Optimal Tax Planning with Mathematical Programming Models

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Abstract Tax planning is a matter of increasing interest to large corporations. Complex tax systems must be optimized using mathematical programming methods. As an example, this paper considers the special tax system in the Canary Islands (REF), which includes incentives for investments in the region and can be used by companies to improve the return of their investments plans. A model for tax planning is implemented and solved with standard software.

Keywords Tax planning · Optimization models

1 Introduction

Optimization techniques are currently used in numerous business areas: production and transport planning, portfolio selection and others (a good example of models for business optimization can be found from Kallrath 1997 to Baker 2011). These techniques have also been used for financial planning since the very beginnings of mathematical programming in business.

Although tax planning is widely used in large corporations, with special emphasis on transfer price optimization (see Klassen et al. 2017), there are very few works on practical tax planning using mathematical programming. Even in one of the foremost collections of articles on tax in Europe (GQTR 2017), it is difficult to find papers on corporate optimization models for tax planning. In a recent work (Dinh 2014), a specific tax planning problem is solved theoretically. Even general manuals on tax planning, (i.e. Schanz and Schanz 2011) do not include optimization models.

This paper describes the implementation of a tax planning optimization model for the Canary Islands special tax system (REF, "Regimen Económico y Fiscal") using standard software to illustrate the advantages of using optimization models in tax

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planning. This example is used to demonstrate a general approach to tax planning optimization problems.

2 The Tax-Financial Model

2.1 Canary Islands Special Tax System

As an example of tax planning optimization, and to establish the general concepts, a multi-period model for REF ("Regimen Económico y Fiscal") is implemented for the Canary Island autonomous region.

The aim of this model is to maximize the cash flow generated (with or without discount, as preferred) in the entire period, making use of the various incentives in the Canary Islands special tax system, mainly "Reserva de Inversiones" (reserves for investments, RIC) and "Deducción por inversiones" (investment deductions, DI). These incentives are regulated by Spanish and European Union laws (Ley 30/1972, 20/1991, 19/1994; RDL 15/2014).

The sample model is built to determine the optimal amounts of RIC and DI each year in the planned period.

The main reasons for using optimization are:

1. RIC is direct-deductible from the taxable profit, thus reducing the tax to pay (general tax rate for firms is 25% in Spain).
2. RIC can reach a maximum equivalent amount of 90% of the undistributed accounting profit which, after taxes, depends on the Corporate Tax rate and the RIC and the investment deduction.
3. Undistributed After-Tax Earnings depend on voluntary reserves and the dividends paid to shareholders.
4. RIC is a reserve that must be materialized in the following three years in various types of investments (Ley 19/1994).
5. DI cannot exceed the ceiling of the corporate taxable profit.

The conjunction of these conditions is perfect for applying optimization models, rather than using trial and error or the more sophisticated simulation procedures commonly used.

2.2 The Mathematical Model

This type of model can be used in budgeting or strategic planning. The model can begin with some pre-tax profit (for example a EBITDA base series for each tax year calculated before including the investment plan), and a pre-defined depreciation for previous and planned investments, could be included in a horizon of several years in

a complete financial-tax planning model. Other data are the investment plan for each year that produces a profit after one year (r), corporate tax rate (t), tax deduction for investments (tdr(i)).

For simplicity, we develop the model without interest costs, and assume all the investments are working throughout the whole fiscal year, and that their effects are included in the pre-tax earnings for the year following the investment (of course the model can be extended by adding more periods (i.e. quarters or months) to improve precision).

To explain the model, let us assume a company established in the Canary Islands (Spain) is deciding its strategic plan for the coming years. It has determined the investments to be made each year, and projected the before-tax profits and the amortizations it will have in the following year. The calculations of these concepts can be included in our optimization model; this is not done here in order to simplify and address the core problem. The model can easily be extended to include debt and its cost, as will be shown below.

Making use of the tax advantages offered by the Canary Islands, the model calculates the optimal amount to be entered as the RIC for each year of the strategic plan. These will be the decision variables for the optimization model. The model includes the investment deductions, an incentive compatible with the previous one. The conditions for the RIC are that it cannot exceed 90% of the undistributed profit. Profits before the tax base series do not include the profitability of the cash for each year, which we will assume to be remunerated at an interest rate i.

The model can be modeled as a linear programming problem as explained below, and solved in any of the available systems. This paper presents one possibility combining Excel with What's Best. This solution allows a combined use of simulation/optimization, and tests different options as shown below.

Model TaxCan

Data:

For i = 1,…, N number of tax years planned

t	Corporate tax rate
r	Interest rate for cash
tdr (i)	Deduction from Investment Tax rate
INV (i)	Investment plan
D (i)	Depreciation
EBT (i)	Earnings before taxes (base)
DIV (i)	Dividends paid in tax year i

(optional parameter that can be imposed by shareholders, considered paid at the end of tax year i)

Variables

RIC (i) Reserve for Investment in Canary Islands

(year i in which RIC is accounted and allows tax deduction)

Dinv (i) Direct investment (not from RIC)

(investments this year that have not been "reserved" in previous years)

TMatRIC (j,i) Materialization of RIC

(the amount to be used as real investment from RIC (i) in tax year j must be only 3 years before)

Objective function:

$$\text{MAX} \sum_{i=1}^{n} \text{CF(i)}$$

Subject to:
(Calculations/constraints for each i, tax year)

$$\text{EFC (i)} = \text{r} * \text{Cash (i} - 1) \tag{1}$$

(Earnings from cash in fiscal year i)

$$\text{EBTC (i)} = \text{EBT (i)} + \text{EFC (i)} \tag{2}$$

(Earnings before taxes computed)

$$\text{INV\_Ded (i)} = \text{DINV (i)} + \text{TMatRIC (i)} \tag{3}$$

(Tax deductible investment)

$$\text{Td (i)} = \text{tdr (i)} * \text{INV\_Ded (i)} \tag{4}$$

(Tax deduction from investment)

$$\text{CCash (i)} = \text{CCash (i} - 1) + \text{EBTC (i)} + \text{D(i)} - \text{Ctax (i)} - \text{Inv (i)} \tag{5}$$

(Cumulative cash at the end of year i)

$$\text{TBD (i)} = \text{t} * [\text{EBTC (i)} - \text{RIC (i)}] \tag{6}$$

(Tax before deduction from investment. Note RIC (i) reduces the tax base)

$$\text{Ctax (i)} = \text{TBD (i)} - \text{TD (i)} \tag{7}$$

(Final Corporate Tax)

$$\text{EAT (i)} = \text{EBTC (i)} - \text{Ctax (i)} \tag{8}$$

(Earnings after taxes, must be positive)

$$\text{CF (i)} = \text{EAT (i)} + \text{D (i)} \tag{9}$$

(Cash flow computation, usually D depends on the investment plan, not considered in this version)

$$\text{INV C (i)} = \text{Dinv (i)} + \text{TMatRIC (i)} \tag{10}$$

(Investment computation: direct investment plus the materialization of previous years' RIC in year i)

$$\text{INV C (i)} = \text{INV (i)} \tag{11}$$

(Investment plan control equation: computed investment must be equal to investment plan)

$$\text{RIC (i)} = \text{SUM for j} = \text{i} + 1 \text{ to i} + 3 \text{ of MatRIC (i, j)} \tag{12}$$

(Entire amount of RIC(i) must be used in the following 3 years)

$$\text{TMatRIC (i)} = \text{SUM for j} = \text{i} - 3 \text{ to i} - 1 \text{ of MatRIC (j, i)} \tag{13}$$

(Actual investment in year i is computed from the previous years' RICs)

$$\text{NDE (i)} = \text{EAT (i)} - \text{Dividends (i)} \tag{14}$$

(Non-distributed earnings, not including legal reserves)

$$\text{RICLimit (i)} = 0.9 * \text{NDE (i)} \tag{15}$$

(RIC limit −90% of non-distributed earnings is computed)

$$\text{RIC(i)} <= \text{RICLimit(i)} \tag{16}$$

(Limit constraint formulation)

There are other objective functions that could be considered, such as CCash at last year and others.

2.3 Implementation of the Optimization Model

This type of model is solved in this case with the What's Best spreadsheet optimization (LINDO 2017) due to its ease for integrating in Excel, but the model can be

	A	B	D	E	F	G	H	I
1	**Model TaxCan**							
2			**2.018**	**2.019**	**2.020**	**2.021**	**2.022**	**2.023**
3								
4	**I. Data (k€)**							
5		Investment plan	600	2.305	1.204	2.712	666	561
6		Depreciation	1.346	1.639	1.870	1.990	2.261	2.328
7		Earnings before taxes (Base)	5.076	2.781	2.790	2.880	2.974	3.076
8		Corporate tax	25%	25%	25%	25%	25%	25%
9		Interest rate for cash	**2%**					
10		Deduction from Investment Tax rate	25%	25%	25%	25%	25%	25%

Fig. 1 Model data

	A	B	D	E	F	G	H	I
1	**Model TaxCan**							
2			**2.018**	**2.019**	**2.020**	**2.021**	**2.022**	**2.023**
3								
12	**II. Equations**							
13		Earnings from Cash		94	41	62	43	81
14		Earnings before taxes Computed	5.076	2.875	2.831	2.942	3.018	3.157
15		Tax deductible Investment	600	2.305	1.204	2.712	666	561
16								
17		Tax Deduction from Investment	150	576	301	678	166	140
18		Cumulative Cash	4.703	2.067	3.090	2.163	4.025	4.275
19								
20		*Corporate tax computation*						
21		Reserve for Investment in Canary Is.	0	0	0	0	0	0
22		Tax before deduction from Investment	1.269	719	708	735	754	789
23								
24		Final Corporate tax	1.119	143	407	58	588	649
25		Earnings after taxes	3.957	2.733	2.424	2.884	2.430	2.508
26		(Must be positive)	>=	>=	>=	>=	>=	>=

Fig. 2 Model equations

formulated in other commercial optimization software such as Frontline (2017) or any other. See Fourer (2015) for a recent survey.

The model can be formulated in an Excel spreadsheet as can be seen below (the underlying equations are explained above), in Figs. 1 (data) and 2 (equations).

Used as a simulator, the Excel model allows all the accounting of the investments to be allocated in the year in which they are going to be implemented: 600 k€ (2018), 2.305 k€ (2019) and so on. No dividends are paid. To aid visualization, only row 33 is shaded in Fig. 3.

In this base case, total cash flow is 28.370 k€ (cell J30).

For the model to give a better solution, we enter new variables related to RIC (shaded cells in rows 34, 36, 37…) in Fig. 4.

Cell J30 is the objective, and the optimal value is now 30.260 k€ when the model uses different levels of RIC every tax year. For example, in 2018 the company will post ("will reserve") 4.476 k€, 593 k€ in 2019 and so on.

(It should be noted that the shaded cells are the model variables.)

	A	B	D	E	F	G	H	I	J	K	L
1	**Model TaxCan**										
2			**2.018**	**2.019**	**2.020**	**2.021**	**2.022**	**2.023**	Total		
3											
27											
28	**III. Decisions**										
29											
30		**Cash Flow**	**5.303**	**4.372**	**4.294**	**4.874**	**4.691**	**4.835**	**28.370**		
31		Investment plan (control equation)	600	2.305	1.204	2.712	666	561	8.047	=	8.047
32		Must be equal to plan	=	=	=	=	=	=			
33		**Direct investment (non from RIC)**	**600**	**2.305**	**1.204**	**2.712**	**666**	**561**	8.047		
34		**RIC accounting**							0		
35		**Materialization of RIC**	0	0	0	0	0	0		Constraint	
36		**RIC from year 2018 in year 2019, 2020 or 2021**		0	0	0			0	=	0
37		**RIC from year 2019 in year 2020, 2021 or 2022**			0	0	0		0	=	0
38		**RIC from year 2020 in year 2021, 2022 or 2023**				0	0	0	0	=	0
39		**RIC from year 2021 in year 2022 or 2023**					0	0	0	=	0
40		**RIC from year 2022 in year 2023**						0	0	=	0
41											
42		**Dividends**	0	0	0	0	0	0	0	=>=	0

Fig. 3 Variables and objective function

	A	B	D	E	F	G	H	I	J	K	L
1	**Model TaxCan**										
2			**2.018**	**2.019**	**2.020**	**2.021**	**2.022**	**2.023**	Total		
3											
28	**III. Decisions**										
29											
30		**Cash Flow**	**6.422**	**4.537**	**4.704**	**4.940**	**4.819**	**4.837**	**30.260**		
31		Investment plan (control equation)	600	2.305	1.204	2.712	666	561	8.047	=	8.047
32		Must be equal to plan	=	=	=	=	=	=			
33		**Direct investment (non from RIC)**	**600**	**0**	**0**	**0**	**0**	**0**	600		
34		**RIC accounting**	4.476	593	1.630	238	509		7.447		
35		**Materialization of RIC**	0	2.305	1.204	2.712	666	561		Constraint	
36		**RIC from year 2018 in year 2019, 2020 or 2021**		2.305	611	1.560			4.476	=	4.476
37		**RIC from year 2019 in year 2020, 2021 or 2022**			593	0	0		593	=	593
38		**RIC from year 2020 in year 2021, 2022 or 2023**				1.152	427	52	1.630	=	1.630
39		**RIC from year 2021 in year 2022 or 2023**					238	0	238	=	238
40		**RIC from year 2022 in year 2023**						509	509	=	509

Fig. 4 Optimal solution without dividends

The 2018 RIC will be used for the real investments in year 2019 (2.305 k€), 2020 (611 k€), and 2021 (1.560 k€), but the 2019 RIC will be completely used in 2020 (593 k€).

The optimization model has improved the cumulative cash flow by nearly 2 M€.

Of course, this model can be used by imposing more conditions. For example, if the dividends request is changed, the model will recalculate the new optimal solution. Let us assume the shareholders claim 50% of the profit after taxes. The new solution is shown in Fig. 5.

The cash flow is worse: 29.402 k€, practically 1 M€ less than the optimum without dividends.

The question is: would be this be the best solution for the shareholders?

The model can be managed in various ways to obtain different solutions.

	A	B	D	E	F	G	H	I	J	K	L	M
1	**Model TaxCan**											
2			**2.018**	**2.019**	**2.020**	**2.021**	**2.022**	**2.023**	Total			
3												
28	**III. Decisions**											
29												
30		**Cash Flow**	**5.805**	**4.480**	**4.579**	**4.911**	**4.810**	**4.818**	**29.402**			
31		Investment plan (control equation)	600	2.305	1.204	2.712	666	561	8.047	=	8.047	
32		Must be equal to plan	=	=	=	=	=	=				
33		**Direct investment (non from RIC)**	**600**	**299**	**668**	**1.492**	**457**	**0**	3.516			
34		**RIC accounting**	2.007	536	1.219	209	561		4.531			
35		**Materialization of RIC**	0	2.007	536	1.219	209	561		Constraint		
36		**RIC from year 2018 in year 2019, 2020 or 2021**		2.007	0	0			2.007	=	2.007	
37		**RIC from year 2019 in year 2020, 2021 or 2022**			536	0	0		536	=	536	
38		**RIC from year 2020 in year 2021, 2022 or 2023**				1.219	0	0	1.219	=	1.219	
39		**RIC from year 2021 in year 2022 or 2023**					209	0	209	=	209	
40		**RIC from year 2022 in year 2023**						561	561	=	561	
41												
42		**Dividends**	2.229	1.420	1.355	1.460	1.274	1.245	8.984	>=	0	

Fig. 5 Optimal solution with dividends

3 Conclusion

Tax planning optimization models can improve the cash flow generated by corporations. This paper presents an example for the special tax system in Canary Islands (REF).

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