Dynamic Behavior of Spur Gearbox with Elastic Coupling in the Presence of Eccentricity Defect Under Acyclism Regime

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Abstract. In this paper, the effect of eccentricity defect on the dynamic behaviour of one stage spur gearbox running under acyclism regime is studied. In fact, acyclism regime is generated by a combustion engine motor which produced fluctuations of load and speed. The motor torque is periodic and it modeled in the force's vector. The rotational speed of the Diesel engine is a harmonic function and it generates a periodic fluctuation of the gear meshing stiffness function. This driven motor is joined to the gearbox through an elastic coupling in which the model of Nelson and Crandall is adopted. The eccentricity defect is introduced in the pinion. This defect produces an additional potential energies and kinetic energy and it is modelled through additional forces. The equation of motion is obtained using Lagrange formalism and the algorithm of Newmark is used to compute the dynamic response of the studied system and the Wigner–Ville distribution shows the dynamic behaviour of the gearbox under this cyclo-stationary regime. Results show the variability of the meshing frequency and its harmonics which excites the system. Also, natural frequencies are observed in the spectrum and Wigner–Ville distribution of the dynamic signal. Nevertheless, these methods fail to detect the frequencies of eccentricity and acyclism.

Keywords: Eccentricity · Acyclism · Elastic coupling · Spur gearbox

1 Introduction

Acyclism is a transient regime. It is generated by a combustion engine and it is characterized by fluctuations of speed and torque.

Many researchers focused on this regime: Barthod et al. (2007a) studied the effect of acyclism on the rattle threshold inside different gearbox configurations. Sika and Velex (2008) used a torsional gear model to study the effect of engine speed fluctuations which is considered as a sinusoidal and multi-harmonic function. Khabou et al. (2011) investigated a spur motored by a diesel engine where its applied torque is considered as a multi-harmonic function.

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In addition to the motor regime, defects are source of excitation. For example, the eccentricity which is due to the non-concentricity between the axis of the pitch cylinder of the gear and the axis of rotation of the shaft is investigated by many researchers:

Driss et al. (2014) studied the dynamic behavior of two-stage straight bevel gear with some defects which are the eccentricity defects, profile error and cracked tooth. They proposed a new method for modeling gear mesh stiffness of straight bevel gear and they introduced these defects in the three dimensional model of the studied system. As results, the spectrum dynamic response shows appearance of sidebands around the meshing frequency excited by the fault and its harmonics. Walha et al. (2011) studied the effect of eccentricity defect on the dynamic behavior of an automotive clutch coupled with a two stage helical gear. They included also three types of nonlinearity which are dry friction path, double stage stiffness and spline clearance. Chaari et al. (2006) studied the influence of eccentricity on the sun gear on the dynamic behavior of planetary gear. They introduced this defect by adding a transmission error modeled as displacement on the line of action of the sun-planet gearmesh.

In this work, effects of acyclism and eccentricity defect on the dynamic behavior of a spur gearbox with elastic coupling are investigated. The system is powered by diesel engine and the second model of Nelson and Crandall (1992) is adopted for the elastic coupling. Excitations due to the fluctuation of load and speed of acyclism regime and eccentricity defect of gearbox are introduced to the dynamic model. The dynamic response is computed through Newmark algorithm and results are shown using Wigner–Ville distributions.

2 Dynamic Model

The studied system is composed by a diesel engine motor and a receiver which are connected through one stage of spur gearbox and an elastic coupling located between the motor and the pinion.

Figure 1 show the corresponding dynamic model which is divided into three blocks. This model was proposed by Hmida et al. (2016, 2017). The pinion and the wheel of gearbox and the diesel engine motor are assumed as rigid bodies. Transmission shafts are assumed massless and have torsional stiffness Kθi and torsional damping Cθi (i = 1, 2, 3). They are supported by bearings which are modeled with parallel springs (Kxi, Kyi) and damping (Cxi, Cyi).

The model of Nelson and Crandall (1992) is adopted for the elastic coupling because this model is best approach to describe the dynamics of elastic couplings (Tadeo and Cavalca 2003; Tadeo et al. 2011). This coupling is modeled with two translation stiffness (K_{xc}, K_{yc}), a torsional stiffness ($K_{\theta c}$), two translation damping (C_{xc}, C_{yc}) and a torsional damping ($C_{\theta c}$). Its inertial effects are included in the first block (I_{12}) and the second blocks (I_{21}).

The degree of freedom vector "q" is defined as following:

$$q = (\theta_{11}, \theta_{12}, \theta_{21}, \theta_{22}, \theta_{31}, \theta_{32}, x_1, y_1, x_2, y_2, x_3, y_3) \quad (1)$$

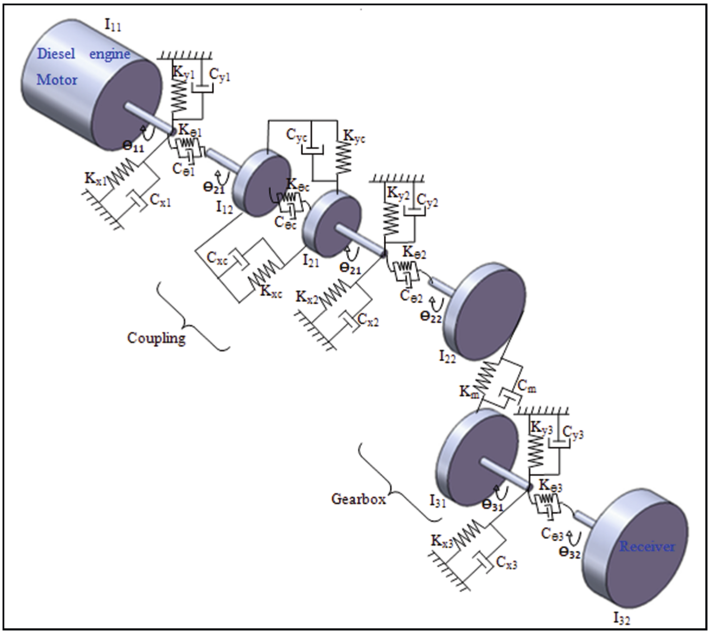

Fig. 1. Dynamic model of spur gearbox with an elastic coupling

2.1 Acyclism Modeling

During the power stroke, the diesel engine generates a variable speed and torque.

The rotational speed of the engine $\Omega(t)$ written by Sika and Velex (2008) as following:

$$\Omega(t) = \Omega_{10}\left(1 + \sum\nolimits_n \rho_n(\Omega_{10})\,\sin(n\Omega_{10}t + \varphi_n)\right) \tag{2}$$

where Ω_{10} is the average velocity.

n is the harmonic of the generated speed function, ρ_n and φ_n are respectively the corresponding amplitude and phase. According to the Eq. (2) limited on the 1st harmonic, the evolution of the rotational speed generated by the diesel engine motor is shown in Fig. 2.

The shape of the rotational speed of the Diesel engine generates a periodic fluctuation of the gear meshing stiffness function (K_m) as shown in Fig. 3.

According to Ligier and Baron (2002), the torque C_m developed by the combustion engine can be written as:

$$C_m \approx \overline{C_m} + \frac{P_{max}}{192} V_{cyl}(0.46\sin 2\alpha_c + 0.24\sin 4\alpha_c + 0.03\sin 6\alpha_c) \tag{3}$$

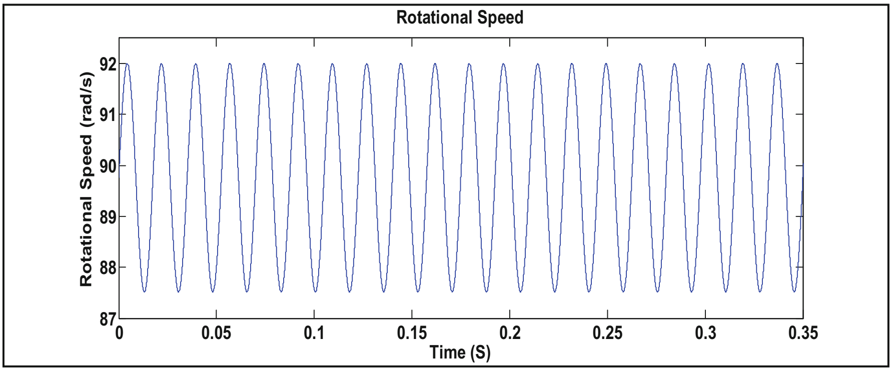

Fig. 2. Time evolution of the engine rotational speed

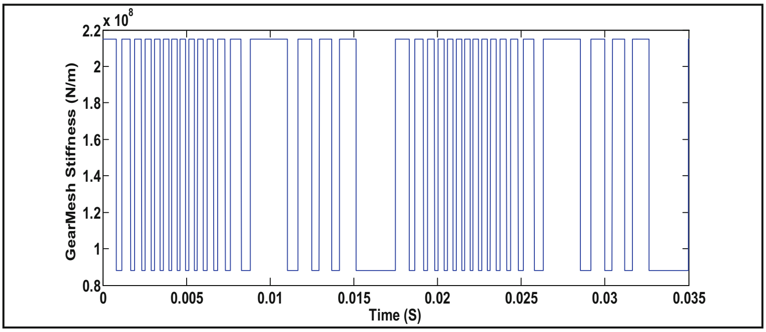

Fig. 3. Time evolution of the gear mesh stiffness

Where $\overline{C_m}$ and α_c are respectively the average of engine torque and the angular position of the crankshaft. V_{cyl} and P_{max} are respectively the cylinders capacity and the maximum pressure inside cylinders.

The applied torque is periodic and it is shown in Fig. 4.

2.2 Eccentricity Modeling

The approach of modelling of the eccentricity is based on an eccentricity error due to a deviation between the center of rotation of the gear and its geometric center. Michalec (1966) considered the case of a single eccentric gear and showed that the transmission kinematic error was a deterministic perturbation of frequency f_d the frequency of defect which is equal to the frequency of rotation f_r of the pinion. The amplitude is proportional to its eccentricity.

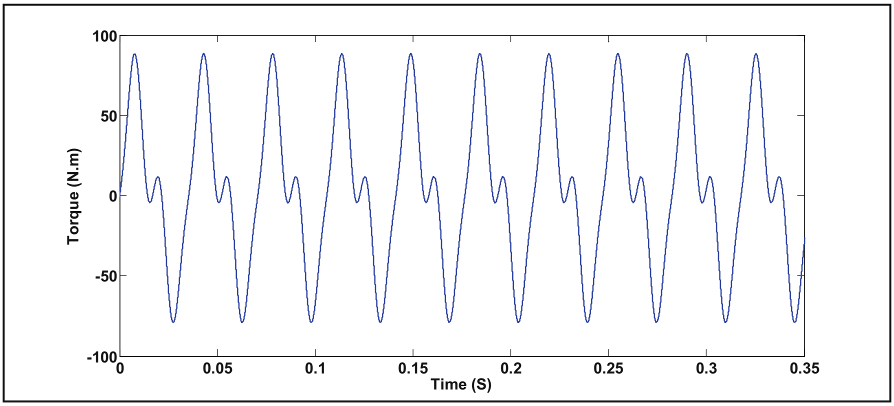

Fig. 4. Time evolution of the engine torque

E_{22} is the distance between the axis of rotation and the axis of inertia of the wheel and expressed by:

$$e_{22}(t) = e_{22}\sin(\Omega_{22}t - \lambda_{22}) \tag{4}$$

Where $\Omega_{22} = 2\pi f_d$ (f_d: frequency of defect)

e_{22} and λ_{22} are respectively the amplitude of eccentricity and the phase of eccentricity. They are shown in Fig. 5.

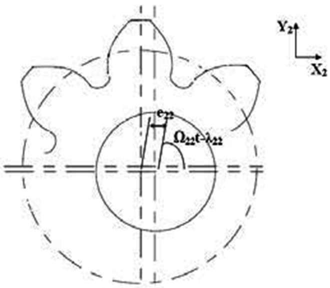

Fig. 5. Eccentricity defect

The eccentricity defect affects the potential energies and kinetic energy. In fact, this defect affects the tooth deflections. So, there is an additional potential energy which is modelled by an additional force:

$$F^{p}_{ecc} = K_m(t)e_{12}(t)\{0 \quad 0 \quad 0 \quad r_{b21} \quad r_{b22} \quad 0 \quad 0 \quad 0 \quad \sin(\alpha) \quad \cos(\alpha) \quad -\sin(\alpha) \quad -\cos(\alpha)\} \tag{5}$$

The additional kinetic energy is also modelled as an additional force:

$$F_{ecc}^{K} = -\mathrm{m}_{22}\mathrm{e}_{22}\Omega_{22}^{2}\{0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \cos(\Omega_{22}\mathrm{t}-\lambda_{22}) \quad \sin(\Omega_{22}\mathrm{t}-\lambda_{22}) \quad 0 \quad 0\} \tag{6}$$

3 Equation of Motion

The equation of motion is obtained using Lagrange formalism:

$$[M]\ddot{q} + ([C_m] + [C_s])\dot{q} + ([K(t)] + [K_s])q = F(t) \tag{7}$$

[M] is the global mass matrix. $[K_s]$ and [K(t)] are respectively the structural stiffness matrix of the system and the time varying mesh stiffness matrix. $[C_s]$ and $[C_m]$ are respectively the structural damping matrix and the mesh damping matrix.

The external force vector [F(t)] is defined as:.

$$F(t) = F_{ecc}^{p}(t) + F_{ecc}^{K}(t) + F_{ext}(t) \tag{8}$$

$$F_{ext} = \{C_m \quad 0 \quad 0 \quad 0 \quad 0 \quad -C_r \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\} \tag{9}$$

C_m and C_r are respectively the motor torque and the receiver torque.

These entire matrixes are defined in Hmida et al. (2016).

4 Numerical Results

In this part, effect of acyclism and eccentricity are induced in the model and numerical results are carried out using the parameters values of the dynamic model presented in Table 1.

Newmark method is used to compute the numerical results.

Figure 6 shows the time displacement signal of the second bloc in the Y direction. The observed fluctuations on this figure correspond to the influence of the meshing and eccentricity phenomena on the dynamic response.

Time acceleration signal of the second bloc in the X direction is shown in Fig. 7. This signal is modulated by the acyclism and the eccentricity defect.

Spectral analysis is the most widely used techniques. Indeed, analysis spectrum of acceleration of the second bloc in the X direction (Fig. 8) shows several peaks in the neighborhoods of the natural frequencies of the system f_i which are resumed in Table 2.

Table 1. Values of the model parameters

Gear box parameters	
Teeth number	$Z_{12} = 20$; $Z_{21} = 50$
Mass (Kg)	$m_{12} = 1.77$; $m_{21} = 2.5$
Pressure angle	$\alpha = 20°$
Teeth module (m)	$m_n = 2 \times 10^{-3}$
Contact ratio	$\varepsilon_\alpha = 1.6$
Average mesh stiffness (N/m)	$K_{moy} = 2.11 \times 10^8$
Coupling's characteristics	
Inertia (Kg m^2)	4×10^{-3}
Mass (Kg)	4.5
Torsional stiffness (Nm/rad)	352
Translation stiffness (N/m)	462×10^2
Engine motor's characteristics	
Inertia (Kg m^2)	4×10^{-3}
Maximum pressure inside cylinders P_{max} (Bar)	49
Average of engine torque $\overline{C_m}$ (N m)	17.5
Cylinders capacity Vcyl (cm^3)	2000
Receiver's characteristics	
Inertia (Kg m^2)	6×10^{-3}
Characteristics of shafts and bearings	
Torsional Shaft stiffness (Nm/rad)	5×10^5
Bearing stiffness (N/m)	5×10^8
Characteristics of eccentricity	
Amplitude of eccentricity (μm)	50
Phase of eccentricity (rad)	$\pi/6$

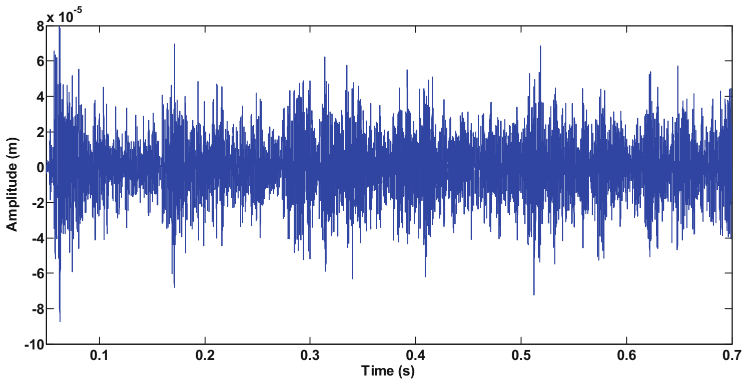

Fig. 6. Time displacement signal of the second bloc in the X direction

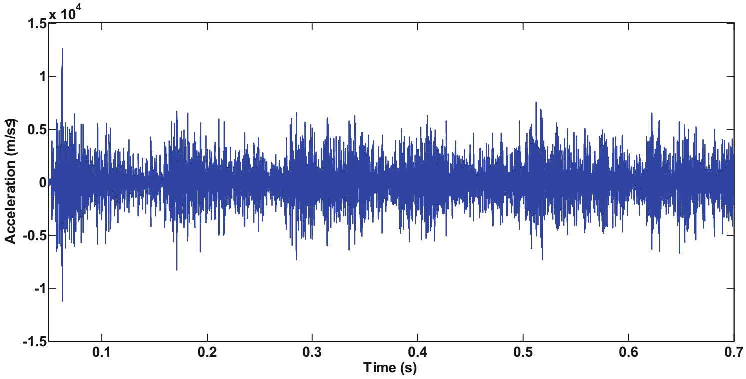

Fig. 7. Time acceleration signal of the second bloc in the X direction

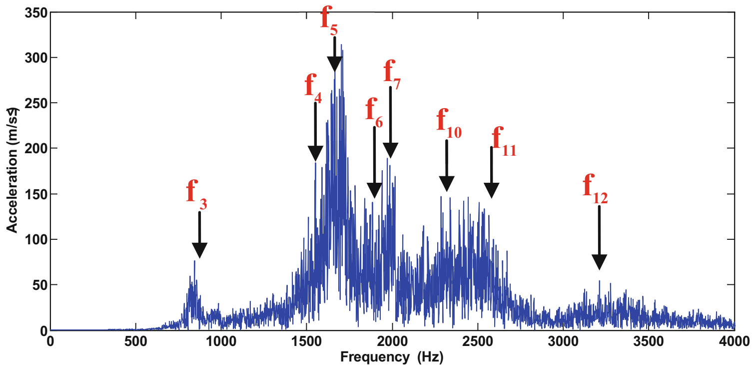

Fig. 8. The spectrum of acceleration of the second bloc in the X direction

Table 2. The natural frequencies

Natural freq	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}
Hz	0	26	891	1583	1664	1867	1902	1974	1974	2344	2459	3292

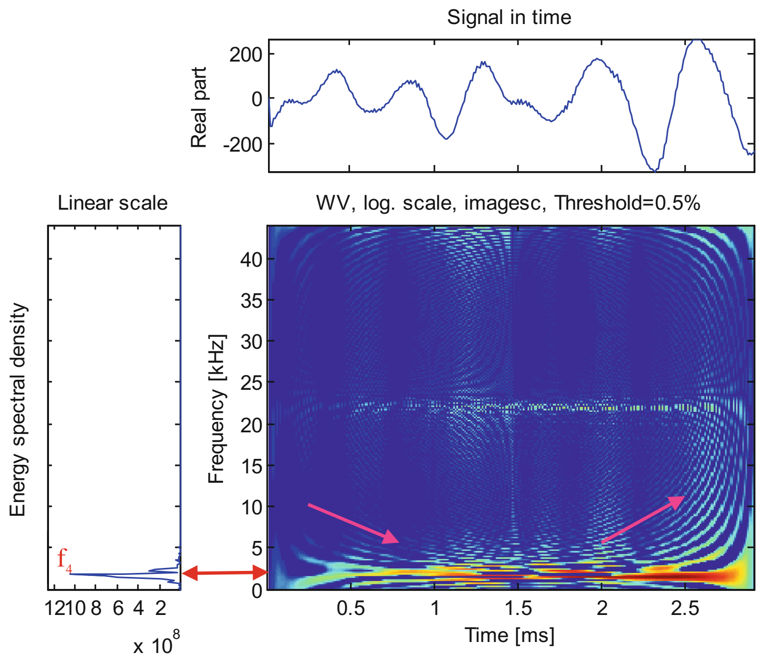

Fig. 9. Wigner–Ville distribution of the acceleration in direction X2

Figure 9 represents the Wigner–Ville distribution of the acceleration of the second bloc in the X direction in order to analyze the non-stationary behavior of the signal.

Due to the acyclism regime, it can be seen the variability of the meshing frequency and its harmonics which excites the system, and horizontal lines represented peaks in the natural frequencies of the system. Nevertheless, the frequency of acyclism and the frequency of eccentricity defect are not observed.

5 Conclusion

In this paper, the dynamic behavior of spur gearbox with an elastic coupling is studied under acyclism regime generated by combustion engine. The eccentricity defect is introduced in the model. Spectral analysis and the Wigner–Ville distribution of the dynamic response are used to provide information about their state. In fact, only peaks in the natural frequencies of the system and the meshing frequencies appear. These methods fail to detect the frequencies of eccentricity and acyclism.

The future study will mainly focus on the decomposition of the dynamic response of the system with eccentricity under acyclism regime in order to extract the frequencies of excitations from the non-stationary signals.

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