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Sketch: Playful Maths

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Maths and play in my opinion go very well together. This may sound odd to many readers, as maths is often perceived as being a dull, boring and very serious subject. However, nothing could be further from the truth! I argue that maths is a highly (if not the most) creative subject and that the best way to discover new maths is to play. What's this about new maths I hear you ask, surely all maths is known and what can be more predictable than the statement $1 + 1 = 2$. Again wrong! Maths as a subject is growing incredibly fast, with many new discoveries being made all the time. As well as being amazing intellectual achievements in their own right, these new mathematical discoveries are transforming the way that we live. For example, the Internet, Google, mobile phones and credit cards are founded on new mathematical discoveries.

So, back to my original theme of why maths is playful and creative. Firstly, maths as a subject takes you well past your imagination. Who could conceive of objects in 22-dimensional space, but such are

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not only studied by mathematicians but have important applications in physics and engineering. Secondly, maths is the subject of many puzzles and games. Sudoku (both ordinary and killer), the Game of Life, logic puzzles, Griddler, magic tricks (as we shall see), jigsaws and mind reading, all rely on maths to work and in some cases (such as Sudoku) were actually invented by mathematicians. Thirdly, to make discoveries in physics you need an expensive and well-equipped laboratory. However to make discoveries in maths, all you need is a pencil and paper, and a bit of time (e.g. waiting for train in Reading station). The reason that maths is so playful is that it is all about finding and learning about patterns, and to find patterns you need to play. In fact, the process of doing maths could best be described as playing to find patterns, generalising and abstracting these patterns and then proving that they are always true. Mathematicians call the best of these patterns *Theorems* and perhaps best of all is Euler's fabulous result $e^{i\pi} = -1$. This is perhaps the most important formula in the whole of maths, and it also lies at the heart of much of modern physics and engineering. You are making use of this formula every time you use a mobile phone, watch TV or flick on a light switch.

Sadly, mathematics is often taught in school as a non-creative subject that comes pre-formed out of a text book. I argue that we should *always* emphasise the creative and discovery aspects of it in all of our teaching. Part of this involves explaining where the mathematical discovery came from (e.g. Euler's formula above). We should also always show the awe and wonder in maths and make clear that it is a subject full of surprises and mystery. My own personal favourite mathematical result is the fabulously useful and beautiful *Gregory's formula* which relates π (which comes from geometry) to the odd numbers and takes the form

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} \cdots$$

What could be more mysterious and/or surprising than that? (Try playing with it to learn even more). It is so mysterious because it links two quite different things, namely geometry and the odd numbers, which seem to have no relation to each other at all. This is rather like finding

that you are directly related to the Queen. Gregory's formula is (to a mathematician) sublimely beautiful because of its elegance and simplicity. It is also extremely useful as it gives us a way to calculate π to any desired precision. And without an accurate value of π, much or modern engineering (such as your mobile phone) would simply not be possible. Thirdly, find the links between maths and other 'creative' subjects. For example, maths is hugely important in music, dancing, art, magic and real life. Above all, show that maths is fun and that good maths is always useful. I personally take this approach in all of my teaching, whether it is to undergraduates, school children or the general public, and I believe strongly that it works. I was once asked what the term was for someone that could not enjoy or understand maths, and my reply was 'a figment of the imagination'.

Let's briefly see how this works by looking at a simple bit of mathematical magic. Brandishing a pack of cards you boldly approach a (friendly looking) person on the street. 'Give me a number between 10 and 19', you say. They reply, for example, 16. You count out that number of cards and then pick them up. You then ask them to add up the digits of their number. In this case, it is 5. You count out that number from the pack in your hand. Finally, you ask them if they like telling jokes. Usually, they say *yes.* Finally, you turn over the next card. It is the *Joker.* This trick is fun, mysterious and relies on a mathematical pattern. If you take *any* number add up its digits and subtract it from the original, then you *always* end up with a multiple of 9. (Remember that someone discovered this by playing around with patterns.) If the number lies between 10 and 19, then the number you get when you subtract off the sum of the digits is always 9. So you simply put the Joker in as the 9th card, and the trick works itself. Bravo and cheers all round. Go maths!