A New Equation for SWCCs of Unsaturated Soils

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Abstract. In the current geotechnical engineering practice, soil-water characteristic curves (SWCCs) are essential for quantifying the mechanical responses of unsaturated soil. The SWCC can be measured from laboratory or estimated from some empirical equations. Measured SWCCs are usually conducted for drying curve by using Tempe cell or pressure plate device. Because of the time consuming associate the absorption, the wetting SWCC can be approximated by estimating the magnitude of the hysteresis loop at the inflection point on the drying SWCC. The SWCC is commonly plotted as a curve between (gravimetric or volumetric) water content and and the logarithm of soil suction. The general shape of the SWCC looks similar to the S-shaped function. An accurate mathematical modelling of the S-shaped curve is usually required for engineering computation such as numerical analyses. In this paper, a new method to quantifying the SWCC, i.e., via the S-shaped equation proposed by Liu et al. (2013), is employed to represent the SWCC. Comparisons between the equation simulations and experimental data are made. Discussions on the equation simulations and characteristics of the mechanical properties of unsaturated soils are also presented.

1 Introduction

Soil-water characteristic curves (SWCCs) have an important role for unsaturated soil, as it can be used to estimate many property functions such as permeability and shear strength functions. The SWCC is commonly plotted as a curve between (gravimetric or volumetric) water content and the logarithm of soil suction as shown in Fig. 1. The general shape of the SWCC looks similar to the S-shape function, in which two points of change in slope along the curve have physical meanings. The first point is defined as air-entry value, where the largest voids start to desaturate as suction is increased. The second point is defined as residual condition, where the removal of water from the soil becomes considerably difficult. The SWCC can be then divided into three zones, namely, the boundary effect zone in the low suction range, the transition zone between the air-entry value and the residual value, and the residual zone at high suctions as depicted in Fig. 1. The SWCC is not a single-valued curve, but it is hysteretic nature

associated with the drying (desorption) and wetting (absorption) as presented in Fig. 2. Consequently, it is difficult to know whether the unsaturated soil is presently on the drying curve, the wetting curve or somewhere in between these two boundaries (Fredlund et al. 2012).
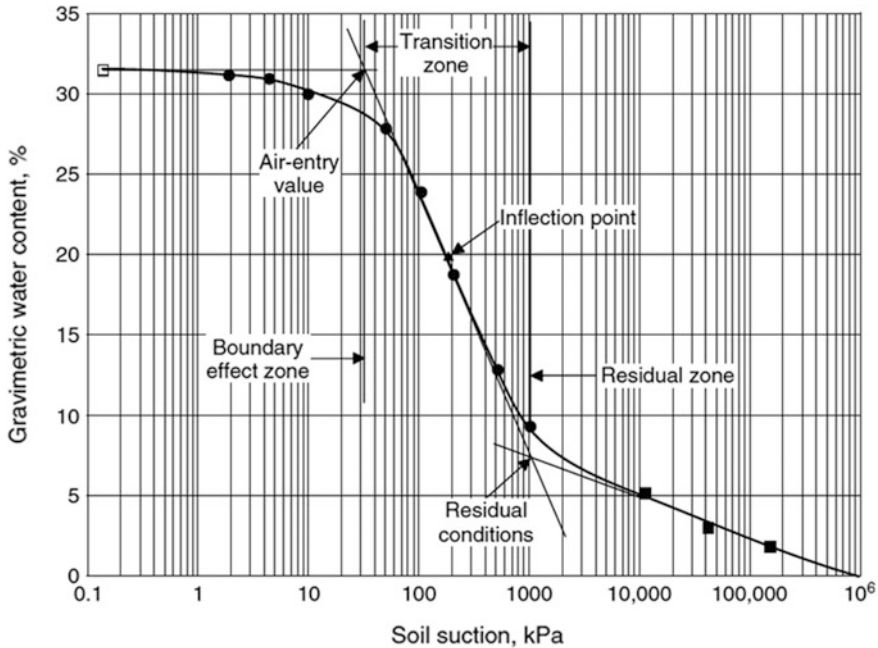

Fig. 1. Typical SWCC (after, Frendlund et al. 2012)

In geotechnical engineering practice, the SWCC can be measured from laboratory or estimated from some empirical equations. Measured SWCC are usually conducted for drying curve by using Tempe cell or pressure plate device. On the other hand, the laboratory tests for measuring wetting curve are considered too costly because of the time consuming associate the absorption. The wetting SWCC can be approximated by estimating the magnitude of the hysteresis loop at the inflection point on the drying SWCC. For simplicity reasons, the effect of hysteresis can be sometime ignored and use only the drying SWCC for the estimation. (i.e., the upper bounding branch of the SWCCs shown in Fig. 2).

There are several empirical equations that have been proposed to estimate SWCCs (i.e., Brooks and Corey 1964; van Genuchten 1980; Fredlund and Xing 1994 and Fredlund and Pham 2006). Most of SWCC equations present the form of a continuous function being asymptotic at both ends. Only the transition zone between the air-entry value and residual suction has sufficient slope for the calculation of soil suction. Fredlund et al. (2011) summarized some commonly used SWCC equations including more recent development in geotechnical engineering. The SWCC equations can be

categorized based upon the range of soil suction conditions that can be well defined by the equations. It is noted that the proposed SWCC equations have been firstly developed and applied in agriculture-related disciplines. However, some mathematics and their physical meanings in geotechnical and geoenvironmental engineering are somewhat different from those in agriculture.
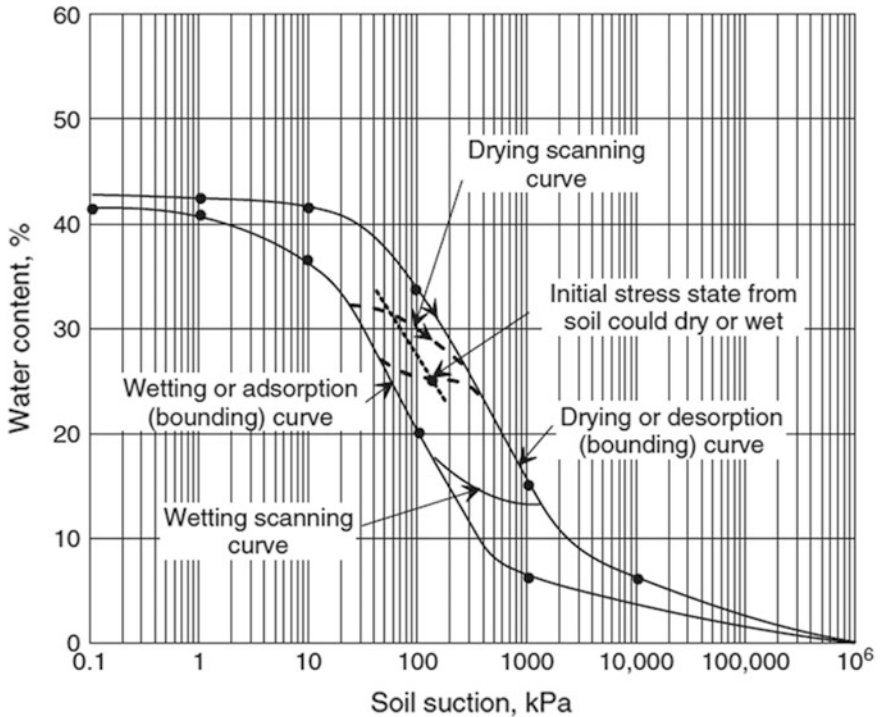

Fig. 2. Hysteresis loops comprising the SWCC (after, Fredlund et al. 2012)

It has been widely observed that the SWCC is *S*-shaped. Based on the mathematical function proposed by Liu et al. (2013), a new method/equation is proposed to describe the SWCC in this study. The SWCCs for different soils under various conditions are simulated, and comparisons between the equation simulations and experimental data are made. Discussions on the equation simulation and characteristics of the mechanical properties of unsaturated soils are also presented.

2 Proposed Equation for SWCCS

Because of the important role of SWCCs in unsaturated soil mechanics, several equations for SWCCs have been proposed and used in research and engineering practice. For examples, they are Brooks-Corey equation (Brooks and Corey 1964),

van Genuchten equation (van Genuchten 1980), and Fredlund and Xing equation (Fredlund and Xing 1994). There are four parameters for Brooks-Corey equation and for van Genuchten equation, and five parameters for Fredlund and Xing equation. All the equation parameters are essentially determined by fitting. However, Brooks-Corey equation and van Genuchten equation cannot represent the SWCC for the entire range of matric suction. Although Fredlund and Xing equation was the best fitting equation among the SWCC equations, the method to determine the parameters was still complicated (see an example in Zhai and Rahardjo 2012). In this paper, a new equation is proposed based on the work by Liu et al. (2013). It is given as follows

$$\theta_w = \begin{cases} \theta_{w,\max} \text{ for } \theta_w > \theta_{w,\max} \\ \frac{a}{\psi}\left[1 + (\psi - 1)e^{-b\psi^c}\right] + d \text{ for } \theta_w \leq \theta_{w,\max} \end{cases} \tag{1}$$

In the equation, θ_w is the volumetric water content, and ψ is the suction with a unit of kPa. There are five parameters in total introduced in the equation. The three main parameters are a, b, and c, defining the curvature of a SWCC. Parameter d defines the asymptotic value of the curve. There is an additional parameter describing the maximum value of the volumetric water content, $\theta_{w,max}$.

Parameters a, b, and c are assumed to be positive. Mathematically, the valid range for variable ψ, which is dependent on c, is given as

$$\begin{cases} \psi \geq 0 \text{ for } c \geq 1 \\ \psi > 0 \text{ for } 0 < c < 1 \end{cases} \tag{2}$$

Parameter c is determined by trial and error. It is suggested that some systematic research on the parameter should be carried out for the convenience of engineering application. It can be seen that the new function has the following features. (1) The asymptotic line for the proposed function is $\theta_w = d$. (2) The value of suction ψ varies linearly with the value of parameter a. This is clearly shown in Eq. (1). Therefore, parameter a is determined by the amplitude of the S-shape curve. (3) The limit value for θ_w, as ψ approaches zero, is as follows

$$\theta_w \longrightarrow \psi \to 0 \begin{cases} a \text{ for } c > 1 \\ a + ab \text{ for } c = 1 \\ \infty \text{ for } c < 1 \end{cases} \tag{3}$$

The function has a singular point at $\psi = 0$ for $c < 1$. The value of a can be determined from Eq. (3) for $c > 1$. For $c = 1$, an equation for parameter a and b is found, i.e., $a + ab = \theta_{w, \text{ for } x=0}$.

3 Simulations

In this section, the SWCCs for eight soils with nine cases are simulated via the proposed equation. The tests are reported in three papers. Values of equation parameters identified are listed in Tables 1, 2 and 3. As seen in Eq. (1), the determination of parameters a, b and

c needs special effort. In this study it is found that parameter c is relatively stable. As can be seen in the values listed in the Tables, it may be assumed as constant for a given soil. Parameter b is usually less than 0.5. It is a variable but usually its value changes gradually with influencing factors such as the value of net stress. Comparisons of equation simulations and experimental data are shown in Figs. 3, 4 and 5.

Table 1. Data for Khon Kaen loess

Tests and figures	θ_{max} (%)	a	b	c	d	Comments
Net stress = 200 kPa, Fig. 3	35.3	53	0.25	0.25	0	SWCCs flat initially
Net stress = 150 kPa, Fig. 3	29.2	78	0.4	0.25	0	

Table 2. Data for a sandy soil, a silty soil, and a clayey soil

Tests and figures	θ_{max} (%)	a	b	c	d	Comments
Sandy soil, Fig. 4	37.4	37.4	0.15	2	0	Influence of soil particle size clearly seen
Silty soil, Fig. 4	57.5	57.5	0.011	0.68	0	
Clayey soil, Fig. 4	91.3	91.3	0.017	0.4	0	

Table 3. Data for SWCCs reported by Mahmood et al. (2016)

Tests and figures	θ_{max} (%)	a	b	c	d	Comments
Soil 1, Fig. 5	41.4	28	0.26	0.5	5.9	For this set of data, $d \neq 0$
Soil 2, Fig. 5	47.3	33	0.2	0.5	8.2	
Soil 4, Fig. 5	31.4	20	0.15	0.5	4.9	
Soil 5, Fig. 5	52.1	36	0.16	0.5	9.5	

The data on Khon Kaen loess were reported by Likitlersuang and Sevilla 2013. The SWCCs for two net stresses are obtained, i.e., 150 and 200 kPa. Comparisons of simulations and data are shown in Fig. 3. The observed data are represented reasonably well by the proposed equation. The initial volumetric water content for the loess is essentially constant, i.e., $\theta_{w,max} = 29.2\%$ for net stress being equal to 150 kPa, and $\theta_{w,max} = 35.3\%$ for net stress being equal to 200 kPa. The maximum volumetric water content changes with net stress.

The experimental data reported by Fredlund and Xing (1994) are simulated. There are three types of soil: sandy soil, silty soil, and clayey soil. Comparisons of simulations and data for the three sets of tests are shown in Fig. 4. The influence of particle size on SWCCs is clearly seen in this set of test data. With the decrease in soil particle size, suction increases steadily, and the range of volumetric water content increases steadily. The experimental data of soil for both clay and sand are described satisfactorily, including different ranges in the variation for matric suctions and volumetric water contents.
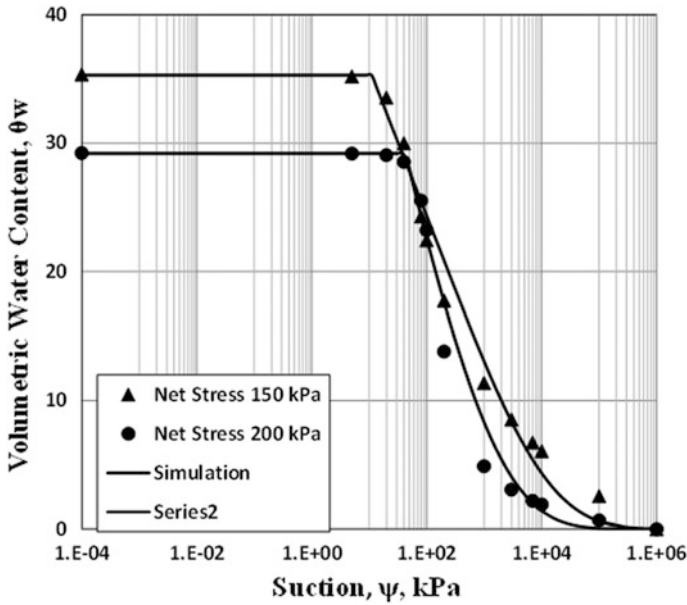
Fig. 3. SWCCs for Khon Kaen loess (data after Likitlersuang and Sevilla 2013)
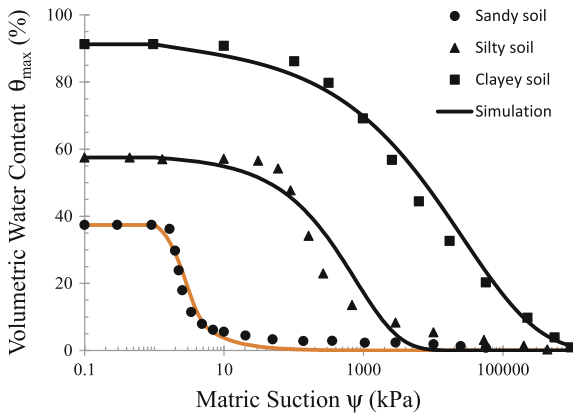

Fig. 4. SWCCs for a sand, a silty, and a clay (data after Fredlund and Xing 1994)

SWCCs for four types of soil reported by Mahmood et al. (2016) are also simu-lated. Comparisons of simulations and experimental data are shown in Fig. 5. The experimental data of these soils are modelled highly satisfactorily. It is also seen that the asymptotic value for this set of data is not zero.
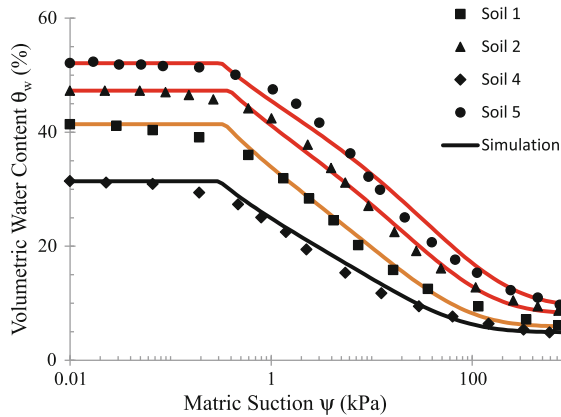
Fig. 5. SWCCs for soils reported by Mahmood et al. (2016)

4   Conclusions

In this paper, a new equation for SWCCs is proposed. The equation is an elemental function and thus can be employed conveniently in mathematical modelling and numerical analyses. There are five parameters in the equations. Two of the parameters, $\theta_{w,max}$ and d, can be determined from the SWCC straightforward, and the other three need to be determined by fitting. The SWCCs for eight soils with nine cases are simulated via the proposed equation, and it is shown that the proposed equation has the capacity to represent the curves of various soils satisfactorily.

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