Study on Structure and Kinematics of Quick-Return Mechanism with Four-Bar Assur Group

A. Fomin and A. Olexenko

Abstract The presented study shows results of structural and kinematic analysis of a planar six-bar quick-return mechanism that is used in shaping and planing machines for transformation of rotational motion of a driving link into prismatic motion of an end-effector. Assur groups of the III and II classes have been separated out from a quick-return mechanism when different driving links have been chosen during a structural analysis. Kinematic analysis has been carried out by grapho-analytical method for the case when a four-bar Assur group is included. Finally, 3D model has been simulated and coordinates of distinguished points of movable links have been found in six positions of the mechanism depending on the rotation of a driving link. The obtained results can be used in kinetostatic and dynamic analysis of the quick-return mechanism. The findings of the study can also be used in a design of planning and shaping machines, in synthesis and analysis of novel planar mechanisms.

Keywords Quick-return mechanism · Assur group · Degree-of-freedom
Kinematic pair

1 Introduction

Various planar mechanisms and manipulators are widely used in different practical applications [1–9]. Their synthesis and analysis are important tasks that allow improving different existing mechanisms, as well as creating new structures. It is well known that any mechanism is designed as a combination of one or several Assur groups with zero degree-of-freedom (DoF) and a driving link with mobility

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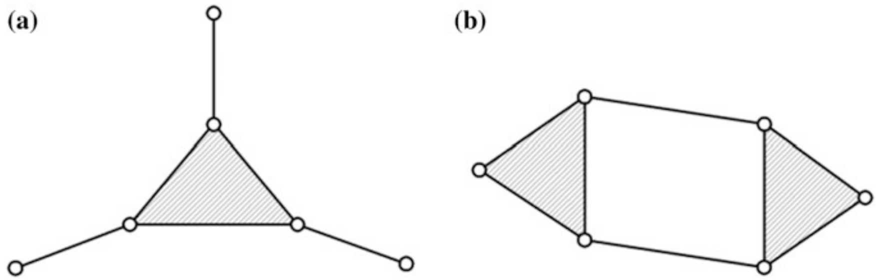

Fig. 1 Planar four-bar Assur groups: **a** of the III class; **b** of the IV class

more than zero [10, 11]. A series connection of Assur groups into a kinematic chain of a mechanism does not change its overall mobility.

The simplest planar Assur group is a dyad, which includes two links with three one-DoF kinematic pairs. Totally five different types of dyads are known [12]. Various mechanisms can be created on their basis beginning with four-bar mechanisms [13, 14]. Four-bar Assur groups are the following by number of links after dyads. They are divided into III and IV class. Assur group of the III class is shown in Fig. 1a, it includes a three-paired link with three levers. Various planar mechanisms and manipulators are designed on the basis of this group [15–18]. Figure 1b shows a four-bar Assur group of the IV class, which includes a closed variable contour [19]. This group is used in designing of planar transport mechanisms, bolt insertion systems, mining machines and other technique.

Links of both Assur groups shown in Fig. 1a, b are connected only by rotational kinematic pairs. However, rotational pairs can be replaced with prismatic pairs. In this case, the mobility of these groups is not changed, it remains equaled zero. At the same time, it becomes possible to design novel mechanical systems on the basis of these groups for performing various technological operations. In the presented study turn to a mechanism that includes a four-bar Assur group of the III class with rotational and prismatic kinematic pairs. Discuss the kinematic scheme of this mechanism and carry out its structural analysis in Sect. 2.

2 Structural Analysis

Figure 2a shows a six-bar quick-return mechanism that includes crank 1, slide block 2, swing lever 3, rocker arm 4, slider 5 and fixed link 6. The number of DoF of this mechanism can be calculated in accordance with the Chebyshev's formula written as follows

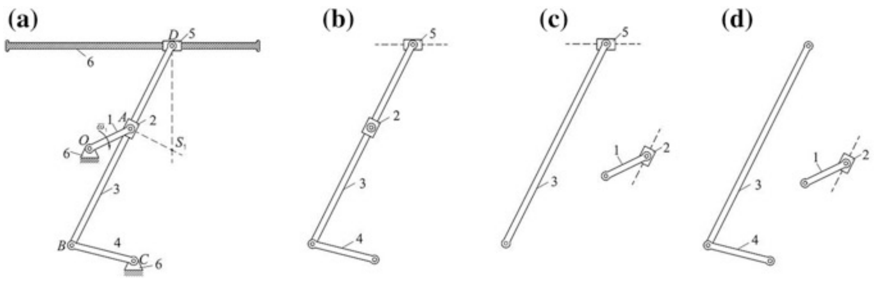

Fig. 2 **a** Kinematic scheme of quick-return mechanism; **b** Four-bar Assur group with three levers; **c** Dyads *RRP*; **d** Dyads *RRR* and *RRP*

$$W_3 = 3n - 2p_5 \tag{1}$$

where W_3—mobility, defining number of DoF of a planar kinematic chain with three imposed constraints, n—number of movable links of a mechanism, p_5—numbers of one-DoF kinematic pairs [20, 21]. The quick-return mechanism shown in Fig. 2a includes $n = 5$ and $p_5 = 7$. According to (1) its mobility equals one ($W_3 = 1$). It means that to have predefined motions of slider 5, it needs to give an input motion to a single link. Such a motion is the rotational and applied to crank 1.

As crank 1 is the driving link in the mechanism, it is possible to separate out only one Assur group, which is a four-bar group of the III class with levers 2 (*RP*), 4 (*RR*) and 5 (*PR*) shown in Fig. 2b. If we chose another driving link, the class of Assur groups included in the mechanism changes. For example, with driving link 4 it is possible to separate out two dyads *RRP* (Fig. 2c), with driving link 5 we can separate out dyad *RRR* and *RRP* (Fig. 2d).

Structural analysis of the six-bar quick-return mechanism allows solving a further kinematic task. Its algorithm depends on the DoF of the investigated mechanism and included Assur groups.

3 Direct Kinematic Analysis

Turn to a kinematic analysis of the six-bar quick-return mechanism. We apply grapho-analytical method to determine directions and numerical values of velocities and accelerations of its links. Draw these diagrams for the position of the mechanism shown in Fig. 3a. Accept angular velocity of crank 1 is w_1 = 60 min^{-1}, the dimensions of the movable links are OA = 74 mm, BC = 104 mm, BD = 364 mm. Links 1 and 2 have identical velocity of point A ($V_{A1} = V_{A2}$), which is calculated as $V_A = w_1 \cdot OA$ = 0.074 m/s. Draw vector pa = 74 mm at the velocity vector diagram shown in Fig. 3b. The scale of the velocity diagram is calculated as $\mu_V = V_A/pa$ = 0.001 (m/s)/mm.

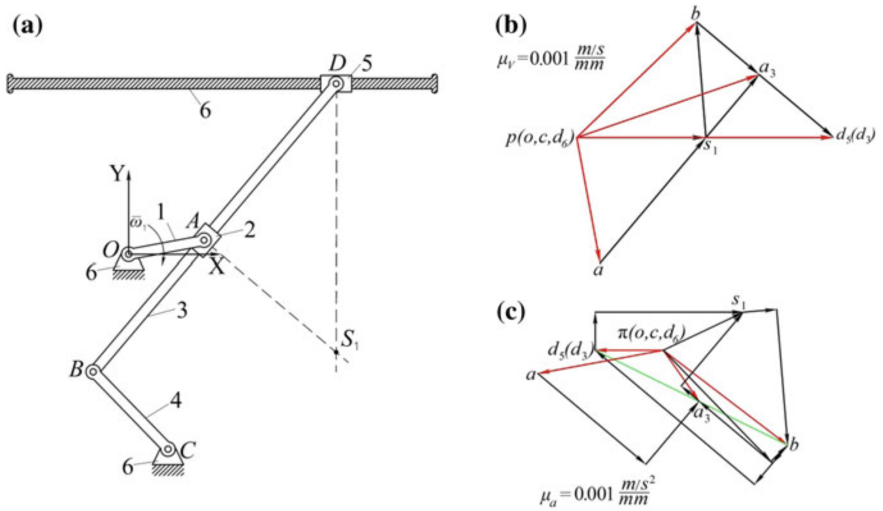

Fig. 3 **a** Chosen position of the quick-return mechanism; **b** Velocity vector diagram; **c** Accelerator vector diagram

As the mechanism includes the three-bar Assur group shown in Fig. 2b, it needs to apply the method of Assur points to solve kinematic task. It is possible to find three Assur points in the mechanism. The first point is S_1, which lies at the intersection of the perpendiculars to links 3 and 6 drawn through points A and D (Fig. 2a). The second point is S_2, which lies at the intersection of the perpendicular to link 6 drawn through point D and an extension of link 4. The third point is S_3, which lies at the intersection of the perpendicular to link 3 drawn through point A and an extension of link 4. We will use point S_1. Turn to its velocity calculation. Velocity of point S_1 is determined from the following system of equations.

$$\begin{cases} \overline{V}_{S_1} = \overline{V}_A + \overline{V_{A_3A} + V_{S_1A_3}}, & \text{where } \overline{V_{A_3A} + V_{S_1A_3}} \perp S_1A; \\ \overline{V}_{S_1} = \overline{V}_{D_6} + \overline{V_{D_5D_6} + V_{S_1D_5}}, & \text{where } V_{D_6} = 0,\ \overline{V_{D_5D_6} + V_{S_1D_5}} \perp S_1D. \end{cases} \tag{2}$$

Then we can define velocities of points B, A_3 and D_5 through equation systems (3–5).

$$\begin{cases} \overline{V}_B = \overline{V}_{S_1} + \overline{V}_{BS_1}, & \text{where } \overline{V}_{BS_1} \perp BS_1; \\ \overline{V}_B = \overline{V}_C + \overline{V}_{BC}, & \textit{where } V_C = 0, \overline{V}_{BC} \perp BC. \end{cases} \tag{3}$$

$$\begin{cases} \overline{V}_{A_3} = \overline{V}_B + \overline{V}_{A_3B}, & \text{where } \overline{V}_{A_3B} \perp AB; \\ \overline{V}_{A_3} = \overline{V}_A + \overline{V}_{A_3A}, & \text{where } \overline{V}_{A_3A} || AB. \end{cases} \tag{4}$$

$$\begin{cases} \overline{V}_{D_5} = \overline{V}_B + \overline{V}_{D_5B}, & \text{where } \overline{V}_{D_5B} \perp BD; \\ \overline{V}_{D_5} = \overline{V}_{D_6} + \overline{V}_{D_5D_6}, & \text{where } V_{D_6} = 0, \overline{V}_{D_5D_6} || D_5D_6. \end{cases} \tag{5}$$

Velocities of points D_3 and D_5 equal to each other ($V_{D3} = V_{D5}$), because links 3 and 5 connect to each other by the rotational kinematic pair. Numerical values of velocities V_{S1}, V_B, V_{A3} and V_{D5} are determined by multiplication of vector lengths ps_1, pb, pa_3 and pd_5 from the velocity diagram by its scale μ_V. Finally velocities equal: V_{S1} = 0.075 m/s, V_B = 0.097 m/s, V_{A3} = 0.112 m/s, V_{D5} = 0.149 m/s.

Angular velocities of slide block 2 and swing lever 3 equal to each other ($w_2 = w_3$), because links 2 and 3 connect to each other by prismatic kinematic pair, their numerical values are $w_2 = w_3 = 0.285$ s^{-1}. Angular velocity of rocker arm 4 equals 0.929 ($w_4 = 0.929$ s^{-1}). Angular velocity of slider 5 equals zero ($w_5 = 0$).

Turn to the calculation of accelerations. Accept that crank 1 rotates with constant angular velocity (w_1 = const). Then acceleration of point A equals $a_A = w_1^2 \cdot OA = 0.074$ m/s^2. Draw vector πa, which describes acceleration of point A at the accelerator diagram shown in Fig. 3c. The scale of the accelerator diagram is calculated as $\mu_a = a_A/\pi a = 0.001$ (m/s^2)/mm. Acceleration of point S_1 is calculated from the following equation system

$$\begin{cases} \overline{a}_{S_1} = \overline{a}_{A_3} + \overline{a}^n_{S_1A_3} + \overline{a}^\tau_{S_1A_3} = \overline{a}_A + \overline{a^k_{A_3A} + a^n_{S_1A_3}} + \overline{a^r_{A_3A} + a^\tau_{S_1A_3}}; \\ \overline{a}_{S_1} = \overline{a}_{D_5} + \overline{a}^n_{S_1D_5} + \overline{a}^\tau_{S_1D_5} = \overline{a}_{D_6} + \overline{a}^n_{S_1D_5} + \overline{a^r_{D_5D_6} + a^\tau_{S_1D_5}}. \end{cases} \tag{6}$$

Accelerations of points B, A_3 and D_5 can be found from equation systems (7–9).

$$\begin{cases} \overline{a}_B = \overline{a}_{S_1} + \overline{a}^n_{BS_1} + \overline{a}^\tau_{BS_1}, & \text{where } \overline{a}^n_{BS_1} || BS_1, \overline{a}^\tau_{BS_1} \perp BS_1; \\ \overline{a}_B = \overline{a}_C + \overline{a}^n_{BC} + \overline{a}^\tau_{BC}, & \text{where } a_C = 0, \overline{a}^n_{BC} || BC, \overline{a}^\tau_{BC} \perp BC. \end{cases} \tag{7}$$

$$\begin{cases} \overline{a}_{A_3} = \overline{a}_B + \overline{a}^n_{A_3B} + \overline{a}^\tau_{A_3B}, & \text{where } \overline{a}^n_{A_3B} || AB, \overline{a}^\tau_{A_3B} \perp AB; \\ \overline{a}_{A_3} = \overline{a}_A + \overline{a}^k_{A_3A} + \overline{a}^r_{A_3A}, & \text{where } \overline{a}^k_{A_3A} \perp AB, \overline{a}^r_{A_3B} || AB. \end{cases} \tag{8}$$

$$\begin{cases} \overline{a}_{D_5} = \overline{a}_B + \overline{a}^n_{D_5B} + \overline{a}^\tau_{D_5B}, & \text{where } \overline{a}^n_{D_5B} || DB, \overline{a}^\tau_{D_5B} \perp DB; \\ \overline{a}_{D_5} = \overline{a}_{D_6} + \overline{a}^k_{D_5D_6} + \overline{a}^r_{D_5D_6}, & \text{where } a_{D_6} = 0, a^k_{D_5D_6} = 0, \overline{a}^r_{D_5D_6} || D_5D_6. \end{cases} \tag{9}$$

Accelerations of points D_3 and D_5 equal to each other ($a_{D3} = a_{D5}$), because links 3 and 5 connect to each other by the rotational kinematic pair. Figure 3c provides accelerator vector diagram from which numerical values of accelerations a_{S1}, a_B, a_{A3} and a_{D5} are determined by multiplication of the vector lengths πs_1, πb, πa_3 and πd_5 by diagram scale μ_a. Finally accelerations equal: a_{S1} = 0.052 m/s^2, a_B = 0.093 m/s^2, a_{A3} = 0.036 m/s^2, a_{D5} = 0.039 m/s^2.

Angular accelerations of slide block 2 and swing lever 3 equal to each other ($\varepsilon_2 = \varepsilon_3$), because links 2 and 3 connect to each other by the prismatic kinematic pair, their numerical values are $\varepsilon_2 = \varepsilon_3 = 0.332$ s^{-2}. Angular acceleration of rocker arm 4 equals 0.878 ($\varepsilon_4 = 0.878\ \text{s}^{-2}$). Angular acceleration of slider 5 equals zero ($\varepsilon_5 = 0$).

Note that points b, a_3 and d_5 (d_3) are placed on one line at the velocity and acceleration diagrams. Such a placement is proved by Fig. 3a where points B, A and D are placed on one line (line BD). When driving link is 4 or 5, the kinematic analysis of quick-return mechanism is simplified as it is not necessary to find Assur points. In these cases the linkage includes only pair of dyads shown in Fig. 2c, d.

4 Simulation Development

Numerical analysis of the quick-return mechanism has been carried out for the position measurements of distinguished points of movable links and tracing their trajectories. Figure 4 presents 3D model of the quick-return mechanism with red-colored trajectories of points A_3, B and $D_3(D_5)$. These trajectories can be varied by using data from the kinematic analysis and changing of link lengths of the mechanism.

Table 1 provides coordinate variation of distinguished points $A_1(A_2)$, A_3, B and $D_3(D_5)$ of the quick-return mechanism depending on the tilt angle of crank 1. The initial position of crank 1 is when it lies in the negative part of axis X, so the Y-coordinate equals zero. Next positions are set after each 60°. The data from Table 1 allows solving inverse kinematic problem, i.e. define parameters of the quick-return mechanism by desired law motion.

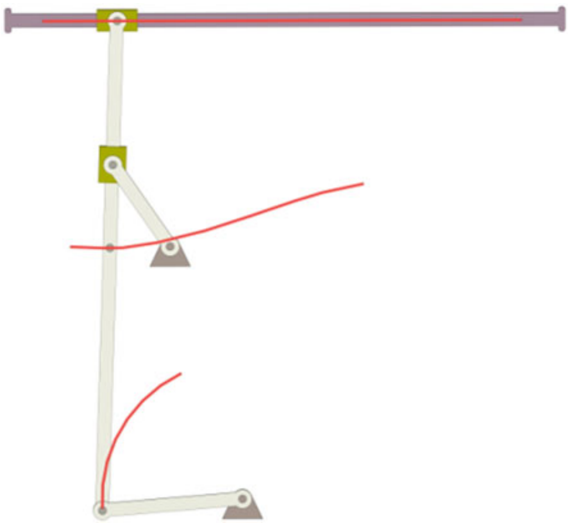

Fig. 4 Three-dimensional model of the quick-return mechanism with motion trajectories of its distinguished points (color figure online)

Table 1 Coordinate variation of the distinguished points of links depending on a tilt angle of crank 1

Point	Position of crank 1[a]											
	1		2		3		4		5		6	
	X (mm)	Y (mm)	X (mm)	Y (mm)	X (mm)	Y (mm)	X (mm)	Y (mm)	X (mm)	Y (mm)	X (mm)	Y (mm)
$A_1(A_2)$	−74.00	0.00	−37.04	64.15	37.04	64.15	74.00	0.00	37.04	−64.15	−37.04	−64.15
A_3	−74.00	0.00	−40.69	−1.01	16.52	9.02	104.66	37.45	143.36	46.25	−30.07	−0.28
B	−50.83	−193.99	−50.76	−196.04	−50.04	−174.53	−19.37	−113.31	7.89	−94.33	−50.79	−194.60
$D_3(D_5)$	−93.94	167.43	−31.61	167.43	74.63	167.43	212.23	167.43	260.79	167.43	−11.55	167.43

[a]Position of crank 1 is determined by the tilt angle between its axial axis and axis X towards the rotation of angular velocity w_1

5 Conclusions

The presented study provides results of structural and kinematic analysis of the planar quick-return mechanism based on the four-bar Assur group of the III class. The structural analysis of the mechanism has allowed defining various Assur groups when different driving links were accepted. The kinematics analysis has been solved by grapho-analytical method for the case when a four-bar Assur group was included in the mechanism. CAD-supported 3D modeling has allowed detecting motion trajectories and numerical values of coordinates of all movable links. The data obtained from the structural analysis can be applied for synthesis of new planing mechanisms, quick-return systems and other types of planar linkages. Results of kinematic calculations can be used in kinetostatic and dynamic analysis of the quick-return mechanism.

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