A Three Dimensional Discrete Constitutive Model for Over Coarse Grained Soil

Yan Zongling[1,2(✉)], Gou Dongyuan[1,2], and Chai Hejun[1,2]

[1] National Local Joint Engineering Laboratory for Road Engineering and Disaster Prevention and Reduction Technology in Mountainous Areas, Chongqing, People's Republic of China
orienty@163.com

[2] National Engineering & Research Center for Highways in Mountain Area, Chongqing, People's Republic of China

Abstract. Over Coarse Grained Soil is widely used as filling material for subgrade in mountainous highway, and the settlement and stabilization of high embankment settlement is critical to operation safety of mountainous highway. The constitutive relationship of over coarse grained soil is theoretical basis for settlement and stability analysis. So a rigid contact model for over coarse grained soil particles is established combined with the distribution of contact force and normal of contact force based on the research of contact characteristics of particles and ignoring the deformation of particle. The local constitutive model is acquired after analyzing the relationship between contact force of particles and local stress in over coarse grained soil. Furthermore, a three dimensional discrete constitutive model is built up. It is proved through discrete constitutive model that the fabric will change during the deformation of over coarse grained soil which results the change of physical and mechanical characteristics. The change of fabric affects the macro mechanical responses characteristics of over coarse grained soil.

Keywords: Embankment subgrade · Over course grained soils
Discrete granular soil particle · Fabric · Constitutive model

1 Introduction

Over coarse grained soil such as gravel soil, soil-rock aggregate mixture, boulder and rockfill is a typical granular media composed of discrete particles within two or three orders of magnitude in dimension. And it is widely applied in embankment in mountainous highway. The physical and mechanical characteristics is quite different from fine grained soil. Over coarse grained soil is a loose and discrete media in substance, and the contact between particles is discontinuous point-contact. From the paper of Luan and Ugai (1999), for over coarse grained soils and other loose granular media, the research of physical constitutive model of deformation and strength characteristics to overcome inherent limitation of mechanical methods of continuous medium with basis of micro mechanics is quite potential.

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Fabric is an important concept in mechanics of granular media which can describe dimensional arrangement feature and interaction of granular particles' (Arthur et al. 1977; Mahmood and Mitchell 1974; Oda 1977). Micro fabric of over coarse grained soil is closely related to particle size, shape, rigidity, gradation, void ratio, contact number between particles and stacking manner. Granular media is compose of large number particles, so fabric has statistical significance.

Macro Characteristics of over coarse grained soil can be described by several key micro fabric parameters. Void ratio is a key physical parameter of over coarse grained soil (Oda et al. 1980; Chang 1990). Direction fabric describes the dominant stacking manner of over coarse grained soil particles in three-dimensional space. Particle Pi contacts to neighbor particle P_1 to P_5, and the contact points are C_1 to C_5, contact normal are $n^{(1)}$ to $n^{(5)}$. The relation between Particle Pi and its neighbor particles includes contact point number and contact normal, as shown is Fig. 1.

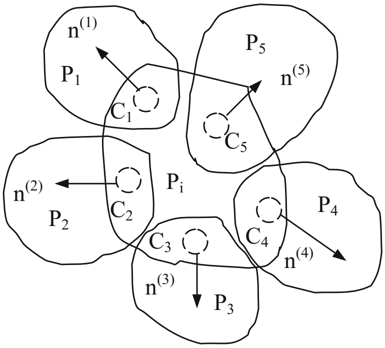

Fig. 1. Particle pi in relation to its neighbor particles

Fabric ellipse is introduced to describe contact normal. Fabric ellipse is a second-order tensor. Probability density function of contact normal $E(\alpha, \beta)$ is closely related to stress ellipse and has specific physical meaning. Probability density function $E(\alpha, \beta)$ changes in loading process so as to change the strength of over coarse grained soil which can result strain-hardening or strain-softening (Oda 1972a, b, c).

It is proved by experiment that probability density function $E(\alpha, \beta)$ can represent the three-dimensional distribution of contact normal (Oda 1972a, b, c). Therefore, particle characteristic of over coarse grained soils, interaction and essential fabric of spatial uniform distribution between particles can be descripted objectively. Based on mathematical function of statistical distribution of the reflected fabric parameters, the relationship could be set up between inherent evolution of microstructure fabric parameters over coarse grained soils and macroscopic mechanics responses.

Granular media mechanics is assumed that over coarse grained soil is composed of solid particles contacted with each other, and interaction between particles obeys the laws of probability.

Granular media mechanics is applied in study the mechanical phenomena on the contact points of particles, and describes the phenomena according to the formula of mathematical statistics. Some mechanical models have been built by different research. However, those models can't interpret the relationship between fabric change and mechanical response of over coarse grained soils under load very well.

2 Rigid Particle Contact Model of Over Coarse Grained Soil

There is a certain gradation within over coarse grained soil particles where large particles act as the skeleton and fine particles fill in the void between the large particles. Both the interaction between particles and between the filled surrounding medium should be belong to solid contact mechanics in terms of the mechanical characteristics (Stake 1983). It is supposed that the number of over coarse grained soil particles is enormous amount, so the macro mechanical parameters have statistical significance. The uneven stress of microcosmic can be described by average stress (Chang 1988a, b). When the over coarse grained soil particle number is infinity, and over coarse grained soil is continuous in the certain space, the summation in the calculation could be changed into integral (Zhong and Yuan 1992).

The particle contact normal density distribution function $E(\vec{n})$ is introduced, where $\vec{n}$ is the contact unit normal vector. The number of contact points within the $\vec{n} \rightarrow \vec{n} + d\vec{n}$ is $E(\vec{n})d\vec{n}$. Supposing $f_i(\vec{x}^{\alpha}, \vec{n})$ is the contact force component located at the contact point $\vec{x}^{\alpha}$ with normal $\vec{n}$. For a certain volume V of the over coarse grained soil, the total contact vectors is zero based on the conditions of static equilibrium.

$$\int f_i(\vec{n})E(\vec{n})d\vec{n} = 0 \quad (i = x, y) \tag{1}$$

where $f_i(\vec{n})$ is the average of contact force within $\vec{n} + d\vec{n}$.

The contact between particles is one of the most fundamental problems in particle mechanics model (Mao 1994). Assumed that the over coarse grained soil particles are rigid, and the deformation of particles is ignored, so the contact of over coarse grained soil particles is elastic contact. Over coarse grained soil appears sliding or rolling possibly under load, so that two adjacent particles can be viewed as two rigid body connected by deformed springs on the contact point (Mindlin et al. 1953). Therefore, the deformation of particles is transformed into spring deformation under the contact force. Choosing two similar particles of P, Q in the over coarse grained soil, it is shown in Fig. 2.

For particle P, the equilibrium equation can be established.

$$\sum_{\alpha=1}^{m} f_i^{p\alpha}(\vec{x}^{\alpha}, \vec{n}')(x_j^{p\alpha} - x_j^{p}) = \sum_{\alpha=1}^{m} f_i^{p\alpha}(\vec{x}^{\alpha}, \vec{n}')(x_i^{p\alpha} - x_i^{p}) \tag{2}$$

where M is the contact number of particle p; $\vec{n}'$ is a unit branch vector, and $\vec{n}' = \dfrac{\vec{x}^{p} - \vec{x}^{q}}{|L^{pq}|}$, $|L^{pq}|$ is branch length.

Putting the summation of all particles into integral form in over course grained soils.

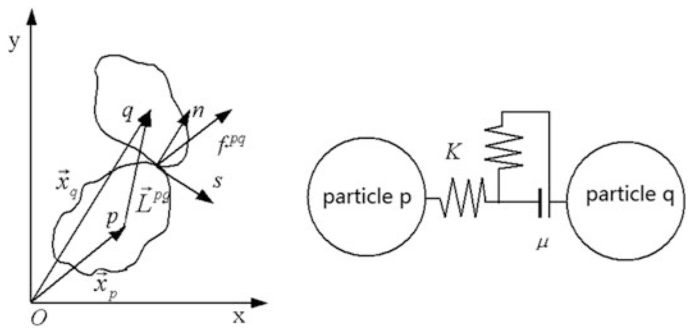

Fig. 2. The rigid contact model of over coarse grained soil

$$\int f_i(\vec{n})L_j(\vec{n}')E(\vec{n})d\Omega = \int f_j(\vec{n})L_i(\vec{n}')E(\vec{n})d\Omega \tag{3}$$

where $L_i(\vec{n}')$ is the average of contact force of the branch vector within $\vec{n}+d\vec{n}$.

Under the contact force, the strain tensor is $\varepsilon_{ij}(\vec{x}^{\alpha})$, the displacement of contact point is $u_i(\vec{x}^{\alpha})$. Ignoring rotation of particles, the equation is shown as below

$$u_i(\vec{x}^{\alpha}) = \varepsilon_{ij}(\vec{x}^{\alpha})L_j(\vec{x}^{\alpha},\vec{n}') \quad (i,j = x,y) \tag{4}$$

The virtual work of contact force in the unit volume is

$$W = \frac{1}{2V}\sum f_i(\vec{x}^{\alpha},\vec{n})\varepsilon_{ij}(\vec{x}^{\alpha})L_j(\vec{x}^{\alpha},\vec{n}') \tag{5}$$

where factor 2 represents each particle counted twice.

The virtual work for average stress is $W = \bar{\sigma}_{ij}\varepsilon_{ij}$.

From Eqs. (2), (5), considering the symmetry of the stress tensor, the equation is obtained as follow.

$$\bar{\sigma}_{ij} = \frac{1}{4V}\sum_{\alpha=1}^{M}[f_i(\vec{x}^{\alpha},\vec{n})L_j(\vec{x}^{\alpha},\vec{n}') + f_j(\vec{x}^{\alpha},\vec{n})L_i(\vec{x}^{\alpha},\vec{n}')] \tag{6}$$

The integral form is

$$\bar{\sigma}_{ij} = \frac{M}{4V}\int [f_i(\vec{n})L_j(\vec{n}) + f_j(\vec{n})L_i(\vec{n})]E(\vec{n})d\vec{n} \tag{7}$$

Equation (7) links the macroscopic average stress tensor and microscopic first-order tensor together. The over coarse grained soil particle size can be classified into 2 classes, and each class has an average particle size $\bar{d}_k$ with N_{k} particles. Assuming that all the over coarse grained soil particle size is $\bar{d}$, then

$$
\begin{aligned}
V_s &= \sum_{k=1}^{l} N_k \cdot \frac{\pi \bar{d}_k^3}{6} = \sum_{k=1}^{l} \frac{\pi \bar{d}_k^3}{6} \cdot \frac{M_k}{\bar{m}} \\
&= \frac{M}{\bar{m}} \sum_{k=1}^{l} \frac{\pi \bar{d}_k^3}{6}, \quad \bar{d} = \left(\sum_{k=1}^{l} \bar{d}_k^3 \right)^{1/3},
\end{aligned}
$$

So

$$
\begin{aligned}
\bar{\sigma}_{ij} &= \frac{3\bar{m}}{2(1+e)\pi} \left(\sum_{k=1}^{l} \bar{d}_k^2 \right)^{-\frac{2}{3}} \\
& \int_0^{\pi} \int_0^{2\pi} [f_i(\vec{n}) n_j + f_j(\vec{n}) n_i] E(\vec{n}) \mathrm{Sin}\gamma \mathrm{d}\gamma \mathrm{d}\beta
\end{aligned}
\tag{8}
$$

where $\bar{m}$ is the average coordination number; e is void ratio, i, j = 1, 2, 3.

For three dimensional form, the particle contact force density distribution function could be described as follow:

$$
f_i E(\vec{n}) = C_i + C_{ij} n_j + C_{ijk} n_j n_k + C_{ijkl} n_j n_k n_l + \cdots \tag{9}
$$

Omitting the higher order term, then

$$
f_i E(\vec{n}) = C_i + C_{ij} n_j \tag{10}
$$

And

$$
\int_{\Omega} (C_i + C_{ij} n_j) E(\vec{n}) dn = 0 \tag{11}
$$

So

$$
C_i = 0 \tag{12}
$$

Therefore

$$
f_i E(\vec{n}) = C_{ij} n_j \tag{13}
$$

By substitution formula (13) into the formula (8)

$$
\bar{\sigma}_{ij} = \frac{4\bar{m}}{(1+e)} \left(\sum_{k=1}^{l} \bar{d}_k^3 \right)^{-\frac{2}{3}} C_{ij} \tag{14}
$$

The three dimensional relationship between integral contact force and stress is

$$f_i = \frac{(1+e)\left(\sum_{k=1}^{l} \bar{d}_k^3\right)^{\frac{2}{3}}}{4\bar{m}E(\vec{n})}\bar{\sigma}_{ij}n_j \tag{15}$$

$E(\vec{n})$ can be expressed as

$$E(\vec{n}) = \frac{1}{4\pi}N_{ij}n_i n_j \tag{16}$$

Substituting formula (16) into formula (15), and the relationship between integral contact force and stress is

$$f_i = \frac{\pi(1+e)\left(\sum_{k=1}^{l} \bar{d}_k^3\right)^{\frac{2}{3}}}{2\bar{m}N_{ij}n_i n_j}\bar{\sigma}_{ij}n_j \tag{17}$$

Assuming that the contact force is f_i (i = n, s, t; n, s, t as the local coordinate), then the relation between force and displacement on the contact point can be expressed as in incremental form.

$$\Delta f_i = D_{ij}\Delta U_j \tag{18}$$

where D_{ij} is contact stiffness tensor.

In local coordinates n, s, t, D_{ij} can be expressed as

$$D_{ij} = D_n n_i n_j + D_s s_i s_j + D_t t_i t_j \tag{19}$$

For macroscopic isotropic over coarse grained soil, the tangential contact stiffness is isotropic on the particle contact plane, so

$$D_{ss} = D_{tt} = D_s \tag{20}$$

Generally, the normal contact stiffness is a function of the normal contact force.

$$D_n = C_1 f_n^{\beta} \tag{21}$$

where C, β functions are associated with the over coarse grained soil characteristic, particle size and surface roughness.

The tangential stiffness is

$$D_s = C_2 D_n(1 - \frac{f_s}{f_n \tan\varphi_{\mu}})^{\eta} \tag{22}$$

where C_2, η coefficients are related to the material itself; ϕ_{μ} is frictional angle between particles.

3 Three Dimensional Discrete Constitutive Model

Over coarse grained soil particles are randomly stacked, so it can be described by fabric density distribution function. According to findings of Rothenburb and Bathurst (1988) and Bathurs and Rothenburb (1988), the density distribution function of over coarse grained soil particle fabric content could be described approximately by three function in the two dimensional case.

Contact normal vector

$$E(\theta) = \frac{1}{2\pi}(1 + a_1(\cos\theta - \theta_1)] \tag{23}$$

Normal contact force

$$f_n(\theta) = f_0[1 + a_2 \cos 2(\theta - \theta_2)] \tag{24}$$

Tangential contact force

$$f_t(\theta) = -f_0 a_3 \sin 2(\theta - \theta_3) \tag{25}$$

where the minus is negative tangential contact force to rotate counter clock wise positive; $\alpha_1, \alpha_2, \alpha_3$ are coefficients reflecting the degree of anisotropy; $\theta_1, \theta_2, \theta_3$ denote fabric shaft angle, maximum normal contact force, average contact angle of maximum tangential contact force respectively.

f_0 is average tangential contact force.

$$f_0 = \int_0^{2\pi} f_n(\theta) d\theta \tag{26}$$

Substituting formula (23–25) into the formula (8)

$$\begin{aligned}\bar{\sigma}_{11} = \frac{\bar{m} f_0}{(1+e)\pi}\left(\sum_{k=1}^{l} \bar{d}_k^2\right)^{-1/2} & [1 + \frac{1}{2}(a_1 \cos 2\theta_1 + a_2 \cos 2\theta_2 + a_3 \cos 2\theta_3) \\ & + \frac{a_1 a_2}{2}(\cos 2\theta_1 \cos 2\theta_2 + \sin 2\theta_1 \sin 2\theta_2)]\end{aligned} \tag{27}$$

$$\begin{aligned}\bar{\sigma}_{22} = \frac{\bar{m} f_0}{(1+e)\pi}\left(\sum_{k=1}^{l} \bar{d}_k^2\right)^{-1/2} & [1 - \frac{1}{2}(a_1 \cos 2\theta_1 + a_2 \cos 2\theta_2 + a_3 \cos 2\theta_3) \\ & + \frac{a_1 a_2}{2}(\cos 2\theta_1 \cos 2\theta_2 + \sin 2\theta_1 \sin 2\theta_2)]\end{aligned} \tag{28}$$

$$\sigma_{12} = \frac{\bar{m} f_0}{(1+e)\pi}\left(\sum_{k=1}^{l} \bar{d}_k^2\right)^{-1/2} (a_1 \sin 2\theta_1 + a_2 \sin 2\theta_2 + a_3 \sin 2\theta_3) \tag{29}$$

Stress is associated with void ratio, the number of contact points and fabric contents. If the relationship between contact force and contact displacement is linear, the formula is obtained as follow by Hooke's law.

$$\Delta\sigma = E\Delta\varepsilon \tag{30}$$

The tangential stiffness

$$D_s = \lambda D_n \tag{31}$$

where λ is a constant, generally $\lambda = 0.1 - 1.0$.

The relationship between strain and stress of over coarse grained soil can't be derived directly, and the strain should be linked to contact force. Over coarse grained soil relationships between stress and strain can be derived when the local constitutive relation is confirmed.

The contact stiffness tensor is

$$D_{ij} = D_n n_i n_j + D_s s_i s_j \quad (i, j = 1, 2) \tag{32}$$

Local contact constitutive relation of incremental form

$$\Delta f_i = D_{ij} \Delta u_j \tag{33}$$

where Δf_i is incremental contact force on the contact point; Δu_j is incremental displacement on the contact point.

For the over coarse grained soil, the number of particles is homogeneous on the large scale, and its displacement is linear distribution without forming shear zone.

$$\Delta u_j = l_i \Delta\varepsilon_{ij} = l(\vec{x}^{\alpha}) n_i \Delta\varepsilon_{ij} \tag{34}$$

where $\Delta\varepsilon_{ij}$ is strain increment.

The relation between contact stress increment and strain increment is

$$\Delta f_i(\vec{x}^{\alpha}, \vec{n}) = l(\vec{x}^{\alpha})(D_n n_i n_j + D_s s_i s_j) n_k \Delta\varepsilon_{kj}, (i, j = 1, 2) \tag{35}$$

Substituting formula (35) into formula (34), the incremental formula of force and strain is

$$\Delta\sigma_{ij} = A_{ijkl} \Delta\varepsilon_{ij}, \quad (i, j = 1, 2) \tag{36}$$

where

$$A_{ijkl} = \frac{2\bar{m}}{(1+e)\pi} \int_0^{2\pi} (n_i n_j n_k n_l + B_{ijkl} D_s) E(\theta) d\theta \tag{37}$$

$$B_{ijkl} = (n_i s_j n_k s_l + n_j s_i n_k s_l + n_i s_j n_l s_k + n_j s_i n_l s_k)/4 \tag{38}$$

$$E_{ijkl} = \frac{1}{4}\left(n_i t_j n_k t_l + n_j t_i n_k t_l + n_i t_j n_l t_k + n_j t_i n_l t_k\right) \tag{39}$$

It is clear that stiffness tensor conforms to symmetry of stress tensor and strain tensor, then

$$A_{ijkl} = A_{jikl} = A_{klij} \tag{40}$$

For three dimensional condition, density function is

$$E(\vec{n}) = \frac{1}{4\pi} \tag{41}$$

Substituting formula (41) into formula (40), and integral is

$$\begin{bmatrix} \Delta\sigma_{xx} \\ \Delta\sigma_{yy} \\ \Delta\sigma_{zz} \\ \Delta\sigma_{xy} \\ \Delta\sigma_{xz} \\ \Delta\sigma_{yx} \end{bmatrix} = \begin{bmatrix} D_{11} & & & & & \\ D_{21} & D_{22} & & & & \\ D_{31} & D_{32} & D_{33} & & & \\ 0 & 0 & 0 & D_{44} & & \\ 0 & 0 & 0 & 0 & D_{55} & \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_{xx} \\ \Delta\varepsilon_{yy} \\ \Delta\varepsilon_{zz} \\ \Delta\gamma_{xy} \\ \Delta\gamma_{xz} \\ \Delta\gamma_{yz} \end{bmatrix} \tag{42}$$

where

$$\begin{aligned} D_{11} &= D_{22} = \frac{D}{5}(12D_n + 3D_s + 5D_t) \\ D_{33} &= \frac{D}{5}(12D_n + 8D_s) \\ D_{44} &= \frac{D}{5}(4D_n + D_s + 5D_t) \\ D_{55} &= D_{66} = \frac{D}{10}(8D_n + 7D_s + 5D_t) \\ D_{12} &= D_{21} = \frac{D}{5}(4D_n + 3D_s - 5D_t) \\ D_{31} &= \frac{4D}{5}(D_n - D_s) \\ D_{32} &= D_{23} = \frac{4D}{5}(D_n - D_s) \\ D &= \frac{\bar{m}}{8(1+e)\pi\bar{r}} \end{aligned}$$

For isotropous fabric, there is

$$N_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{43}$$

And the density function obeys the formula

$$\int_{\Omega} E(\vec{n})d\Omega = 1 \tag{44}$$

So

$$N_{11} + N_{22} + N_{33} = 3 \tag{45}$$

Stiffness matrix can be expressed as

$$\begin{bmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} C_{11} & & & & & \\ C_{21} & C_{22} & & & \text{symmet -} & \\ C_{31} & C_{32} & C_{33} & & \text{rical} & \\ C_{41} & C_{42} & C_{43} & C_{44} & & \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} = \begin{bmatrix} Q & & & & & \\ S & Q & & & \text{symmet -} & \\ S & S & Q & & \text{rical} & \\ 0 & 0 & 0 & R & & \\ 0 & 0 & 0 & 0 & R & \\ 0 & 0 & 0 & 0 & 0 & R \end{bmatrix} \tag{46}$$

There are three equivalent coefficient for stiffness tensor.

$$Q = \frac{\bar{m}}{10(1+e)\pi\bar{r}}(3D_n + 2D_s) \tag{47}$$

$$S = \frac{\bar{m}}{10(1+e)\pi\bar{r}}(D_n - D_s) \tag{48}$$

$$R = \frac{Q-S}{2} = \frac{\bar{m}}{20(1+e)\pi\bar{r}}(2D_n + 3D_s) \tag{49}$$

There are only two independent coefficient in 12 coefficient, it indicates that the stress-strain characteristics of over coarse grained soil is isotropous when its fabric is isotropous.

The average bulk modulus $\bar{K}$, shear modulus $\bar{G}$ and Young modulus $\bar{E}$ are respectively

$$\bar{K} = S + \frac{2}{3}R = \frac{\bar{m}}{6(1+e)\pi\bar{r}}D_n \tag{50}$$

$$\bar{G} = \frac{\bar{m}}{20(1+e)\pi\bar{r}}(2 + 3\xi) \tag{51}$$

$$\bar{E} = \frac{\bar{m}}{2(1+e)\pi\bar{r}}\frac{2+3\xi}{4+\xi} \tag{52}$$

Poisson's ratio

$$\upsilon = \frac{1-\xi}{4+\xi} \tag{53}$$

It is proved that the bulk modulus and shear modulus of over coarse grained soil is related to void ratio and coordination number.

The bulk modulus is only related to the normal contact stiffness, shear modulus is both related to the normal and tangential contact stiffness, Poisson's ratio is only related to contact stiffness, coordination number, void ratio and particle size, but other parameters are related to particle size and its' distribution, the relationship are shown as Figs. 3, 4, 5 and 6.

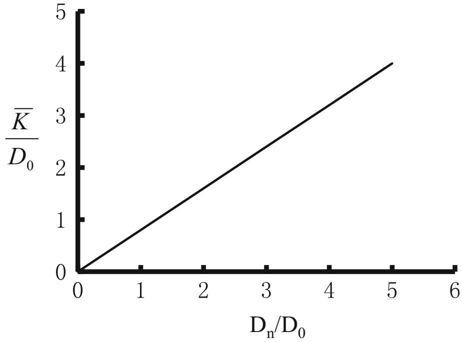

Fig. 3. The relationship between bulk modulus and normal contact stiffness ratio

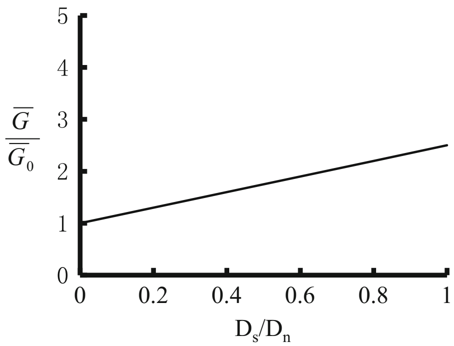

Fig. 4. The relationship between shear modulus and contact stiffness ratio

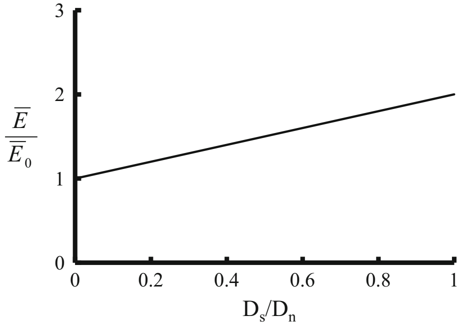

Fig. 5. The relationship between Young modulus and contact stiffness ratio

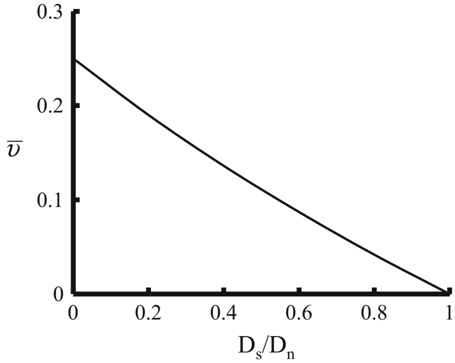

Fig. 6. The relationship between Poisson's ratio and contact stiffness ratio

4 Conclusions

The physical and mechanical properties of over coarse grained soil are closely related to particle spatial stacking mode, void ratio and its' spatial distribution. The relationship between average force and contact point is set up by analyzing the force between particles and normal direction of contact based on the fabric characteristics of over coarse grained soil. Due to fabric change during the process of deformation, relationship between force and strain is nonlinear as same as relationship between stress and contact force, the relationship between stress and contact force is affected by the fabric. The mechanical response of over coarse grained soil in the large scale proved that the changes of fabric contents play an important role on the deformation characteristics of materials.

The rigid contact model of over coarse grained soil is established based on the microstructure of over coarse grained soil particles. Local constitutive relation of over coarse grained soil is acquired by the connection between contact forces of local stress between particles. Based on the local constitutive relation of over coarse grained soil, the constitutive relation of three dimensional granular of over coarse grained soil has been set up. These studies provide foundations for the further research.

Particle flow code (PFC 3D) method is a feasible method for experimental verification of three Dimensional Discrete Constitutive Model. The author will write other paper to discuss.

The application of basic theories of discrete mechanics and particle mechanics is a promising method to study stress-strain characteristics of over coarse grained soil. And the contact force of over coarse grained soil particles and contiguous normal distribution need further research.

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