



# A Novel Dynamic Mathematics System Based on the Internet

Yongsheng Rao<sup>1</sup>, Hao Guan<sup>2,3</sup>, Ruxian Chen<sup>1</sup>, Yu Zuo<sup>4</sup>, and Ying Wang<sup>5</sup>(✉)

<sup>1</sup> Institute of Computing Science and Technology, Guangzhou University,  
Guangzhou, China

`rysheng@gzhu.edu.cn`, `chrxfan@qq.com`

<sup>2</sup> University of Chinese Academy of Sciences, Beijing, China

`gducas@qq.com`

<sup>3</sup> Chengdu Institute of Computer Application, Chinese Academy of Sciences,  
Chengdu, China

<sup>4</sup> Institute of Mathematics and Computer Science, Guizhou Normal College,  
Guiyang, China

<sup>5</sup> South China Institute of Software Engineering, Guangzhou University,  
Guangzhou, China

`wying@sise.com.cn`

**Abstract.** In this paper, we introduce a novel dynamic mathematics system called NetPad for teaching and learning mathematics in elementary and secondary school. NetPad is a product of Internet Plus Education and can be launched directly from the internet using a web browser. It combines the Internet with dynamic geometry, computer algebra, and automated reasoning technology. NetPad distinguishes itself from other dynamic geometry systems by being an open, internet-based and sharing oriented intelligent system. NetPad is not only a tool but also a cloud platform for creating and sharing. Since NetPad is developed in HTML5, it is platform independent, runs on every operating system and intelligent device, and can be seamlessly integrated into other websites, PowerPoint and other software. The resources of NetPad can be shared to various social networks directly. The functions of NetPad include dynamic geometry drawing, symbolic computation, programming, automated reasoning in geometry, and so on. NetPad was published in March, 2016. Nowadays, there are more than 100,000 users and 30,000 mathematical resources on the NetPad website.

**Keywords:** Dynamic mathematics · The Internet  
Mathematics education · NetPad

---

This work was supported in part by the National Natural Science Foundation of China (11701118 and U1201252), the Guangdong Provincial Engineering and Technology Research Center ([2015]1487), the Specialized Fund for Science and Technology Platform and Talent Team Project of Guizhou Province (QianKeHePing-TaiRenCai[2016]5609), the Guangdong Provincial Key Platform and Major Scientific Research Projects (2016KQNCX238).

# 1 Introduction

The international education community has reached a consensus that the impact of dynamic geometry on education is positive [1]. Many dynamic geometry systems (DGSs) are used in mathematics education. The popular DGSs include The Geometer's Sketchpad (GSP) [2], Cabri [3], and Cinderella [4]. However, there are not only geometry, but arithmetic, algebra, analysis, programming, and proof systems used in mathematics education at elementary and secondary school. In order to satisfy the demands of mathematics education, our dynamic geometry system has been developed into a dynamic mathematics system (DMS) by integrating computer algebra and other technology. GeoGebra(GGB) [5] and Super Sketchpad (SSP) [6] are popular DMSs in elementary and secondary school.

With the development of mobile Internet, various devices and operating systems are widely used in education. For better integration of the Internet with mathematics education, we developed a novel dynamic mathematics system based on SSP, called NetPad. NetPad combines the Internet with dynamic geometry, computer algebra, and automated reasoning technology. Therefore it is not only a dynamic mathematics teaching tool, but also a cloud platform for creating and sharing educational resources. Since NetPad is developed in HTML5, it is platform independent. NetPad can be launched directly from internet and is freely available at [www.netpad.net.cn](http://www.netpad.net.cn). Figure 1 is its homepage, and Fig. 2 is its user interface.



Fig. 1. Homepage of NetPad

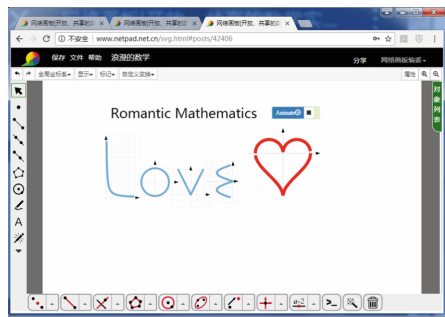


Fig. 2. User interface of NetPad

# 2 Functions of NetPad

The functions of NetPad are listed below.

## 2.1 Geometric Drawing

The geometric drawing function of NetPad includes geometric figures, conic curves and function graphs, geometric transformations, customized coordinate systems, animation, graphic loci, iterations, variables, built-in functions, and so on. For example, Fig. 3(a) contains two iterative figures, the square and triangle.  $B_1$  is generated rotating point  $A$  by  $150^\circ$  around point  $B$ . We make a further iteration from  $(A; B)$  to  $(A_1; B_1)$ . The iterations of the two iterative figures are shown in Fig. 3(b) where the iteration depth is 5. This resource can be found via ID 27858 at [www.netpad.net.cn](http://www.netpad.net.cn).

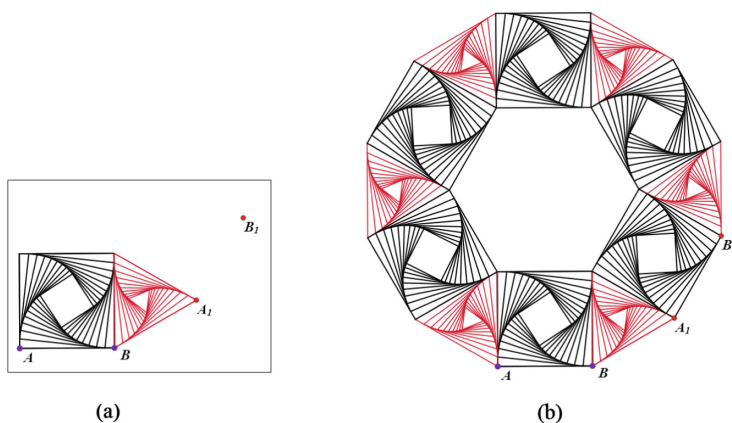


Fig. 3. The iteration of the iteration figures

The Intelligent Pen is a drawing tool based on context-aware technology. With it, users can construct about 20 kinds of dynamic geometric figures with the mouse alone, without toolbar buttons or menus, allowing the construction of dynamic geometric figures in an accurate and efficient manner. Further details are explained in [7]. With the Intelligent Pen, NetPad only needs 8 mouse operations (clicks or movements) to draw the orthocenter of a triangle (Fig. 4) and

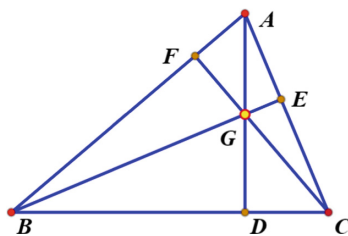


Fig. 4. The orthocenter of a triangle

requires no switching between the graphic window and menu. But it takes 22 mouse operations and 8 window switches in GSP, or 19 mouse operations and 2 window switches in Cinderella.

NetPad can conveniently construct intersection points of almost all linear geometric figures. The types of intersection points that can be constructed in NetPad(NPD), GSP, GGB and SSP are shown in Table 1. “Y” means this type of intersection point can be constructed, “-” means this type of intersection cannot be constructed.

**Table 1.** Comparison of intersection points function

	S - S	S - C	S - P	S - V	C - C	C - P	C - V	P - P	P - V	V - V	V - L	L - L
GSP	Y	Y	-	Y	Y	-	Y	-	-	Y	-	-
GGB	Y	Y	-	Y	Y	-	Y	-	-	Y	-	-
SSP	Y	Y	-	Y	Y	-	Y	-	-	-	-	-
NPD	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

S means Straight Line, C means Circle, P means Polygon, V means Curve, L means Locus.

### 2.2 Symbolic Computation

NetPad supports symbolic computation. The feature can assist students in learning some basic concepts about algebraic operations as well as in carrying out mathematical calculus and finding the results of computations, as shown in Fig. 5.

```
Factor(80501181299);
>>(7)^2*(11)^5*(101)^2

Factor(x^6*y^4-1);
>>(x^2*y^2-x*y+1)*(x^2*y^2+x*y+1)*(x*y+1)*(x*y-1)

Diff(a*x^2+ln(x),x);
>>(2*x^2*a+1)/(x)

(18*a^5*b^4)^(1/2);
>>3*a^2*b^2*(2*a)^(1/2)

(x^3-1)/(x^2-1);
>>(x^2+x+1)/(x+1)
```

**Fig. 5.** Symbolic computation

```
Dichotomy(a, b, d) {
  u = f(a),v = f(b);
  while( u * v < 0 && b - a > d ) {
    c=(a+ b) / 2;
    w= f(c);
    if( w == 0 ) { u=w; }
    else{
      if( u * w < 0 ) { v = w; b = c; }
      else{ u = w; a = c; }
    }
  }
  (a + b) / 2;
}
>>Dichotomy(a, b, d)

f(x) { e^x + x; }
Dichotomy(-1, 0, 0.000001);
>>f(x) -(1134285926818847)/(2000000000000000)=-0.567143
```

**Fig. 6.** Programming

### 2.3 Programming

NetPad provides a simple interpretive language for programming whose syntax is similar to C. It includes assignment, conditional branching, loops, and definition of functions. Figure 6 shows the definition of a program for finding the zero of a function  $f(x)$ ,  $x$  in  $(a, b)$  using the dichotomy method, where the error of the zero is less than  $d$ .

### 2.4 Automated Reasoning in Geometry

Automated reasoning in geometry is highly useful in mathematics education [8]. Based on our automated reasoning technologies [9–11], NetPad can solve most elementary geometric problems. Figure 7 shows the five-circle theorem. A readable proof of the theorem is generated and shown in the text box. The reasoning system generates 2276 pieces of information via the detailed derivation process, including 30 pieces of information about similar triangles, and 480 pieces of information about equal angles.

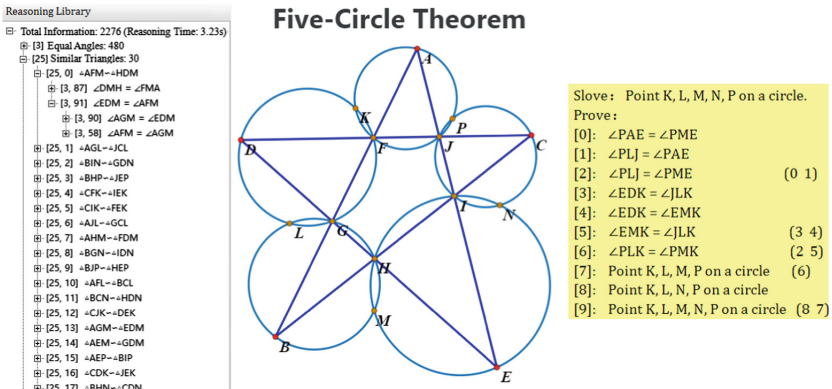


Fig. 7. The automated reasoning of the Five-Circle Theorem.

## 3 Features of NetPad

### 3.1 Convenient Platform to Share and Communicate

One notable feature of NetPad is webpage hyperlinks, which can be shared on various social networks such as Twitter, Facebook, WeChat, Microblog, etc. via the resource link or a QR code. For example, via its QR code (Fig. 8), a resource can be viewed in WeChat on a cell phone (Fig. 9). Furthermore, users can download resources from NetPad and the desktop application at <https://www.netpad.net.cn/en/index.html>, then use the resources offline without accessing the Internet.



Fig. 8. The QR code

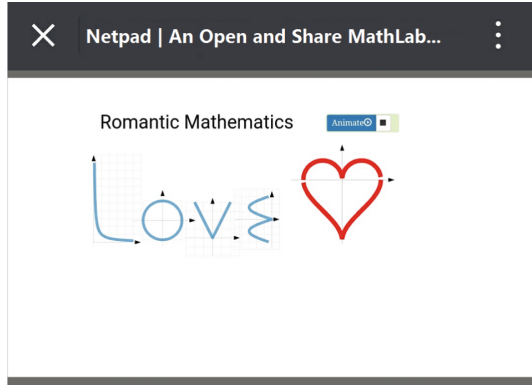


Fig. 9. NetPad in a cell phone

NetPad is also a community with users and resources. All resources created by users are stored in NetPad’s resource cloud. Users can also ask for help through this platform. Through the time of publication of NetPad (March, 2016) to the time of writing this article, NetPad has acquired more than 100,000 registered users and 30,000 resources. The resources of Fig. 10 classified by knowledge topics are a part of the resource cloud.

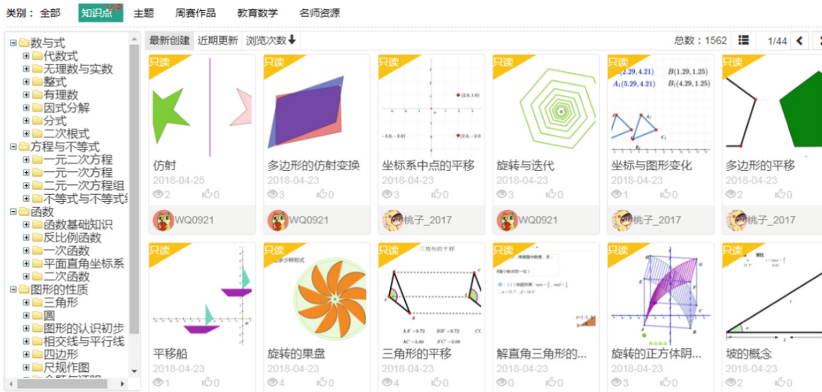


Fig. 10. Resources classified by knowledge topics

### 3.2 Seamless Integration with Other Systems

Microsoft SharePoint is a collection of web-based tools and technologies that help people store, share, and manage digital information. With SharePoint Add-ins Technology [12], NetPad becomes available in the Microsoft Office Store, such that NetPad can be embedded into PowerPoint in only two steps. The first

step is searching for NetPad in the Office Store with the keyword “NetPad” and adding it into a PowerPoint document, as shown in Fig. 11. Because the resources are stored in the cloud in the web-archive format, the second step is searching for a resource in the resource cloud by keyword, author or resource ID. In Fig. 12, the resource is embedded using the key words “The Properties of the Linear Equation”, then users can change the variables  $k$  and  $b$  by dragging the sliders dynamically in the slide.

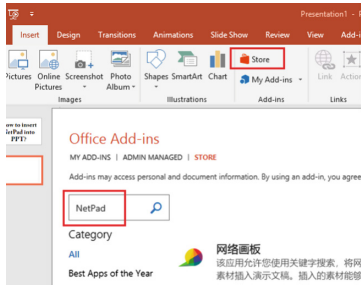


Fig. 11. Search for NetPad in the Office Store

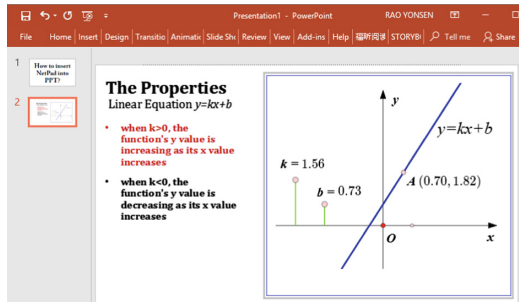


Fig. 12. Insert NetPad into PPT

Because each resource of NetPad has a webpage hyperlink associated with it, it can be seamlessly integrated into other websites using its ID and NetPad APIs. For example, the resource “Romantic Mathematics” can be embedded into any website with the following code, in which 42406 is its ID.

```
<iframe src="https://www.netpad.net.cn/thirdInnerPad.html#posts/42406" style="width:800px;height:600px;"></iframe>
```

## 4 Conclusion

NetPad is a novel dynamic mathematics system for teaching and learning mathematics in elementary and secondary school. It is a product of Internet Plus Education and is not only a teaching tool, but also a cloud platform for creating and sharing resources. The most important characteristics of NetPad are that it is open, sharing oriented, intelligent, internet-based, and that it features rich functions. The resources of NetPad can be shared to various social networks directly, seamlessly integrated into other websites, PowerPoint and other software. The functions of NetPad include dynamic geometry drawing, symbolic computation, programming, automated reasoning in geometry, and so on. NetPad was published in March 2016. Nowadays, there are more than 100,000 users and 30,000 resources on the NetPad website.

In future work, we will enrich and optimize the functions of NetPad, improve its running efficiency and stability, and release a 3D version.

**Acknowledgements.** We are grateful to Masataka Kaneko for proposing many good suggestions and Zak Tonks for improving the English.

## References

1. Zhang, J., Xiong, H., Peng, X.: Free software SSP for teaching mathematics. In: Symbolic Computation and Education, pp. 115–135 (2014)
2. The Geometer's Sketchpad. <http://www.dynamicgeometry.com>
3. Cabri. <http://www.cabri.com>
4. Richter-Gebert, J., Kortenkamp, U.: The interactive geometry software Cinderella 2. *Am. Math. Mon.* **107**(8) (1999)
5. GeoGebra. <http://www.geogebra.org>
6. Super Sketchpad. <http://ssp.gzhu.edu.cn>
7. Wang, Y., Rao, Y., Zou, Y., Huang, Y.: An algorithm for dynamic geometric intelligent drawing based on context awareness. In: Proceedings of IEEE the 2nd International Conference on Computational Intelligence and Applications, pp. 547–550 (2017)
8. McCharen, J.D., Overbeek, R.A., Wos, L.A.: Problems and experiments for and with automated theorem-proving programs. *IEEE Trans. Comput.* **25**(8), 773–782 (1976)
9. Jingzhong Zhang, L., Yang, X.G., Chou, S.: Automated generation of readable proofs in geometry. *Chin. J. Comput.* **18**(5), 380–394 (1995)
10. Zhang, J., Gao, X., Chou, S.: The geometry information search system by forward reasoning. *Chin. J. Comput.* **19**(10), 722–727 (1996)
11. Zhang, J., Gao, X., Chou, S.: Geometric Invariant Methods of Geometric Theorem Proving. The Science Publishing Company, Beijing (2015)
12. SharePoint Add-ins. <https://docs.microsoft.com/en-us/sharepoint/dev/sp-add-ins/sharepoint-add-ins>