Vitória da Conquista, 2007, p. 20). Furthermore, "an educational proposal based on the cultural reality of the subject is essentially problem-based, mediated by dialogue between their authors. The essence of the word is the activity spurred by the reflection" (Vitória da Conquista, 2007, p. 20).

In search of dialogicity in the educator-educating pedagogical relationship, Freire and Shor (2006) understand this as an indispensable action in the liberating educational process, in which the dialogue is a necessary posture for comprehending reality which is created and recreated by human action and, because of this, it is necessary to perform critical reflection for the transformation of this same reality.

In the analysis of the pedagogical proposal we found the Freirean assumptions: dialogicity, problematization, meaning and world comprehension in its theoretical foundation, as well as methodology. Regarding mathematics, the proposal professes that his "problem-solving teaching offers an opportunity for the educator to work on ways of reasoning, problematizing, inquiring and questioning" (Vitória da Conquista, 2007, p. 29) having as the objective to stimulate curiosity, the investigative spirit, the capacity to generalize, predict and abstract, contributing effectively to the development of the citizen.

The proposal directs that the reading of the world of the learner should serve as the beginning of the problematization in the classroom, hence to be respected and developed in search of awareness for the perfection of mathematics, so that the student "must be the subject of action, focusing the vision of reality in search of liberation from the oppressive situation, that is, the pupil must be aware of the real situation in which he is inserted, acting on it, transforming it" (Vitória da Conquista, 2007, p. 20).

The proposal clarifies that class organization should consider the problem based mathematical content, having in view the conscious action about reality. In such a way that "the orientating thematic planning should be created in an intellectual framework (formal content) and in an action framework (critical, political vision), revealing the concrete situation of men towards which educational work is directed" (Vitória da Conquista, 2007, p. 20). In the same way, Freire (2013, p. 32) questions the education that does not value local knowledge of the students' reality and he questions:

> Why not discuss with the students the concrete reality to which the discipline whose content is taught should be associated, the aggressive reality in which violence is the constant and the coexistence of people is much greater with death than with life? Why not establish a "intimacy" among the students' fundamental curricular knowledge and the social experience they have as individuals?

Therefore, Freire (2013) does not deny the need for curricular content, instead, questions the decontextualized work of the reality which is intended to be transformed by this action. It is in the problematization of the mathematical contents in the middle of the social context in which the students are immersed, that the effective action-reflection can be realized.

As for the approach to teaching mathematics, the analysis revealed the option for dialogue as a problematizing element of reality, reflection on mathematical structures and reality at work with the resolution of problems focused on significant themes. These include: The world comprehension when proposing the work directed to the mental construction in the diverse situations of the quotidian; The problematization by favouring the structuring of thought and logical reasoning with a view to a greater criticality in the various fields of activity, thus evidencing the articulation between the Freirean premises and the studies of critical mathematical education and the specificities of Youth and Adult Education sector defended by Fonseca (2007).

10.5 Pedagogical Practice in Mathematics in the Education of Youth and Adults Sector in a Freirean Perspective: Iza's Lessons

In order to understand the data and get a glimpse of the Freirean premises in the pedagogical practice of teachers, we organize the analysis by taking the dialogicity, the problematization, the meaning and the world comprehension as they are explained and understood in the studies of Freire (2000, 2005, 2013). In the dynamics of the organization of a class, such premises were interwoven, in a dialectical relation of interdependence.

In this study, we present the mathematics classes held by Iza with students about planning an interdisciplinary project: "Water, our greatest asset!", with the significant theme: "Water Economy". Mathematics teachers, in a pedagogical meeting at the school, identified mathematical contents about information processing, percentages, decimal numbers and equations (Researcher's field diary, 09/10/13).

The choice of teacher Iza was motivated because her degree is in mathematics and she works in the municipal school system in the Youth and Adult Education sector. Moreover, she was willing to participate and cooperate in all phases of the research.

Initially the teacher asked students to take with them their water bills. She started the class talking about the problem of water supply in the city, the reason for the lack of water was due to scarce rainfall in the region and also the waste. So she asked questions to encourage the participation of students:

Iza: How is the water supply in your house?
Students: Water is lacking!
Iza: Lack of water in all city neighbourhoods?
Student 1: In the neighbourhood of the rich it lacks least. They have large boxes to store it.
Iza: Is that fair?
Student 2: It is not, teacher, but they do not matter! At home it's missing for four days.
Iza: What can we do then?
Student 3: Yeah, we can economize and complain to Water Management Society that water is not enough!
Iza: Let's look at a graphical chart on the amount of water available for use.
Iza: There is plenty of water on Earth's surface? [Points to the chart]
Iza: Why are we rationing?
Student 2: We waste a lot of water, soon we won't have any.
Iza: What is the percentage of water on Earth's surface?
Students: Seventy-five percent!
Iza: Is that a lot?
Student 5: I think it's a lot.
Iza: But we're saving. Look at the information in the graphs, you need to read and interpret. They cannot portray the reality we live in. It is a general thing. Although much water on the surface we are experiencing problems here. You must learn to read the information that is here on the chart.

Source: Information collected during observation of group of the Module II (2013)

School:
Student:

Water: our largest well

What does graph 1 show? And the graph 2?

Observing the graph 1, is it possible to affirm that there is much more water than dry land in our planet?

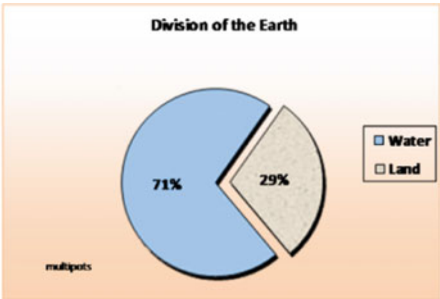
Just with the information of the graph 1 it is possible to affirm that will never have problems with the water in our planet?

From the title is it possible to understand graph 1?

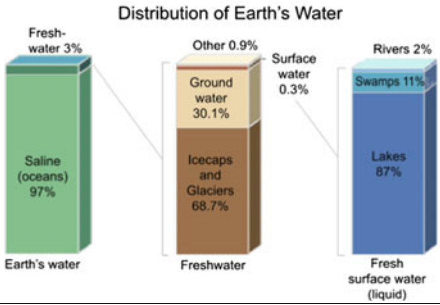
So that it serves the legend below the graph 2?

Graph 1is separated in two colors, why?

Graph 1



Graph 2



In the organization of Iza's classes the dialogue begins with the invitation to question the significant issue, made in the form of formulated questions based on student answers. She mediated the dialogic situation in order to get the students to focus questioning on the mathematical object, without losing sight of the main theme.

Iza's posture in insisting on the response of the students on how they understand their reality, seeking to provoke their curiosity, whereupon she organizes the object of knowledge, seems to be a feature of an democratic educator (Freire, 2013), who is curious, includes himself in the search, who stimulates inquiry and critical reflection on the question itself, rather than giving ready answers to the questions of the students.

In this sense, we understand that Iza is aware of the organization of her pedagogical practice, proposing work with significant themes and she values moments of dialogue with the students, in the search for the problematizations necessary to unveil reality with the support of mathematical contents, when she says that:

> We planned the mathematics class within a project of a significant theme, we cannot bring everything ready to the room, put the content on the board and go immediately explaining, giving the answers and wanting to hear from the students what we think is correct. (Interview with Iza in the focus group, 11/27/2013)

Thus, the approach that Iza takes to teaching mathematics for young and adults, in keeping with the educational proposal of Vitoria da Conquista for this type of education, matches the pedagogical proposal of Vitoria da Conquista for this type of education, according to which, in mathematics teaching "work with problems involves discussion, analysis, leads to the formulation of conjectures, favours, above all, reflection and questioning" (Vitória da Conquista, 2007, p. 33).

Organizing mathematics classes in order to provide a space for questioning and discussions in a systematized way, in search of the students' knowledge, as we perceive in this episode of classes, corresponds to the search, through dialogue, to explore students' perspectives on the Water Economy and try to help them express their knowledge around the significant theme.

By insistently questioning the students in the search for problematization of the meaningful subject, Iza also performs in the dialogue a search for their perspectives on the subject in question and challenges them to position themselves critically on the problems posed. We understand that the questions asked by her in search of the object of the investigation, that is, of the problematization on the meaning, were possible through the dialogue that we envisage. The dialogue about the meaning allowed the students to problematize their existential experience with the help of mathematical objects and to envision a world comprehension focused on concrete action on reality, in this case, the Water Economy and the actions carried out with the water bill.

On the work with the significant themes, Iza clarifies that the situations experienced in the students' daily life provide rich materials for the mathematics classes, and it falls to the teacher to use their creativity to select those more suitable for use in the classroom.

> We work with supermarket price pamphlets, water and electric bills, parking price charts, bank lending pamphlets, magazine and newspaper reports, and more. These materials are rich for working with various mathematical content, and it is the teacher who needs to be

> creative and sensitive to the needs of the students by engaging in meaningful learning activities so that their young and adult learners can use mathematical knowledge to solve their problems in the contexts in which they happen. (Interview with Iza in the focus group, 11/27/2013)

By using situations present in the reality experienced by the students regarding the need for saving water, Iza interprets the work with the meaning as guided by the Pedagogical Proposal for the Education of Youth and Adults (Vitória da Conquista, 2007), which provides for the work with mathematics based on solving problems and establishing connections with life. So, when proposing the critical analysis of charts and tables related to the problem of water shortage, the teacher made use of an excellent pedagogical material (D'Ambrosio, 2007), in addition to complying with the guiding principles for Young People and Adults Education sector that advocates an educational process focused on collective problems, such as saving water.

As presented in her class, Iza has the autonomy to organize the mathematics class under a significant theme, as well as to select the materials that she believes are the most appropriate to foment the problematizations of the significant theme. In order to do so implies that the teacher's willingness to be sensitive to the students' wishes and commitment in the planning of the classes is necessary in order to provide moments of problematization in which students feel motivated to mobilize mathematical knowledge to solve problem situations in their social context.

In the classes observed, Iza began problematizing with questions about where students stood on the problems of water supply and how they should claim their rights regarding this. With the help of graphics, Iza questions are geared towards a questioning as to reading and interpreting graphs and tables and the large amount of water available on the planet, through the graphical data, warning that, however, the city it is in crisis, so that the mathematical models do not attempt to represent reality in all its particularities (Skovsmose, 2012).

Using specific mathematical content, Iza continues to question the students and they answer, analysing the water bills corresponding to the consumption of their families, thereby calculating the conversion of volume measurements (m^3) to capacity measures (*l*) and verifying the corresponding consumption in liters. Similarly, they are tasked to calculate how many liters are required for daily tasks, based on a table.

Iza: How much water are you using? [at home]
Student 7: Cubic meter, what is that? [The student said looking at his water bill]
Iza: It's called a measure of capacity. It is the amount of water in a box of 1 m long and 1 m high and 1 m wide. Hence it is called cubic meter. Take note how many cubic meters each of you spent last month.
Students: We spend too much. It's a lot of water, but it is good to know the liters!
Student 3: At home we spend a lot! Should be in the bath!

Next, Iza presents a table and asks for the attention of the students:

Based on the data in the table above, the teacher writes a problem situation on the board and asks students to solve:

Angela watered garden plants with an open hose for 10 min. Knowing that the garden has a 10 m^2 area, with the amount of water she spent, how many times could she have watered the garden using a watering can?

Iza: First we have to know how much water she used! Based on the information in this chart, how many liters has Angela used?
Students: 186 L because the hose was left open for 10 min.
Iza: That is enough to solve the problem?
Student 4: We need to know how many full watering cans would be used to water a garden with that size.
Iza: Yes! See the table one full watering can wets 2 m^2 spending 20 L of water.
Student 2: So to wet the ground we would have to used 40 L?
Student 1: No! If a can is watering 2 m^2 and the ground is 10 m^2, let's divide. I know this because we divide the amount of vaccine that we have to give to the cows by their weight!
Student 6: Oh, so it would give 5 sprinklers! It will have used 100 L of water.
Iza: Perfect! And did we already solve the problem?
Student 4: We need to know how many times she can water the garden with 100 L of water. Hence it does not come twice, but almost twice!
Iza: Very good! Angela would save almost half the water she used if she used a watering can rather than a hose! Can we now solve more situations like this and also check the consumption of water in your house?

After verifying how much water they consume in their houses in Table 10.1 and in the water bill, students were able to reflect on the amount of water they could use and the amount they actually used. Also, they realized that the data organized in graphs and tables need a critical interpretation, not faithfully portraying reality in a homogeneous way. In addition, students were able to express the conclusions and actions they can take to effectively change their behaviour in the face of excessive water use.

Iza: So, data from a table and graphs, portrays a general reality. We have only a sense of the numbers. It is not always possible to make sure that everything is correct. For mathematics, it was necessary to find a standard measure that is not always our standard.
Iza: We use the information as a standard. They do not accurately portray the reality because there are many individual differences. Mathematics does not consider everything, but we have an idea of the amount of water you can save with small actions that give results.
Student 6: It is true, teacher! I'll talk at home to everyone to save water.

Table 10.1 Water: "Our greatest good!"

1. Brush teeth	
With tap on 5 min With 4 cups of water	12 L 1 L
2. Wash face	
With tap on for 1 min	2.5 L
3. Shave	
With tap on for 5 min	12 L
4. Shower	
Shower on 15 min Shower on 5 min	135 L 45 L
5. Flush toilet	
With discharging valve 12 s With small water tank With bucket	20 L 10 L 6 L
6. Wash dishes	
With tap on 15 min With half sink full two times and closed tap	117 L 6 L
7. Water the garden	
With hose on 10 min Using sprinkler in 2 m^2 of garden.	186 L 20 L
8. Washing the car	
With hose on 30 min With 2 buckets of 10 L	216 L 20 L
9. Washing sidewalk	
With strong jet 15 min	279 L
10. Washing clothes	
In washer with 5 kg of clothes	135 L

Source Reproduction of activity proposed by Iza in the class of Module II (2013)

With world comprehension, the theme after the problematization with the assistance of mathematical content was revealed and the students could understand the obstacles that did not allow them to perceive a possibility of change in reality. Now, more critical and with the help of mathematical content, they have become more aware about the water consumption and the need to read and interpret data arranged in graphs and tables.

We understand that the problematization of the meaning on the Water Economy allowed the students, starting from an initial naive world comprehension, to proceed to a more elaborate, critical and, not only systematized at the level of experience of students' world comprehension, but in the problematization through the mathematical object.

Still in an early reading of the world, the students expressed their views on the theme using real life experiences, injustices as to the problem of supply being greater in the poor neighbourhoods of the periphery and the way they interpreted data in charts and tables. At this level, the world comprehension is still naive because students do not glimpse the difficulties within the problematic and are accommodated to the reality as it is.

In a more elaborate world-in-process reading, the significant topic after being problematized with the aid of mathematical objects was unveiled and the students were able to perceive the obstacles that did not allow them to envisage a possibility of change in reality. At this level, more critical and in possession of the learning of mathematical objects, students became aware of water consumption and the need to read and critically interpret data arranged in graphs and tables.

We therefore consider, in the presentation of Iza's classes, that Freirean premises influenced the organization of her pedagogical practice by making possible, through the dialogicity present in the discussions, the significant theme on the Water Economy being problematized with the reality of the students, through the approach of mathematical objects, allowing the critical comprehension of the world by the awareness about the conscious consumption of water and the need to read and critically interpret data arranged in graphs and tables. Also, to solve the proposed problem situation knowledge about measurements (time, volume, area, …), numbers, operations and reading charts was needed.

10.6 Discussion

In this research we work with the Pedagogical Proposal for Youth and Adults in the city of Vitoria da Conquista—Bahia—Brazil (Vitória da Conquista, 2007), which recognizes the adolescents, young adults and adults in the concreteness of their lives, so that in the problematization of their existential reality, in a critical-transforming process, they expand their knowledge and overcome the naive forms of everyday knowledge.

In the analysis of the pedagogical proposal and the focus group meetings it was evident in the discussions of teachers that this proposal is based on Freirean assumptions. In class, this also happens in the field of orality, mobilization of students' previous knowledge, reading and reflection of reality and the tendency to approach at the moment of carrying out the activities. The teachers made clear the difficulty in planning the activities according to a significant theme of the way the pedagogical proposal foresees. The teachers made clear the difficulty in planning the activities, according to a significant theme as provided in the educational proposal since it requires elaboration of critical thinking, knowledge of the reality of the teaching subject.

Our analysis of the pedagogical proposal was based on the pedagogical principles found in the Freirean studies. In this way, we find in a theoretical/practical construction of pedagogical actions, the Freirean presuppositions: the significant themes that were worked as themes present in the students' reality and as the beginning of the organization of mathematics classes; problematization as a form of immersion in social problems in a critical way, although this problematization arose from the needs of the learners, or broader context situations, as the city, the world etc. (Water, Energy, etc.); and world reading, while the students' perceptions about their contexts, work, day-to-day problems, and as the beginning of the questioning.

In mathematics classes with adult learners, dialogicity around the substantial theme allowed inquiring about their experience with the help of mathematical objects (information processing, percentages, decimals, 1st degree equations, volume measurement and capacity) and a world comprehension focused on the action of the reality, in this case, saving water.

In this episode of Iza's classes, we conclude that Freire's premises influenced the pedagogical practice of this teacher in a class in which mathematical content was explained starting from the substantial theme "Water Economy" (mathematical content), caught in the socio-cultural reality of the students (world comprehension) and through dialogue (dialogicity) expressed in classes allowing the process in the construction of knowledge, organization and systematization of pedagogical action considering the highlighted Freirean premises.

Thus, we believe that Freire's premises: dialogicity, meaning, problematization and world comprehension can indicate ways to assist the educator to find balance between his view and understand mathematics in life and educational activities and so better coordinate their teaching.

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Chapter 11
"I've Never Cooked with My Maths Teacher"—Moving Beyond Perceived Dualities in Mathematical Belief Research by Focusing on Adult Education

Sonja Beeli-Zimmermann

Abstract The affective domain of mathematics education, ranging from beliefs and attitudes to emotions, has been a frequent topic of discussion. However, in spite of some decades of research in this field, there are still a number of gaps and open questions, among them the field of adult education. This contribution focuses on one specific finding from a qualitative study describing the mathematical beliefs of eight adult education teachers in Switzerland. Illustrating three dualities identified in the participants' data confirms the complexity of mathematical beliefs and it is argued that a focus on adult education teachers as well as a reflected use of research methods and instruments would be beneficial for the advancement of mathematical belief research, particularly the change of beliefs.

Keywords Mathematical beliefs · Adult education · Qualitative methods

11.1 Introduction

Teachers' beliefs matter—not least of all because they are decisive for what is happening in the classroom and therefore have an impact on students' learning. Indeed, they have become important enough in the past decades that the first ever *International Handbook of Research on Teachers' Beliefs* has been published recently (Fives & Gill, 2014). Within the field of belief research, mathematical belief research is considered to be quite advanced, however, neither the seminal work by Leder, Pehkonen, and Törner (2002) nor a later encompassing publication by Maasz and Schlöglmann (2009) contain any contribution focusing on adult education. Similarly, many shorter thematic overviews contain summaries for

S. Beeli-Zimmermann (✉)
PH Bern, Bern, Switzerland
e-mail: Sonja.Beeli@phbern.ch

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specific groups of teachers, such as primary, middle and secondary school teachers (Forgasz & Leder, 2008) or preservice and in-service teachers (Cross Francis, Rapacki, & Eker, 2014), but they never contain a specific section on adult education. Assessing the field of adult education, Taylor (2003) states that there are only a handful of studies addressing adult education teachers' beliefs in general. Therefore, FitzSimons, Coben, and O'Donoghue (2003) assessment of adult mathematics education being an "under-researched and under-theorised domain" (p. 112) is particularly true for the intersection of adult mathematics education and belief research. However, as it has been shown that teachers' mathematical beliefs influence young students' mathematical achievements (Gales & Yan, 2001; Staub & Stern, 2002), it seems relevant to know whether adult education teachers' beliefs can be compared to those of other teachers. The study, from which the data presented here is taken, therefore aims at contributing to closing this gap by examining the mathematical beliefs of a group of Swiss adult education teachers. The following paragraphs outline the theoretical and practical background, including a brief presentation of relevant research, before focusing one main finding, namely the participants' divided views of mathematics—as it is also embodied by the quote in the title of this contribution. The paper concludes by arguing that focusing on adult mathematics education teachers and using specific research methods and instruments in a reflected and professional manner could contribute to the advancement of belief research.

11.2 (Mathematical) Belief Research—Concepts, Issues, Methods and Results

11.2.1 The Challenge of Conceptualising Beliefs

Beliefs are one of many cognitive concepts that are not always easy to differentiate and the term is often used interchangeably with others such as conceptions (Thompson, 1992) or world views (Grigutsch & Törner, 1998; Bulmer & Rolka, 2005). In fact, there is such a plethora of terms that Mason (2004) managed to find one for every letter of the alphabet. Other authors express the difficulty of conceptualising beliefs by assigning it attributes such as "messy" (Pajares, 1992) or "murky" (Fives & Buehl, 2012). According to Richardson (1996) beliefs develop over a lifetime and are influenced by three kinds of experience, namely personal experience, experiences in schooling and other forms of instruction as well as experience with formal knowledge. Very generally, they relate to contents and processes of learning (epistemological beliefs), teachers and students (personal beliefs) and school and society (contextual beliefs) (Reusser, Pauli, & Elmer, 2011). They function as "filters" (Pajares, 1992) or "lenses" (Philipp, 2007) through which the world is perceived and according to which individuals act. When discussing the concept of beliefs, different authors therefore deliberate their characteristics and areas to which beliefs relate as well as their functions (see Pajares, 1992; Calderhead, 1996 or Fives & Buehl, 2012 for discussions of the concept in general and Op’t Eynde, de Corte, & Verschaffel, 2002, Furinghetti & Pehkonen, 2002 or Pehkonen, 2008 for mathematical beliefs in particular).

With regard to mathematics education, one of the most referred to conceptualisations of beliefs is Ernest’s (1989), who, like others, distinguishes them from knowledge and attitudes. With regard to the characteristics of beliefs, he acknowledges that beliefs may be implicit, however, he assigns all affective components such as liking, enjoyment, interest or “mathephobia” to attitudes (p. 24). But rather than discussing conceptual differences between these constructs, he focuses on describing the respective areas to which beliefs—a term he uses synonymously with conceptions and views—relate, namely: the nature of mathematics, its teaching and learning as well as principles of education. On the basis of philosophy of mathematics, Ernest describes three views, namely the instrumentalist, the Platonist and the problem-solving view and he argues that each view of mathematics corresponds with very specific views of teaching and learning mathematics: the view of mathematics as problem solving corresponds with the teacher acting as facilitator, whereas a Platonist view of mathematics sees the teacher as explainer and the instrumentalist view is associated with a transmission model of teaching. Much of mathematical belief research refers to these perspectives[1] and has been motivated by fundamental reforms in mathematics education with respect to their failure: “despite many educational reforms, a large number of teachers still perceive mathematics in traditional rather than in progressive terms; that is, as a discipline with a priori rules and procedures, ‘out-there,’ that has to be mechanically discovered rather than constructed” (Handal, 2003, p. 54). Furthermore there is evidence that Many authors attribute this perceived lack of progress to missing conceptual clarity (for example Pajares, 1992; Op’t Eynde et al., 2002; Fives & Buehl, 2012 or Skott, 2014).

As it seems unlikely, that a generally agreed upon definition of the concept will be established in the near future—there are also authors who argue that this is not desirable, for example Furinghetti and Pehkonen (2002)—many authors have continued the approach already taken by Abelson (1979, cited by Nespor, 1987), namely to identify a number of features which need to be considered when discussing beliefs, rather than providing a specific definition. Fives and Buehl (2012) have identified the following dimensions: their (i) implicit versus explicit nature, (ii) degree of stability, (iii) situated versus generalised nature; (iv) relation to knowledge, and (v) existence as integrated systems or individual propositions.

[1]It is worth noting that while many studies refer to this threefold perspective, Ernest has neither been the first nor the only one to describe it. He himself refers—among others—to Thompson’s work (Thompson, 1984), another author is Dionne who described the traditional, the formalist and the constructivist perspective (Dionne, 1984, cited by Pehkonen & Törner, 2004) and Törner and his colleagues talk about the toolbox, the system and the process aspect (Törner & Grigutsch, 1994; Pehkonen & Törner, 2004). However, “all these different notions correspond more or less with each other” (Liljedahl, 2008).

In the field of mathematics, Furinghetti and Pehkonen (2002) also suggest to consider degrees of stability, however their other criteria include objective and subjective knowledge (with beliefs belonging to the latter), affective factors (to be distinguished from cognitive ones if necessary) and to take note of the context. Goldin, Rösken, and Törner (2009) argue for ontological, enumerative, normative and affective aspects to be included in the discussion of beliefs. It is noteworthy, that while the issue of affective aspects named by both author groups belonging to the field of mathematics education, it is absent from Fives and Buehl's dimensions. Indeed, they assert that "scant attention is paid to the emotional nature of beliefs" (Fives & Buehl, 2012, p. 490). This might be true for general discussions of the concept, however, in the field of mathematics education affective aspects of beliefs seem to be more prominently and regularly represented—even though the discussion is often limited to negative emotions, particularly mathematics anxiety (Evans, 2000). So, while it is not possible to present a generally accepted definition of what beliefs are, a minimal requirement for any study would be to describe the concept or assumptions upon which one bases her/his work. This seems to not always have been the case and has been criticised by many (for example Op't Eynde et al., 2002 or Forgasz & Leder, 2008). However, conceptual clarity is fundamental, not least of all to facilitate the interpretation of obtained results and relating them to other work and/or theoretical considerations.

11.2.2 The Relationship Beliefs-Practice and Belief Change

How closely conceptual issues are related to specific research results is not least of all illustrated by the results which have been achieved with regard to the two main issues of belief research, namely the relationship between beliefs and practice on one side and belief change on the other. The former is also known as the congruity thesis (Skott, 2014) and is based on the assumption that specific beliefs correspond to specific behaviour and if that is not the case, an explanation is needed. Studies discussing this relationship also refer to belief enactment (Woolfolk Hoy, Davis, & Pape, 2006; Beswick, 2004). The most accurate summary of the numerous studies carried out on the topic of congruence of beliefs and practices has been given by Fives and Buehl who assessed that "it seems that for every study that offers evidence to support the relation […] an equal number suggest that beliefs are not consistent with practices" (Fives & Buehl, 2012, p. 481). As the relationship between beliefs and practice is not the focus of this study, this issue is not further elaborated—for an overview of research addressing this question in the field of mathematical beliefs see Thompson (1992), Philipp (2007) or more recently Cross Francis et al. (2014). However, what is worth noting in this context is that there are also a number of authors who argue that inconsistencies can be explained with conceptual and/or methodical weaknesses: Skott (2009), for example, argues for beliefs as a social rather than an individual construct; Speer (2005) identifies a lack of shared understanding among researchers and teachers as a cause of inconsistencies; similarly Leatham (2006) postulates that what seems contradictory to the researcher might make sense to the individual teacher. Consequently, they demand different conceptualisations for beliefs, namely to see them as a social rather than an individualised construct or to consider the organisation of beliefs the way it makes sense to the individual rather than the researcher.

With regard to the second large issue of belief research, namely that of belief change, empirical evidence is equally inconclusive. Research carried out in this field focuses particularly on pre-service teachers, who are at the same time novices in their profession, yet have extensive knowledge of and experience with the school system. They can be considered "insiders […] who view their new experiences through their 'old eyes'" (Cross Francis et al., 2014, p. 340). Factors which have been identified to facilitate change include field experience or practice which is supported by opportunities for reflection (Richardson, 1996 or Swars, Smith, Smith, & Hart, 2009). For a systematic discussion of stability of beliefs in mathematics education see Liljedahl, Oesterle, and Bernèche (2012).[2] From a qualitative methodical perspective the lack of absence of long term studies is noteworthy: While it is agreed upon that beliefs form over a long time, most studies cover only a few weeks or months—the "longitudinal study" covering three semesters by Swars and her colleagues is a notable exception (Swars et al., 2009), as does a similarly long one by Brosnan, Edwards, and Erickson (1996) or different case studies by Skott (2001, 2009, 2013). Similarly, there are only few studies addressing teachers' biographies to try and trace the development of beliefs (for example Millsaps, 2000 or Kaasila, 2007). Overall, while published research on mathematical beliefs is "massive" (Liljedahl et al., 2012, p. 102), it has not always provided the answers that were hoped for some decades ago. This is not only due to conceptual issues, as has been mentioned, but is also influenced by some specific methodical issues as will be seen in the next paragraphs.

11.2.3 From Quantitative Surveys to Rich Case Studies

Methods employed in belief research need to defy the challenge of operationalising a multidimensional construct which—as has just been shown—is difficult to grasp. Particularly if beliefs are considered to be partially implicit or unconscious, specific methods used need to be able to detect and analyse these aspects. It is therefore not surprising that the methods employed in belief research reflect the changes in belief conceptualisation and encompass the entire spectrum of quantitative and qualitative methods (for general overviews see for example Calderhead, 1996 or more recently Schraw & Olafson, 2014; for mathematics education see

[2]This meta study is also an excellent illustration of how conceptual clarity is not only relevant for beliefs themselves, but how other key terms such as stability respectively change are interpreted differently by different researchers sometimes resulting in contradictory findings.

Leder & Forgasz, 2002; Pehkonen & Törner, 2004 or Speer, 2005). Initial studies often followed the quantitative tradition of standardised belief inventories, questionnaires or large surveys—reflecting the cognitive approach dominant at the beginning of belief research and resulting in descriptive classification and categorisation schemes. The field later on recognised the limitations of this approach and moved on to the use of interviews, observation, videos[3] and other diverse materials such as teachers' journals, lesson plans or images which reflect a broader understanding of the concept. These usually smaller qualitative studies tend to focus on the teacher's perspective and processes such as the change of beliefs or understanding a teacher's practice in a given situation (Forgasz & Leder, 2008; Goldin et al., 2009). And while it is true that questionnaires and interviews are frequently used methods, the impression that the overall the range of employed methods is "fairly limited" (Forgasz & Leder, 2008, p. 187) does not seem true, particularly not for those studies using several sources of data.

There are indeed a number of authors who use a combination of instruments and/or methods, such as Pehkonen and Törner (2004) or all those listed in what follows. Frequently found combinations are the use of interviews and video (Aguirre & Speer, 1999; Speer, 2005), but more often multiple sources of data, many of which include questionnaires/surveys and interviews are used: survey, observation and interviews (Barkatsas & Malone, 2005); questionnaires, interviews, self-estimations (Pehkonen & Törner, 2004); questionnaire, observation, different types of interviews (Skott, 2009), observations, interviews, lesson plans and field notes (Cross, 2009) or questionnaires, pictures, texts and interviews (Rolka & Halverscheid, 2011)—to name a few only![4] Unfortunately, the decision of and arguments for using different methods and/or instruments are rarely reflected or communicated explicitly, rather one has to deduct from the context what the purpose was for using different methods or multiple data sources. And while general references to concepts such as mixed methods or triangulation[5] are found in some instances, methodical implications thereof are seldom discussed. Such a limitedly reflected use of methods, however, is a potential pitfall, particularly when it comes to contradictory results—an issue that is not irrelevant for belief research, as seen above. The potential problem relates to the fundamental question of how to deal with contradictory results originating from different sets of data. Depending on the purpose for the use of mixed methods or triangulation, contradictory results have differing

[3]However, it should also be noted that recent technological developments have greatly facilitated the use of different methods and it has become much easier to include film or other visual images in research designs.

[4]It is not always clear which sources of data were used for a specific publication, as often only a select aspect of a larger study using various sources was reported (for example Skott, 2009; Kaasila, 2007 or Speer, 2008).

[5]An in-depth discussion of these two terms and how they relate to each other is beyond the scope of this contribution, however, mixed methods is understood as combination of qualitative and quantitative methods whereas triangulation refers to a plurality in perspectives which may include different methods (Burzan, 2016).

functions and need to be dealt with accordingly (see Tashkkori & Teddlie, 2008 for an overview of different purposes for using mixed methods and Kelle, 2001 for a discussion of different understandings of triangulation and their implication). So while some authors indicate that they use qualitative methods in order to compensate for weaknesses or limitations of quantitative ones (among them Di Martino & Zan, 2011), others imply that the use of different methods ensures a more complete picture of the phenomenon of beliefs (Pajares, 1992, for example, talks about "additional insights", p. 327).[6] In short: belief research is not only challenged by the fact that its core concept is difficult to grasp, there are also methodical issues which are directly linked to conceptual ones and impact the results obtained so far.[7]

11.2.4 Relevant Research Results

The overarching research question of the reported study was: What mathematical beliefs do adult basic education teachers hold? Therefore, there are two main topics in which relevant results can expect to be found: studies focusing on describing the areas of mathematical beliefs and studies focusing on adult education. As has been said before, there are hardly any studies which address both issues, with the exception of the following which are briefly presented:

- Henningsen and Wedege's (2003) study focuses on values and mathematics amongst 212 Danish teachers in adult mathematics education. The participants were asked to describe mathematics in their own words, provide biographical information and associate 18 value items such as abstraction, puzzle or creativity with mathematics. The authors found evidence "that teachers with no specialised education in mathematics and teachers with students on the lowest level tend to see mathematics as closed and undemocratic" (p. 116).
- Swan (2006) developed an instrument for the assessment of an intervention for 64 teachers of further education colleges in England on the basis of

[6]An interesting example in this context is the study reported by Pehkonen and Törner (2004) who in their discussion conclude: "The original underlying hypothesis of our research question […] namely that our methodical approach is to be understood as a triangulation, had to be revised in part. The collected data is only partly redundant, although it merges into a complete picture that could not have been drawn in such detail through any of the three approaches alone. In other words, the results of the various methods complement each other" (no page). It implies that their initial motivation for the use of triangulation was to increase the data's validity and use the various sources to confirm each other, however, it seems they then changed to seeing the purpose of triangulation as complementary in the sense that it helped them to get a more complete picture.

[7]While the previous discussion implies that methodical issues relate mainly to studies using multiple sources of data, this is not the case. Particularly studies referring to qualitative research methods often merely make superficial references, for example claiming to use a grounded theory approach when working with deductive or open coding, even though other basic premises of grounded theory such as a circular research design are at best partially implemented (see for example Bulmer & Rolka, 2005 or Di Martino & Zan, 2011).

transmission, discovery and connectionist views of mathematics teaching and learning. While he found that the questionnaires were also useful for promoting discussion and reflection, he concluded that teachers “appeared to make a clear distinction between the transmission orientation and the remaining two constructivist orientations” (p. 61).

- Schlöglmann (2007) compared beliefs held by adult students and their teachers on the basis of a questionnaire which was completed by 400 students and 40 teachers in Austria. In many respects, their views are quite similar, however what is striking is the contrast with regard to the second view (“Mathematics is a rather non-transparent field where one must simply believe many things”), which 86.8% of the teachers reject, but only 36.5% of the students. Similarly, 43% of the students agree with the statement that “Mathematics is a collection of calculation rules”, but only 18.4% of the teachers do.
- Stone (2009) presents three teachers working in very different contexts, namely in the British armed forces, in a health and social care programme and with English as a second language students. All three individuals teach basic mathematics. Stone traces the individual’s experiences and concludes that specific environments not only shape an individual’s mathematical beliefs, but are equally influential in future employment choices.

Apart from these few studies, there are numerous aspects from other studies that are potentially relevant when discussing the following results, for example that perceived needs of students plays an important role in shaping teachers’ actions (Sztajn, 2003 has shown that for two elementary teachers; Dirkx & Spurgin, 1992 for adult basic education teachers). Therefore, select results of other studies will be referred to in the presentation of results, whenever relevant.

11.3 Contextualising the Study—The Instruments Used, the Situation in Switzerland and the Study Participants

11.3.1 Situating and Implementing the Study

As there is little known about mathematical beliefs of adult education teachers, this study uses an exploratory approach and focuses on describing the mathematical beliefs of eight adult education teachers. It relies on visual and verbal data generated by the participants independently and in interview situations and aims at reconstructing their perspectives. It therefore follows the qualitative imperative of trying to understand the individual and uses data triangulation with the aim of creating a more complete picture of the phenomenon of mathematical beliefs held by adult education teachers. This qualitative approach was also imperative in the choice of specific instruments: In addition to conducting two semi standardised interviews, the study participants were asked in advance to create an image of what mathematics means to them.[8] The first of the two interviews was determined by the discussion of the image that the participants completed and sent to the author beforehand. Apart from asking a few standardised questions such as how they went about when creating the picture or whether there was something that they would have liked to include but could not, they were free to talk about the picture any way they wanted to. Similarly the second interview was dominated by a large narrative part, in which the participants were asked to talk about their educational biographies with a particular focus on mathematics. Again some standardised questions were added afterwards focusing on their teaching practice and use of materials.

The method of using participant generated images has previously been used for mathematical belief research by Rolka and colleagues with students of different age groups (see Rolka & Halverscheid, 2011 for an overview of their different studies). They developed a scheme for the analysis of the created pictures on the basis of Ernest's (1989) threefold perspective of mathematics, which therefore also serves as theoretical framework of this study. However, contrary to Ernest who assigns affective issues to attitudes, this study is based on an understanding of beliefs as containing both cognitive and affective elements. Furthermore, beliefs are considered to be implicit and explicit, to be structured and therefore relate to other mental constructs such as knowledge. Also they are subjective perspectives and they have been formed by the participants' education and work experience. Moreover, the terms beliefs, concepts and views will be used interchangeably throughout the text.

With regard to the analysis of the data, the method of qualitative content analysis (Mayring, 2014) was used. While being less known in the English speaking social science community, this approach is well established and frequently discussed in the respective German speaking community, particularly for the analysis of verbal data (see for example Kohlbacher, 2005 or Gläser & Laudel, 2013). It is based on a systematic use of a set of categories which can be both defined in advance (deductive categories) and developed on the basis of the available data (inductive categories), therefore providing, methodical rigour as well as flexibility towards emerging issues. Preserving the systematic procedure is central to the approach, which is also why it is imperative for each category to be clearly defined and corresponding coding rules to be established. This contributes to the method's objectivity, reliability and validity. Similarly, Rolka and Halverscheid (2011) say that they have performed a "qualitative analysis" (p. 523) of their pictures.[9]

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[8]Ten days before the first interview they received a letter asking them to create a picture as follows: "Imagine you were an artist and have accepted the following contract work: What is mathematics? A personal view. Present your views in a pictorial, creative manner, working with materials and techniques of your choice (coloured pencils, watercolour, collage, etc.)." Together with this task they received an A3-format piece of paper, which they had to use for the creation and presentation of their picture.

[9]They also used text and in some cases interviews as multiple sources with the aim of increasing the "trustworthiness of scientific results" (Rolka & Halverscheid, 2011, p. 527), however the focus of their analyses were pictures and the other material was mainly used in order to facilitate the classification of the picture.

They describe the development of their categories throughout the different studies and present an analytic framework which largely corresponds to what Mayring postulates: They name two characteristics for each of Ernest's dimensions (which can be considered to be the three categories with the two characteristics as their subcategories) and they provide a description for each of the characteristics thereby effectively describing coding rules. It can therefore be argued that while Rolka and Halverscheid do not call their approach a qualitative *content* analysis, their analysis would meet the requirements specified therefore by Mayring (2014).

In addition to the theoretical considerations made with regard to belief research, the following comments need to be made with regard to mathematics—a term used for both the discipline and the school subject, meaning that mathematical beliefs can relate to either.[10] However, the issue is even more complex in the field of adult education, particularly adult basic education which is the context of the study. In this context mathematics is often used synonymously with other terms such as numeracy, quantitative literacy or it is qualified by the terms "basic" or "everyday". While it is beyond the scope of this paper to discuss these terms in detail (this is done for example by Steen, 2001; O'Donoghue, 2002, or Condelli, 2006), it is important to note their usage in this paper which is based on the following reflections: A distinction is made between mathematics as the discipline and everyday mathematics, as the subject which the study participants generally teach and have been trained for. However, the terms everyday mathematics[11] and numeracy are used synonymously. Similarly, maths and mathematics are used synonymously, even though they have slightly different connotations in Swiss German.[12]

11.3.2 *Adult Education in Switzerland*

Contrary to many English-speaking countries, adult education, particularly adult basic education, does not play an important role in Switzerland's educational field overall. The term is generally used synonymously with further education,[13] continuing

[10]Relatively few studies address this differentiation, among them Beswick (2012).

[11]The term everyday mathematics is also a literal translation of the German term "Alltagsmathematik" which is widely used in adult basic education in Switzerland and therefore denotes the local context.

[12]"Mathe" generally denotes a more colloquial and everyday context, whereas "Mathematik" is understood to have more formal connotations and refer to the discipline as such, see also Kaye (2015) for a discussion of the terms mathematics, maths and numeracy in the English context.

[13]This is not only clearly expressed in the new Federal Law on Further Education which has come into effect on January 1, 2017 and which includes basic skills, but it is also reflected in the name of the national umbrella organisation, the Swiss Federation for *Adult* Learning SVEB which in German, French and Italian is always translated as the Swiss Federation for *Further* Education (own emphasis).

education and training or is subsumed in other fields such as vocational education. However, it should be noted in this context that vocational education enjoys a very high status in Switzerland with two thirds of all young people coming out of compulsory education enrolling in vocational education. Furthermore, completion rates for upper-secondary education are equally high: 90% of young people under the age of 25 have successfully completed upper-secondary education (educa.ch, 2016). This leads to generally high levels of education, particularly in numeracy where merely 8.6% of the population between 16 and 65 years performed at Level 1 the Adult Literacy and Life Skills Survey (ALL)—compared to 19.5% in Canada or 26.8% in the USA (Statistics Canada and OECD, 2005). There is, therefore, limited demand for courses for adults focusing on basic skills.

The Swiss educational system overall and teacher training in particular is marked by the fact that the main responsibility for educational matters resides with the 26 cantons. While most of the teacher training has been moved to the tertiary level in the 1990s and standardised in this process, the training of adult education teachers remains fractured. Various institutions offer certificates and diplomas with many specific foci, however there is no standardised training for adult basic education teachers. However, within its focus on basic skills, the Swiss Federation of Adult Learning (SVEB) is running different projects to professionalise the training of adult basic education teachers.[14] One such project was a numeracy course for adult educators. The course design allocated a lot of time to the development and discussion of specific numeracy tasks and emphasised a transfer of the participants' experience in language teaching to the teaching of numeracy (oral communication by the course leader).

11.3.3 Study Participants

Data were collected in the summer of 2012 in Switzerland. All eight participants (4 male and 4 female, average age at the time of the interviews: 48 years) responded to a call which was distributed through the Swiss numeracy network.[15] While their educational backgrounds were quite varied, all of them were working—at least partially—as adult education teacher, private tutor or mathematics teacher in various institutional contexts. Five of them were teaching German as a second

[14]Unfortunately, the recently redesigned SVEB homepage no longer contains any information in English, however, the site http://swisseducation.educa.ch/en contains comprehensive information in English on the Swiss education system in general. (accessed 12 July 2018).

[15]The "Netzwerk Alltagsmathematik" as it is called in German brings together some 125 individuals interested in everyday mathematics/numeracy in an informal manner. Many of them work as course leaders, in further education institutions or in academia. As the network provides no personal information apart from their names, it is not possible to describe the network members more systematically. More information on the network (in German) can be found at: http://www.netzwerk-alltagsmathematik.ch (accessed 3 March 2016).

Table 11.1 Key information about the study participants

Participant	Sex	Age	Education	Work experience	Current position(s)
TP1*	F	57	Commerce diploma, translator, music school, adult educator	Secretary, company owner, translator, mother, adult education teacher	German as a second language teacher with low qualified people
TP2*	M	53	Chemical technician, forestry manager, adult educator	Chemical technician, craftsman in France and Spain, outdoor social worker, father, adult education teacher, case manager, trainer	Course leader for low qualified people
TP3*	F	52	Teacher training, speech therapist	Teacher, various small jobs, speech therapist, mother, adult education teacher	German as a second language teacher, speech therapist, trainer
TP4*	M	43	Teacher training, public relations and communication specialist	Radio journalist, self employed consultant, trainer for adult education teachers	Self employed public relations consultant, trainer
TP5*	M	45	Commercial apprenticeship, social worker, adult educator	Banking, IT coordination, social worker, adult education teacher	German as a second language teacher, independent developing of numeracy courses
TP6	F	28	Teacher training	Private tutor, work various jobs, course leader for adults, scientific collaborator	Course leader for adults, scientific collaborator for e-learning
TP7	F	49	Psychologist, career counsellor	Career counselling, prevention of addiction, mother	Counselling and training in debt prevention
TP8	M	58	Geometer, adult educator	Geometer in Switzerland and abroad, sometimes with training responsibilities, father	Geometer, mathematics teacher

language to immigrants and addressed basic mathematics in this context. Table 11.1 provides a short overview of their key characteristics.

Within this group of participants, TP1 to TP5 (marked with an *) constitute a somewhat more homogenous group, as all of them work with either students of German as a second language or with low qualified people and have completed the eight day numeracy teacher training described in the previous paragraph.

11.4 Key Results and the Duality of Mathematics

11.4.1 Key Results

Many of the findings obtained in the course of the study are reported elsewhere (Beeli-Zimmermann, 2014, 2015). Key results include the following: Referring to Ernest's (1989) perspectives, the participants' views of mathematics contain elements of both the instrumental and the problem-solving views, while the Platonist view is almost completely absent. Furthermore, a number of additional aspects emerged that could not easily be captured by the deductively developed codes based on Ernest's framework, for example a repeated comparison of mathematics with language, the participants' own experiences with mathematics (as students, but also outside of the classroom, for example as parents), affective issues or characteristics of mathematics, such as its fundamental role in life or its duality. While it is not surprising that affective issues cannot be captured with a framework that does not contain any aspects thereof (as mentioned before, Ernest assigned all affective issues to attitudes), it is worth noting that some of the issues appear almost exclusively in the verbal data, for example affective issues, but also the similarity between mathematics and language or personal experiences. Others—such as its duality—can easily be traced in both the verbal and visual data, as the following paragraphs will illustrate.

11.4.2 Dualities of Mathematics

Duality, which is understood in a very basic sense, namely as having two parts, emerged as a frequently mentioned or implied topic in the interviews[16] and was also clearly visible in some of the participants' pictures. While the principle denotes a simple dichotomy, such as either negative or positive emotions towards mathematics, a closer look reveals that the identified dualities are much more complex, as they are first of all not always easy to separate and secondly link to other issues such as the participants' personal experiences, various characteristics of mathematics or their teaching preferences. The aim of the following paragraphs is therefore to describe these different aspects by illustrating them with quotes from the interviews. While quantitative statements could also be made (for example on the basis of the number of codes), this will not be done, as the aim of this contribution is to be illustrative of possible connections and processes rather than focusing on static descriptions.

[16]Verbal indicators of possibly dualities are contained in phrases like "at the same time", "it is also", "on one hand … on the other" or adjectives such as "ambivalent", to name a few.

11.4.2.1 A Division Between What the Participants Like About Mathematics and What They Do Not Like

This division comes as no surprise, as negative and positive affects towards mathematics are a recurring research theme (see for example Evans, 2000). What might be more interesting is the fact that none of the participants displayed an entirely positive or negative attitude towards mathematics—as is sometimes implied by earlier research (which often focused on a possibly causal relationship between attitude and achievement, see also Ma & Kishor, 1997 for a meta-study on this issue). Throughout the conversation the participants referred to a variety of emotional notions or related memories indicating that this topic is not as simple as it is sometimes implied by certain questionnaires. Overall, a number of positively connoted affects dominate their statements, the most important of which include:

– *Fun and enjoyment* when working on specific problems: "This crossword [an element on the participants' picture] symbolises enjoyment. It stands for enjoying mathematics as well. It symbolises that I can also do mathematics with a relish." (TP1) And: "I have a playful relationship with mathematics […] in the supermarket it is fun, to know what the total is before the check-out person hits the button. Sometimes I just calculate for fun." (TP4)
– *Pleasure* when being successful: "I'm good at basic arithmetic because we practised extensively in primary school. It is something that remained and that somehow also was a pleasure. Really, one was pleased that there was some routine." (TP3)
– *Trust and reliability* in its concepts: "The regularity, the structure which makes it possible to feel at home. Some calculability – you know exactly where the boundaries are." (TP1)
– *Fascination* for specific aspects of mathematics: "Mathematics also has something artistic, but I really don't understand that. I'm not interested enough in the concepts behind that. Mathematics in nature, the classic sunflower story and so on – that I find really fascinating." (TP4) Or very simply: "I find maths fascinating." (TP5)

This diversity illustrates a difficulty that other authors have also identified, namely what exactly constitutes positive affects towards mathematics (see for example Di Martino & Zan, 2011 or Evans, 2000 for emotional diversity in mathematical thinking in general). It is interesting to note that different extents of activity are involved in these statements—one does not only have to be successful at mathematics to experience it positively, one can also enjoy the beauty of it without understanding it. However, there is also some display of indifference: "If starting tomorrow it [mathematics] would no longer exist, I'd not regret it. It's like, it's a reality and I'm the type of person who says, ok, if it is a reality I try to get something from it rather than being upset by it every day." (TP4)[17] Or: "Mathematics is not necessarily, well, it is a part of life, but not an overly important one. […] I can't always take it very seriously" (TP6).

While the participants show a broad variety of positively connoted affects, their negative associations are more homogenous. However, they are not directed at mathematics itself, rather at the social function of and general expectations towards mathematics. Negative references are almost exclusively made with regard to specific events or teachers at school, where mathematics usually enjoys a high status. Its associated practices in the classroom, where teachers often focus on the correct result instead of how students are getting there, were experienced as discriminatory and revealing by those who fail: "In fifth grade we had a teacher who, I don't remember exactly, it was about mental arithmetic […] you had to get up and when you got the right result you could sit down, if you got it wrong you had to remain standing […] my friend still says, she always had to remain standing, she still says that she is no good at it." (TP8) Or: "We had a maths teacher who held a high rank in the army[18] and no day went by without him expressing that. He always sorted the maths test according to grades, the worst came first. And usually, anything that was below a 3.5[19] landed on the floor." (TP4) More generally, the participants harbour mainly negative feelings towards the selective function of mathematics: "It is an image of maths that is qualifying. Either you know how to do it or you don't, and if you don't, you're obviously dumb. […] It was a main subject and the grades had double weight for promotion." (TP2) Its gatekeeping function is even seen as a potential trigger for an individual's downfall: "With those who are really bad [at maths], where it goes very far down the hill, that leads to a catastrophe – at least in relation to one's self esteem. And that is why I'm somewhat ambivalent, I'd almost say, maths is a possible road to ruin for some." (TP2) It is interesting to note that in spite of some personal negative experiences at school—which all participants reported for themselves and TP2 also for his son!—their affective references are dominated by positive emotions: "There [high school] it [mathematics classes] became relatively good again and that is when I also started to work as a private maths tutor. It seems that at that time I've managed the change from the negative maths experience, something which was negatively connoted, to a positive one. And it's right, at no moment in time, when I listed all the terms [which were entailed in the participants' picture] did I think of something negative." (TP8)

Overall, it seems that one of the most important aspects related to this division is the insight that an individual's emotional relationship with mathematics is not necessarily either merely positive or negative, but that the two affective aspects

[17]This statement is also an interesting indicator of how other aspects such as personality traits may shape mathematical beliefs.

[18]Switzerland's armed forces are largely militia based which means that its members work in other professions such as that of a teacher for the most time of their lives.

[19]Grades in Switzerland are awarded from 1 to 6 with 4 being a pass and 6 being excellent.

coexist in the same person and link to different issues such as the individuals' experiences but also her/his self-concept or different areas of mathematics. A second important aspect which is also closely related to affective issues is that of change and it is worth noting how some participants can identify key moments which relate to changes in their emotions towards mathematics. Noteworthy is also the positively connoted fascination for specific areas of mathematics which relate to strands of mathematics beyond an individual's understanding. The described affective division therefore presents relations to a variety of issues beyond the individuals' personal experiences and competences. It is also interesting that the participants are aware of this complexity and use it for the planning of their classes: Even though they might not like some aspects of mathematics, they know that there are other aspects which they like and these positive elements are used as reference or anchoring points for their classes: "I try to teach my students exactly what I have on my picture. On the one hand I try to convey a little bit of fun, that it [mathematics] is a brain teaser. […] then I want them to experience the flow of solving a problem" (TP1). Furthermore, they claim to not make use of practices which are often associated with negative emotions in mathematics classes (and which many of them have experienced themselves!), for example exposing students in front of an entire group, insisting on formalisms or on the right result rather than the problem solving process. The latter aspect is particularly interesting as the general perception is that most maths teachers teach the way they were taught (the traditional way) and not the way they were trained (the progressive way, Handal, 2003), as has also been described at the beginning of this contribution.

11.4.2.2 A Division Between What They Call Applied Mathematics and Abstract Mathematics

This division is particularly well illustrated in the following two images in Fig. 11.1 which are both marked by a clearly visible split without any connections between the two perceived halves. Both compositions consist of one half which is characterised by the use of images and colour and a second, very sober, half consisting of words and/or numbers.

Both the numbers and words can be considered to be abstract representations of concepts, whereas the colourful bits and pieces represent and are taken from the participants' life (TP4 said that he simply emptied his wallet for the construction of his collage). They are symbols for applied mathematics. The visually perceived division is also reflected in TP7's quote which serves as the title of this contribution: The statement "I have never cooked with my maths teacher" is not only interesting because it separates what the mathematics teacher does and what someone in the kitchen does, but also because by doing so it indicates an awareness that aspects of mathematics are contained in both instances. TP7 realises that cooking contains mathematics and would be an opportunity to practice or apply some aspects of it and she considers the lack of cooking with the mathematics teacher a missed opportunity. She elaborates this as follows: "Mathematics is often

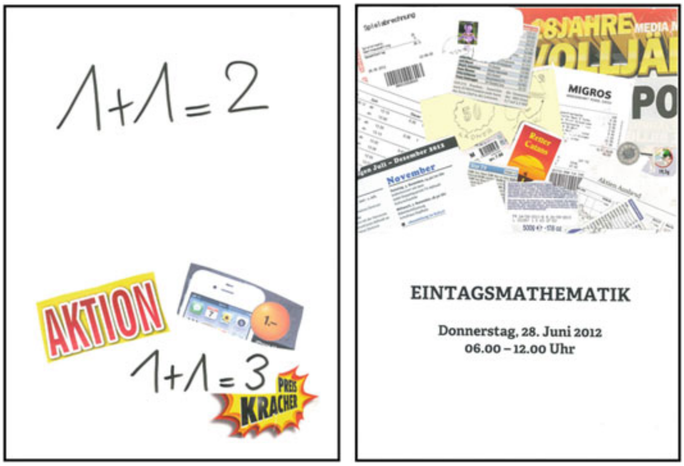

Fig. 11.1 Picture created by TP7 (left) and TP4 (right)

regarded as very theoretical, but if you look at it in everyday life, then there is a rupture. And to some extent this rupture also exists at school. You calculate beautiful stuff, but it doesn’t hold up in everyday life.” (TP7) Or as another participant expressed it: “There are the two sides, mathematics as instruction knows it, like one and one, the concept really. And then the hidden application, which occurs quite frequently, but which has nothing to do with instruction.” (TP5)

The dividing line between these two fields of mathematics seems to be that of everyday life (of which school interestingly does not seem to be a part, if one reads the previous quotes carefully!). Given their training as numeracy teachers and the situations in which they work, the following statements are not surprising: “Everyday mathematics is really the application. Very simple, being able to use a ruler, interpreting distances, knowing weight units, being able to guess volumes, to use scales and being able to interpret numbers.” (TP5) Or: “Everyday mathematics is an expression of what I do in my daily life.” (TP3) Many of the participants also talk about “the world of things” (TP2, TP3 and TP7) and “the world of concepts” (TP5). It is interesting to note that when talking about this division most references to language are made, highlighting the following parallels between the two: “Language helps to cope with life, and communication, and so does mathematics, it helps to cope with life.” (TP1) And: “Maths is like grammar in language, it explains how something works.” (TP5)

Overall, this division between applied or situated and abstract mathematics reflects a dichotomy that has previously been described by numerous other authors, such as Lave (2000), Nunes, Schliemann, and Carraher (1993) or Wedege (1999). What is different about the described group of people is that they are aware of the fact that some of the tasks they encounter(ed) in everyday life are of mathematical nature, that they can name the mathematical concepts behind and therefore link the two areas: “The first time I came in contact [with maths] was when I was catching mice[20] […] there we had a little booklet, where you noted how many tails, and that’s what I did during my holidays.” (TP2) This statement indicates that TP2 has an awareness of the fact that there were some mathematical concepts hidden behind his bookkeeping. Or: “For some time I was very interested in pre-Christian cultures in which calendars are very important […] and in some way that is nothing else but maths. They wanted to solve a problem, wanted to survive so they needed to know when to harvest and when to snow.” (TP3) This awareness and the related ability of knowing where and how to identify mathematics in specific contexts or situations seems an essential feature of the study’s participants and constitutes the basis of their teaching philosophy: TP2 often teaches at his students’ place of work, for example in restaurant kitchens or outside in maintenance depots and he uses this context to work on specific mathematical tasks, for example by starting to ask about the volume of saucepans or piles of wood and then arriving at the identification of a problem which is of interest and relevance to the students. Many participants start from situations their students describe and the statement of “The goal of my class is of course to relate to practice.” (TP6) seems to be more than an empty phrase.

11.4.2.3 A Division Between What the Participants Are Able to Do and What Is Beyond Their Abilities

Of course the previously described approach to teaching only works if the teachers are capable of responding flexibly to specific situations, meaning that they need to identify the hidden mathematics and know what concepts are useful to solve a particular problem. It is crucial for numeracy teachers to be able to identify and solve mathematical problems occurring every day and this is also understood by the participants: “Helping them to integrate into Switzerland that is my task […] and certain mathematical things are a part of that. Time management or dealing with money. That’s why we talked about household budgets, that is something essential. Dealing with money.” (TP1) This perspective includes a very strong focus on problem solving activities as overarching skill—something which is mentioned by many of the participants: “To teach them the dramaturgy of such problems, I also see this as my job. Apart from technical aspects.” (TP4)

It is not surprising that this division is also closely linked to the participants’ biographies: “I only practiced maths until upper secondary school, in a sense. Afterwards I thought that it really was too difficult for me, but of course not every day mathematics. Because what I know about everyday mathematics, there I’m absolutely with it.” (TP3) A majority of the participants mentioned a similar moment during their schooling when they no longer understood what was

[20]In order to reduce the number of mice in the fields, many villages issued a small fee as reward for each caught animal. In rural areas of Switzerland catching mice was therefore a possibility for children to earn pocket money. They could bring a part of the mouse (tail or specific paw) to their village administration and got a little money for each animal they caught.

happening in the classroom: "I always really liked mathematics at school and I was good at it, as long as it was related to logic, as long as I could envisage it. And when it was about higher mathematics, like algebra with I don't know how many unknowns and stuff, then, at some point, I gave up." (TP1) Or: "Being a linguist, I've quite a high affinity for math. I had that at school as well. I was left behind when it was not connected to everyday life." (TP4) This estrangement was reflected in many stories, however, it seems not to have left long lasting negative effects—as has been described above, none of the participants expressed a purely negative attitude towards mathematics, at best they were ambivalent. But in most cases the participants' initial interest in and like of mathematics prevailed: "I obtained the diploma afterwards and suffered for a while that I didn't get straight into uni […] actually it is the fault of mathematics that I couldn't take the direct way. But I'm not mad at mathematics." (TP1)

This specific division suggests a close link to what other authors have called mathematical identity (Kaasila, 2007). The fact that both TP2 and TP3 stress explicitly that they are neither mathematicians nor mathematics teachers, but that their area of expertise is everyday mathematics, confirms that the issue of identity also plays an important role for this group of people. It would be interesting to explore to what extent the dividing lines (if they can be drawn clearly!) between the previous duality of applied and abstract mathematics overlaps with the (again hypothetical) dividing line of this dichotomy. Would it be possible to identify something like a grey area which the participants are able to do but which are not applied? Their personal stories suggest that the division of applied and abstract mathematics is very much related to their personal experiences, however, it is unclear to what extent that also reflects their current mathematical abilities and knowledge. Either way, it seems that this perspective of having experienced themselves that some aspects of mathematics can be difficult, is central for the participants to relate to their students: "I like doing mathematics and where it is beyond my capacity, there it can become stressful […] it was stressful once, and I can imagine that my students, that they are stressed." (TP1) This empathy, an understanding of what it feels like when learning is difficult and the conviction that everybody has her/his personal boundaries, enables the participants to relate to the feelings and needs of their students and is therefore a basis for their student centred teaching approaches.

11.5 Discussion and Concluding Remarks

11.5.1 Beyond Dichotomies in Characterising Mathematical Beliefs

The previously identified divisions of mathematical beliefs are by no means new. Very similar areas have been described for example by Di Martino and Zan (2011) who identified emotional disposition, vision of mathematics and perceived competence in mathematics as three dimensions in essays written by more than 1600 students on the topic of “Me and Maths”. However, it seems a shame that after describing a wealth of data and advocating for “non-traditional methods” (p. 475), the authors again reduce the characterisation of the identified dimensions to three simple dichotomies: vision of mathematics (relational/instrumental), perceived competence (high/low) and emotional disposition (positive/negative) and focus on correlations between these six elements. Yet one of the key insights from the above descriptions of the three dichotomies is the various explicit and implicit references to possible instances of change of the participants’ mathematical beliefs. They are linked to successful as well as discriminatory moments at school, to interactions with their students and not least of all to their training as numeracy teachers. In order to learn more about the development and change of beliefs, it would therefore be desirable to conduct more longer term studies spanning ideally not only the period of teacher training, but including also other contexts of the participants’ personal life. Working with visual data could be particularly beneficial for capturing specificities of different contexts. Another aspect which emerged from these descriptions is that of the participants’ awareness of their views of mathematics and how they detect mathematics in many aspects of life. In addition to the quote in the title of this contribution which reflects this awareness, the following statement underlines this aspect: “And then I had a gap year during which I did all kind of funny things which had nothing to do with maths. Well, I mean I didn’t study maths – as I know today, probably everything has something to do with maths, of course!” (TP3) Part of the awareness is certainly due to the methods of the study and the fact that they were asked to present their views of mathematics. However, the previous statement indicates that there were other instances in the lives of the participants when they became aware of specific characteristics of mathematics, such as its ubiquity. And while some factors which contributed to the participants’ current views of mathematics are specific for this group—not least of all their work as adult numeracy teachers in Switzerland and their training therefore—there are other aspects, such as their experiences as students and not being successful in certain fields of mathematics, as parents or as adult educators in general, which they share with other groups. In this respect it would be interesting to extend mathematical belief research beyond the classic target groups of teachers and students and include parents or other groups of the general population. Just like the complementary intention of visual and verbal data has been met in this study (with some issues being only visible in one set of data), focusing on different groups might shed light on specific aspects of belief change.

11.5.2 Limitations

As with any study, there are a number of limitations also to this one, most of them relating to the study design. The following points will underline the importance of conceptual and methodical clarity which not only means explicitly declaring how beliefs are understood and defined, but also stating the purpose of the chosen methods and reflecting the extent to which the intended purpose was met:

- The overall study design was an exploratory one, therefore limiting its usefulness to contribute to the further development of theoretical or conceptual issues as it largely makes another descriptive inventory of the mathematical beliefs of a specific group of teachers. Yet the results indicate that the chosen target group could be informative, particularly for the study of belief change.
- The study relied to a large extent on a method elaborated by Rolka and Halverscheid (2011) and by doing so also adopted Ernest's (1989) concept of beliefs (in spite of its missing inclusion of affective issues). Furthermore, the analytical scheme used for the images neglects visual specificities such as colour or composition (see Kress & van Leeuwen, 2006 who argue that visual structures contain semantic dimensions) and makes therefore an incomplete use of this specific type of data. While the visual data did complement the verbal data and triangulation was successful in this respect, another study would need to be more considerate of the specificities of visual data.
- The relationship between the participants' beliefs and their classroom practice is a recurring theme that confirms the close relationship of mathematical beliefs and beliefs about its teaching. A future study should also envisage including classroom visits in order to obtain information that goes beyond of what is reported by the participants.

A final issue which is both a limitation and a distinct potential are the participants who represent a very specific group of people: In many other countries adult education teachers would be a more homogenous group, depending particularly on the extent to which their training is standardised. However, it is the diversity of this group of people in terms of both their educational and vocational experiences which presents unique opportunities for studying the change of beliefs and trying to identify particularly relevant settings in this context. Their training as numeracy teachers, for example, included a targeted focus on changing their beliefs about mathematics and linking them to their beliefs about languages. At the same time their focus on everyday mathematics provides a clear entry point to possible effects of experiences made outside of the classroom on the change of beliefs. A more systematic and possibly longitudinal study of this or similar groups of adult education teachers could therefore be very informative for a deeper understanding belief change in general.

11.5.3 Concluding Remarks

The key points to derive from these three sketches of different dualities in adult education teachers' views of mathematics is certainly that there are no straight forward relationships between various elements of mathematical beliefs. Moreover, simple descriptions of correlations will not contribute to an in depth understanding of the phenomenon. Affective and cognitive aspects are closely interrelated and interact via a number of variables, including but not limited to personal experiences and specific strands of mathematics. It seems important to bear in mind that descriptive inventories are snap-shots and that an individual continuously makes new experiences which modify her/his views—be it in the context of further training, in their everyday interaction with students or in any other role or function of which one holds many. In order to further advance the understanding of the complex nature of mathematical beliefs, it would be desirable to continue researching the beliefs of very specific groups, such as adult basic education teachers, using a variety of methods purposefully and over a longer period of time.

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Chapter 12
Maths Eyes—A Concept with Potential to Support Adult Lifelong Mathematics Education

Terry Maguire and Aoife M. Smith

Abstract Maths Eyes is an evolving construct that is gaining popularity in adult education, schools and community in Ireland and internationally. Maths Eyes works on the premise that if individuals are supported to look at familiar things through the lens of mathematics, they begin to see that mathematics is all around them, and they can build confidence in their own ability and practice. This paper reviews the way in which the concept of developing maths eyes has been adopted and used both in Ireland and internationally and suggests that Maths Eyes can be used to make connections across and between 'big ideas' to develop a profound understanding of mathematics that underpins adult lifelong mathematics education. The findings of the study provide recommendations for how best teachers can support adult learners of mathematics through developing their own 'Maths Eyes'; to complement this, the key components of professional development provision for adult mathematics teachers are discussed.

Keywords Adult · Mathematics · Education

12.1 Introduction

The environment in which we now live and work is becoming more digital, and now more than ever mathematics plays an increasingly important role in helping us negotiate this dynamic world. However, mathematics teaching and learning often leaves individuals with a narrow and incorrect view of mathematics based on their school experience which is, in large, partly composed of facts, formula and

T. Maguire (✉)
National Forum for the Enhancement of Teaching and Learning in Higher Education, Dublin, Ireland
e-mail: Terry.maguire@teachingandlearning.ie

A. M. Smith
EPI·STEM, The National Centre for STEM Education, University of Limerick, Limerick, Ireland

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processes that are often abstract. Consequently, many adults have a negative view of mathematics and its usefulness, and do not think it has any relevance in their real life (O'Donoghue, 2000).

Effective mathematics teaching has the potential to change these negative views of mathematics and to fundamentally change adults' relationship with mathematics. Effective teaching requires shifting adults' perception and increasing their engagement with mathematics by making the discipline have more meaning and relevance to their lives.

The *Maths Eyes* concept was developed as a central component of a professional development model for adult numeracy tutors[1] in Ireland (Maguire, 2003), and was inspired by the work of Ritchhart (1997). The intention was to help teachers of adult mathematics move away from a view of mathematics as a decontextualized set of abstract skills and formulae, towards a view of mathematics as an integral part of their own and their learner's lives. Much of mathematics is about abstract ideas, but for the vast majority of people it is accessible only if it can be understood in a context with which they are familiar (Eastway & Wyndham, 2002, p. ii). Mathematics that surrounds people in their everyday lives, for the most part, remains 'invisible' (Coben, 2000). Maguire (2003) suggested that adult numeracy teachers needed to develop their 'mathematical eyes' to enable them to see this 'invisible' mathematics in order to pinpoint appropriate starting points for introducing numeracy knowledge and skills, that were relevant to the lives of the individual learners.

Viewing the world from a mathematical perspective involves having a sense of quantity, an ability to recognise connections and interdependencies, an awareness of functional or cause-and-effect relationships, and the ability to construct visual images to capture these relationships (Ritchhart, 1997). It enables individuals to look at their world e.g., an event happening in sequence, a structure e.g., a building, a bridge, or a fir branch and query, is there any mathematics to help me understand this better?

Maths Eyes is now an evolving construct that is gaining popularity in adult education, school and community in Ireland and internationally. *Maths Eyes* works on the premise that if individuals are supported/challenged to look at familiar things through the lens of mathematics, they begin to see that mathematics is all around them, and they can build confidence in their own mathematics. The level of mathematics that an individual uncovers by developing their Maths Eyes will reflect the level of engagement of the individual, including the level of mathematics the individual brings to the process, whether that is basic or more advanced. The most effective starting points for developing an individual's Maths Eyes is the use of contexts that are familiar to the learner whose reaction to the stimulus (typically a picture, poster, or news bulletin) includes not only their level of mathematics but

[1]Although the word 'tutor' is the term used in Ireland to describe those who teach in the Further Education Sector, it distinguishes them from school 'teachers'; for clarity the word 'teacher' will be used throughout the paper.

also their communication, motivation, attitudes and beliefs, life and work experience. Consequently the mathematics of their real world begins to become more visible to them. This visibility of the mathematics in their everyday life challenges their own view of mathematics as being located in the school context (Steen, 1997). The view of numeracy that underpins *Maths Eyes* is a broad view that sees numeracy 'as not less than mathematics but more' (Johnson, Marr, & Tout, 1997).

12.2 Evolution of Maths Eyes

Since the initial introduction of *Maths Eyes* as a key component of the model of professional development put forward by Maguire (2003), *Maths Eyes* has remained a key aspect of the national approach to professional development for teachers of adult numeracy in Ireland (NALA, 2015). Developing *Maths Eyes* has also been incorporated into adult mathematics provision through its inclusion in the curriculum for adult vocational mathematics (Quality Qualifications Ireland, 2015), and has been integrated into primary and post-primary secondary teacher education programmes.

In tandem with the inclusion of *Maths Eyes* in adult numeracy provision and professional development, the concept was also embraced by the primary and post-primary secondary schools sector. A resource pack for developing *Maths Eyes* was developed with teachers, facilitated by the lead author, Institute of Technology Tallaght and Dublin West Education Centre [DWEC is a network of Association of Teachers/Education Centres of Ireland (ATECI)]. A number of workshops were held nationally to support both primary and post-primary secondary teachers to develop their maths eyes and a *Maths Eyes* trainer's network was established. The early success of the *Maths Eyes* concept in Irish adult mathematics education and the interest of schools led to a community-wide initiative. '*Looking at Tallaght with Maths Eyes*' was launched in June 2011 (O'Sullivan, Maguire & Robinson, 2012).

This community-based initiative aimed to develop the maths eyes of the community in Tallaght, (Dublin, Ireland), to help individuals to make the link between mathematics and the real world, as well as develop a positive view of mathematics. The event aimed to empower the community and build their confidence in their own mathematics knowledge and skills (empowered parents are more confident in supporting their children's learning, more confident citizens can make more informed evaluations of the information that inundates them every day, and have a better understanding of the impact of their actions and decisions in their life, work and leisure). In particular, a key focus was to encourage people in the community to use their maths eyes when they were thinking about their water usage and water conservation. Since then, many other national and international community initiatives, school and collaborative projects and competitions based on the *Maths Eyes* concept have been undertaken (Maths Eyes Website, 2016).

12.3 Scope of the Maths Eyes Initiative

Along with encouraging individuals to consider the mathematics in their real world, *Maths Eyes* has been used in a number of different ways. The range of *Maths Eyes* initiatives in Ireland have been described by Smith, O'Meara, and Maguire (2015). These include, for example, re-engaging adult learners in learning mathematics (Read Write Now, Television Programme); tackling cultural diversity (Maths Eyes in Cork's Northside); supporting the integration of numeracy across the school and vocational education curriculum (Newbridge Community School); bringing school and community together (DunLaoghaire-Shankill Cluster Project); developing a dialogue between parents and children that is not about mathematics homework (Dominican Campus Project); providing opportunities for primary and second level students to collaborate (Dominican Campus Project); used with first year students to ensure they appreciate the usefulness of mathematics in the real world to underpin the junior cycle (lower second level) mathematic curriculum (St. Kevin's Community College), and critical numeracy (Looking at Tallaght Through Maths Eyes). In addition, *Maths Eyes* has also been used by transition year students [the year between ending post-primary secondary junior cycle and beginning senior cycle (upper second level)] to re-engage and motivate students in their study of mathematics prior to undertaking the senior cycle curriculum (St. Lawrence's College). Further, *Maths Eyes* has been used to build a resource for communities that can be used by families (Clondalkin Maths Trail, Maths Eyes Waikito Trail New Zealand). The National Maths Eyes and GeoGebra competition that has run since 2012 has increased its entries each year. In 2015, over 75 schools, adult education and community groups participated. A sample of the entries to the Maths eyes and Geogebra competition have been included in Appendix.

12.4 Level of Engagement with Maths Eyes

Maths Eyes provides access to opportunities for discussion and problem solving for learners at all levels. It builds confidence in an individual's ability to engage in mathematics. An analysis of the entries in the 2015 national *Maths Eyes* and GeoGebra competition entries highlighted that the largest proportion of entries were linked to the Quantity and Number and Shape and Space at a basic level across all age categories. Developing *Maths Eyes* is being facilitated by both specialist mathematics teachers, non-specialist mathematics teachers and non-teaching individuals. One of the challenges with the process of developing maths eyes is encouraging parents, school teachers and adult education teachers to encourage learners to through a wider range of mathematical lenses (Quantity and Number, Shape and Space, Pattern and Relationship, Data Handling, Problem Solving). A second problem is getting those that engage with *Maths Eyes* to deepen their level of engagement. At a national level the entries to the Maths Eyes competition for under 18 category in particular, showed a superficial approach to developing maths eyes and did not reflect the level of mathematics that these individuals would be expected to have mastered at this stage in their education cycle. The output from these projects in terms of community exhibitions and poster submissions to the Maths Eyes competition or local Maths Trails demonstrate that teachers who support the development of the maths eyes of their pupils may need to be offered opportunities to develop their own maths eyes. Many of the Maths Trails developed by teachers are often written as school mathematics book-type problems that learners are asked to complete outdoors. Clearly constructed Maths Trails, provide a wonderful opportunity for developing number sense and problem solving strategies. They can help challenge the view that mathematics problems have one right answer only, and only one particular way that they can be solved a view that is often one that is developed from the school mathematics textbooks and more traditional teaching practices.

The accessibility of *Maths Eyes* and the extent of engagement, by a diverse cohorts of learners and facilitators underlines the merit of the concept. Those who have engaged with *Maths Eyes* have reported that they enjoy developing their maths eyes, have indicated that their confidence in mathematics has increased and their awareness of the real world mathematics that they do each day has become more visible. Most importantly they report that they had fun. Further, *Maths Eyes* encourages discussion and provides opportunities for collaboration which are often limited in the mathematics classroom. *Maths Eyes* seems to initiate what Frid and White (1995), describe as a 'success cycle.' Through *Maths Eyes* an individual achieves some success in mathematics, and this contributes to the development of a more positive attitude towards mathematics. Deeper levels of engagement with *Maths Eyes* has the potential to be used as a starting point for the further development of an individual's mathematics, which will in turn drive the individual to persevere with mathematics. This perseverance in mathematics is likely to lead to further development of the individual's mathematics, which will again lead to better attitudes towards mathematics, and so this positive cycle continues. Maguire (2003) proposed a process model of numerate behaviour which results 'from the internal, dynamic interaction of an individual's mathematics with the other elements'. These elements include communication, mathematical skills and knowledge, life experience, values, beliefs, motivation, attitude, personal and social development. Further it has been argued that adult mathematics education provision that supports adult lifelong mathematics education '*must be underpinned by this dynamic view of numeracy and must be delivered by teachers who can manage 'big ideas' and who can cross the boundaries of mathematics, communication/literacy, and technology*' (Safford-Ramus et al., 2016, p. 21). Engaging with and developing maths eyes impacts on the numerate behaviour of the individual because it can effect the elements of an individual's numerate behaviour e.g. their motivation, communication, beliefs and attitude, mathematical confidence and competence.

As individuals engage with developing their maths eyes they begin to build confidence in their mathematics and its relevance to their lives and their mathematical confidence and competence develops further having maths eyes enables them to see more, better and below the surface of the world in which they live and learn. They begin to 'see intellectually' how to make connections and develop understanding.

Although using the concept of *Maths Eyes* can be facilitated and be successful when used by non-specialist mathematics teachers or facilitators, the potential of *Maths Eyes* to enable learners to reach a more profound understanding of mathematics requires teachers to have an insightful understanding of mathematics themselves. Research suggests that the relationship between teacher qualification and student achievement in school is significant. It also shows that students achieve higher standards in mathematics when taught by qualified mathematics teachers when compared to students taught by unqualified mathematics teachers (OECD, 2009).

Teachers who support lifelong adult mathematics education need have maths eyes; need to understand mathematics and need to be excellent communicators to be able to engage in a meaningful way with their learners, to support their learners to develop their maths eyes, to start making connections and to develop a better understanding of the mathematics of the learner's real world context. The need for teachers to understand mathematics and to make appropriate connections has also been highlighted by Dixon (2015). Providing opportunities for teachers to develop an understanding of the big ideas in mathematics has the potential to develop teachers' abilities in a way that equips them to be able to develop lifelong mathematics learning opportunities in adults.

12.5 Big Ideas in Mathematics

The focus on 'big ideas' has its origins in the work Bruner (1960) who described four key functions of concepts including their facility to provide structure for a discipline, a framework for understanding, to facilitate transfer and as a result support further learning. Charles (2005, p. 10) defines a 'big idea' as 'a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole'. He contends that 'big ideas' are important because they enable us to see mathematics as a 'coherent set of ideas' that encourage a deep understanding of mathematics, enhance transfer, promote memory and reduce the amount to be remembered (Charles, 2005, p. 10). Clark (2011, p. 32) defined a concept as a 'big idea' that helps us make sense of, or connect lots of little ideas. 'Big ideas' provide a good foundation for mathematics content knowledge, teaching practices, and the mathematics curriculum (Charles, 2005).

There is no universal agreement in relation to what the 'big ideas' of mathematics should be. Charles (2005) described 21 'big ideas' for elementary school mathematics in the US. In Ireland, the 'big ideas' at primary school level include;

Number, Algebra, Shape and Space, Measure and Data (NCCA, 1999). At post-primary secondary school level the 'big ideas' are Number, Algebra, Geometry and Trigonometry, Functions (picks up from pattern at primary level), Statistics and Probability (NCCA, 2013). The Irish vocational mathematics curriculum is more granular in terms of content and is less focused on 'big ideas'.

Teachers who understand the 'big ideas' of mathematics are more able to represent mathematics as a coherent and connected enterprise (NCTM, 2010, p. 16). Together the development of the teachers' own *Maths Eyes* and their understanding of the big ideas in mathematics has the potential to enable teachers to progress from seeing 'superficially' (e.g., looking at a building and seeing the shape of the windows) to seeing 'intellectually' (e.g., the shape of a house to maximize usable living space). Figure 12.1 attempts to capture how developing maths eyes, increasing understanding of the big ideas in mathematics can improve confidence and competence in mathematics and develop numerate behaviour. Individuals can have different starting points with different levels of engagement with mathematics. An individual whose starting point is very basic mathematics can develop some level of deep understanding through a similar process as someone whose starting point may be a higher level of mathematics. In all circumstances, the development seems to be closely linked to and involve the development of an individual's numerate behaviour.

If we want teachers to develop their maths eyes and have a profound understanding of mathematics, to be able to make connections and integrate mathematical concepts we need to ensure that opportunities are available for teachers to develop this ability to see and understand mathematics through their teacher training programme or professional development opportunities.

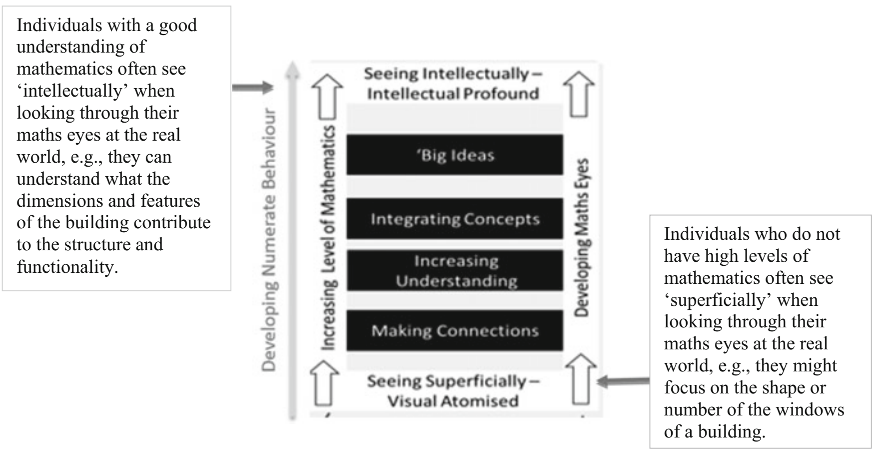


Fig. 12.1 Using the maths eyes concept and 'big ideas' in mathematics to support teachers to develop their profound understanding of mathematics

12.6 Integrating Maths Eyes in the Professional Development of Teachers

The way in which the maths eyes and increased mathematical understanding is developed in teachers, through professional development, should model the approaches these teachers might use to support their own learners to develop a similar level of understanding. However, in the absence of appropriate professional development many teachers of adult mathematics in some countries rely on their personal understanding of the often decontextualized and abstract mathematics they learnt in school and teach the way they were taught (Maguire, 2006). Chisman (2011) highlights the need for teachers to know and understand mathematics differently from that which they experienced in school and highlights the difficulty in identifying teachers with these abilities.

A brief review of those that teach mathematics in Ireland across all levels of education supports Chisman's findings. Primary school teachers are generalist rather than mathematics specialists. A study in Ireland of those that teach post-primary mathematics found that 48% of teachers did not have a mathematics teaching qualification (Ní Ríordáin & Hannigan, 2011). In particular the evidence of the level of unpreparedness of adult mathematics teachers in Ireland is problematic. A survey conducted by the lead author in 2001 on the level of mathematics of adult mathematics teachers in Ireland in the Further Education and Training sector (Maguire, 2006) highlighted that teachers:

- own histories in learning mathematics and their educational experiences influence their view of mathematics and subsequent teaching practice concerning adult numeracy;
- may not be mathematics specialists and may have come to teaching adult numeracy via a variety of career paths;
- may not always have a background in mathematics;
- may not have had training in teaching mathematics;
- may or may not have completed training in teaching adults;
- may or may not have formal teaching qualifications;
- may be teaching without having any training at all.

This survey was repeated by the lead author in partnership with the National Adult Literacy Agency in 2012 (NALA, 2013). The percentage of those who responded who had a qualification in mathematics had not changed (8%). Although the 2012 survey showed an increase in the proportion of adult mathematics teachers with a third level qualification (although not in mathematics), the findings showed only small changes in the profile of those who teach numeracy to adults. Most of the (very limited) continuing professional development (CPD) completed focused more on teaching adults generally rather than teaching mathematics specifically. Over 50% of the tutors indicated that their current teaching is influenced by their own school experience. One of the most notable changes observed when the findings of both surveys were compared was the reduction in the number of full time (42% vs. 28%) and volunteer tutors (23% vs. 12%) and an increase in the number of tutors on part time contracts (46% vs. 60%) recorded in the 2012 survey (p. 39). A 16% decrease was observed in the number of tutors delivering non-accredited customized programmes. One of the main challenges faced by adult mathematics teachers reported in the survey was how to remain learner-centered and still meet the demands of a packed curriculum strongly focused on accreditation (NALA, 2013).

In both surveys almost all tutors who completed CPD reported a positive impact on their teaching practice. However, in responding to the type of CPD that would best suit their needs, the 2012 survey highlighted that only 24% of adult mathematics teachers felt it was important to develop a profound understanding of elementary mathematics; this was a reduction of 20% when compared to the 2001 survey. In terms of the CPD provision identified, teachers wanted CPD to focus on resource development (58% down from 71%), applying maths in different contexts (53% little changed from 51%) and in using technology to teach mathematics (51% increased from 34%). Over 60% of tutors reported that they did not have enough training in teaching mathematics to adults and of these, 15% reported that they had no training at all in this area (p. 38).

The overall picture of adult mathematics teachers in Ireland does not suggest a well-trained, competent work force equipped to support lifelong mathematics learning for adults.

One of the main challenges faced by adult numeracy/mathematics teachers is how to remain learner-centred as a more accredited provision develops for adult learners. The pressure on teachers to meet the demands of a crowded curriculum for adult mathematics especially if the programme is linked to accreditation, means that mathematics is often delivered in a linear and in an unconnected way. Clark (2011) suggests that this approach does not facilitate adults to transfer mathematics knowledge from one context to another. This lack of transfer can impact on an individual's ability to deal with the mathematical demands of the workplace:

> *Work contexts often require multi-step solutions to open-ended problems, a high degree of accuracy, and proper regard for required tolerances. None of these features are found in typical classroom exercises.* (Steen, 2004, p. 55)

Safford-Ramus et al. (2016) argue that adult mathematics education provision needs to be conceptualised and reconceptualised to meet the needs of adults at different stages of their lifelong learning journey to ensure high use value/high exchange value and caution against the homogenisation of mathematics education provision.

Clearly the way mathematics is taught and the ability of mathematics teachers to teach mathematics impacts on the way students experience mathematics and their ability to develop lifelong mathematics learning (Chisman, 2011). However there has been only limited discussion of what constitutes appropriate teacher education for those that teach adult numeracy/mathematics (Morton, Maguire, & Baynham, 2006).

12.7 A Model of Professional Development for Teachers of Adult Mathematics Education

Maguire (2003) put forward an evidenced-based model of professional development for those teaching adult mathematics education. The model had as its objective the aim of developing an adult mathematics teacher who was a professional, autonomous, and confident lifelong learner. The author in partnership with the National Adult Literacy Agency reviewed the model in 2015 and subsequently developed a national framework for meeting the professional development needs of teachers of adult numeracy in the Irish further education and training sector. This framework currently guides the content of professional development provision for teachers of adult mathematics learners in Ireland (Fig. 12.2).

The author has subsequently reviewed and adapted this framework to incorporate the need for teachers of adult mathematics to be good communicators and to understand the big ideas in mathematics as a framework for developing their own understanding and competence in mathematics.

The key components (Fig. 12.3) of professional development provision for adult numeracy/mathematics teachers to support adult mathematics lifelong education would in the authors' opinion include, *inter alia*:

1. **Providing opportunities to develop Maths Eyes and a profound understanding of 'Big Ideas' in mathematics**
   Adult mathematics teachers should be required to develop their maths eyes to 'see intellectually' and acquire a profound understanding of the 'big ideas' that are appropriate in their teaching context. They should be able to make connections between and across the components of these 'big ideas'.

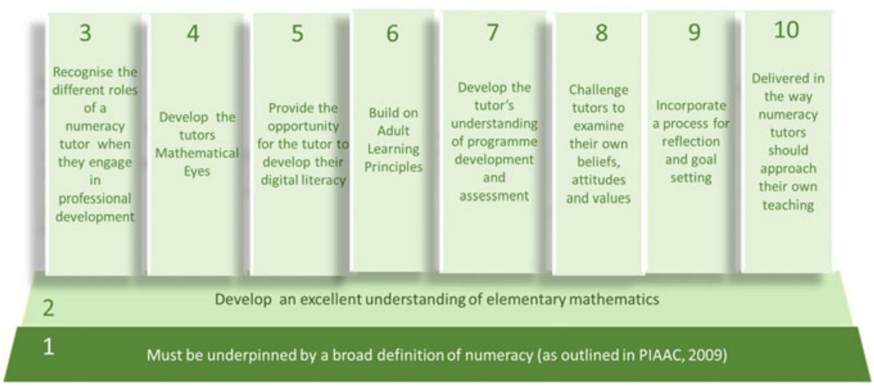


Fig. 12.2 Core components of professional development for tutors of adult numeracy in further education in Ireland

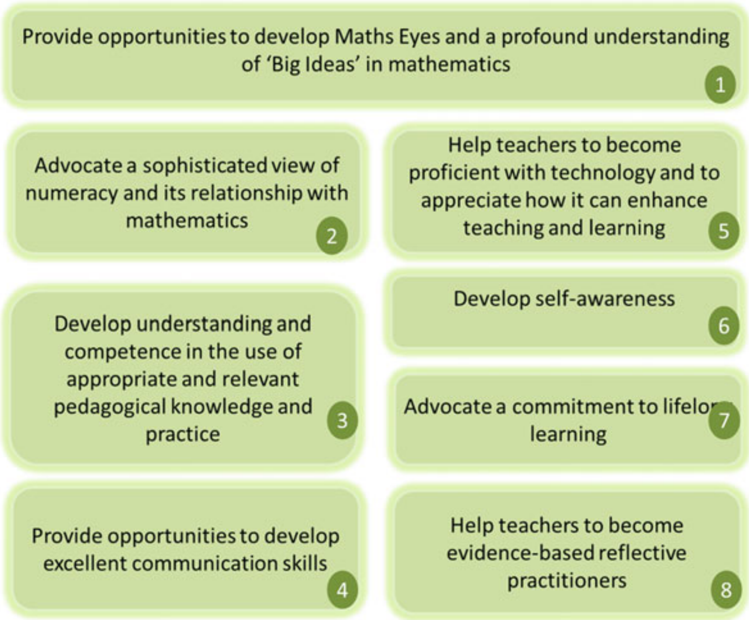


Fig. 12.3 Eight components of professional development provision for an adult numeracy/mathematics teachers to support adult mathematics lifelong

2. **Advocating a sophisticated view of numeracy and its relationship with mathematics**
   The way in which adult mathematics/numeracy teachers interpret the concept of numeracy, numerate behaviour and mathematics will influence the kind of numeracy provision they deliver to their adult learners. Adult mathematics teachers should hold a sophisticated view of the concept of numeracy (Maguire & O'Donoghue, 2004). They need to recognise that developing an individual's numerate behaviour incorporates not just mathematical development, but must take account of the individual's communication, personal and social development, attitudes, beliefs, values, life experience and motivation and the individual's mathematics and that 'one size does not fit all'.
3. **Developing understanding and competence in the use of appropriate and relevant pedagogical knowledge and practice**
   Adult mathematics teachers need pedagogical knowledge that is learner-centred and equips them to teach adult learners in a way that facilitates connectedness and deep understanding to support their further learning of mathematics.
4. **Providing opportunities to develop excellent communication skills**
   Excellent teachers are excellent communicators (National Forum, 2016). Teachers of adult mathematics need to be able to communicate mathematical

ideas and concepts to adult learners enabling them to find meaningful starting points to develop the mathematics of their learners.

5. **Helping teachers to become proficient with technology and to appreciate how it can enhance teaching and learning**
   We now live and work in a digital world and teachers must be equipped to support their students to develop numerate behaviour that will enable them to live and work in this rapidly changing environment:

> *Being numerate in the 21*st *century means being able to cope with the aspects of the world as we encounter it, which includes digital and technological aspects. The reality that techno-mathematical aspects of the workplace and society are now ubiquitous must be acknowledged.* (Tout et al., 2017)

   Teachers of adults need to be comfortable in their personal and professional use of technology. They need to be confident in identifying ways to harness the potential of technology to enhance their teaching and its impact on student learning. They need to be aware of the technologies that have relevance in mathematics learning.

6. **Developing self-awareness**
   Adult mathematics teachers need to be self-aware and consider the aspects of their character and personality that can enhance or impede their teaching. Teachers' personal beliefs and attitudes can strongly influence their teaching practice (Bishop, 1988). Consequently, adult mathematics teachers need to be aware of personal beliefs and attitudes that they bring to their practice.

7. **Advocating a commitment to lifelong learning**
   Those who teach need to consider and review their professional development needs in the context of their rapidly changing learning environment. Consequently, adult mathematics teachers need to recognize that professional development is not a once off activity, but should be actively engaged in throughout their working career. They need to have a commitment to their own lifelong learning.

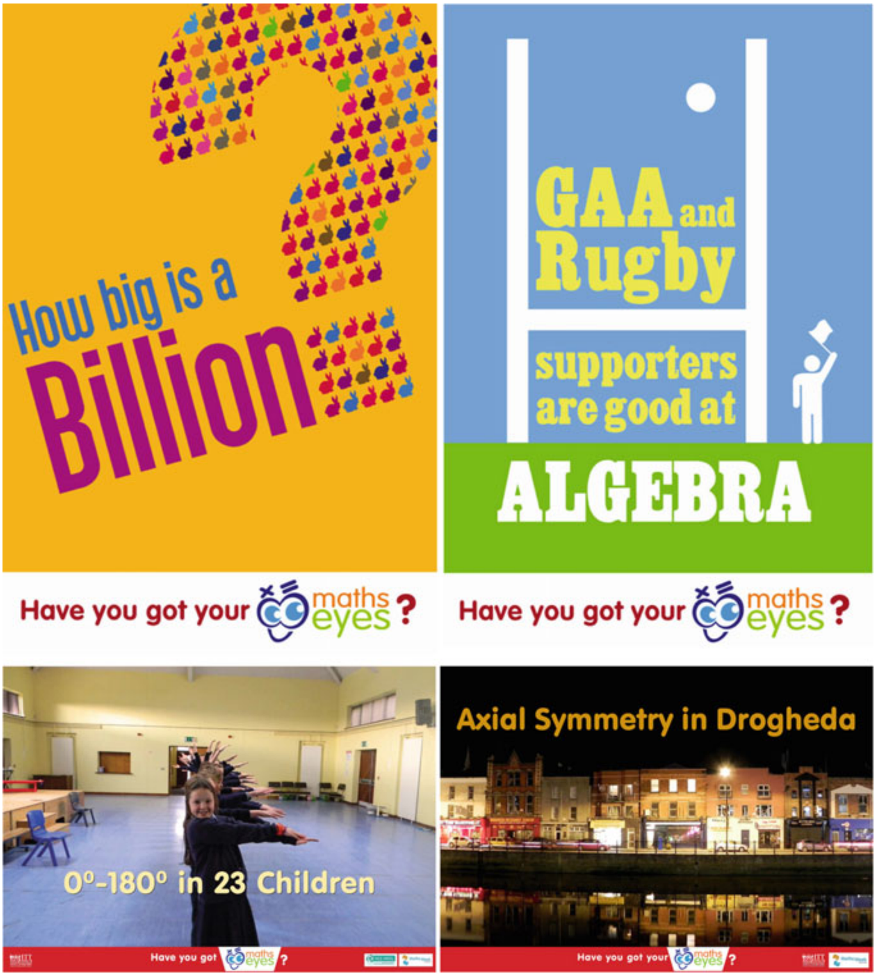
8. **Helping teachers to become evidence-based reflective practitioners**
   Approaches to professional development of teachers has increasingly been suitated around the concept of on-going individual critical reflection and evidence-based review of practice. Adult mathematics teachers need to develop and engage in evidence-based reflection on their practice to ensure that over the lifelong learning process, their approaches are renewed and refreshed regularly.

12.8 Conclusion

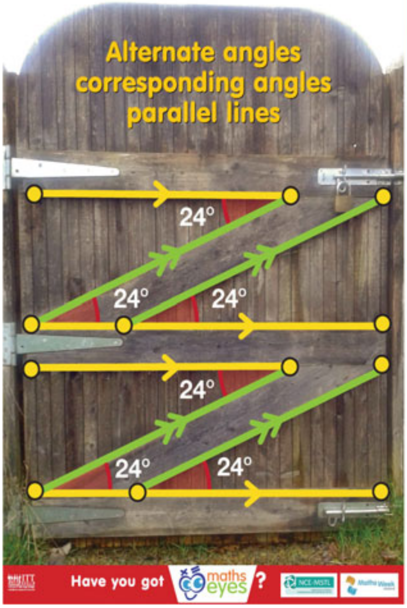
Developing an individual's maths eyes has a role to play in provision that supports adult lifelong mathematics education. Maths eyes provides an accessible entry points to developing mathematics confidence and competence that reflects the current mathematical skills and knowledge of each adult learner, it enables learners to engage in mathematics in contexts that are familiar to them. Adults lifelong mathematics education requires adults to see mathematics learning as being relevant to them. Maths eyes provides an effective approach for linking mathematics with an individual's life experience so mathematics has meaning for them in the context of their own lifelong learning journey.

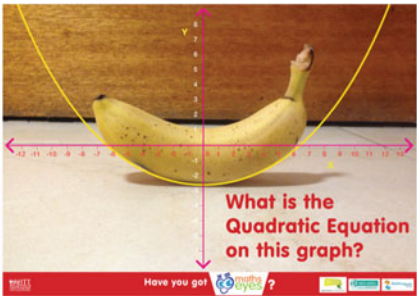
A significant challenge to enabling adult lifelong mathematics education is the need to have highly skilled teachers to support the learning process. Eight key components of professional development provision for adult mathematics teachers that will enable teachers to support adult mathematics learners to live and work in a rapidly changing and increasingly digital world has been proposed. The development of the Maths Eyes of teachers and their understanding of 'big ideas' in mathematics is regarded as a core component of effective professional development provision.

Appendix: Some Examples of Maths Eyes Posters




Symmetry in Motion
Have you got maths eyes ?
How far back is Alex?
Have you got maths eyes ?
How many teeth are in our class?
Have you got maths eyes ?
spiralling out of control
Have you got maths eyes ?
What is the modal number of our 1st Year Group?
Have you got maths eyes ?





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Chapter 13
Danish Approaches to Adults Learning Mathematics—A Means for Developing Labor Market Skills and/or for Bildung?

Lena Lindenskov

Abstract This paper presents a picture of two Danish formal instruction settings with mathematics designed for adult participants: One setting is labor market training for unskilled and semi-skilled workers (adult vocational training). The other setting is preparatory adult education (PAE) as a second chance for adults who did not acquire what is regarded as sufficient mathematical knowledge and skills in lower secondary school. The long-standing tradition of adult learning in Denmark has as its core values to empower adults and encourage them to participate in democracy and to offer instruction which appeals to participant' active involvement. The paper explores some challenges in mathematics instruction and how the challenges are met in aims, organization and national teaching guidelines in the two settings. Also, it analyzes how instruction strategies, which are recognized internationally, are mirrored in the national teaching guidelines.

Keywords Adult mathematics education · Adult vocational training
Preparatory adult education · Values

13.1 Introduction

This chapter analyzes the aims and organization of, and teaching guidelines for adults learning mathematics in Denmark. This chapter focuses on two settings: One setting is labor market training for unskilled and semi-skilled workers (adult vocational training); the other setting is preparatory adult education (PAE) as a second chance for adults who do not have what is regarded as sufficient mathematical knowledge and skills from lower secondary school. My leading research question is, how do ideas of adult mathematics education as labor market preparation and ideas of Bildung for personal development and citizenship seem to influence each of the two settings and their recent development? It may be

L. Lindenskov (✉)
Danish School of Education, Aarhus University, Copenhagen Campus, Denmark
e-mail: lenali@edu.au.dk

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immediately apparent that mathematics in labor market training is driven solely by industry needs, and mathematics in preparatory adult education is driven soleby by Bildung for personal development and citizenship, but I reveal a more complex picture.

First, the justification and Danish national aims for mathematics in the two settings are analyzed. There are significant differences between the two settings. Second, organizational aspects of the two settings are compared. Similarities in terms of access and payment questions are shown, whereas embeddedness, teacher qualifications, and the use of tests differ. Third, national teaching guidelines for the two settings are compared. The findings are summarized by discussing the complex picture of the two Danish settings' balance between labor market training and Bildung for personal development and citizenship. Information in English about public education in Denmark can be found at http://www.eng.uvm.dk/.

13.2 Theory

Theories applied in this exploration of Danish approaches to teaching mathematics to adults come from the philosophy of educational aims, and from educational theory concerning what seems to work in mathematics instruction for adults.

13.2.1 Philosophy of Educational Aims

Education is seen as a cornerstone of Danish welfare society today, even as some shifts towards neo-liberalism are evident in recent educational discourse and politics. Two aspects still make education a cornerstone of a welfare society. One aspect concerns access: free access to basic education for all is a value shared by citizens and political parties. Other aspects concern the justification problem in mathematical education on purposes, reasons, motives, and arguments for providing mathematics education to a given category of students (Niss, 1994, p. 373) with broad or narrow educational aims. The problem of justification and the problem of breath are made evident by the co-existence of two terms for education in the Danish language, *uddannelse* and *dannelse*. The two terms indicate two perspectives on education, which are perceived as fundamentally different. *Uddannelse* is a term for all kinds of institutional teaching and learning, whereas *dannelse* goes beyond the teaching and learning of facts, knowledge, and procedures. *Dannelse* always include personal and social aspects that deepen the learning and help the participant to become a person and a citizen. However, *dannelse* is interpreted slightly differently in other countries where two such terms exist, for example, the German *Erziehung* and *Bildung*, the Dutch *onderwijs* and *opvoeding*, and the Swedish *utbildning* and *bildning* (see Gustavsson 2004, on Swedish interpretations of Bildung).

The English language differs from Danish, as the term *education* covers both perspectives, but does not clearly distinguish between them. Therefore, an explicit definition of the terms used in an this English language text is important. In this chapter the English term *education* will be used as a synonym for the Danish *uddannelse*, and the German term *Bildung* will be used as a synonym for the Danish *dannelse*.

Even today, Danish interpretations of "Bildung" and "education" are influenced by the clergyman, philosopher, social reformer, and hymn-writer N. F. S. Grundtvig (1783–1872), who lived during the transition from absolute monarchy to democracy in Denmark. Grundtvig is known as a pioneer of Danish folk high schools, originally intended for young peasants. The following one of his hymns elucidates his philosophical ideas of Education and Bildung. The first verse goes like this:

> Is enlightenment just for scholars?
> No, the sky gives to all.
> Light is a gift from heaven,
> and the sun rises with the peasants,
> not at all with the scholars,
> and enlightens best from toe to top,
> who is most at work.

From the last verse:

> Enlightenment should be our desire,
> also when it is just about reeds.
> But first and last, with the people's voice:
> enlightenment about human life. (author translation[1])[2]

How are the foregoing verses to be interpreted in the contexts of adult learning in Denmark, and adults' learning mathematics in Denmark?

The use of the sun as a metaphor associated with enlightenment implies that enlightenment is essential to all, and that all, not just the scholars, have a right to it. Actually, enlightenment should start to educate the less educated, the peasants. They are the ones who wake up and go to work earlier in the morning than do the wealthy elite. So, the first idea is that adult learning should be accessible to all.

The verses say that first and last, enlightenment should be about human life, culture, and society, and about getting to know yourself and others. Most importantly to discussions on adults learning mathematics, even when you learn about reeds, enlightenment should be your desire. Grundtvig himself did not embrace

[1]In Danish: Er lyset for de lærde blot—til ret og galt at stave?—Nej, himlen under flere godt—og lys er himlens gave—og solen står med bonden op—slet ikke med de lærde—oplyser bedst fra tå til top—hvem der er mest på færde. Oplysning være skal vor lyst—er det så kun om sivet—men først og sidst med folkerøst—oplysningen om livet.

[2]This hymn was sung at the People's Political Festival 2015: https://www.youtube.com/watch?v=5scdFX-1YS4.

mathematics and aritmetics. So, the second idea is that when you acquire recognized knowledge and procedures about reeds or about mathematics and numeracy, knowledge and procedures should also become personally meaningful to you and have social significance.

Personal meaningfulness was an important part of Grundtvig's folk high schools. An emphasis on participants' emotional connection to learned content may be seen as inspired by Rousseau's ideas. Teaching and learning were mainly seen as communication with interaction, co-operation, discussion, and dialogue. Grundtvig repeatedly emphasized the importance of living interaction between past and present, between teacher and student, and among the students. La Cour's books on historical mathematics (1899) intended to motivate participants in folk high school courses to patiently dwell on numbers, geometry, and algebra by presenting concepts and tools that helped to solve problems in the historical contexts in which concepts and tools were invented. Grundtvig advocated for people's critical voices, regardless of the power in the hands of a king or of an elected Parliament. Human rights, such as freedom of speech, were seen as insufficient. A competent and enlightened population was regarded as necessary, too.

Enlightenment was—and still is—perceived as both an individual and a social process. By giving "the people" access to both new knowledge and new skills, and also to a new kind of collective self-awareness, enlightenment helped to build a bridge between "the elite" and "the people" (Korsgaard, 2002). I suggest that—for better or worse—this bridge-building remains a basis for the relatively high trust you see among people of different social classes in Danish society, between participants and teachers, and between people and politicians, compared to what is seen in other countries.

To sum up, the philosophical ideas in the present Danish approaches to adults learning mathematics that I explore are equal access for all, and the provision of conditions that make it possible for all to engage in the learning processes, making them personally and socially meaningful. Is free and equal access to education for adults still widely valued? Is Bildung part of the approaches, so that knowledge and procedures becomes personally meaningful and embrace human life, culture, and society?

13.2.2 Educational Theory of What Seems to Work: Internationally Recognized Instruction Strategies

Through the years, a fund of knowledge concerning instruction strategies that seem to work well for teachers and adult learners has been explored and compiled in publications and at *Adults Learning Mathematics* conferences (Coben & O'Donoghue, 2014). Kaye (2014) sums up part of the recognized fund of knowledge and describes how some ongoing instruction may contradict this knowledge.

Kaye's summary of existing knowledge regarding adult education first focuses on learners' existing knowledge. Although the research literature advocates taking students' experience into account, in practice this seldom occurs, and it is insufficient when it is done. Kaye's data is from Great Britain, but similar practices may be seen in other countries, too. Secondly, Kaye focuses on context. Placing calculating techniques into context is considered good practice, but some programs begin with the four arithmetic operations, proceed through whole numbers, then decimals, fractions, and percentages, and then proceed to contextualizing. Kaye underlines the need to emphasize context as particularly important, when working with students in vocational programs. Thirdly, Kaye discusses how to best organize the domain of mathematics in courses for adults. He recommends organizing mathematics courses around broad topics concerning numbers, measurements, shapes, space, and data handling, instead of conventional mathematical strands such as arithmetic, geometry, and algebra.

Organizing broader topics better allows students' experience to be taken into account and provides the opportunity to use examples relevant to students' lives or the other programs they are studying. According to Kaye, it is worth mentioning that measuring and using data are areas with which adult learners may be familiar from family or work situations. Finally, Kaye focuses on the need to prepare the students for further vocational training. Specific mathematical techniques may be sufficient in a specific vocational practice (Kaye mentions the use of the length of the radius to divide a circle into six equal parts), but may not be sufficient for further vocational training, where some vocational study plans present more generalizable mathematical methods, such as dividing the circle into 360°.

The four abovementioned instruction strategies to be taken into account are further supplemented by Marks, Hodgen, Coben, and Bretscher (2015) study of nursing students' experience of learning numeracy for professional practice. Through individual interviews with eight student nurses, light was thrown on possible disjunctures between the ways in which numeracy is taught and assessed in programs to train nurses, and the broader context in which calculations are carried out by nurses in practice.

A fifth recognized instruction strategy is the need to explicitly explore and relate to authentic mathematics. The construct "invisible mathematics" (Coben, 2002, p. 55) is a specific challenge for adults learning mathematics. The construct refers to "the mathematics one can do, which one does not think of as mathematics—also known as common sense." When giving the same kind of drug many times, then you may overlook that mathematics is involved. Supporting the development of skills as they will be used in professional practice is regarded as good vocational training. But how to teach in order to develop these skills is challenging for mathematics instructors: It is difficult for the instructors to understand how math-containing tasks are handled in professional practice.

Marks et al. (2015) emphasize that the need for authenticity relates not only to the given tasks, but also to the broader context of nursing calculations, including the use of colleagues' advice, the use of a calculator and other machinery, estimation, and the safeguards of double and triple checking. Programs for future nurses should explore practicing nurses' methods in greater depth, and investigate the mathematics underlying these, in order to develop a deeper understanding of the mathematics actually used in practice. Then the job-related invisible mathematics would be visible. Authentic medications, and tools such as protocol books, reference materials, and equipment should be used in mathematics programs for nurses. Finally, new technology needs to be continuously taken into account. These recommendations with respect to a very broad perception of authenticity are further supported by Wedege's work (2010). Wedege has found that research into adults' mathematics in the workplace is limited and the fund of research-based knowledge of authentic mathematics *in* work is limited. Not much is actually known about mathematics in work per se, because most research is done for the sake of instruction and learning mathematics *for* the workplace. This may be a reason for some instruction's focus on formal mathematical ideas and techniques, and their use in some semi-authentic tasks.

The five above-mentioned, recognized instruction strategies focus on learners' existing experience and knowledge, on contextualization, on how to organize the domain of mathematics, on the need to ensure the necessary prerequisites for further instruction, and on the authenticity problem. Ginsburg (2012) adds to these five strategies, by elaborating remarks on learners' experience and existing knowledge, as he found that in many programs, adults' experiences are thematized as problematic deficits. Ginsburg notes the need to recognize, acknowledge, and address prior negative experiences, too, and their resulting negative effects. Ginsburg also adds that the goal of uncovering and building on students' experience and existing mathematical knowledge is to help them to make meaningful connections between the existing and the new knowledge. Finally, Ginsburg addresses not only the individual student's funds of knowledge, but also those of his/her surrounding communities.

Also, Ginsburg adds three more instruction strategies. The strategies added by Ginsburg concern meaningfulness, collaborative work, and ICT. One further recognized strategy—number six—is to help the learner to expect that the mathematics to be encountered and learnt will make sense to the learner. The learner has the right to be able to see the mathematics as meaningful. Another strategy that Ginsburg adds—number seven—is to arrange classroom activities for collaborative, small-group work. Finally—number eight—Ginsburg mentions the use of ICT to support adults learning mathematics.

In the following sections, the research questions are explored together with the educational philosophy concepts and the eight instructional strategies. The research is based on a range of documents, from official national curriculum and teaching guidelines and reports, to information for students and potential students on institutional websites. No observation data from classrooms or shopfloors is included.

13.3 Analyses

I use the above-mentioned philosophy of educational aims and educational theory to investigate education for adults in the two Danish settings available. First 13.3.1–13.3.3, I investigate the justification and Danish aims for mathematics: Do they seem to promote Bildung, or just education/instruction, and education/instruction for what? Then 13.3.4–13.3.8, the organizational aspects of the two settings are compared.

13.3.1 Justifications for, and Aims of PAE

Let us first briefly consider the general adult education system. General adult education aims to strengthen adults' opportunities for further education, and offers the prerequisites needed for active participation in a democracy. Consequently, it is expected that students will be able to apply their acquired knowledge and attitudes to act in local and global contexts. General adult education includes all subjects from compulsory school, and examination materials are equivalent to those for compulsory schools, but teaching and examination materials are designed to accommodate, and produced for, adult participants (Ministry of Education, LBK 1073, 2013). All kinds of general adult education are public, and offered to all who are at least 18 years old, and wish to improve their general knowledge. Teenagers under 18 may participate only if this education is included in the participant's so-called "education plan," agreed upon by both the participant and a representative from a public educational or social institution.

Since 2001, the most elementary levels of Danish and Mathematics in general adult education have been organized under the heading of "Preparatory adult education," or PAE (In Danish: "Forberedende voksenundervisning," or FVU). PAE—preparatory adult education—offers only two subjects: Danish and Mathematics. PAE is available to all adults who are 18 or older, who wish to improve basic in reading, writing, spelling and mathematics skills, and are pre-tested to have an appropriate skill level. Teenagers under 18 years of age may participate only if they also participate in adult vocational training or in a company-addressed program, or participate in training in institutions run by the Ministry of Justice (e.g. prisons).

PAE is regulated by different legislation than general adult education, and has specific other aims. PAE's aims include the idea of active participation in society. PAE aims to provide instruction that enables adults to improve and supplement their basic skills in reading, spelling, and written work, and number skills, arithmetic, and basic mathematical concepts related to qualifying for further education and active participation in all aspects of society. Participants receive a certificate for the mathematics programs after passing a national digital exam (Ministry of Education, LBK 96, 2017a).

The aim of PAE mathematics is described as assuring participants the opportunity to recognize, improve, and supplement their functional arithmetic and mathematical skills. The program improves the participants' ability to oversee, handle, and produce math-containing data and materials. The goal is described as improving participants' numeracy, which consists of the functional mathematical skills and understanding that, in principle, all citizens need (Lindenskov & Wedege, 2001). Participants' individual, working methods for calculation and problem solving should be used and in evidence in the programme's teaching lessons (Ministry of Education, BK 1776, app. 4, 2017b).

PAE consists of two stages. PAE Stage 1—*Figures and quantity* has a duration of minimum 30 h and maximum 60 h. The aim is:

> that the participants develop their numeracy by clarifying, improving and supplementing their number sense and functional arithmetics for everyday practical use and personal organization. The education is to ensure participants the possibility of developing their mathematical awareness to overview, process, evaluate, and produce math-containing information and materials, as well as being able to communicate about these things.

PAE Stage 2—*Patterns and Connections* has a duration of minimum 45 h and maximum 60 h. It includes includes also Form, Dimension, Data and Chance. The aim is:

> that participants further develop their functional skills to interpret, produce and reflect numerical, statistical and graphical information as well as the ability to communicate this knowledge. The teaching ensures the participants the possibility of further developing their mathematical awareness for instance as preparing for possible further education.

13.3.2 Justifications for, and Aims of Adult Vocational Training

The Danish system of adult vocational training (in Danish: "arbejdsmarkedsuddannelser" or AMU) is public and provides short training programs for skilled and less-skilled workers on the labor market. The system is directed at societal requirements for basic labor market skills. The aim is to maintain, develop, and improve participants' qualifications in accordance with labor market demands and to contribute to participants' further skills development. The aim is also to help alleviate transition and adaptation problems in the labor market, in line with labor market needs in the short and longer term. Finally, the aim is to provide adults with opportunities to improve both professional and personal skills. Participants receive a certificate in the mathematics programs after being assessed by the teachers. There are no exams (Ministry of Children, Education, and Equality, LBK 226, 2014).

The 2-day *basic vocational arithmetic* (in Danish: "Grundlæggende faglig regning") is one of approximately 2900 programs (Ministry of Education & Møller, 2014a). The goal of basic vocational arithmetic is described as:

> Participant strengthens and develops his/her skills in vocational arithmetic related to solving vocational tasks in industry or related to adult vocational education. (author translation)

The 3-day *basic vocational mathematics* [in Danish: “Grundlæggende faglig matematik”] is another program (Ministry of Education & Møller, 2014b). The goal of basic vocational mathematics is described as:

> Participant strengthens and develops his/her skills in vocational mathematics and geometry related to solving vocational tasks in the industry or related to adult vocational education. (author translation)

13.3.3 Summary of Aims

The two settings differ in relation to aims and justification: Bildung is included in the official aims and justifications for PAE by the terms of the guidelines that mention active participation in society. PAE does not exist just to improve qualifications for subsequent education. Adult vocational training aims to create opportunities to improve personal skills, as well as professional competence, but personal skills may be improved as a side effect of the opportunities in the program to obtain basic formal competence.

It is beyond the scope of this paper to analyze mathematical concepts and tools included in the two settings. Just this: although some mathematical concepts and tools are included in both settings, concepts and tools are presented and utilized differently, as different contexts are relevant for the two settings.

13.3.4 Access and Payment

The government covers almost all costs to both Danish adult education systems by redistributing what citizens and enterprises pay in taxes. The Danish labor market model implies that much work is regulated by collective agreements among social partners. As part of the collective agreements the social partners agree to establish an economic fund for skills development, which may cover educational fees, transportation, materials, and wages, when employees attend adult education. Businesses pay EUR 30–150 into the fund annually for each full-time employee, and any kind of adult education enterprise may receive economic support from the fund for. For participants, all programs are free, or cost a maximum EUR 35 per day for adult vocational education, and EUR 150 for a whole program of general adult education. All programs are free for unemployed adults when these programs are agreed to by a public job centre. Unemployed, employees, and self-employed adults may keep their normal social benefits or salary, or they may apply for public educational support, set at a maximum of EUR 90 daily.

Of special interest in this chapter on mathematics is that all mathematics programs in both Danish adult education systems are free for all participants. That programs—part-time and full-time—for adults are mostly paid by the government is in line with attitudes of Danish citizens. In 2011, a European survey showed that Denmark was the European country with fewest people factoring cost into their decision about vocational education and training (30%). Also, in 1995 a European survey showed that 59% of Danes thought the state should fund continuing education programs. Incidentally, this was the second highest level in Europe (Europeans and their attitudes to education and training) (http://ec.europa.eu/public_opinion/archives/ebs/ebs_112_en.pdf).

The conclusion is that free and equal access to education is still a widespread value when it comes to Danish approaches to adults learning mathematics. All mathematics programs in PAE and in adult vocational training are provided free of charge to the participants. Incidentally, other programs are paid mainly by the government, with only minor participant contribution.

The conclusion is that the two settings, PAE and adult vocational education, are similar with regard to access and payment structure.

13.3.5 Embeddedness of Mathematics Programmes into Other Programmes?

PAE consists of a number of reading/writing and mathematics programs. Mathematics is organized as two separate programs. Neither is embedded in other programs.

Around 2900 adult vocational training programs exist, and around 200 new programs are developed each year, in cooperation with the social partners and the Ministry of Education. Enterprises may supplement this by purchasing programs that are not approved by the Ministry, from other providers. Unlike PAE, mathematics is embedded in almost every program, in one way or another. Estimation, exact calculation, and assessing whether calculated results are reasonable for specific contexts are key skills when addressing work-related questions such as, which kinds of, and how many materials should be used? How may the quality of working processes and products be ensured? How should offers tailored customers' needs be designed? What about safety regulation, wages, time plans, days off, holidays? How should technical aids be used? How should figures and symbols in manuals and on machinery be interpreted?

Part of the educational philosophy of Danish adult vocational training is that relevant mathematical competence and numeracy are best motivated and learnt when they are embedded in vocational contexts. This is in line with the above-mentioned, recognized instruction strategies that advocate learning mathematics in authentic contexts. In order to support embedded mathematics, the Ministry of Education has developed two mathematics programs. Parts of them may be used to qualify embedded mathematics, but the 3-day *basic vocational mathematics*, and the 2-day *basic vocational arithmetic* may be used as individual programs.

The conclusion regarding the embeddedness aspect is that in adult vocational training, mathematics is embedded in perhaps thousands of programs, and there are two individual programs of non-embedded mathematics: Basic vocational arithmetics; basic vocational mathematics. In PAE, mathematics consists of two non-embedded programs: PAE Stage 1—*Figures and Quantity*. PAE Stage 2—*Patterns and Connections, and includes also Form, Dimension, Data and Chance*.

13.3.6 Teacher Qualifications

For adult vocational training, teachers must complete a minimum of vocational education degree program, and 3 years of relevant employment in order to start as a teacher. During the first 4 years as adult vocational teacher, you must complete a 1-year, full-time degree in andragogy at university college or university. The Ministry of Education recommends that the degree includes two programs called “Adult numeracy” and “Adults’ mathematical difficulties.”

For preparatory adult education, teachers must complete a minimum of a 4-year, full-time teacher education degree program at university college, and a four-month full-time degree program at university college or university, including the two programs, “Adult numeracy” and “Adults’ mathematical difficulties.”

I conclude that the required teacher qualifications for the two settings of Danish adult education differ. But the intention for each setting is that the teachers’ formal qualifications will provide relevant skills to understand and act on the setting’s official aims and recommended instruction strategies, and to engage in contextualized mathematics which is relevant to the setting.

13.3.7 Testing

The use of testing differs in the two Danish adult education settings. Before beginning PAE, participants complete a written pre-test to determine whether participation in PAE programs would be beneficial and relevant, and to determine the correct initial level and class. Many adult learning centers offer weekly pre-tests, and programs start no later than 1 month after test referral.

PAE participants receive a certificate in the mathematic programs after passing a written ICT-based national exam.

Participants in adult vocational training may complete the same pre-test as is used for PAE, but it is not compulsory, as it is for PAE. According to a law passed in 2007, all participants in adult vocational training are offered an assessment of their basic skills in mathematics. The aim of this is to plan programs and materials for the individual participants, to guide the participants to relevant mathematics programs in adult vocational training and in PAE, and to guide the adults towards a relevant combination of adult vocational training and other offerings, such as PAE.

Many adult vocational training programs with embedded mathematics award a certificate to successful participants. Participants in approximately 120 adult vocational training programs, may take an exam and receive a recognized, formal, qualifying certificate, such as "crane operator." Participants in the two un-embedded mathematics courses, basic vocational mathematics, and basic vocational arithmetic may also receive a certificate after being assessed by their teachers; there are no exams. After the program is completed, all participants and some of their employers are asked to evaluate the program using an online questionnaire at www.viskvalitet.dk.

13.3.8 Summary of Organizational Aspects

The two settings for Danish adult education are similar in terms of access and payment framework, as both are free. The two settings differ in terms of testing. Access to PAE is determined by a compulsory pre-test, whereas a pre-test is recommended for adult vocational training, although it is not compulsory. Also, examination tests differ. The two settings in terms of embeddedness. While in PAE no mathematics programme is embedded in other programmes, some mathematics programmes are embedded and others are not in vocational training. The two settings differ in terms of formal teacher qualifications, as the Ministry of Education requires 3 years of relevant employment in order to start as a vocational teacher, but not as a PAE teacher. The following full degree program at university college must for PAE teachers include "Adult numeracy" and "Adults' mathematical difficulties," which is not required, but recommended for vocational teachers.

13.4 Evaluation of Instruction According to Internationally Recognized Instruction Strategies

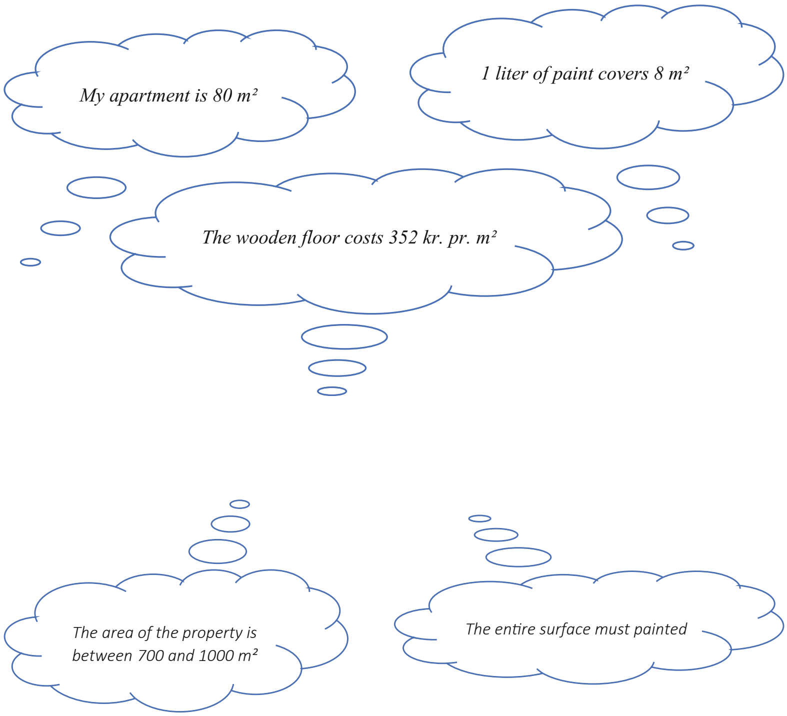
For both Danish adult education settings, national teaching guidelines are established and authorized by the Ministry of Education. The following exploration includes how the internationally recognized strategies are included in the national teaching guidelines:

- Take participants' experience into account, and expect the participants' to make meaningful connections between their existing knowledge and new knowledge
- Place techniques and tasks into contexts
- Organize mathematics courses around broad topics
- Ensure vocational training gives the necessary mathematical prerequisites for further training and education
- Explore authenticy *in* practice of tasks, technical tools, and collaboration structure
- Help the participantls to find that the mathematics to be encountered and learnt makes sense to them
- Arrange classrooms activities for collabative, small-group work
- Let ICT support participants' leaning

Participants' experience is taken into account in the guidelines for both Danish adult education settings. Participants' existing knowledge is a primary focal point of the teaching guidelines Repeatedly assessing participants' experience is recommended. It is explicitly stated that participants' experiences should be thematized as potential, and not just as problems. This concurs with Ginsburg (2012), who found that in many programs, adults' experiences are thematized only as problematic deficits. Both participants' difficulties and their strengths should be identified according to the national guidelines. Teachers should emphasize what participants already know and can do, and teachers should help the participant to recognize it him/herself. Participants need the teacher's support to describe why they find some tasks difficult. Participants's difficulties vary significantly, although some difficulties are frequently seen. In response to a common participant difficulty, teachers often focus on how to sort information and textual problems, and they often let the participants color the passages that relate to the questions to be answered.

Techniques and tasks are to be put into contexts, as contextualization is a focal point of both Danish adult education settings. In PAE and adult vocational training, teachers recognize that mathematics is everywhere, and may serve as a tool in many everyday situations away from, and at work. The frequent use of embedded mathematics in adult vocational training shows the serious concern for contextualization. Whether embedded or not-embedded, when vocational trading instruction includes for instance diagrams, they should be chosen and interpreted in a way that is relevant to the industry in question. In PAE, personal context is included. For example, in a teacher-written instructional text, the concept of area is preceded by the statement, "We often talk about area without using the term. We may say that my apartment is 80 square meters, the wooden floor costs DKK 352 per square meter, the entire surface must be painted, 1 L of paint covers 8 square meters, the area of the property is between …." Participants are encouraged to identify the words that indicate that we are talking about area, and to sense the magnitude of the area by drawing a square centimeter and a square decimeter. Societal contexts are also included, such as the statement that Denmark is a small country, followed by the question, how big is Denmark?

In a PAE instruction material it is illustrated in the following way:
We often talk about areas without using the word:



1. *Which words indicate we are talking about area?*
2. *Denmark is a small country. What is the size of Denmark?*

(Nielsen, 2000, p. 67).

The domain of mathematics is organized differently in the two Danish adult education settings. As mentioned earlier, organizing broader topics instead of conventional mathematical strands is recommended in the international literature, to better allow examples to be relevant to participants' lives or the other programs they are studying.

In PAE, two programs in mathematics exist. The two programs are called "figures and magnitudes" and "patterns and connections," which show the intention of organizing mathematics in broader topics. The mathematics in each program is further described based on social activities that implies the use of mathematical concepts and procedures. These social activities are drawn from Alan Bishop's work (1988) about counting, measuring, locating, and playing. Each program is also described with authentic data, for example about time, price, and weight. Finally, some conventional mathematical operations, concepts and discourse are chosen based on empirical investigations into authentic contexts: such as specific fractions and percentages, and such as bigger than or equal to.

In adult vocational training mathematics is organized as classic disciplines and as broader topics. Basic vocational arithmetic is described as a list or menu of vocational texts and tasks: calculate (add, subtract, multiply, divide), use ICT tools and estimates, use percentage (per 100) and per mill (per 1000), calculate fractions, use proportional reasoning, understand measurements. Basic vocational mathematics is described as a list of vocational texts and tasks: solve mathematical tasks, know and recognize geometric shapes, use formulas, read and understand the meaning of prefixes, interpret a variety of diagrams.

The vocational guidelines take into account the location where learning takes place. Even the un-embedded mathematics programs are not always located in a classroom. The guidelines refer to the fact that teachers value the fact that programs may be located among cranes and other machines and materials available at the vocational school site. A number of vocational training teachers were interviewed when the guidelines were formulated. Many citations from these interviews are included in the guidelines. A citation from a teacher goes: "It is important for us in all possible ways to show the participants that mathematics equals reality."

Vocational training aims for giving the necessary mathematical prerequisites for further training and education, so does PAE. This is in line with Kaye's fourth recommendation. PAE Stage 2 *Patterns and Connections* program and Basic vocational mathematics include generalizable methods, and may be directed more or less at further training and education, depending on the participants' further progress in a vocational or other program.

The fifth question of authenticity is challenging for PAE and for adult vocational training, as most research is done for the sake of instruction and learning mathematics *for* the workplace. The intentions for both adult learning settings is to address authentic use of mathematics *in* the workplace How is it possible to address the disconnect between the taught and learnt in programs, and how problems are solved by calculating and interpreting *in* practice? Each of the two adult learning settings has its specific potential: In vocational training it is relatively easy to find authentic contexts and teachers and participants may explore what is actually used *in* the workplace. Vocational training centres have their own workshops with cranes and other machines and materials, and teachers and participants are invited to bring their own examples and techniques. In PAE the teachers and participants are free to choose contexts, questions, and tools that they personally find motivating and interesting, and teachers encourage participants to bring problems and materials *in* their own lives to the course. Most PAE courses are held at adult learning centres and vocational training centres, but economic incentives are given by the government for PAE courses held at factories, other workplaces, associations, and culture houses (Ministry of Education, 2011). Ginsburg identifies meaningfulness as a sixth point. Institutions and teachers must help the participants to expect that mathematics will make sense to them. This is stressed in all information material on vocational training and PAE in Denmark. According to the teaching guidelines, Danish vocational teachers state that instruction includes the reasons that specific calculations are important. The teachers say that they illustrate to the participants how what they learn is important and relevant to their specific industries, or to the specific vocational training programs they wish to attend. They say that mathematics should not be abstract, but concrete and tightly connected to their professions. So, although there is a case for rapid recall of mathematical facts, no participant should calculate just to calculate.

Specific calculations—for instance, the area and volume of specific geometric shapes—do have an effect on the number of resources being used in practice, on waste, on safety, on economy. Unskilled and semi-skilled workers may be responsible for making their own decisions, and for participating in decisions made at Danish workplaces, characterized by minimal hierarchy and autonomous tasks. Mathematics may very well be part of decision-making.

A seventh point relates to Ginsburg's recommendation to arrange classroom activities for collaborative small group work. This is recommended in both adult education settings' guidelines. An example from basic vocational mathematics suggest letting participants draw specific geometric shapes, insert measurements of lengths, angles, and so on, and finally, present and compare their drawings with other participants' drawings. This is meant as preparation for calculating the area of a trapezoidal building lot, or a metal plate or tabletop; calculating the circumference of a concrete pipe, or the length of a triangle's long side when cutting fabric for clothes; calculating the weight of various objects. It is noted that the abstractness of formulas is difficult for many participants, so it is recommended that teachers make formulas meaningful by letting the participants set up formulas for everyday materials, for instance, the average weights of a number of materials, and the average height of the participants. Participants should also be allowed to choose formulas for problems they have encountered in various circumstances and areas of specific concern, and to explain visual images (graphs) to each other.

A final point relates to the recommendation in the guidelines for both adult education settings to use ICT to support adults learning mathematics, and to learn what is relevant in practice.

To sum up, the teaching guidelines are in line with internationally recognized teaching strategies.

13.5 Recent Developments: From Citizens' Right to Education, to Citizens' Right to, and the Requirement for Education

In recent years, mathematics has been increasingly recognized as a basic skill relevant for all in the 21st century. This conviction is evident in the privileged status given to mathematics, and to reading and writing the Danish language. Basic mathematics programs in PAE and vocational training are free to all, and it participation is now a citizen's right, as it is a citizen's right to vote and to receive social benefits as pensioner. In recent years, not only are free basic mathematics programs a citizen's right, basic mathematics competence is also a requirement that citizens have to meet. One type of requirement concerns access to upper secondary education. New requirements have been introduced for teenagers. For teenagers to be allowed to start any educational program at upper secondary level, minimum scores must be achieved on national lower secondary written and oral exams. Also, minimum final grades assigned by the teachers have been introduced. These requirements apply specifically to the two subjects of mathematics and Danish, and not to subjects such as history, geography, and German. Also, there are special requirements for unemployed Danish citizens under 25 years of age, and unemployed citizens below 30 years of age. The so-called active labor market policy for unemployed citizens has several general objectives:

– to assist jobseekers to find a job;
– to offer services to private and public employers who are looking for workers, or wish to retain their workforce;
– to help persons who are receiving social assistance, or the so-called "start help" for refugees and immigrants to find a job quickly, so that they are able to support themselves and their families;
– to help persons who, owing to reduced ability to work, have a special need for assistance in finding a job.

The Danish system differentiates between two groups of unemployed citizens. The first group comprises unemployed, uninsured persons, and they may be entitled to receive social benefits administered by the municipalities. The second group comprises persons insured under a voluntary scheme administered by the unemployment insurance funds. Most funds are connected to labor unions, but unemployment benefits from the funds are also largely financed by the State. The second group is assured economic benefits for a maximum of 2 years of unemployment.

Both the public employment service and the insurance funds are responsible for measures related to unemployed persons who are receiving unemployment benefits. According to social welfare laws, since 2014 both groups of unemployed citizens are obligated to pursue some form of education. This means that a citizens' right to education is supplemented by state requirements that for citizens to actually pursue education, if they are under 25 years old and lack vocational or further education.

Unemployed persons between 25 and 30 years old must meet with the municipal authorities for an overview of possible further training, and the unemployed may be told to start an educational program or communal job as a prerequisite to obtaining social benefits. In particular, when citizens under 30 years of age do not meet the requirement of the minimum scores mentioned above, they will be told to start and complete preparatory adult education in mathematics or some adult vocational training programs with mathematics embedded, or a single non-embedded program.

The new requirements are challenging for both citizens and adult learning centers and vocational schools. When an adult who did not succeed in school mathematics as a child is forced into a mathematics program, possibly against her will, the institutions providing the mathematics program face huge challenges in terms of motivating such adults to start and to complete the programs. How does one motivate these adults to actually go into the classroom and to engage mathematics, which may have previously caused them trouble, and hurt their self-confidence? The new political requirement puts the educational institutions in a completely new situation, compared to the previous political framework that presented mathematics programs as a right, and offered equal access and free education/training to all.

It is evident that Danish adult education institutions and participants are challenged. Many participants have reacted by choosing to not participate, and not complete programs. Education institution websites show some of the institutional reactions. First the free access is emphasized:

PAE is freely available to all adults, employed or unemployed, who wish to become better at reading, spelling, writing, understanding numbers, and arithmetic and learn mathematics. (author translation)

The broader future possibilities are then underlined:

You will be allowed to start a vocational education or a (theoretical) upper secondary education, when you complete all programs of preparatory adult education in mathematics and in Danish. You will be allowed to start higher education, when you complete theoretical upper secondary education will allow you to go on to higher education. (author translation)

A website targeting citizens receiving social benefits says:

Programs in adult vocational training give your skills a boost and make you more attractive as job applicants. At the same time you will develop a clearer sense of the direction you wish to take. When you have a clearer sense of your desired direction, it is easier to determine which skills you need. (From: http://www.veucentervest.dk/jeg-er-paa-kontanthjaelp.html) (author translation)

Thirdly, flexibility is underlined:

You can follow instruction in one are both subjects in PAE. You can combine it with vocational training or education. We assist you and your employer in determining how to combine and review financial matters. (author translation)

In some cases, specific programs are addressed:

PAE Danish language program is for those who want to become better at writing, reading, and spelling at the workplace and to help your children with their homework. The PAE Mathematics program is for those who want to become better at arithmetic, to know everyday numbers, to learn basic mathematical elements, and to help their children with their homework. (www.veucenterost.dk/shared/fvu-forberedende-voksenundervisning.html) (author translation)

In some cases, a website has businesses as its target audience:

You will find that it will be easier for your employee to complete documentation tasks and everyday calculations such as waste calculation, percentage calculations, material use etc. (www.vucvest.dk) (author translation)

Individual participant narratives are also shown. Many of the stories start with general comments, such as the foregoing, concerning reading and numeracy skills as prerequisites for most education, and as facilitators of everyday life at work and at home. Also, information about the opportunities for adults to enroll in different kinds of courses is given. Then, the personal aspects of the stories follow:

Fortunately, even when you are an adult it is possible to improve reading and writing skills. PAE participants, Carina Langholz and Victor Reipur, attend PAE in order to be able moving toward their educational goals.

Carina Langholz had no further education or training after leaving compulsory school. Now she wants to train to become a pre-school teacher or a social worker. A prerequisite for this is the completion of a specific training program. Furthermore, before entering the required training program, she must improve her skills and knowledge in mathematics and Danish language at PAE.

Carina Langholz enjoys studying again. She says "It is different, because the participants are adults. The atmosphere is pleasant, and we are being respected and taken seriously. I am super happy to get skills that make me able to start an education."

Five years ago, Victor Reipur dropped out of compulsory school without taking any exams. He has tried to follow a variety of programs without success. After that, he started at PAE, which seems to be a better fit for him. Victor Reipur says: "Here at PAE I come everyday to learn something. You need to have an education. I actually enjoy going to school and learning again. Next summer I will start general adult education, corresponding to compulsory grade 9. Then I wish to continue at grade 10, and then at grades 11 and 12 in higher secondary school [Danish: HF]. I don't yet know what I wish to be, but I might dream about a technical education related to space research." (http://www.veu-midtost.dk/shared/i-skole-som-voksen.html) (author translated)

13.6 Summary

The two settings for adult education in Denmark offer opportunities to improve basic and/or vocational mathematics skills, conceptual mathematical understanding, and problem-solving skills at no cost. Organizational aspects such as access and payment are the same in both settings. Access to mathematics programmes is free for all adults who can benefit from the programmes and the programmes are free of charge. Embeddedness issues, the use of testing, and teacher qualifications, differ. Teacher qualifications include specific full degree program at university college as requirement or as recommendation.

The philosophy of education for all seems to be met by the PAE and in the Danish adult vocational training system, when any adult wishes to upgrade his or her mathematics skills for any reason, or in order to improve labor-market-oriented skills.

The philosophy of Bildung is less evident in the nationally stated aims and justification. Active participation in society is included in PAE aims. In adult vocational training, only the development of personal skills is included, which may just be for the sake of industry. Looking at explicated aims and justifications, it is clear that mathematics instruction for labor market training is directed solely at industry needs, whereas mathematics in preparatory adult education is directed at Bildung for personal development and citizenship, and towards further training and education.

The existence of national aims does not ensure that they are deployed. This is mirrored by the three-fold character of intended curriculum, implemented curriculum, and learnt curriculum (Bauersfeld, 1980).

A complex picture emerges when considering teaching in Denmark. To a large extent, the teaching guidelines are in line with internationally recognized instructional strategies. The focus on meaningfulness, personal experiences of mathematical difficulties, collaboration, and relevant contexts in working life and private life suggest that Bildung is possible in the two settings. Mathematics should not be only some symbols separate from human life, culture, work, and society. Mathematics programs are also about knowing yourself and others. Learning about reeds, as in Grundtvig's strophes, or learning mathematics, numeracy and procedures, acquiring knowledge, must be personally meaningful, and have social significance.

The recent development where obtaining mathematics skills is not only a citizen's right, but also a citizen's requirement introduces new features for lifelong learning and give rise to new questions to be handled by participants, teachers, government, social partners, and researchers: Will participants' motivation to participate in mathematics programs be undermined or strengthened if you participate, not of own free will, but in order to avoid losing social benefits? Will teachers focus more on labor-market training or on Bildung? Will government and social partners interfere with organizational aspects and the use of internationally recognized teaching strategies? And how will research investigate the risk of tipping the balance between labor-market training, and Bildung for personal development and democratic citizenship, in favor of the former.

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Part IV
At the Crossroads—Overarching Themes

Chapter 14
A Tale of Two Journeys

Barbara Miller-Reilly and Charles O'Brien

Abstract Two decades ago we met: Charles, a young business man needing assistance with a debilitating fear of mathematics; Barbara, an experienced teacher of maths-avoidant adults, in the early stages of research for her doctorate. Both of us were embarking on big challenges. An initial six-month course, set up for our mutual benefit, enabled Charles to progress from viewing mathematics as "the most disgusting, unappealing building" to one "with form, balance and symmetry" and, on the other hand, the metaphors gathered from Charles became an illuminating part of Barbara's Ph.D. thesis (Miller-Reilly in Affective change in adult students in second chance mathematics courses: Three different teaching approaches. University of Auckland, Auckland, New Zealand, 2006). Recently Charles asked Barbara to teach him again, trying to meet the mathematical prerequisites for entry to a post-graduate business degree. The aim of this paper is for each of us to reflect on our respective journeys over two decades. Each of our narratives is presented in four sections: firstly, relevant background experiences before we met; secondly, two decades ago when the six-month course occurred; thirdly, some recent study; and finally, our reflections over two decades.

Keywords Adult · Mathematics · Education

14.1 Introduction

Two decades ago we met: Charles, a young business man needing assistance with a debilitating fear of mathematics; Barbara, an experienced teacher of maths-avoidant adults, in the early stages of research for her doctorate. Both of us were embarking on big challenges. A one-to-one course of seventeen sessions spread over six

B. Miller-Reilly (✉)
University of Auckland, Auckland, New Zealand
e-mail: barbara@math.auckland.ac.nz

C. O'Brien
Business Consultant, Auckland, New Zealand

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months, set up for our mutual benefit, enabled Charles to progress from viewing mathematics as "the most disgusting, unappealing building" to one "with form, balance and symmetry". On the other hand, the metaphors gathered from Charles about his personal conceptions of mathematics became an illuminating part of Barbara's Ph.D. thesis (Miller-Reilly, 2006). Recently Charles asked Barbara to teach him again, trying to meet the mathematical prerequisites for entry to a post-graduate business degree. The aim of this paper is for each of us to reflect on our respective journeys over two decades.

What could be the possible theme of this joint paper? Charles and Barbara met to discuss this issue. Charles' initial reaction: it could *not* be two decades since we first met! It felt "like yesterday"[1] because "it's real". The knowledge gained then is "applied everyday". Charles said he feels calm now when mathematics arises whereas he would have felt fear and panic. The six-month course had been profoundly beneficial with long-term benefits. It was like "facing a demon", he recalled, not a phobia because it was "not specific enough". It was "all encompassing" because mathematics "is in everything". It was not a "disability", this would be an "overstatement" because he has "ability". It took hard work, he faced "the struggle head on" and to address it was "massive" for him. Other people need this opportunity also, Charles emphasized. There would be "massive benefits" if these teaching methods ("professional skills and knowledge; innate ability to connect as a person; ability to understand the problem") were delivered by teachers with patience, enthusiasm and kindness. He remembered that Barbara did not say 'do more' or 'apply yourself more', that is, you just need to work harder—he had experienced this advice and believes it is a myth! He believes that adult numeracy is very different from adult literacy and that both need to be addressed—Barbara agreed. She recalled his incredible fear and how it seriously affected his career. Then also she remembered her amazement at his ability to express his fears as graphic metaphors and how this inspired her research which she hoped would inform further generations of teachers.

Four topics emerged from this meeting: our concern for all the other adults (and children) who have little mathematical confidence and competence and often suffer from the debilitating effects of this lack; he remembered the helpful characteristics of her teaching method and emphasized that others would need this type of teaching [described in Miller-Reilly (2008)]; he tried to describe the nature of the difficulties he had experienced which had motivated him to seek help two decades ago; and we talked about the long term benefits of our work together.

Each of our narratives is presented in four sections: firstly, our relevant background experiences before we met; secondly, two decades ago when the six-month course occurred; thirdly, some recent study; finally, our reflections over two decades. Overall conclusions complete the paper.

[1]Quotes in this section are from notes taken at our discussion.

14.2 Literature Review

For many adult students "a major life change, transition, or developmental task is probably involved in the decision" to return to study (Smith, 1990, p. 50). This is a rich source of their "readiness to learn", one of the six key assumptions underlying andragogy (Merriam & Brockett, 1997). Merriam (2005, p. 5) suggests that "our adult life can be seen as a structure consisting of periods of structured maintenance and stability, alternating with periods of change and transition" and that "all types of transitions hold a potential for learning and development" (ibid, p. 4). A common theme among adults learning mathematics is that it is a "means to achieve future change" (FitzSimons & Godden, 2000, p. 19). This paper investigates specific transitions in the life cycle: for Charles, tackling his debilitating fear of mathematics; for Barbara, undertaking the huge task of a Ph.D.

Drawing on their theoretical study of women's development and ways of knowing, Belenky, Clinchy, Goldberger, and Tarule (1986) developed the theory of connected teaching, which Renne (2001, p. 166) succinctly describes as a "shared conversation in which the teacher and students collaborate to jointly construct new understanding". Morrow and Morrow (1995, p. 19) discuss the aims of connected teaching in mathematics: students' "confirmation of self in the learning community"; "learning in a believing mode of communication and questioning"; "taking on challenges with support"; "gaining a sense of their own voice in mathematics"; and "becoming excited about possibilities of posing their own problems and inventing new knowledge". These ideas inform their SummerMath program for high school students. Jacobs and Becker (1997) suggest that connected teaching will help students who have not previously been successful in mathematics. Barbara's research of her teaching practice, using Charles' responses to aspects of her practice to illustrate each goal of connected teaching, confirms that it is a powerful and lasting experience for such students (Miller-Reilly, 2008). Buerk (1986) and Kalinowski and Buerk (1995) write about an undergraduate class/course called The Writing Seminar in Mathematics, informed by the theory of connected teaching, where students' development as mathematical thinkers is visible in their journals which are written throughout the course. In all her undergraduate mathematics classes, Buerk (1996, p. 29) tries "to create environments to help these students to connect with their own mathematical intuitions and ideas, and to gain confidence in their own abilities to do mathematics". She includes "extensive use of writing, of both cooperative and collaborative learning activities, of open-ended questions with individual student feedback, and of mathematical situations that yield surprise or doubt" (Buerk, 1996, p. 30).

Women's relative lack of doctoral degrees means that they are over represented in the lower ranks of academia, where they make large contributions to teaching and outreach/initiatives within and outside the department and university but without the material or status rewards that come with a research profile. Women more often participate in doctoral study later in life when they are likely to have adult children (Reyes and Stanic 1998 in Harding et al., 2010, p. 395). Mura (1987) found that undergraduate women feel less confident than men in their ability to undertake Ph.D. study in mathematics. Becher, Hendel, and Kagan (1994 in Harding et al., 2010, p. 395) found that mature students enrolled for doctoral studies for personal development. Harding et al. (2010, p. 10) have used the personal narratives of seven women from several countries who completed Ph.D.s later in life, and found that their stories detail "the strong intellectual appeal of asking and answering a burning question and the personal and professional growth that can result from achieving the terminal degree". They conclude "no matter how late a Ph.D. is obtained, the qualification has a direct and beneficial influence on women's career paths" (ibid, p. 416).

Important mathematical skills and competencies in the workplace are "an ability to think in a mathematical way and to make decisions based on the interplay of a mathematical and situational sense of a problem", an aspect of 'mathematical literacy' (Noss & Hoyles, 1996, p. 4). Mathematical literacy is defined by the OECD's Programme of International Student Assessment (PISA) as follows:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2003, p. 24).

A large report about the mathematical skills needed in several sectors of the workplace suggests that education for adults in the workplace should "reflect mathematical literacy, rather than decontextualized skills" (Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002, p. 14.) Techno-mathematical literacies are an extension of mathematical literacy. In workplaces, as much as in the broader culture, this kind of necessary mathematical literacy is supplemented by the ubiquity of the computer: the new forms of computational technology that are used for doing mathematics are connected with new 'mathematical literacies'. The most complete description and examples of techno-mathematical literacies is to be found in Hoyles, Noss, Kent, and Bakker (2010).

14.3 Method

At the beginning of the two-decade time period for this paper, qualitative and quantitative data was gathered as part of Barbara's large study (Miller-Reilly, 2006) in which she investigated several teaching approaches used for adults returning to study mathematics. Additional data was gathered for the limited number of students who were interviewed, in particular, metaphors about mathematics were gathered using the Mathematics Metaphor Questionnaire (Buerk, 1996; Gibson, 1994). All the teaching sessions between Charles and Barbara were audiotaped and Charles responded to the questionnaires completed by participants in the large study.

Literature on metaphors, particularly in educational research, seems to fall into a number of categories—for this paper three categories are relevant. One category is when metaphors are used to enable students to become aware their personal conceptions of mathematics (Buerk, 1994, 1996; Dooley, 1998; Gibson, 1994), and sensitive ensuing discussions create a supportive classroom environment. Another category is as qualitative data when exploring attitudes towards, and beliefs about, mathematics (Leder & Forgasz, 2006; Miller-Reilly, 2006, 2010). A third category is the use of metaphors to illuminate teachers' personal perspectives in both pre-service teacher education and in their teaching practice (Algar, 2009; Briscoe, 1991; Bullough, 1991; Chapman, 1997). In much of this literature the term metaphor is used in a broad sense, referring to similes or metaphors.

Buerk (1996) reports how she first "became conscious of mathematical metaphors" when she realised how often she "heard metaphors" as her "math-avoidant teaching colleagues responded to mathematical situations" in her earlier research. Gibson (1994) and Buerk (1996, p. 26) devised a "protocol for the collection of metaphors" in 1988, a protocol which Barbara adapted and used (Miller-Reilly, 2006)—the Mathematics Metaphor Questionaire. It allows students to write their metaphors about their conceptions of mathematics, which are often not easy to describe literally (Bowman, 1995). Both Gibson and Buerk use their protocol regularly in their classes which are informed by the theory of connected teaching; Gibson in her high school mathematics classes and Buerk in her undergraduate mathematics classes. "Collecting metaphors had a profound effect on me", wrote Gibson (1994, p. 9). The sense of powerlessness and frustration evident in so many of her students' metaphors motivated her to create a "positive environment in which students could risk small doses of frustration" and "meaningful lessons and activities that involved all students and relied more on cooperative learning" (Gibson, 1994, p. 9). Buerk (1994) suggests that metaphors give clues about students' learning strategies and their conceptions of mathematics. Writing their metaphors for mathematics is also another way that students "become aware of their own thoughts and the extent to which their teachers value this independent thinking" (Buerk, 1994, p. 46), which is particularly important for adults returning to study mathematics. In addition, gathering students' metaphors is valuable because the experience and ensuing discussion broadens mathematics to include language, imagery and reflection (Buerk, 1996).

In their review of research on affect and mathematics education Leder and Forgasz (2006, p. 409) list projective techniques as an approach "adopted by those who favour qualitative approaches to the measurement of beliefs and by those concerned that respondents to Likert items may not express the beliefs they actually hold but those they consider socially acceptable". Miller-Reilly (2006, 2010) found that sensitively introducing metaphors into interviews for her research allowed energized and emotional accounts of mathematics learning experiences, providing rich additional qualitative data for her study of teaching approaches used for adults returning to the study of mathematics.

Chapman (1997, p. 209) found that teachers unconsciously constructed personal metaphors and that these became the "basis of their conceptualization of problems and made sense of their teaching". Metaphors "frame the meaning one assigns to events", a way of understanding "how we think about things, make sense of reality,

… a perspective or way of looking at things" (Schön 1983 cited in Chapman, 1997, p. 209). Dooley (1998, p. 105) suggests that pre-service teachers can discover "unarticulated beliefs and assumptions" from an exploration of the "unique meanings that they have constructed for their metaphors and images" about teaching. Bullough and Stokes (1994, p. 197) furthered the development of pre-service teachers by gathering their personal teaching metaphors as a means of exploring their conceptions of teaching. Algar (2009) studied changes in teachers' beliefs over their career span by asking experienced teachers to select from six of the most common conceptual teaching metaphors (from the literature on teaching) one which indicated their current beliefs, as well as one that illustrated their beliefs early in their careers. Algar (2009, p. 743) found that while "very experienced teachers began teaching with teacher-centered conceptual metaphors", over the course of their careers about two-thirds "moved toward student-centered metaphors".

The stories of the journeys of Barbara and Charles over two decades seem to be examples of life stories or personal narratives, in particular for Charles, it is a mathematics life history (Coben, 2000). "We know or discover ourselves, and reveal ourselves to others, by the stories we tell" (Lieblich, Tuval-Mashiach, & Zilber, 1998, p. 7). "In many studies … the narrative is used to represent the character … of specific subgroups in society … minorities whose narratives express their unheard voices" (Lieblich et al., 1998, p. 5). Each of the four sections of our narratives, our experiences at four different stages of our journeys over two decades, will be "laminated". Just as lamination can provide more strength and stability in the resulting material, it is hoped that laminating our narratives will strengthen the tale of our two journeys.

The first stage of each of our journeys addresses each of our background experiences. Barbara writes about some of her experiences over the three decades since she had completed her tertiary education at bachelors and masters levels. Charles includes his mathematics autobiography, written two decades ago, describing his background mathematical experiences before we met.

14.4 Background—Barbara

My professional life and family commitments were interwoven. In common with many other women of my generation, having a career, although important to me, was secondary to embarking on family life. During my family's pre-school years I left my teaching position at the university and became involved in many community and parenting activities as well as teaching adults at night school. One class which stands out, and which influenced my later work, was when a maths-avoidant friend and I developed a new adult education course called "Maths Anxiety" which we taught as part of a community education program. When my youngest child started school, I came back to paid work at the university (job-sharing). Many interesting challenges emerged: developing the mathematics and science program in a new centre within the university, the Student Learning Centre; initiating, with others, a day-long careers seminar for girls in the mathematical sciences which became an annual event; initiating, with others, a network of teachers from the tertiary, secondary and primary sectors (EQUALS Maths/Science Network) that worked toward making the classroom a more inclusive place to learn mathematics. Five years later, research and study leave in the U.S. allowed me to study programs for encouraging women in computing, mathematics, and science, and to present my first paper at an international conference, describing our equity initiatives in NZ. I incorporated many of the ideas and resources from EQUALS into my own teaching practice, that of teaching mathematics to intellectually-able maths-avoidant adult students in the fledgling Student Learning Centre at the University of Auckland. Evaluations from students indicated my teaching was successful. The support of colleagues and friends in the Auckland EQUALS group was important to me, as my mathematics teaching practice was atypical within the university. My concern about the under-representation of other groups (for example, Māori and Pacific Nations' students) within the university saw me join the Bi-cultural Staff Group where, for several years, we organized workshops for staff (faculty) within the university to share information about special initiatives taken by a number of university departments to support such groups. My academic research began at this time. A colleague and I started a quantitative study considering gender differences in achievement in the mathematics examinations in the last year of high school. The findings were presented for the first time at the 1989 NZ Association of Mathematics Teachers Conference. I found it satisfying being able to do research as well as teach. In the early 1990s I was asked to supervise teachers' research projects, which expanded my knowledge of the link between teaching and research. An invitation to attend the 1993 ICMI Conference on Gender and Mathematics Education in Sweden provided another turning point career-wise, and with the family leaving home, I decided to start doctoral studies, partly motivated by the limited promotion possibilities for me in my academic career without a Ph.D. I did not fully realize how huge a challenge this would be for me, partly due to many unforeseen circumstances in the family, and the size of the Ph.D. enterprise.

14.5 Background—Charles

During the initial stages of the 6-month course of study, at Barbara's request, I wrote my mathematics autobiography (Table 14.1).

The second section of our journeys describes parts of the six-month course, particularly at the start (two decades ago) and then when it was completed. These details, and Charles' quotes, were taken from the transcribed audio-tapes of the classes.

Table 14.1 My mathematics autobiography written in week 3 of our course

Age	Experiences
13 years	IQ test for streaming[b], no problems whatsoever with English and comprehension, knew I was hopeless at maths, and therefore ended up in a lower stream than that which really I should have been in, which pulled me back enormously. On all other subjects I was bored because I could do them. I wasn't being extended. I didn't realise that because I was poor at maths, though good at English, that I'd been held back—this affected my self-confidence
15 years	I was working to apply myself but failed School Certificate Maths. Homework was a complete nightmare. Despite listening attentively and taking the necessary notes, I could never[a] apply the concepts to a different set of numbers. It was totally frustrating! Despite assistance from Dad, I never really improved
16, 17 years	The same applied as per age 15, failed UE and Bursary maths and nobody seemed to understand. I failed every maths exam throughout my secondary school career. Therefore my association with and attitude towards maths was completely negative. Application was not the issue, understanding was!
20 years	After the completion of my 1st degree, majoring in English literature, I was unable to complete my 2nd degree (Commerce) because of the mathematical component
21 years	I had to leave a fabulous job, sharebroking, trading and providing advice for clients on equities, fixed interest. I was offered an opportunity to become a partner at 21, but I had this total fear of maths and recall breaking out into a sweat at the prospect of undertaking this work. I was surrounded by numbers, and I just thought, I can't do this, this is ridiculous. But, in reality, they could see skills that were good for them in me, which I can recognise now but …
33 years	An experience I had in the past month: sensitivity analysis, and a feasibility study on a large block of land. I recall thinking 'how can I do this?' When I'd complete any part of the exercise I'd review it and think 'how did I get this result? Is it correct? I'm not sure.' Then I'd review it again and again … In other words, complete insecurity in my own ability. In general my analysis was correct, however it took me forever to reach a result and the task seemed quite daunting. It took an unbelievable amount of energy. It's so tiring Continued failure with maths is frustrating, hurtful and demeaning. I wonder whether I have the maths version of dyslexia

[a]Underlining was that done at the time I wrote this
[b]Terminology for grouping by 'ability' varies by country: commonly called streams (in NZ), tracks (in US) or sets (in UK)

14.6 Two Decades Ago—Barbara

14.6.1 At the Beginning of the Six-Month Course

The year before I met Charles, I had embarked on doctoral studies, researching mathematics courses set up to teach adults coming back to the study of mathematics.

Comparison of two different bridging/access/second-chance courses was the topic. I had started gathering data from students using a questionnaire, which I had developed and trialed. I was planning to interview some students from key groups in each course, identified through analysis of the questionnaire data, and to interview the teachers of these courses. Some open questions about how students had experienced each course were included in the questionnaire at the end of the course.

When Charles contacted me, wanting some mathematical help, I decided that researching my own teaching practice, which I had not examined in this way before, would be an interesting third comparison, another teaching approach with adults returning to the study of mathematics. I agreed to address his mathematical needs and he agreed to be part of my research project.

With Charles' permission, I audiotaped all our classes, and he completed the questionnaire I had developed for my doctorate. He also wrote his mathematics autobiography (see in the previous section of this paper) so that I became aware of his past mathematical experiences. Charles had completed a degree in English literature in his twenties. He was now 33 years old, a business man, and he felt at a turning point in his career because of his debilitating fear of mathematics.

My aim was to understand his mathematics avoidance and fear and let Charles experience mathematics as an enjoyable, creative, pattern-searching discipline, with connections to his context. I believed in taking plenty of time in the first few classes to acknowledge a student's feelings about mathematics so I asked Charles if he would complete the Mathematics Metaphor Questionnaire (Buerk, 1996; Gibson, 1994).

I was very fortunate that Charles had great ability in language and creativity as well as high motivation. His answers to the metaphor questionnaire were amazing: graphic, but profoundly negative, metaphors. For example, in answer to the question *What does using or doing maths feel like?* he said "Skiing on blue ice, with no edges, blindfolded". [Many more of his metaphors are listed in my other work (Ocean & Miller-Reilly, 1997; Miller-Reilly, 2006, 2008, 2010).] In all my experience of teaching maths-avoidant adults, I had never encountered this level of fear in a student. His earlier mathematical experiences were also very negative. I acknowledged them, took care not to blame him (Zaslavsky, 1994; Taylor & Shea, 1996), and promised that I would not give him any similar experiences. After the first few weeks I was so pleased when Charles said to me "you're the first person I have come across who can genuinely understand—it's a huge relief". I had already started gently and carefully doing some mathematics with him using mathematical games, manipulatives and explorative materials. I saw changes in his confidence and his mathematical knowledge as we worked together.

14.6.2 At the End of the Six-Month Course

At the start of the fifth month of the six-month course, I introduced algebra. We explored word and number patterns, using 'function machines' as a model (Langbort & Thompson, 1985). Charles commented that he had "gone a long way—especially on the fear side—I think I'm going to enjoy algebra". I started introducing a spreadsheet for generating (linear) functions since spreadsheets are commonly used in the business world. We entered formulae into the spreadsheet, 'filling down' to generate a sequence of numbers, which led to the need to introduce the convention for the order of operations (using the mnemonic BEMA). His reaction was: "I am enjoying this—this is the best thing that has happened to me this year!" For our last session, our seventeenth session, he said the course had been "rewarding—very" and he had a "great feeling of accomplishment". He now "knows he can (do maths), given time—before it felt impossible". The change in his metaphors amazed me.[2] I was very pleased. We had both worked very hard! He wrote:

> Barbara approached my problem, which was very real, in a thoughtful, gentle and completely encouraging manner. She was able to empathise with me and fully understand what had been an on-going and seemingly never ending horrible experience.

I asked how it affected his day-to-day work and he replied:

> Oh just general confidence when it comes to using maths. When it comes to business matters, it's not a problem, but when it's something complex, some spreadsheet which I haven't done, it doesn't concern me if at first I don't see what's happening, because I know if I analyse it, slowly I will. So my attitude, that's what has changed. It's a confidence thing. You look at it more objectively, feel more at ease. It's a huge difference—enormous! I'm just so grateful.

I also was very pleased, and very grateful for his thoughtful clear statements, and told him so.

14.7 Two Decades Ago—Charles

14.7.1 At the Beginning of the Six-Month Course

During my first appointment with Barbara (two decades ago) I mentioned some of my strengths and discussed other reasons why I wanted to study mathematics. I said I "loved language, economics, creativity, colour, form, business", "especially colour and form and trading, they're instinctive".[3] I continued:

> On the one hand there's this business trading thing and on the other is this creative side. To really succeed on the business side, I think that I really need to understand maths. Now I could run with my creative side, and I may still do so, however if I do it now I'm doing so without a choice.

[2]"Black Holes and Beginning Teachers" (Ocean & Miller-Reilly, 1997), a paper jointly written by Jude Ocean and me, published in a journal for teachers, lists many of Charles' metaphors and alerts beginning teachers to what some of their students may be feeling about mathematics. It is now required reading for pre-service teachers in the University of Auckland teacher education program.

[3]All the quotes come from the transcripts of the audiotaped teaching sessions two decades ago.

The second time we met I again talked about the lack of choice of career options because my mathematics was “abysmally weak” and continued:

> In order to make a valid career choice I have to either say ‘OK, it is, for want of a better description, that’s a horrible black hole, or it’s not’. But if it is that horrible big black hole, then I have to say ‘it’s there Charles, it’s a reality, you’ve got to live with it, you’ve got to work with it’. I’ve reached an age (32–33) that I have to start making some pretty serious choices. OK, I do have other skills which are quite strong and are undeveloped. Can you see the importance of this? I mean it’s actually quite a big life decision.

Later, during our second appointment, I again checked with Barbara if she realised how significant this chance was for my career—she replied that she understood. Barbara also emphasized that it would take a lot of work on my part—I understood that and was prepared to put in a lot of effort.

My metaphors were profoundly negative. I felt they very effectively captured the nature of my fear and negative experiences. I felt that mathematics was most like a hyena, “a scavenger-predator, rearing its head when I least want it; always succeeding in removing my self-confidence and sense of self. I hate it.” If maths had been a way to communicate I thought it was “a forgotten dialect, making two-way communication impossible”.

14.7.2 At the End of the Six-Month Course

At the end of the 6-month course my metaphors were very different. In answer to the question *If maths were a building what kind of building would it be?* at the start of this course I described mathematics as “the most disgusting, unappealing structure in history—maybe a prison, white, grey and ugly” but at the end “it would have a lot of form, it would have a symmetry, quite classical, pleasing to the eye—I would never have said that before. I can now see how a mathematician and an artist can be one and the same.”

Barbara asked if I could describe what it was about the course that had contributed to these positive changes. I wrote this statement:

What Has Made The Changes

In the very first instance, actually facing up to the problem and seeking advice from a psychologist. Had the said psychologist told me in the very first session that I didn’t have a problem with mathematics I would have thought him inept. However, by way of a process he was able to illustrate to me that in all likelihood I had the inherent capacity to cope with maths. Stage two was making contact with Barbara, who had taught students of a similar nature. What exactly has Barbara done which has been so enlightening? I need to list the points and ideas which have made the change as there have been so many.

1. Identifying the gaps.
2. Really starting with the basics. Letting me know it was fine not to understand some rudimentary concepts with certainty. Previously I had been greatly embarrassed by my lack of certainty.
3. Encouraging my memory and actually saying on many occasions: "you'll probably find you realise more than you first thought".
4. Approaching problems in bite-sized pieces.
5. Patterns. These were a major as they illustrated a concept (an alien one at that) in a visual fashion which I could relate to.
6. Building on knowledge and the basics as I learnt engendered great confidence.
7. Being completely encouraging and motivating. What an amazing teacher.
8. Letting me know it was OK if I didn't get it first hit.
9. Once the logic emerged from the patterns it was fun. Now I thoroughly enjoy maths and would like to spend more time discovering it. In some instances it's like reading a fabulous book, interesting and expanding your perceptions. At other times it's like feeling a cog turn in your brain for the very first time.

Four years later, when I asked Barbara if we could discuss some mathematical problems again, I wrote this statement:

> Prior to commencing tuition with Barbara I would have described my pure mathematics skills as severely limited. This limitation had, from my own perspective, been a significant impediment. Despite my abilities with basic and business mathematics I was acutely aware of my shortfall in what I referred to generally as "maths". This shortfall manifested itself as a lack of self-confidence when approaching most situations involving mathematics. This mindset was frustrated by an innate knowledge that this should not be the case, as my other skills were well developed. In a way fear and frustration reigned.
>
> I am delighted to say that I now approach mathematics with a degree of pleasure and relative confidence.
>
> Business mathematics seldom becomes extremely complex, other than for say a business analyst, as such some principles provide a guiding hand and the whole process is manageable. An appropriate example of the great value of this tuition is my ability to understand, use and enjoy algebra.
>
> The knowledge gained from expert tuition has enabled me to tackle such problems without fear and in a reasonably timely fashion. The method of teaching adopted by Barbara has opened up doors within my brain which were firmly closed and enabled me to progress with a greater sense of confidence.
>
> I would rate the knowledge gained and consequent confidence as the highlight of the last four years.

Now, in the third section of our journeys, we consider our recent tuition and discussions, two decades after the first course of study.

14.8 Two Decades After the First Course

14.8.1 Recent Tuition and Discussions—Charles

By changing my inadequate maths experience to an adequate maths experience two decades ago I was prepared to embark on a new journey, to try to meet the entry requirements to enter a leading business school in Australia for post-graduate business studies. However, the institution under-pitched the reality of their requirements. Even though Barbara and I worked on some of these requirements over several months, the additional algebraic knowledge that was required was not made clear until after I took the Harvard pre-entry mathematics test (for MBA). They then informed me that I required a score of at least 80% in this test. It was a massive disappointment to me when I found out about this requirement after I did not make this score. I had worked hard to try to enter this university for more tertiary study. However, on the positive side, without the work done two decades ago I would not have even attempted to get entry.

14.8.2 Recent Tuition and Discussions—Barbara

I also felt very disappointed for Charles. Is this another example of mathematics being used as a gatekeeper (FitzSimons & Godden, 2000), where an inappropriate level of mathematics knowledge is required for entry? I am not familiar with this course so do not know. Familiarity with several chapters of a finance textbook were part of the requirements for entry to this graduate course so we worked our way through these chapters. I became aware again of the fact that Charles, coming from part of the financial world, spoke a different language from me—the discourse of the financial world. I was coming from the mathematical world and was familiar with the mathematics register. I found that, as did Noss and Hoyles (1996, p. 9), "financial instruments which, from a mathematical point of view, seemed to be more or less the same phenomenon, have different names" depending on the context. Charles became the teacher at times to make this classification of financial instruments more understandable to me. I learned about this classification of financial instruments while bringing the attention of Charles to the relationships between their underlying structures, as Noss and Hoyles (1996) also did in their study of the mathematics of banking.

Charles often mentioned how valuable learning about the convention for the order of operations (BEMA) has been for him. The process of searching for patterns and discovering "principles" (Charles), which we did so frequently in the six-month course, has also been very useful and this knowledge has been used many times over the years.

Reflections over the previous two decades are the final aspects of our journeys.

14.9 Reflections of Our Journeys

14.9.1 Barbara's Reflections

The last two decades have been dominated by the journey to complete my doctorate then by my continuing teaching and research, and by my growing family—the birth of my doctorate as well as the births of my nine amazing grandchildren!

Teaching Charles, and collecting his responses to my teaching practice, was intertwined with the challenging journey, more than 10 years ago, to complete my doctorate. Charles' ability with language and his creativity, which produced such amazing metaphors, brought my Ph.D. experience to life. His responses were inspirational and hugely energizing for me both as a teacher and as a researcher. They provided the motivation for me to write about the importance of the utilization of metaphors in research (Miller-Reilly, 2010).

As a teacher, it was a powerful and satisfying experience for me to successfully help an adult with such a debilitating fear of mathematics. Reading about the theory of connected teaching I recognized most of my goals as a teacher. I also saw these goals realised in Charles' metaphors and in his statement "What has Made the Changes" (in a preceding section of this paper). I researched my teaching practice using the theory of connected teaching, using Charles' responses to illustrate each goal (Miller-Reilly, 2008). Charles' comments confirm Morrow and Morrow's (1995, p. 20) statement that "gaining a sense of their own voices in mathematics", as a result of connected teaching, is a "very powerful experience" for a student. Gibson (1994) and Buerk (1996) had similar experiences as teachers of their mathematics classes, described in a previous section of this paper.

My role metaphor would be a swimming teacher who gets into the water with the student, holding the student, keeping their head out of the water, as they slowly feel more confident in the water, knowing when to let go, knowing when to let them take risks with their new-found confidence to gain new skills and knowledge. This metaphor is similar to Algar's (2009) final (sixth) teaching metaphor which is student-centred, involving equitable distribution of power and the ethic of care.

In common with other women who do doctorates at a later stage of their lives, I agree that "a major benefit of doing a Ph.D. late in life is the personal growth experienced" (Harding et al., 2010, p. 416). Completing this degree also has given me added recognition in my academic environment and has positively influenced my retirement, enabling me to continue doing research in an honorary position within my Department.

14.9.2 Charles' Reflections

You presented pure mathematics from a considered position. In the first instance you acknowledged my fear and approached the subject gently, this created a sense of comfort and a platform for learning, critical to the acquisition of knowledge. Then you proceeded to enlighten me in respect to the subject by first presenting it as a series of principles and patterns versus rules. I found this enlightening and entirely understandable. This foundation enabled us to embark on a series of subjects which I have been able to utilise on both a specific and general level.

Broadly my confidence when approaching subject matters with a general pure mathematics component has ensured I address the same with a high degree of calm, a stark contrast to my previous position. Adoption of the principles learnt and application of a step by step approach has been a significant guide.

The benefits of the study have enabled me, in part, to maximise returns via sound due diligence—invariably with a mathematical component, strategy, implementation and leadership. As a consequence I have relished completing some large commercial transactions—significantly increasing the probability of success via the methodologies and collaboration I employ throughout the process. I agree with Noss and Hoyles' (1996, p. 4) statement that "what matters (in a workplace situation) is an ability to think in a mathematical way and make decisions based on the interplay of a mathematical and situational sense of a problem".

On a specific basis I have applied algebra when developing spreadsheets, considering problems with a number of variables and building discounted cash flows plus simple valuations. The application of our study has also been relevant when reading and interpreting balance sheets; inherently this has provided me with the confidence to apply my more natural lateral thinking skills, to interpret high level information and look for connectivity based on principles. So the union of my learnt mathematical skills and more innate strategic skills has proven most beneficial. A simple example of the same was balancing the primary costs, administration, with a primary income stream and seeking to achieve parity between the two by the application of an airline model requiring 95% occupancy. Whilst simplified this assessment was core to the development of a strategy which, combined with a series of other components, led to a significant financial turnaround in the affairs of the entity in question.

Had I not embarked on the journey with Barbara life would have been different, better or worse I'm not sure but certainly the study and positive consequences have created comfort in a subject area that was previously faced with fear and dread. I think it would be reasonable to say that this comfort was a key contributor in some of my more recent career achievements for example:

(a) Achieved an overall recovery rate of NZ$0.95 in the NZ$1.00, including a New Zealand transaction record, for a series of property receivership transactions.
(b) Delivered a negotiated capital gain of NZ$7.0 m, within the confines of the Public Works Act.
(c) For the financial year ended 30 June 2015 achieved a return of 13.1% vs. the NZX Gross at 10.8% for a diversified managed funds portfolio.
(d) Set a four year New Zealand transaction record for an international property divestment programme inclusive of a NZ$50 m capital raise and capital gain of NZ$12 m.

My learning experience with Barbara was overwhelmingly positive with significant constructive spin-offs. I'm most grateful to have had this opportunity and sincerely hope that the lessons learnt will have application within further learning environments.

14.10 Discussion

Charles' experience supports Parker's (1997, cited in Safford-Ramus, 2004, p. 57) conclusion, that "overcoming mathematics anxiety during adulthood" is a "transition of major magnitude", an important "life event". Parker identified this process in a number of adults, interviewed in her research, who were mathematics anxiety success stories. The first five stages are clearly visible in Charles' story: his "perception of a need", his "commitment to address the problem" by taking "specific actions to become more comfortable with mathematics", his recognition of a turning point having been reached", which has resulted in a change in his "mathematical perspectives". It created comfort in a subject area that Charles previously faced with fear, with beneficial outcomes at work. Charles' concern that others have the same opportunities that he has found so valuable illustrates that he is in the final stage of the six-stage process identified by Parker (1997, cited in Safford-Ramus, 2004, p. 57).

Barbara felt a great personal achievement completing her doctorate, as it was the culmination of many of her previous experiences both in life and in her profession. It also resulted in further professional recognition, supporting Harding et al. (2010) conclusion that such doctorates result in personal and professional growth.

We each took on big challenges during these two decades and both feel very positive about the outcomes. Remarkably our lives have intersected in ways that have been markedly mutually beneficial, a level of reciprocity that had not been anticipated. The lamination of the narratives of our two journeys (Lieblich et al., 1998) has created our unusual and inspiring tale. We come from different subgroups in our society: an academically-able maths-avoidant adult (Charles); and a mature woman undertaking doctoral studies (Barbara). We were both undertaking different transitions (Merriam, 2005): for Charles, tackling his debilitating fear of mathematics; for Barbara, undertaking the challenging task of a Ph.D. Unexpectedly we each provided a crucial component of the other's journey.

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Chapter 15
Lifelong Mathematics Learning for Adult Learners and Open Educational Resources

Pradeep Kumar Misra

Abstract Lifelong mathematics learning is a necessity of our times but adult learners still face a number of challenges to practice it. Efforts are needed at different levels (personal, societal, institutional, and governmental) to overcome these challenges. Open Educational Resources (OER), a byproduct of open access movement offers one such opportunity. OER has emerged as one of the most innovative teaching and learning practices to support education in form of freely available resources that can be accessed, reused, modified and shared by users. The other notable aspect is that OER are available in different languages and increasing day by day. Considering all these characteristics, it may be argued that use of OER will be a value addition to promote lifelong learning of mathematics among adult learners. Extending this argument, present chapter details about benefits and global initiatives to use OER for mathematics learning; and proposes potential strategies to use OER to achieve lifelong mathematics learning for adult learners.

Keywords Lifelong mathematics learning · Adult learners · Open educational resources

15.1 Introduction

By a rough estimate, almost half of the world population falls under the ages of 25–64. Therefore, it is obvious that this population, generally referred as adult learners is vital for world prosperity and socio-economic development. Adult learners are usually defined as a very diverse group (typically ages 25 and older) with a wide range of abilities, educational and cultural backgrounds, responsibilities and job experiences (Southern Regional Education Board, 2015). Keeping this vast number of adult learners engaged, economically active and productive on continuing basis is a key challenge before policy makers and governments across the world.

P. K. Misra (✉)
Department of Education, Chaudhary Charan Singh University, Meerut, India
e-mail: pradeepkmisra@yahoo.com

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Talking about the importance of learning in the life of adults, an UNESCO position paper on education emphasizes that young people and adults across the world acquire relevant and recognized functional literacy and numeracy skills that allow them to identify, understand, interpret, create, communicate and compute (UNESCO, 2015a, p. 6). In fact, adults of today are ready to be the learners throughout their life as a survey conducted by Pew Research Center in America reports that 73% of adults consider themselves lifelong learners (Horrigan, 2016). This trend is more or less prevalent in different parts of the world as more and more adults are embracing lifelong learning whether that means gathering knowledge, reading something new or improving their job skills.

15.2 Practicing Lifelong Mathematics Learning: Significance for Adult Learners and Challenges

Talking about the importance of learning in societal context, Cresson and Dean (2000, p. 87) observe,

> Learning is seen as the answer to many of the today's crucial issues. It is essential that global learning society be created to address and rectify such problems as unemployment, world peace and poverty.

Lifelong learning is an extension of this thinking as it emphasizes on the total personal, intellectual, emotional, social, and spiritual development of an individual (Faure, 1972). The concept of lifelong learning stresses that learning and education are related to life as a whole—not just to work—and learning throughout life is a continuum that should run from cradle to grave (Smith & Spurling, 1999). According to this concept, lifelong learning refers to all kinds of formal education and training (whether or not they carry certification); and can occur anywhere including education or training institutions, the workplace (on or off the job), the family, or cultural and community settings. Lifelong learning, according to Royce (1999, p. 149),

> Aims to give students the skills to go on learning throughout life and also positive attitudes towards learning which accept and even welcome change and new learning.

In this sense, lifelong learning supports the development of knowledge and competences to enable adult learners to adapt to the knowledge-based society and actively participate in all spheres of social and economic life (Misra, 2011). In an European Commission (2001, p. 9) document, Lifelong Learning (LLL) is defined as,

> All learning activity undertaken throughout life, with the aim of improving knowledge, skills and competence, within a personal, civic, social and/or employment-related perspective.

This definition points towards extrinsic motivation for an adult to practice lifelong learning and subsequent benefits. The other definition given by Jarvis (2006, p. 134) sees lifelong learning from a holistic perspective and claims that it helps the adult learners to keep changing and evolving at different stages of life and in many ways,

> The combination of processes throughout a life time whereby the whole person-body (genetic, physical and biological) and mind (knowledge, skills, attitudes, values, emotions, beliefs and senses) – experiences social situations, the perceived content of which is then transformed cognitively, emotively or practically (or through any combination) and integrated into the individual person's biography resulting in a continually changing (or more experienced) person.

Essence of all these definitions help us to claim that lifelong learning offers different opportunities for adult learners to learn in a variety of contexts—in educational institutions, at work, at home and through leisure activities (Misra, 2012). World Education Forum of UNESCO (2015b) declared that every person, at every stage of their life should have lifelong learning opportunities to acquire the knowledge and skills they need to fulfill their aspirations and contribute to their societies. Therefore, provisions of lifelong learning to adult learners will help them to continue developing on a personal level, having greater individual autonomy and making a more active and productive contributor to society. The role of lifelong learning in the life of adults is clearly visible from a study reported by O'Brien (2009),

> When a group of older adults, age 55-75, were asked if they would be interested in lifelong learning and living in a college atmosphere, more than half of the respondents said they like the idea of retiring to a home on a college campus.

The adults of today need many types of lifelong learning to keep themselves productive and engaged, and mathematics learning is one of them (Safford-Ramus, Misra, & Maguire, 2016). Talking about the importance of mathematics learning in one's life, a report from U.S. Department of Education (2015) observes that mathematics skills are a gatekeeper for further education and training, and significantly affect employability and career options. Even for jobs requiring post-secondary education, employers seek employees who are proficient in mathematics, as well as reading; use math to solve problems; and communicate effectively. In this report, it has also been highlighted that if people miss out on mathematics at school or had a bad experience the first time round, discovering mathematics later in life can be really important in achieving their potential. The report further emphasizes that since today's decisions are based on data, it is equally important for learners to develop and strengthen skills in mathematics. Keeping these benefits in sight, different communities, associations and organizations are looking for better ways to make lifelong learning of mathematics easier, engaging and productive for adult learners.

Instead of these multi-faceted benefits, adult learners still feel reluctant to lifelong mathematics learning. The reasons are many. First among them is negative perception about mathematics. Many adult learners approach math with anxiety and frustration. Negative previous experiences with math instruction create legitimate barriers for many adult learners (U.S. Department of Education, 2015). Mathematics in particular is often associated with negative memories, and so people try to avoid using mathematics in their everyday or vocational lives. This leads to a problematic affective situation in adult-educational mathematics courses (Schlöglmann, 2006, p. 15). According to Klinger (2010, p. 7),

> A major challenge for practitioners in adult mathematics education is to achieve effective learning outcomes in the face of prevailing negative attitudes in their students, often present as a consequence of unsatisfactory early mathematics learning experience and flowing from the well established connection between adult innumeracy and mathematics anxiety.

Second, adult learners everyday competences do not count as mathematics (FitzSimons, 2002). Adult learners practice different types of mathematical activities in their everyday life. But learners themselves, employers and societies hardly recognize these activities as mathematical competences. Talking about this tendency of adult learners, Wedege (2010, p. 89) comments,

> People simply do not recognize the mathematics in their daily practice – as mathematics. They do not connect the everyday activity and their own competence with mathematics. Most of them only associate mathematics with the school subject.

In other words, adult learners do not pay enough attention to improve their mathematical learning by practicing their routine activities. Third major challenge is procedures of learning surrounded by a popular belief that mathematics is the subject about which students cannot ask "why." In words of Chisman (2011, p. 7),

> The greatest concern of math reform advocates is that most instruction in this field consists of memorizing rote procedures for solving math problems.

In fact, too much emphasis on memorizing procedures and too little on conceptual understanding lead to a situation where learners started hating mathematics. The other issue is ability of school teachers teaching mathematics. Teaching mathematics based on rigorous, focused, and coherent standards requires teachers to know mathematics in ways that are likely different from how they were taught. Such teaching requires an understanding of the mathematics taught but also the mathematics that comes before and after that content so that appropriate connections can be established (Dixon, 2015). But finding teachers having these types of mathematical abilities is getting more and more difficult. In nutshell, it may be argued that mathematics education is facing number of challenges and these are equally applicable to adults learning of mathematics.

In this backdrop, policy makers, governments and researchers from different parts of the world are looking for new teaching-learning models that can connect and integrate a variety of tools to meet lifelong learning needs of the adult learners of mathematics. The development of the information society and the widespread diffusion of information communication technologies give rise to new opportunities to support adult learners to achieve lifelong learning of mathematics. Internet based and freely available Open Educational Resources (OER) offers one such opportunity. OER, a byproduct of open access movement can help adult learners a lot to continuously improve their foundational skills and depth of knowledge in mathematics and keep them learning mathematics throughout their life. This argument is based on the fact that OER based education supports different learning styles and offers the opportunity to learn from countless sources as per their ease and time.

15.3 OER: Concepts, Characteristics and Benefits

Looking from 'learning resources' angle, the most widely accepted explanation defines OER as,

> teaching, learning and research materials in any medium – digital or otherwise – that reside in the public domain or have been released under an open license that permits no-cost access, use, adaptation and redistribution by others with no or limited restrictions. (The William and Flora Hewlett Foundation, 2017)

In fact, the "free use" benefit makes OER as one of the most sought learning resources in the world. Here the meaning of 'free' is wide ranging and also includes freedom from financial obligations to access resources, freedom from getting permissions to use resources, and freedom to use resources at mass scale. The other notable feature is that users of OER have the rights to: revise (adapt and improve the OER so they better meets your needs); reuse (use the original or your new version of the OER in a wide range of contexts); remix (combine or "mashup" the OER with other OER to produce new materials); and redistribute (make copies and share the original OER, or your new version, with others) (Collins & Levy, 2013). In nutshell, OER offer the four most important rights to the user:

- Reuse—the right to use the application without changing the original form
- Review—the right to adapt, adjust, modify, or change the resource
- Remix—the right to match the original or adapted resource with other resources to create something new
- Redistribute—the right to share copies of the appeal, adaptations or remixes.

Clements and Pawlowski (2012) see OER as resources for the purpose of learning, education and training that are freely accessible. This includes literature and scientific resources (open access for education), technologies, and systems (open source for education), and open content (actual learning materials/contents) as well as related artifacts (such as didactical materials or lesson plans), full university courses, interactive simulations, electronic textbooks, elementary school and high school (K–12) lesson plans, worksheets, activities, quizzes, and fun games. However, definitions differ as to whether OER consists only of digital resources, whether it constitutes resources produced specifically for educational purposes and whether these resources should be in the public domain (Misra, 2014). Therefore, conceptually we can say that the distinguishing feature of OER when compared to other resources is the freedom with which it may be used, reused and repurposed thanks to its open license (Camilleri, Ehlers, & Pawlowski, 2014).

OER can be reused and repurposed to suit different needs and could be available in any medium, print, audio, video or digital. One key difference between OER and other educational resources is that OER have an open license, which allows adaptation and reuse without having to request the copyright holder (Kanwar, 2015). Talking about the benefits of, one can strongly argue that OER offer an exclusive advantage when compared to other educational resources. They allow legal and extensive use of the resources free of charge, and offer in exchange greater openness in access to education, increased quality, innovation, creativity and sustainable use. The benefits of using OER are not limited to this but also includes drastic savings in the cost of education, granting access to more quality choices, helping in preparation for course and retention of knowledge after course, and re-using someone else's material with peace of mind (Chae, 2015). On a different note, the purpose of using OER in education is to enhance learning, notably a kind of learning that enables the development of both individual and social capabilities for understanding and acting (OECD, 2007).

Studies conducted in different parts of the world speak in favour of the adoption of OER for learning of different subjects including mathematics. In a study, conducted to understand the impact of Open Educational Resources (OER) on first-year mathematics students at the Instituto Profesional Providencia (IPP) in Santiago, Chile, it was reported that (i) Students who used Khan Academy resources (OER resources) obtained statistically significantly better exam grades than those who used the proprietary resource, (ii) the qualitative and quantitative data confirmed the assumption that OER can be relevant and useful, and (iii) "openness" does not necessarily produce an impact in and of itself, but is instead part of a greater set of tools and practices in which many variables exert an influence (Juárez & Muggli, 2017). Similarly, Chiorescu (2017) noted that OER have the potential to save money to our students with almost no significant effect on learning outcomes. And, Hilton (2016), who has analyzed 16 research studies involving OER (across a variety of subjects) reported, "students and faculty members generally find that OER are comparable in quality to traditional learning resources, and that the use of OER does not appear to negatively influence student learning" (Hilton, 2016, p. 588).

In addition, it has also been observed that OER presents a number of opportunities for adult learners. A study reviewing the current use of Open Educational Resources in adult education reported, "OER encourages learning by seniors who do not wish to, or cannot, travel to formal or non-formal study locations; seniors can collaborate in formal and non-formal programmes with learners of other ages" (Bacsich et al., 2015 p. 42). OER presents a number of opportunities for adult learning of STEM subjects (Science, Technology, Engineering and Mathematics) as evident from following observation,

> Quality OER can be powerful tools to enhance learning and instruction. The topics that make up STEM areas are especially well-suited for the use of OER in instruction because they require specialized content and explanations. Well designed and explained OER can fill the gap between what instructors can do with existing materials in the adult education classroom and the deeper content they want students to learn. (U.S. Department of Education, 2017)

Considering these promises, a number of institutions and organizations have already started OER based mathematical learning initiatives to help different target groups including educators, students, and lifelong learners.

15.4 OER for Lifelong Mathematics Learning: Initiatives Across the Globe

In global context, a number of initiatives using OER for mathematical learning have emerged. For example, OER in mathematics professional development project takes advantage of the potential of OER, combined with training and appropriate technological hardware, to support classroom technology integration that improves grades 7–12 student mathematics achievement and technological literacy. The professional development also fosters the creation and ongoing use of a statewide mathematics learning network. While, Humboldt State University's free and open textbook alternatives project offers a number of free online teaching and learning materials related to mathematics. Similarly, Karnataka Open Educational Resources portal of India provide a number of useful resources including quizzes, articles, books, lab activities, fun games and interesting news about mathematics for students, teachers and anyone interested in mathematics. University Libraries OER Initiative from University of Oklahoma provides a number of journals, databases, primary content, and e-books for free for anyone interested in mathematical learning.

OU Libraries provides access to content such as online books, journals and image collections that can be used to lower or remove student textbook costs for OU students. The K–12 OER Collaborative, a coalition of eleven U.S. states and eight organizations, is creating comprehensive, high-quality, open educational resources (OER) supporting K–12 mathematics and English language resources aligned to state standards and accessible under the most open Creative Commons license. MERLOT) a program of the California State University provides a curated collection of free and open online teaching, learning, and faculty development services contributed and used by an international education community. In its Mathematics Portal, one can find some of the best mathematics resources hosted on the Internet. WeBWorK, an open-source online homework system for math and science courses comes with a National Problem Library (NPL) of over 20,000 homework problems. Problems in the NPL target courses include college algebra, discrete mathematics, probability and statistics, single and multivariable calculus, differential equations, linear algebra and complex analysis. The Open Textbook Library offers a number of free, open-source, peer-reviewed, and high-quality text books of Mathematics and Statistics. These books can be downloaded for no cost, or printed at low cost. All textbooks are either used at multiple higher education institutions; or affiliated with an institution, scholarly society, or professional organization.

The primary goal of the Community College Consortium for Open Educational Resources (CCCOER) is to create awareness of OER and help colleges to identify, create and or repurpose existing OER to improve teaching and learning and make education more accessible for all learners. OER Commons helps educators, students, and lifelong learners avoid time-consuming searches and find exactly the right materials. With a single point of access, everyone can search, browse, and evaluate resources in OER Commons' growing collection of over 50,000 high-quality OER. Similarly, Classroom Aid offers a large number of free resources for teaching and learning of Mathematics. National Council of Teachers of Mathematics offer free resources for teaching mathematics including activities, lessons, standards, and web links on illuminations. One can freely use these online activities and manipulative to teach or play mathematics. The OER STEM Project, aims to strengthen science, technology, engineering and math (STEM) instructional content and practice in adult education specifically through the use of OERs. The project engages adult educators to serve as user group members who locate, use, evaluate, and share science and math OERs that are appropriate for adult education classes. The project also developed online professional development courses for teachers on how to use OERs for math and science instruction in their adult education classrooms.

A look on all these initiatives helps us to draw some conclusions. First, OER offer a number of opportunities to improve the affordability and availability of mathematics education, and above all, can help adult learners to engage in lifelong mathematics learning. Second, among these initiatives, only few focus on adult learning of mathematics. Third, new initiatives and efforts to use OER in different ways and formats to support adult learners to practice lifelong learning of mathematics is expected to yield rich dividends. Fourth, promoting use of OER for adult earning of mathematics is a mammoth task that requires specific policies and interventions at different levels. Fifth, considering the benefits attached to using OER for lifelong mathematics learning among adult learners, it becomes imperative that main stakeholders and policy providers, namely International organizations/institutions (UNESCO, COL, WORLD Bank, ICDE, European Union, etc.), governments, mathematics education institutions, and adult learners themselves must come together to offer and promote OER supported lifelong mathematics learning. In backdrop of these conclusions, proposing specific strategies to use OER for lifelong mathematics learning of adult learners become imperative.

15.5 Using OER for Lifelong Mathematics Learning of Adult Learners: Potential Strategies

A project carried by American Institutes for Research reports three key findings related to use of OER for STEM teaching and learning in adult education— (i) adult educators need time and training to effectively incorporate OER into instruction,

(ii) adult learners need support in using and learning from technology, and (iii) there are limited numbers of OER available for specific use in adult education (Shaewitz, 2017). These observations are equally applicable in terms of adult learning of mathematics. In fact, use of OER for lifelong mathematics learning has still not drawn the attention of adult learning providers and adult learners as well. There are some probable reasons to it. First among these is that only a few initiatives of anecdotal nature of using OER for mathematics learning is available. Second, many adult learners lack awareness and knowledge about the OER and their use for mathematics learning. Third, enough attention has not been paid at policy and institutional level to tap the potential of OER for lifelong mathematics learning. Fourth, not many efforts have been made to search possibilities of using OER for lifelong mathematics learning. In a way, the potential of OER for mathematics learning of adult still remains untapped. Considering these probable reasons, following specific strategies may be of some help to policy planners, organizers of existing OER based leaning initiatives, and all those who would like to take benefit of OER for lifelong mathematics learning of adult learners.

15.5.1 Design OER-Based Courseware and Textbooks of Mathematics for Adult Learners

As first initiative, learning communities across the globe can come-up with OER-based courseware for adult learners of mathematics. The main difference between traditional courseware and OER-based courseware is that former uses licensed and copyrighted material while later uses different type of open digital publication of high quality educational materials. Keeping this feature in mind, the international organizations of mathematics education are supposed to come forward to develop and release adult learners' centric OER-based courseware of mathematics learning. These OER-based courseware of mathematics learning will help the institutions, learning providers, and adult learners in many ways. First of all, these OER based courseware will almost be free and offer the flexibility of addition, deletion or revision with ease. Second, it will be easy for institutions and organizations to promote and disseminate the courseware for adult learners without any financial and copyright issues. Third, it will be very easy for any educational institution or organization to localize the courseware according to need, level and language of targeted adult learners.

The available OER can also be used to create text books of mathematics for adult learners. Currently, there are sufficient numbers of high quality OER in mathematics and need of the hour is that education providers must come forward to initiate and fund some projects to develop OER-based text books for adult learners of mathematics. These OER-based text books will especially help adult learners to keep learning mathematics without much financial burden. The other benefit of OER based textbooks will be its easy accessibility via different electronic devices like desktops, laptops, tablets, e-readers or mobiles. It will also be easy to edit and revise OER-based textbooks and come-up with newer versions on periodic basis. One can hope that ease of localization and adoption of these textbooks in different contexts will make them a popular choice among adult learners to practice mathematics learning on continuing basis. In general, OER-based text books will be a valuable resource for those adults who intend to practice lifelong learning of mathematics with interest and ease.

15.5.2 Promote and Produce OER-Supported Mathematics Learning Programme for Adult Learners

As other potential use, OER may be employed as an additional support for study of mathematics. Usually adult learners demand something more than that they have already got in classrooms from their teachers. This more may be defined in terms of adult learners' desire to learn the content in a different way than presented in the classes when they were young. OER can play important role to fulfill this wish of adult learners. Explaining the use of OER as additional support to study, a write-up from DARCO/EADTU (2012) suggests,

> As these materials are published with open licensing tutors, teachers and lecturers are also recommending resources as part of a formal course. Alternatively, an open resource might just be used by a casual learner who is curious about a subject and just wants to understand more.

In other words, use of OER as support material for mathematics learning will also be helpful to draw the attention of adult learners from different fields and will promote informal learning of mathematics.

The other notable observation is that majority of available OER for mathematics learning has not been produced specifically for adult learners. Usually, a number of individuals produce different types of OER by using different instructional approaches and learning styles. Whereas, it is a well-known fact that adult learners have distinct learning needs and styles. Therefore, these OER, produced for diversified group of learners is supposed to have less impact on adult learners. In comparison, OER specifically produced for adult learners is supposed to make better contribution in terms of learning. To make this happen, all those having knowledge and expertise in mathematics education must be required to come forward and develop different OER supported learning programs specifically for adult learners. Once developed, these OER based learning materials will be a valuable support for adult learners to learn mathematics according to their learning needs and styles.

15.5.3 Adapt OER-Supported Mathematics Learning Material in Local Languages

A look on the global adult population reveals that majority of adult learners live in those countries where English is not a native language. In other side, language of majority of available OER is English. This situation creates a language barrier and deters majority of adult population to take full benefit of available OER. Therefore, adoption of available OER in different regional and local languages will be a useful step to promote lifelong learning of mathematics. This language support will help learners from different sections, localities and societies to take full benefit of OER for mathematical learning purposes. In other side, language adoption will also be helpful to showcase and detail mathematics learning styles and practices of different cultures and societies at global map. In addition to these benefits, adoption of available OER in different languages will be a significant move to popularize adult learning of mathematics across the globe.

Here a very importunate question arises that which section of society will come forward to adopt as well produce OER for mathematics learning of adult learners in different languages. As per the available information, individual teachers and researchers use and produce OER on their own initiative and most of them do not get any monetary benefits for this work. Therefore, it is expected that teachers and researchers from different countries will come forward to develop locally suited OER-based mathematics learning programmes for adult learners of their regions. Besides, it also seems that adult learners will themselves take initiatives to contribute for development of such programmes for their fellow learners and support each other to convert existing OER in mathematics in their own languages. In a way, this strategy will not only help in adoption of OER in local languages but also bring fellow adults to come together and help each other to practice mathematics learning on continuing basis.

15.5.4 Train Teachers to Use OER for Mathematics Learning of Adult Learners

The argument to train teachers and instructors to use OER for adult learning of mathematics is based on the assumption that this type of training will be helpful in filling the huge gap in demand for skilled teachers, particularly for adult learners. The other assumption is that being OER-based training, it will be easy for teachers across the globe to adapt these resources to different cultures and languages. Talking about the benefits of training teachers to use OER, Misra (2012, p. 3) argues,

> use of OER for various teaching learning purposes can support teachers in many ways like making their teaching meaningful; accelerating changes in the traditional teaching learning process; and developing a culture of independent study among their students.

While, Park (2009) suggests that OER training will help teachers to learn how to use OER according to its license status, and realize that the commons of open educational resources is vast and global, open to be adapted, derived, and remixed with other OER on the Internet.

The organizations and institutions associated with adult learning across the globe are required to train teachers and instructors about the use of available OER for adult learning of mathematics. During this training, they can be taught about available OER, benefits of using, and modalities of using OER for adults learning of mathematics. This training can be organized in face to face or online mode. Those successfully completing the training may be certified as 'trained for using OER for mathematical learning teacher'. It is expected that this training would enable teachers to see open courseware as part of a larger world of open materials and communities, rather than as an institutional initiative. One can hope that these trained teachers will employ available OER via different modes and modalities (offline, online) to teach, guide and counsel adults to become lifelong learners of mathematics. The other advantage is that these trained teachers will also act as a master trainer for all those who are interested to promote adult learning of mathematics.

15.5.5 Create OER Repositories of Mathematics for Adult Learners

There are number of OER available about different branches of mathematics. These OER are available electronically and includes items such as lecture notes, reading lists, course assignments, syllabi, study materials, tests, samples and simulations. But these OER are scattered and one has to search a lot to find them. Creation of mathematics specific OER repositories for adult learners will be helpful to overcome this situation. Repositories of OER (ROER) have been defined by McGreal (2011) as digital databases that house learning content, applications and tools such as texts, papers, videos, audio recordings, multimedia applications and social networking tools. Through OER repositories, resources are rendered accessible to learners and instructors on the World Wide Web. In this context, mathematics specific OER repositories will be a value addition to promote lifelong learning of mathematics.

These OER repositories will be helpful for adult learners of mathematics in many ways. First, these repositories will help learners to access different types of mathematics learning materials at one place and free of cost. It is expected that with the help of these repositories, adult learners across the globe will be able to access different kinds of mathematics content and resources at one place by simply clicking a mouse or tapping on the phone. The easy availability of resources through these repositories will motivate adult learners to learn mathematics by different ways and techniques. These repositories will also help adult learners to edit or modify the available resources according to their needs and purposes.

In nutshell, creation of OER repositories of mathematics will be a very useful step to promote lifelong learning of mathematics among adult learners.

15.5.6 Establish OER-Supported Mathematics Learning Communities for Adult Learners

Establishment of learning communities is a new trend now. Learning communities in general are supposed to help learners living across different geographies and locations to connect each other and share content, ideas, and materials related to a specific field. Following this pattern, creation of OER supported mathematics learning communities for adult learners will be another useful step to promote lifelong learning of mathematics among adult learners. These communities may be established online as well as in the form of traditional organizational establishment. The role of OER experts and organizations will be to establish and provide technical support to these communities, while adult learners will be required to take care and run these communities. These establishments will act as connecting link for adult learners to share their mathematics learning needs, concerns, problems and doubts.

It is common knowledge that adult learners of mathematics live in different places and have distinct learning needs. Being adults, often these learners live and learn alone. Sometimes, this loneliness deters one from practicing something new or learning on continuing basis. The OER supported mathematics learning communities will be a very helpful platform for all those adult learners feeling de-motivated or hesitant to learn mathematics. In addition, these establishments will also provide a forum for adult learners to showcase their expertise and experiences related to mathematics learning. In other words, these learning communities will support adult learners across the globe to connect as well share their mathematics learning needs and expertise for larger benefit of individuals and societies. This connectedness will ultimately make adult learners more motivated and committed to learn mathematics.

15.6 Conclusion

Lifelong mathematics learning is a necessity of our times. Promotion of lifelong mathematics learning among adult learners offers multiple benefits ranging from personal to social to economic gains. Efforts have been made in different parts of the world to realize this potential but success still eludes. In other words, the challenge for policy makers and practitioners in adult mathematics education is to find effective ways to break through the prevailing barriers so that adult learners may experience success. Need of the hour is that efforts should be made at different levels (personal, societal, institutional, and governmental) to overcome these challenges and promotion of lifelong mathematics learning among adult learners.

Researcher hopes that governments, policy planners and educationists across the globe will pay much more attention and find ways to implement above proposed strategies to use OER to promote lifelong mathematics learning among adult learners.

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Chapter 16
Learning from Research, Advancing the Field

Katherine Safford-Ramus

Abstract Much of the research in adult mathematics education resides in doctoral dissertations and conference proceedings. At the Thirteenth International Congress on Mathematics Education (ICME-13) The topical survey for Topic Study Group 6 (TSG06) presented a summary of that research and the themes that surfaced as the research was reviewed. This chapter highlights a subset of that work, namely the doctoral studies that recorded the voices of the students and teachers. A discussion of the international implications for the work is presented. The chapter concludes with suggestions for the direction that future research might take.

Keywords Adult · Mathematics · Education

16.1 Introduction

The literature review for Topic Study Group (TSG)-6 published prior to ICME-13 identified several areas of adult mathematics education research where a substantial amount of work has been published (Safford-Ramus, Misra, & Maguire, 2016). At the same time, it exposed shortcomings in published research to date, areas that are ripe for exploration or currently under-reported. Focusing on research reported in doctoral dissertations, this paper will revisit the work discussed in the topical survey, spotlight those topics that call for priority attention, and speculate on ways that the adult mathematics education community can accomplish this task.

The dissertation research summarized in this chapter originated in the United States and at first might seem parochial to that nation. I suggest, however, that it is increasingly relevant at a time when globalization has rendered national economies interdependent. Furthermore, in the quarter century that I have spent immersed in the international adult mathematics community I have witnessed globalization trends among providers of education to adults. Around the world, at the lowest level

K. Safford-Ramus (✉)
Saint Peter's University, Jersey City, NJ, USA
e-mail: ksafford@saintpeters.edu

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of instruction, refugees need the services of adult basic education providers. At the other end of the spectrum in countries where university education was once limited, the concept of lifelong learning has expanded to include tertiary studies.

16.2 Methodology

The research in adult mathematics education is reported in three major depositories: doctoral dissertations and the publications of Adults Learning Mathematics—A Research Forum (ALM) consisting of the proceedings from their annual conference and an electronic journal. These are not the only sources but do constitute the bulk of published work in the field. While the latter two are readily accessible on the ALM website, http://www.alm-online.net, doctoral dissertations are more difficult to retrieve as they are stored in a fee-based database, http://www.proquest.com/products-services/dissertations whose typical subscribers are universities.

As part of an ongoing research project, the dissertations from the past two decades have been downloaded and read. To keep track of the findings, a database was built that captured pertinent data about each document including an index of primary and secondary themes that are the foci of the work, http://www.alm-online.net/useful-links/resources/postgraduate-us-dissertations-on-alm-99-2016. This theme categorization was a qualitative determination and another researcher might choose other designations. Six major categories of themes emerged in the end, among these were student and teacher specific issues (Safford-Ramus, 2017). This subset of the greater body forms the basis of this paper. Those studies that attended to the student voice, either quantitatively or qualitatively, are presented first followed by the smaller group of studies that examined teacher characteristics. Taken together they offer suggestions to consider when refining one's teaching of adult mathematics students.

16.3 Student Issues

16.3.1 Math Anxiety

Perhaps the affective factor that is best documented in the literature about affect is *math anxiety*. Thirteen of the 122 doctoral dissertations reviewed focused on this phenomenon. It comes as no surprise that students who are anxious avoid taking courses in mathematics. Doing so precludes the pursuit of degree programs that require higher levels of college mathematics. This, in turn, locks them out of jobs that require mathematics, science, or engineering degrees. The problem is prevalent in, but not restricted to, women returning to study. Quantitative studies often found an inverse relationship between math anxiety and self-efficacy.

Using a case study methodology, Yuen investigated Mathematics Anxiety Learning Phenomenon (MALP) with six adult students taking a developmental mathematics class. Five major themes evolved from her analysis of the interview transcripts and student journals. The first theme concerned the learners' beliefs about mathematics, namely, that it is a series of prescribed steps that should be memorized and duplicated and those individuals who can deviate from the steps using alternate methods and problem-solving strategies are able to do so because of innate ability. The second theme concerned roadblocks to learning mathematics. These include, but are not limited to, incomprehensible texts, impatient instructors, fast-paced syllabi, and demeaning responses to questions. A third theme recognized student perception of a duality in mathematics. Material was either clear or incomprehensible, knowledge was either conceptual or procedural, work and solutions were either right or wrong. The fourth emergent theme was the belief that learning mathematics required, and was completely dependent on, good rote memorization skills. Finally, a fifth theme focused on student efforts to take control of their learning despite the frustrations implicit in the second theme.

Based on the plentiful findings that emanated from her data analysis, Yuen suggested the following action list for developmental instructors intent upon decreasing the MALP of their students:

- Create a learner-centered environment that encourages active participation by the students.
- Help learners to re-conceive mathematics in a dual manner—both conceptual and procedural—respectively as rational and functional approaches (Yuen, 2013, p. 165).
- Foster and sustain positivity in the learning environment (Yuen, 2013, p. 166).
- Contextualize the mathematics using problems rooted in real-world situations.
- Support learners' transition to defining mathematics as a reasoning and problem-solving discipline (Yuen, 2013).

16.3.2 Perspective

Cohen examined the affective factor of *perspective* in a case study that explored in depth the "ways of knowing" of three adult women in a basic mathematics course offered at a university. Using interviews and individual teaching sessions she gained insight into the beliefs, and sometimes contrasting performance, of the subjects and their ability to do mathematical tasks. Cohen situated her findings in the work of Baxter Magolda and Kasworm as well as Graham and Donaldson and Skemp. Like the subjects in Yuen's study, the women in this study expressed the belief that you are either good at mathematics or you are not. Cohen's subjects, however, also felt that one can increase performance in subjects, including mathematics, through hard work although they saw that more clearly for other subjects. The participants had difficulty seeing the connection between the word problems in a typical textbook and their lives outside the classroom. They viewed the teacher-learner relationship in the mathematics classroom traditionally and did not feel that they were free to participate in discussions in the way they could in other disciplines at the university (Cohen, 2002). These summaries do not do justice to the rich discussions in both dissertations—Yuen and Cohen. Readers interested in either topic would benefit from reading the primary sources.

Using semi-structured interviews, Kimura explored the perspectives of students and teachers in a developmental mathematics program at a large community college. The courses at all levels were offered in traditional lecture format and the lowest level course was also offered as a self-paced, lab-based format. Sixteen students and thirteen faculty members participated in the study. After analysis, she found that the data could be categorized under three major umbrella themes. The first theme, which encompasses hatred of math, includes the familiar litany: a deep hatred of math, negative self-concept and low self-efficacy, a belief in the genetic ability to do or not do math, some evidence of learning disabilities, responsibility for previous failure as well as the surety that success is important, and criticism of the pace and instructor attitude in the developmental classroom. The second theme (whose title I loved) she termed "Magical Thinking and Logical Fallacies" (Kimura, 2012, p. 147). Included here are:

> These beliefs are seeded in a realm that go beyond a simple misapplication of logic; rather, they refer to ideas that involve: absolution of past performance; an assumption that college and high school are similarly structured; a belief in last-minute grade resuscitation; an overestimation of personal skills; an aversion to seeking help; a fixation with grades over achievement; and a belief that education is purely an economic transaction. (Kimura, 2012, p. 148)

The final theme that Kimura identified was a binary sense of doom on the part of the students and a resistance to failure. Despite a good degree of evidence to the contrary, i.e. poor grades and repeated coursework, some students in the study expressed confidence that they would prevail and advance to college level mathematics work. The literature review of this dissertation contained an excellent history of the community college movement in the United States as well as the resultant need for developmental programs to compensate for deficient scholastic credentials.

16.3.3 Self-efficacy

Albert Bandura did the seminal research on *self-efficacy* (Bandura, 1997). Fourteen of the doctoral dissertations had self-efficacy as their primary or secondary focus. Three studied it as a predictor of success and, not surprisingly, found that students' higher levels of self-efficacy were more likely to persist and succeed in a mathematics course. While not a proven causal relationship, a holistic view of the research suggests that a decrease in math anxiety caused by or resulting in an increase in self-efficacy is likely to boost adult student success, or the success of any student for that matter.

The TSG6 topical survey shared research results from some of the studies on self-efficacy interventions (Safford-Ramus, Misra, & Maguire, 2016). A theme that repeats through much of the research reviewed is that of the pivotal role of the teacher. Rawley, in a dissertation that studied anxiety and confidence in women at a community college in the United States formed a 17 point recommendation for teachers:

(1) Teachers who show that they care about the students and are willing to listen to them and answer their questions have a positive impact on learning.
(2) Teachers should take care to connect new concepts to material taught previously.
(3) Sometimes learning multiple strategies can be overwhelming, knowing one good way to solve a problem might be better for adult students.
(4) A teacher who was caring and helpful was key to performance and persistence.
(5) Sometimes students reach an impasse in their progress and encouragement from the teacher is critical to forward movement.
(6) The pace of the course should reflect the student pace, not a pre-determined schedule.
(7) Large classes can be overwhelming. Small class size is more beneficial.
(8) Teachers need to tackle head-on any hostile environment issues. Demeaning remarks from fellow students can be so distressing that a student might drop the course.
(9) Stress detracts from performance. Open discussion of stress and math anxieties should be undertaken if necessary.
(10) Teachers should be aware that adult students may be seeing topics in the course for the first time. They need to be patient and understand that students are grappling with novel concepts.
(11) Be non-threatening, approachable. Develop a rapport with the students.
(12) Math tutoring centers should have evening hours and be staffed with friendly, knowledgeable tutors. Creative materials like manipulatives are a plus.
(13) Students bring negative experiences in mathematics classes to the adult classroom. Women should be encouraged to join in discussions and develop problem-solving skills.
(14) Connect the classroom learning to real-world situations.
(15) Students felt that traditional teaching in an organized, step-by-step process helped them to develop understanding of the underlying concepts.
(16) Teachers should be learners too.
(17) A special institutional program that focuses on the adult learner, adult women in this study, would provide support and encouragement for learners (Rawley, 2007, pp. 178–189).

These points are not unique to this study but appear as subsets in many of the studies presented throughout this chapter.

16.3.4 Success

While the literature about math anxiety paints a depressing picture, there are success stories and they provide hope and a roadmap to achieving a goal for students and instructors who counsel them. Parker narrates the results from a study of twelve adult students who had overcome their math anxiety and prevailed to succeed and identified six stages that they passed through on that journey:

> First, adults *perceived a need* to become more comfortable with math. Recognition of the need was followed by *making a commitment* to address the problem. Third, the math-anxious adults *took specific actions* to become more comfortable with math. Learning how to get the most out of math, they refined their study techniques, used learning tools, attended tutoring sessions, and applied relaxation techniques. These time-consuming actions required them to make sacrifices. Fourth, the adults recognized that they had reached a *turning point* and were no longer math anxious. The adults' *mathematical perspectives changed.* Finally, the adults became *part of the support system* for others seeking help with math, just as others had helped them overcome their math anxiety. (Parker, 1997, ii–iii)

Torvicia focused on the "high-end" student, that is adult women who returned to college to study for STEM (Science, Technology, Engineering, and Mathematics) degrees and who succeeded in their ventures. Her qualitative study looked at the lives of five women using interviews, journals, artwork, and observation to discern the paths that they took on their journeys to a degree. The subjects shared common traits—they had high self-efficacy and strong mathematics backgrounds. After an exhaustive analysis of the data, she identified themes that describe both barriers and supports that the women experienced. The barriers come as no surprise: financial concerns, personal problems, and work-time constraints. Descriptors of supports, positive influences on their successes, include spouses, peers, work colleagues, and university faculty and mentors. Family encouragement over the lifespan laid the groundwork for their adult success. A third and final theme that Torvicia derived was one she termed "harmony." She describes it this way "For each of the women, all the pieces are finally in harmony. The goals they are heading for are in alignment with their values. They did not take the traditional path, but they are right where they want to be" (Torvicia, 2012, p. 225).

16.3.5 Classroom Methods

Adults study mathematics across a broad continuum of educational levels. Some methods are more appropriate to one level than to another. This section of the chapter presents work from the most basic arithmetic instruction through mathematics content courses at tertiary institutions.

16.3.5.1 Adult Basic Education

Johnson explored the impact of active learning in the adult basic education mathematics class. She found that, despite initial discomfort with non-traditional methods, most students came to appreciate the opportunity for growth as learners of mathematics afforded by cooperative learning activities (Johnson, 2010, p. 118). Project-based activities offered opportunities to develop skills that contributed to academic success as well as employability (Johnson, 2010, p. 120). Luke inquired into the utility of using manipulatives in the adult numeracy classroom. She found no benefit from their use. Furthermore, some students found them "babyish" and one student stated that she used a debit card rather than cash (Luke, 2012, p. 61). This last comment opens the door to re-examining what truly are the numeracy skills needed in 21st century society.

16.3.5.2 Adult Secondary Education

Martin explored the effectiveness of Integrated Learning Systems (ILS) in an adult secondary school setting. She compared individual delivery versus cooperative (dyad) delivery and found no significant difference in the effect of either on performance or attitude (Martin, 2005, p. 68). Wharton experienced success incorporating Computer Aided Instruction (CAI) in a General Educational Development (GED) class but not without initial resistance. Her experience highlights the disparity between the technological skills of younger adult students versus older, returning students. Using cogenerative dialogue, "discussions among teachers and students regarding shared experiences in the classroom to identify and review practices that afford or constrain the teaching and learning of mathematics" (Wharton, 2010, p. 10), Wharton bridged the generational divide and constructed a course that was student-directed, a true example of adult learning theory in practice. Class time and activities were altered as instruction responded to student needs. Immediate instruction took place in what she termed "huddles" as well as mini-lessons derived from the need to clarify CAI questions. Students came to value the computer as the true assistant that the "A" in CAI implies (Wharton, 2010).

16.3.5.3 Tertiary Education

Developmental Mathematics

In the United States, many students entering tertiary institutions are under-prepared for university level mathematics courses. They are required to enroll in non-cdit-bearing courses termed *developmental* before they can advance to the university courses. Gilbert explored the effects of different course formats in an elementary algebra course: online, weekend, short-term, and computer aided instruction on student retention and performance. Comparing the alternates to traditional course format, he found that retention was higher for weekend and short-term courses and lower for online and computer-aided instruction. (Gilbert, 2010, p. 84) When he looked at performance, however, computer-aided instruction had the highest test results followed by weekend courses when compared to traditional courses. Short-term courses were less successful and online courses showed the poorest results (Gilbert, 2010, p. 88). Gilbert concluded that “As such, adult learners have an advantage over younger learners. They are aware of their own strengths and limitations and can adapt to their learning environments in ways that see them achieving higher average grades than their younger counterparts” (Gilbert, 2010, p. 89).

Moulton investigated the implementation of mediated instruction in developmental mathematics courses offered online. He concluded that “Frequent communication, prompt feedback, and instructor flexibility in working with students were primary factors contributing to effective online teaching. Interaction and self-motivation were primary success factors for students taking these courses” (Moulton, 2014, no page). Success was fostered by self-discipline and good time management skills. Some students wanted a hybrid component that offered the opportunity to meet the professor in person to work on particularly difficult material. Moulton (and I) expected students and instructors to cite the input keying of mathematics symbols as a challenge but that was not the case (Moulton, 2014, p. 100).

Baugher looked specifically at adult learners in developmental mathematics classes where online tutorials and quizzes were utilized. This was a mixed methods study. The quantitative portion revealed no significant achievement advantage to the online tutorial and a negative impact on student attitude measured at the end of the course (Baugher, 2012, p. 151). There was no age-related benefit or detriment due to the online tutorial. Seven themes emerged from interviews with six students. Three of the interviewees preferred the online tutorial over homework from the textbook. The other three were either neutral or preferred the textbook problems. All the students saw benefit from using the online program for homework completion. “The homework hints and step-by-step instructions, the ability to redo homework problems many times for practice, and the chance to raise the grade of the student were benefits for the online tutorial mentioned by students” (Baugher, 2012, p. 161). There was frustration expressed concerning the rigidly structured responses required from the students. They felt that a human grader would recognize the variations that a correct response could take. Sadly, the positive qualitative responses to the online tutorial did not translate to greater success on the course final exam nor result in improved attitude and confidence (Baugher, 2012, p. 155).

Garrett used a case study methodology to explore the effect of dynamic computer systems on the internal representations of mathematics by adult developmental students. Based on conversations with two students within teaching interviews, she surmised that the use of technology aided students’ familiarity with multiple representations of functions and helped them build representations based on their own thinking (Garrett, 2010, pp. 196–197). Corey Legge compared three different course offerings: Individualized, Traditional, and Mandatory Supplemental Instruction (MSI) in a developmental math course. She found no significant difference between the last two but a substantial success gap for the individualized course (Corey Legge, 2010, p. 72). Students in the MSI course did find value in the support, especially near test deadlines (Corey Legge, 2010, p. 73). Rambish also looked at developmental mathematics at the basic level but her study contrasted a conceptual approach versus a procedural emphasis while exploring the impact on results by ethnicity. She found that the conceptual methodology benefitted students regardless of race although African-American students experienced larger increases in course grades than White students (Rambish, 2011, p. 85).

Groce looked for a relationship between developmental algebra course outcomes, mode of instruction, and enrollment status for non-traditional students. She found no significant differences but does note that the percentage of students who achieved a higher passing grade was higher in live courses, that is, those not offered online. Also, students who were part time chose the online courses more often (Groce, 2015, p. v). Henley also focused her attention on online courses, but in her case the investigation compared standard online provisions versus the incorporation of video based instruction. The results were inconclusive due to substantial drop in student numbers during the course duration. The first of the three modules had the only sample size large enough from which to allow any result and that showed no significant difference between versions (Henley, 2015, p. 49).

In a study that spanned the community colleges within one state in the United States, Butler found that instructional changes that implemented research-based practices had a significant effect on student success. She recommended that institutions require entering developmental students take a study skills course (Butler, 2014, p. 100) and encourages the use of cooperative and collaborative learning methods in conjunction with a mastery learning component. On an institutional level, she advocates that non-traditional course structures like individualized instruction, mathematics refresher courses, immersion and bridge programs be made available to returning students (Butler, 2014, p. 100). Regardless of the course structure, Butler's study implies that the improvement of the critical thinking skills of the student will contribute to success. Outside the classroom she argues for strong academic support programs including early intervention and tutoring services (Butler, 2014, p. 101).

Moody surveyed students, instructors, and manufacturers to design an Open Educational Resource (OER) for use by individuals interested in polishing up their basic mathematics prior to taking a developmental mathematics course or beginning employment. He envisioned software that was free and accessible to anyone with access to the Internet and *Blackboard* software. The course that he describes would include fractions, geometry, and algebra, would incorporate visual tools, mind maps, pictures, images, and structured navigation, real-life examples across a span of age groups, tutorial videos with constant feedback on every problem (Moody, 2015, pp. 43–44).

Williams investigated student attitudes towards learning mathematics in a technological environment. This quantitative study sought to determine the relationship between six attitudinal variables as well as age, gender, and experience with the use of available technology in developmental mathematics classes. Performance expectancy, effort expectancy, social influence and facilitating conditions, attitude towards technology, and behavior intention all proved to be significant predictors of technology usage. Age and experience in technology usage were also significant but gender was not. Williams points out that performance and effort expectancy are the most critical variables that influence student usage of technology and suggests that instructors "demonstrate the benefits of the technology and communicate how a technology will prove useful for the particular subject" and "help new users by providing time before the introduction to the new technology" (Williams, 2012, pp. 68–69).

Watson focused on success, not failure, in a qualitative study of non-traditional remedial students who had soldiered through remedial math. The participants identified quality faculty as pivotal to success and described such an individual as "providing thorough explanations, being engaged, and being a good communicator; being accessible to students; providing supplemental study material; and being encouraging and/or supportive to students" (Watson, 2015, p. 149). A second line of support was tutors, peers, and study groups.

Frodsham used a case study methodology to determine if and how an intervention program tailored for women helped the participants to prevail over math anxiety and successfully complete remedial algebra classes at a community college. The project, *Project Independence,* is more than 20 years old. Frodsham strove to identify the ways in which the women in the program acquired the study, coping, and learning skills that improve their math performance in both the short and long-term (Frodsham, 2015, p. 8). The strongest support in this project derived from the presence of a "cohort" of students who progressed together and helped each other throughout their studies. All the students in the cohort had taken a math-anxiety class at the outset of their studies. The need for study skills and self-motivation were cited. Matching student learning style to teacher instructional style was one key to success. Believing that the instructor is approachable, willing to answer questions, able to use multiple approaches, and a positive role model were all characteristics valued by the women. In the end, students acknowledged that they are ultimately responsible for their learning and that the goal of the math-anxiety class was personal empowerment.

Using a case study methodology, Esposito-Noy describes the impact of a different intervention program, one that reflects a social modeling construct. Her study looked at the impact of embedding a peer role model in a basic mathematics course at a community college. Five of the six students who participated in a semi-structured interview found the presence of the peer model very helpful. Four themes arose from the interview transcripts, the first of which echoes the nurturing role of the teacher:

> One consistent theme was significant for nearly all of the participants: the significance of feeling cared for. The sense of feeling cared for informed the student-peer model relationship for nearly all of the participants and from this theme emerged two others: the importance of acquiring college know-how, and the importance of setting, maintaining, and

> understanding expectations. The fourth and unexpected theme that emerged was the significance of faith. These themes served as the foundation for the embedded interventions and informed how students experienced the peer model. (Esposito-Noy, 2013, p. 92)

In a 2016 dissertation, Dawes explored ways to effectively incorporate an online homework and testing software package into an elementary algebra course. In a mixed methods study, he examined the impact of multiple learning tools from the online preparation and rigorous enhancement platform (OPREP) WebAssign™ on student success and satisfaction. Three course configurations were studied: traditional, hybrid, and an Accelerated Study in Associate Program (ASAP). A quantitative analysis of the results of homework assignments, quizzes, and practice examinations showed a positive correlation between each tool with the final examination grade with the strongest correlation on the latter two. The qualitative data, drawn from student comment sheets, student evaluations, and the responses from students who completed a questionnaire, was analyzed using grounded theory. It revealed student appreciation for the tools linked to homework assignments that allowed them to view video clips stepping them through solutions. They felt that the immediate feedback provided by the OPREP was its most important feature. "Combining the gradebook with extensions facilitated self-directed learning and self-regulated learning in an online environment where students could control the trajectory of their performance" (Dawes, 2016, p. 132). The companion role of the teacher continued to be valued by the students with the learning tools seen as a positive supplement to classroom instruction, not a replacement.

Credit-Bearing Courses

Hosch examined the connection between teaching style and the reduction of math anxiety in students taking their first university mathematics course. She found that students whose teachers employed a student-centered approach, serving as a guide rather than a leader, were more confident and less anxious at the end of the course. There was an inverse relationship between teacher-centered instruction and math anxiety. The bad news is that roughly 60% of the teachers fell into that category (Hosch, 2014, p. 66).

Parsons looked at the effect that a writing component would have on performance in a pre-calculus course. She found that "many students made significant changes in approaches to learning and also made deep and meaningful conceptual connections as a result of *Writing to Learn Mathematics*. It also was apparent writing in mathematics and about mathematics encouraged students to reflect on what they were learning and facilitated meaningful connections about content and themselves as learners (Ray Parsons, 2011, p. iii)." Bellaire designed and taught a sophomore-level statistics course for traditional age college students. She found that the students could not be typified by Knowles definition of the adult learner yet they did respond positively to andragogical teaching methods. They journaled positive feedback on learning activities that were participatory, hands-on, problem-centered, and experiential (Bellaire, 2005, p. 108).

Adams explored the difficulty that students have learning the concept of limit in a calculus class. She situates her study in the theories of Piaget, Inhelder, and Skemp about student construction of knowledge and her recommendations reach back to foundational work in middle and secondary school. The dissertation contains specific recommendations for instructors in the university calculus class as well as sample lesson plans. Adams presents a strong argument for the importance of language and definitions in the formation of student understanding and sees a link between effective reading instruction and mathematics learning. I was reminded of the connection I saw between the phonics versus whole language and procedures versus concepts arguments that raged in the 1990's when I was pursuing doctoral studies. At the same time, the evolution of literacy teachers into numeracy teachers came to mind. Could methods used in the first sphere inform the second? (Adams, 2013).

Roubides looked at the potential for OER use in a College Algebra course at a community college. His quantitative study explored the impact of three self-regulatory behaviors on the potential for successfully completing the course. Of the three behaviors he examined: goal setting, self-efficacy, and time and task management, only the last showed a significant influence on student grade outcomes. The particular course he appraised was an accelerated, online course intended to provide students with the opportunity to advance academically in half the usual time, eight weeks versus the standard US sixteen week semester. Such a course appeals to the adult student who wants to move along in his degree program. Earlier in this paper it was reported that online courses had the lowest retention rates of the courses studied. Roubides work underscores the need for adult students to assess frankly their available time for study and their skill at managing that time before embarking on a mathematics course that is online, particularly one that is accelerated (Roubides, 2016).

The studies summarized in this section cover a broad range of questions and sometimes present contradictory conclusions. The themes that bind them together are the need to identify course structures that offer both emotional and instructional support to assist adult students as they advance their confidence and self-efficacy with the goal of diminishing math anxiety and achieving success in mathematics. The interventions examined operated at both the instructor and institutional levels with the common goal of improving the mathematics education offered to the adult student.

16.4 Teacher Issues

16.4.1 Teachers

If the description above posits good teaching, what does research tell us about the reality of teachers of adults? The answer lies along a continuum. Teachers of adult basic education may be well-intentioned volunteers with training in neither mathematics nor educational theory. Paid positions are often staffed by individuals who are certified to teach elementary school children. At the other end of the spectrum, tertiary instructors at the developmental level may have master's degree outside of mathematics while university faculty might have a doctorate in mathematics but no coursework in educational theory.

Kantner interviewed four community college instructors in order to explore their view of mathematics and mathematics education. She found that each had a different philosophy of mathematics and that philosophy drove their teaching across courses. The four categories she identified were mathematics as the study of patterns, mathematics as the action of mathematicians, mathematics as an axiomatic system, and mathematics as the study of connected relationships (Kantner, 2008). McManus also studied mathematics faculty at the community college level. Her study quantified their beliefs, feelings and behaviors in light of adult education theory. She found that respondents were average or below average in their application of andragogical principles, younger faculty (less than 35 in age) had lower empathy for adult learners while faculty with a doctorate or professional degree had higher empathy. The same pattern held for the issue of trusting learners and the younger faculty were less likely to apply andragogical principles in their planning and delivery of instruction or accommodation for student uniqueness. Faculty holding a doctorate or professional degree were least insensitive to learners while those with the rank of associate professor were most insensitive. Individuals holding the rank of assistant professor were most likely to use experience-based learning techniques. Individuals under 35 were least likely to use evidence-based learning techniques and more likely to offer teacher-centered instruction. (McManus, 2007).

Hamilton interviewed developmental faculty from six Nebraska community colleges to determine their reactions to changes being suggested, and in some cases already implemented, to improve developmental education success. He found that few were aware of national trends or research in their field. Most were skeptical about a broad-based move to accelerate the courses and to replace classroom instruction with computer software. In fact, they felt that one-to-one instruction was the most effective route to student success. Where offered, they did value the contributions of student support programs and tutoring labs (Hamilton, 2014).

16.4.2 Diversity

Easley investigated STEM faculty perceptions of diversity. The findings indicate that faculty of color, women faculty, and faculty that identify as GLBT framed their engagement in intellectual exchange and collaboration in diversity through their personal identities while white male faculty framed their engagement through their personal lens of their professional background and position or rank. This finding indicates that lived experience and identities inform faculty engagement in diversity issues, as well as the university type and the culture of the discipline. In decision making processes for diversity, tenured faculty reported that they struggle with an internal battle whether they should try to fit in the academic culture or fight for others while pushing against the academic culture. Depending on faculty's experience, faculty members decided to either address the issue of diversity or attempt to fit in the academic culture. The final major finding is that every participant understood mentoring was important and that mentoring for diversity can be complicated. In mentoring relationships each person may not share identities, but they must desire success of the mentee, particularly if the mentee is a diversity hire (Easley, 2013, no page). Raney studied the beliefs and practices of community college developmental faculty towards culturally responsive instruction. She found inconsistencies between espoused beliefs and classroom practices that impede student success (Raney, 2013, p. 77).

16.4.3 Professional Development

Patterson sought to identify the methods of professional development that would best assist teacher transition to the standards promoted by the National Council of Teachers of Mathematics and to instructional practices that encourage students to pursue learning as a lifelong activity. Hers was a mixed-method study that began with a survey of 30 secondary school teachers that polled their attitudes towards the following PD methods: collaborative coaching and learning, collegial coaching, cognitive coaching, mentoring, lesson study and study teams. This was followed by interviews with six of the survey respondents. The six unanimously chose collegial coaching, a process in which faculty or teacher pairs (colleagues) assist one another by enriching and sharing their knowledge and expertise (Patterson, 2009. p. 8).

Since 2009, more than 40 states in the United States have implemented uniform standards for mathematics achievement by grade level from elementary to secondary education, the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices, 2010). While the impact on tertiary instruction has been minimal, a subset of those standards that reflect adult student needs, the *College and Career Readiness Standards for Adult Education*, should eventually affect adult basic and secondary education (United States Department of Education, 2013). Kniss studied professional development efforts aimed at elementary school teachers that may inform PD for Adult Basic Education (ABE) and Adult Secondary Education instructors charged with the instructional shift to the new standards and their assessment. Her subjects favored collaboration as an effective tool to implanting change in their view of, and implementation of, the *CCSSfM*. She states:

> Participant comments seemed to indicate that participants did not value the collaborations simply because they were granted the authority to step into a leadership role (although this may have been an aspect of their collaborative experience), they expressed that they valued the opportunity to use the collaborative setting to deeply examine the content they were to teach. (Kniss, 2015, p. 75)

Two dissertations looked specifically at pre-service student populations. In a qualitative study that employed interviews and classroom observations, Wheeler compared traditional and non-traditional pre-service elementary teacher candidates' thinking about mathematics and mathematics teaching. She found that the adult students tended to hold traditional beliefs about the nature of mathematics and maintained those beliefs despite completing a trilogy of mathematics content courses that were constructivist in methodology. On a positive note, the non-traditional students were more confident in their ability to teach students in grades K–6 than the traditional students (Wheeler, 2009). Johnson focused on proportional reasoning in pre-service teachers, both elementary and secondary candidates. She found that the students were positioned on one of four levels of understanding and through intensive interviews/lessons could transition across levels. This dissertation contains several problems in the appendix that could be used in either pre or in-service professional development to spark discussion about proportion (Johnson, 2013).

Unobskey tells a cautionary tale. He implemented a year-long intervention with teachers in grades six and seven to determine the effect of collaborative planning on teacher practice where the support of struggling students was paramount. While the sixth grade cohort showed progress in working together, the results from the seventh grade were disappointing. Unobskey attributes the shortfall to reluctance on the part of collegial teachers to criticize each other and an innate belief that teaching style is a personal construct, He acknowledged that change takes more than a one year intervention and

> Clearly, establishing school norms and a culture of relentlessly searching for improved practice is crucial. However, during the Leadership Project, the Principal/Researcher also learned that teachers needed to experience the ups and downs of carefully comparing each other's work several times before they could see the benefits of this collaboration for their students. Clearly, a principal needs time to build these beliefs and habits before teachers have the capacity to carry on independently. (Unobskey, 2009, p. 143)

Teacher issues pose a two-pronged problem—in some cases the teachers themselves lack a depth of conceptual knowledge of the subject and even those who are accomplished mathematicians may not possess effective strategies to meet the needs of adult mathematics students. These few studies have barely scratched the surface of the research questions that should be posed and investigated in order to improve the current state of adult mathematics instruction.

16.5 Discussion

The following discussion is informed by my work in the adult mathematics education community over the past twenty-five years. The United States has a long history of "second chance" tertiary education for adults dating back to the Great Depression but accelerated dramatically with the end of World War II and the *Servicemen's Readjustment Act of 1944* that provided educational benefits to roughly 8 million returning veterans. In the 1960s the expansion of the community college network offered open access and low tuition options for anyone with a high school degree or its equivalent. The open access facet of these institutions necessitated the creation of developmental education programs, many of which were the site of the research reported earlier in this chapter. Chapters within this volume attest to the presence of "second chance" opportunities in other countries. While the programs may still be housed in traditional tertiary or vocational education settings, the need to upscale mathematics skills for adult students is present and the research findings gleaned from the doctoral dissertations may provide insights to instructors, particularly those specified as developmental provisions.

At every level of provision, it would seem that providing experiences that build self-efficacy will help to decrease math anxiety, opening the door to the pursuit of higher level mathematics studies as well as other STEM disciplines. Studies reported earlier in the chapter point to the following as critical considerations for success in an adult mathematics education course:

- Multiple platforms should be offered as no one "type" of course insured success for all students.
- Adult students need sound selection advice when choosing a course format. While an intensive course format and/or online class might seem preferable, research has shown that this often sets the student up for failure as adult responsibilities impose time constraints that clash with the ambitious program selected. Placement advisors should encourage adult students to make realistic choices. While not included in any of the cited research, perhaps a personality assessment tool could be created to assist the advisor and student with this decision.
- Ongoing support needs to be available for adult mathematics students once the course is underway. This can take the form of regular cohort seminars, individual counseling, or math support centers specifically targeted to a non-traditional population.

Technology has come of age over the past three decades. Early drill and practice software has been replaced by embedded tutoring videos tailored to assist students in understanding the possible steps to a solution. One of the papers presented in a TSG06 session addressed at length the potential for Open Educational Resources (OER) to enhance classroom experiences as well as homework assignments. Textbooks are now available in electronic formats, allowing students to study anywhere without the need to carry a heavy text with them. Public WiFi spots enable access to Internet resources on a variety of devices virtually anywhere as do wireless phone service providers.

Challenges and unanswered questions still exist. There is very little research that focuses on issues of race or gender yet women and minority adults are over-represented in the adult mathematics classroom. Do they need specific classroom methods or has age smoothed the differences that may have impeded their youthful learning experiences? An enormous question surrounds the need for professional development at all levels of adult mathematics education. How do we communicate the research results to those who would most benefit from the information? Many studies point to a need for long-term interventions if meaningful change is to occur. At times the task seems like pouring a cup of tap water into the ocean in the hope of altering the salinity. The following section suggests an agenda to move the field forward.

16.6 Setting an Agenda

There is now a solid four decades of research in adult mathematics education upon which to fashion an action agenda. Certain things are clear. The teacher is pivotal to student success. In repeated studies, students have described the characteristics of a good teacher. Teachers have a dual perspective—they are learners seeking a deeper understanding of mathematical concepts and teachers desirous of honing their expository craft. Technology has evolved dramatically over the past twenty years yet we have not effectively reaped the benefit of those improvements in courses or professional development offerings.

The research reported in these dissertations and a presentation made at the Topic Study Group 6 sessions has caused me to re-think the materials I use to teach and to consider a radical change. First, students repeatedly reported minimal use of the textbooks used in courses despite the substantial price paid for them. Even in the study where they reported great satisfaction with the electronic assistance provided, the e-text was rarely referenced by the students (Dawes, 2016). In my own dissertation work, we authored a text with minimal content and provided practice problems using a slender commercial workbook text (Ramus, 1997). The time may have come to consider dispensing with a commercial text for my current courses and providing students with meaningful classroom experiences and lectures with OER links for homework assignments and supplementary expository material. Secondly, this research has persuaded me to explore the tools available within the classroom management software provided by my institution. At the moment I only utilize the e-mail and assignment options. Quizzes, required or optional, provide immediate feedback to students and there is a facility to link to other sources that could provide access to appropriate OER work. Finally, the weather in the Northeast part of the United States often causes canceled classes in January and February. Technology provides an opportunity to substitute online content, creating a temporary hybrid class when face-to-face is pre-empted by the whims of Nature. Until recently it could not be expected that students have Internet access off-campus but increasingly their phones provide it so the classroom management software becomes a reasonable vehicle of communication.

ICME offers the marvelous opportunity to foment a global plan to improve the professional development offered to mathematics teachers of adults. I offer the following two suggestions to jumpstart the conversation. First, the time has come for an advanced degree specific to adult mathematics education. Within that curriculum there could be a subset of courses available for individuals who want a lesser degree of credential. While the faculty could reside anywhere, the degree should be centralized. Our numbers do not justify multiple institutions and would be diluted if spread around.

Second, professional development efforts should focus on incorporating technology in ways that reflect the recommendations of the teachers in the cited studies. For example, perhaps a local cohort of teachers could gather for video conferencing with an instructor who is remotely located. Conferencing software exists that allows individuals in scattered locations to communicate visually and orally. Assigned readings, online video instruction, and discussion boards could be utilized at the convenience of the individual participants. The possibilities seem to be limited only by our experience, not our imaginations. We started the conversation in Hamburg and let us continue it henceforth.

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Chapter 17
Conclusions and Looking Ahead

Jürgen Maaß

Abstract Looking back to our Topic Study Group 6 "Adult and Lifelong Learning" at ICME-13 in Hamburg, looking around (at what is going on in the area of adults learning mathematics), and analysing the situation of billions of adult learners, I will try to outline some major important aspects of future work like better communication within groups represented in ICME conferences, better qualification for teachers, more research about specific characteristic of adult learners, etc.

Keywords Adult · Mathematics · Education · Numeracy

17.1 Conclusions

While you are reading these words millions of adults in all countries of the earth are learning mathematics. Some of them have just started to learn because they never attended any school. Some of them took part in eight or more years of mathematics lessons at school but did not understand much (we have about 15–20% mathematical illiterate people in countries like USA or Western European countries). Some of them lost their job and need general or special qualification courses to have better chances for (re-)employment. Some of them want to get a better (paid) job, and need better mathematical qualification (often not named as mathematics in the course but as containing mathematics knowledge like bookkeeping, statistics, programming computers etc.). Some of them have high paid jobs like a chemist or engineer who needs special knowledge about statistics for doing quality control. Some of them are parents who wish to help their children at school. Finally, we like to mention all those people who have to learn some mathematics auto-didactically if they try to use the MENU function of their new electronic device (computer, TV, smartphone, …) to get access to all the fantastic features of their new machine. The menu functions are ordered along mathematical structures and algorithms. Problems

J. Maaß (✉)
Universitaet Linz, Linz, Austria
e-mail: juergen.maasz@jku.at

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while using them are often problems of understanding mathematics. Even more people have to understand tricks with statistics used in the media to convince them to buy useless things, to pay higher taxes, to spend more money for insurance, or to agree with crazy new laws and so on.

Everyone is learning mathematics every day? Lifelong learning is not only one of the often used advertising words? But who is teaching all those adults learning mathematics that are not learning auto-didactically? Who is training all these teachers? Please have a look at your university: Is there a special teacher education for those who will teach adults? No? Looking all over the world we know very few universities and university colleges offering such study. You like to know something about such a study? Please have a look at https://www.daea.dk/themes/adult-learning-in-denmark/!

Looking around in many other countries we find that typical teachers for adults are doing this in addition to their job as a teacher at school for younger students, as an experienced job owner (giving courses for beginners in this type of job), as a retired person and so on. All these teachers want to do the best for the learning adults, but they have no special training on a university level to do this type of teaching. They teach some hours a week for some seasons. This means that they have no time and no interest to read journals about teaching adults or to do mathematics education research about their courses. In most countries, we find no colleague in mathematics education who is engaged in research in the area “Adults Learning Mathematics” because this area is not seen on the university level. Our thesis based on this experience is simple: If no person is engaged in a special aspect of mathematics education in one university this aspect is not a research topic at this university. We know some part time hobby teachers for adults that worked at university but after they finished their PhD they had to look for a job for the rest of their life. And there was no job at a university for specialists in mathematics education concentrated on adults learning mathematics in their country. Therefore they changed their centre of research to other aspects, something more traditional concerning mathematics in school.

Having understood this background of our Topic Group 6 maybe you will be astonished that there is such a topic group with so many participants. We think that the main reason is that there is the group “ALM” (Adults Learning Mathematics http://www.alm-online.net/) that is organizing all people all over world that are working in this area. Looking at the history of this topic group and the history of ALM you find a lot of persons as chair or participants (and authors) of both.

Looking at the work done in the last decades and presented at the conference in Hamburg we see some progress and a lot of open questions. You will find an overview in the introduction of this book. Katherine Safford—Ramus is our expert for overview. If you want to start a new research project in this area, begin by reading her overview to get an initial orientation on the research situation.

Considering future, we have the feeling that the main impulse forward is written by David Kaye (who is chair of ALM now). There is a strong need for more contact to other groups in ICME or (in other words) to other aspects of mathematics education. This will be very good for our topic group because some questions about learning mathematics (like geometry, statistics, mathematics at workplaces …) are more reflected in the specialized groups. On the other hand, the groups concentrated on teaching mathematics at school could get a much better perspective on their field if they include what we know about results of the teaching some years after leaving school (keyword: sustainable teaching and learning). We think that ideas and research done about "good" teaching of different aspects of mathematics will get a new and better perspective if the situation after leaving school is included.

Looking at actual courses for adults learning mathematics are much more (and much faster than years ago) influenced by political and economic changes in the world than schools. Many of the courses for adults are in market situations—they need paying customers, money from a company or the government. If a new government decides to change the rules for funding courses—as it happens every day!—to support the economy better, the general aim for courses might change from "educating people to become critical citizens" to "qualifying people to fulfil the job needs that are declared by industry". It might be a result of the so called "Globalization" that this way of redirecting courses by governments is seen in many countries. One of the research questions arising now is: What is really changing in the courses for adults learning mathematics? Is there any research about it? We have the impression that the mathematical kernel in old and new types of courses is very similar, "only" the ideological frame seems to be different. Maybe the words in a task to calculate percentages are changed, but these words are not important for the learning process. Is this correct? Is learning the four arithmetic operations, proceeding through whole numbers and then decimals and fractions and percentages and so on independent from the ideological frame? We hope to hear some answers when we meet again!