Drawings and Models in English Perspective Treatises of the XVII and XVIII Centuries

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Abstract. What do some children's books today have in common with English perspectives treatises of the seventeenth and eighteenth centuries? The answer lies in the fact that they both use alternative strategies for three-dimensional spatial rendering. These treatises, in fact, contain some mobile inserts that transform two-dimensional drawings into true 3D models, allowing the reader to immediately understand the spatial nature of a phenomenon such as the generation of perspective projections. English perspective literature arrived somewhat later in embracing the advances achieved in previous centuries in Italy and France on perspective. However, it does reveal a distinct approach in the development of graphic apparatus, essential for understanding the contents of this field of study. These approaches are useful in explaining the fundamentals of perspective but also in seeing the effects of the mutual movement of reference elements, subjects and observer. The most direct ancestor of these perspective texts can be found in books that belong to different fields as Geometry and Cryptography, using moveable elements to clarify or add new significances to the treatises they host.

Keywords: History of representation · Perspective · Moveable books
Pop-up · Descriptive geometry · Perspective treatises

1 Books and Moveable Elements

In today's educational literature for children we can enjoy the striking effect of beautiful illustrations, which sometimes use moveable elements to create transformations and three-dimensional images that look fascinating and at the same time effectively explain complex content when necessary.

So-called moveable books were used in different fields (Harris Taylor and Larson Bluemel 2003) and in science, these stratagems have been adopted since at least the thirteenth century (Crupi 2016), when Matthew Paris (1200 ca.-1259) used layers of sheet in his manuscript *Chronica majora*, to better explain maps of the routes for religious pilgrimages.[1]

[1] The present study is part of a Research Project of the University of Genoa (2017) entitled "Representation for simulation in architecture", scientific director: C. Càndito. Photos by the author were taken at the British Library, London.

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These mobile elements take different shapes including volvelles: paper circles which revolve, allowing the reader to make calculations or view combinations of elements. Matthew Paris seems to have been the first to introduce the volvelle (*Chronica*, *majora* f. 1v), widely used in astronomy books, such as the publication by the German Petrus Apianus (*Astronomicum Caesareum*, 1540), who used it to explain the movement of celestial bodies (Crupi 2016). The volvelle is also employed to formulate oracular predictions or encrypted languages, as can be found in Giovanni Fontana's *Secretum de thesauro* (1430 ca.), in Leon Battista Alberti's *De componendis cyfris* (1466 ca.), as well as in Giovan Battista Della Porta's *De occultis literarum notis* (1593) (Crupi 2016). As is known, these authors also share an interest in optics and perspective, and their different artistic and scientific aspects, as part of the wide group of disciplines that characterized the Renaissance and partly survived until the Enlightenment.

Dynamic elements can also transform two-dimensional drawings of a book into 3D models, allowing the reader to immediately understand the spatial nature of a phenomenon. In this way, moveable books became an interactive instrument of learning generating forms of hypertextuality.

Sometimes, the textbooks were accompanied by models, as in Martin Cortés's volume (1510–1582), *Breve compendio de la sphera y de la art de navegar* (Sevilla 1551), translated into English (Eden and Tap 1630). In both versions, the publication contains templates for astronomical instruments and volvelles (Cunningham 2004) such as the one that can be found in the third part (*Entreating of the composition and the use of instruments*, p. 169), which can be used as an instrument for orientation (Fig. 1).

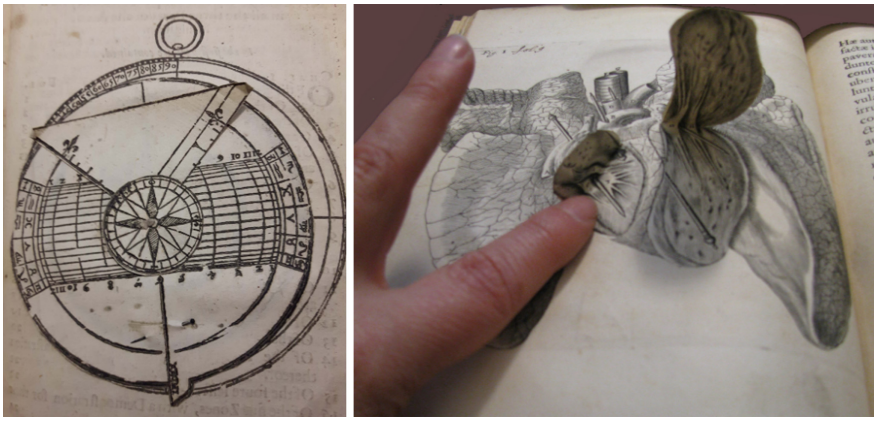

Fig. 1. Left: volvelle in Eden, Tap 1630. Right: flap in Descartes 1662 (photos by the author)

Moveable elements, like flaps are found in some anatomy texts, such as that of Andreas Vesalius (*De humani corporis fabrica*, Basel 1543), which comes with two hundred engravings. Illustrations were made by artists from the circle of Titian in Venice and then transported and included in a publication that would have a great influence on the teaching of the subject (Ackermann 1991, 143–144).

De homine by René Descartes (1662) belongs to the same scientific field; in this publication, the illustration of the heart has been created with flaps and different layers to better describe it (Fig. 1).

Flaps are also used in ancient and modern narrative texts, to illustrate changes in characters in the same scene, as in the *Queen Mab or The Tricks of Harlequin* by Robert Sayer (1771) or the recent *Cent mille milliards de poèmes* by Raymond Queneau (1961), in which combinatorial art makes it possible to achieve different narrative compositions (Fig. 2).

Fig. 2. Left: Sayer 1771, flaps to illustrate the transformation of spaces and characters (http://privatelibrary.typepad.com). Right: Queneau 1961, combination of flaps (http://disquiet.com)

2 English Treatises on Perspective

English treatises arrived somewhat later in embracing the advances achieved concerning perspective in previous centuries in Italy and France. However, they do reveal a distinct approach in the development of graphic apparatus. In some cases, there are moveable elements, which facilitates the understanding of the phenomenon, especially in its early stages of identifying the reference elements and their positions in the space.

Paper items sticking out from book pages have been found in the perspective treatise (Moxon 1670) by Joseph Moxon (1627–1691), hydrographer and mathematician (Vagnetti 1979, p. 412; Sgrosso 2001, p. 301). In chapter II, page 7 (Fig. 3) the perspective of a figure belonging to the Ground Plane is described by the rabattement on a vertical position of two moveable pop up elements: the Picture Plane (π) and an outline of the observer integrated with a plane that contains one projecting line (VoC). The Picture Plane is represented by a frame, to see both the figure in perspective (the square on the Ground Plane), and the spatial perspective system (Càndito 2017).

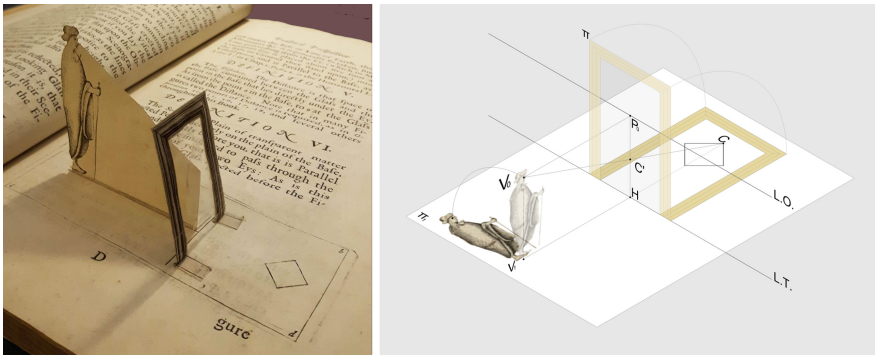

Fig. 3. Moxon 1670. Left: the image with pop up elements (chapter II, page 7). Right: virtual model (photo and image by the author)

Eighty-four years later, the perspective teacher of the Academy of Painting, Sculpture and Architecture, John Joshua Kirby (1716–1774), also wrote his perspective treatise (Kirby 1767), which follows in the steps of Brook Taylor (1685–1731) (Loria 1921, 48–49, Vagnetti 1979, 449; Baynes 1750, 108–127; Andersen 2007, 55–56; Kemp 1990, 171–172; De Rosa 2013, 91–93; Giordano 2001, 61–62). Kirby used a pop up element in the Fig. 2 of plate V (Fig. 4) which represents the horizontal plane passing through the Horizon Line (Càndito 2017). The elements are designed to easily visualize the operations of rabattement of the elements that allow recognition of their projective relations, in line with Brook Taylor's theory. The use of this stratagem and the fifty illustrations accompanying the text, many drawn by the author himself, demonstrate how important representation of the phenomenon described is for Kirby. Making three-dimensional inserts in textbooks was and still is a challenging task because it requires coordination between author and typographer, during creation and printing processes.

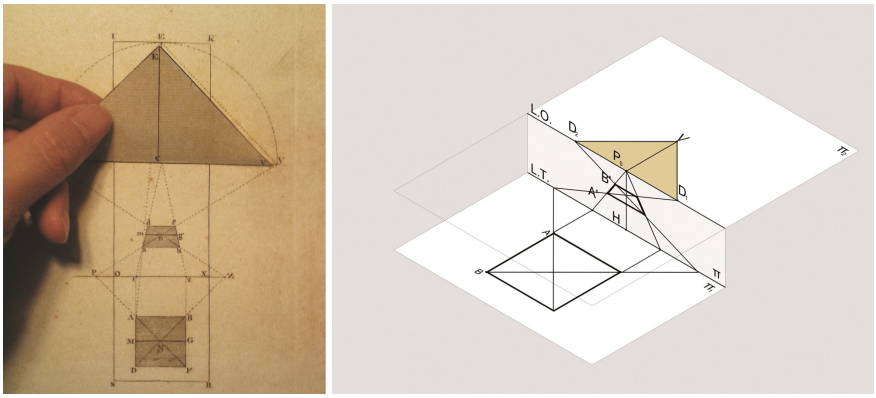

Fig. 4. Kirby 1754. Left: the image with flap elements (plate V, Fig. 2). Right: virtual model (photo and image by the author)

Similarly, in Thomas Malton's treatise on perspective (1726–1801) (Malton 1775), many plates demonstrate a great educational commitment centered on perspective, in compliance with article XII of the *Instrument of Foundation* by London's Royal Academy, established in 1768 (this book is dedicated precisely to the President of this Academy) (Wiebenson 1979, p. 234).

Malton became popular for his architectural drawings and for his ability to spread perspective and geometry, the latter explained in a textbook published a year before his book on perspective (Malton 1774). The first edition of his perspective volume (Fig. 5) was followed by a 1778 edition, due to a fire that destroyed the remaining stock of the first edition, and another one from 1779. The 1775 edition consists of four books: the first dedicated to Optics and Vision, the second to the Theory of Perspective, the third book to the Practice of Perspective and the last and fourth book to Shadows.

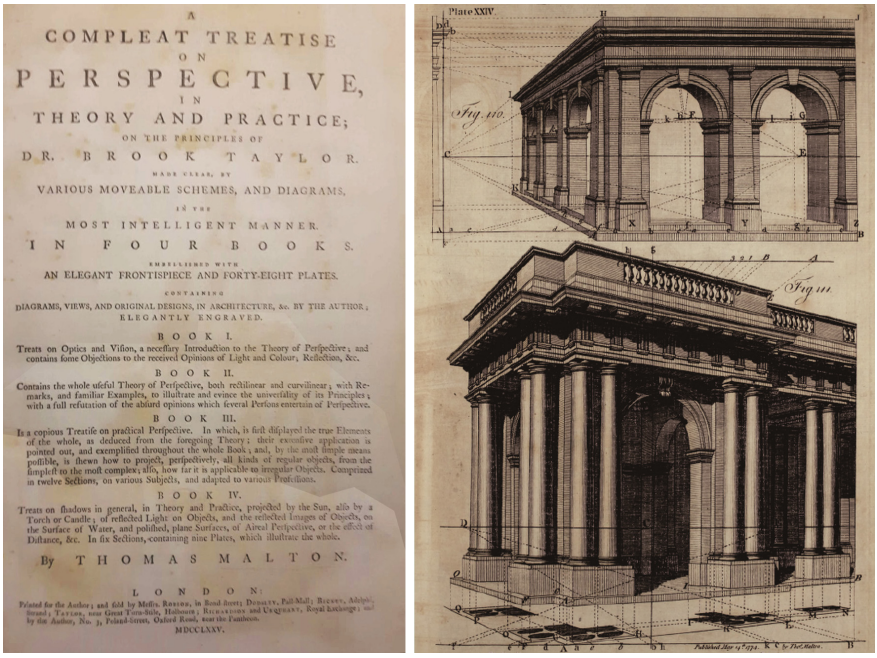
A
COMPLEAT TREATISE
ON
PERSPECTIVE,
IN
THEORY AND PRACTICE;
ON THE PRINCIPLES OF
D.R. BROOK TAYLOR.
MADE CLEAR, BY
VARIOUS MOVEABLE SCHEMES, AND DIAGRAMS,
IN THE
MOST INTELLIGENT MANNER.
IN FOUR BOOKS.
EMBELLISHED WITH
AN ELEGANT FRONTISPIECE AND FORTY-EIGHT PLATES.
CONTAINING
DIAGRAMS, VIEWS, AND ORIGINAL DESIGNS, IN ARCHITECTURE, &c. BY THE AUTHOR;
ELEGANTLY ENGRAVED.
BOOK I.
Treats on Optics and Vision, a necessary Introduction to the Theory of Perspective; and contains some Objections to the received Opinions of Light and Colour, Reflection, &c.
BOOK II.
Contains the whole useful Theory of Perspective, both rectilinear and curvilinear; with Remarks, and familiar Examples, to illustrate and evince the universality of its Principles; with a full refutation of the absurd opinions which several Persons entertain of Perspective.
BOOK III.
Is a copious Treatise on practical Perspective. In which, is first displayed the true Elements of the whole, as deduced from the foregoing Theory; their extensive application is pointed out, and exemplified throughout the whole Book; and, by the most simple means possible, is shewn how to project, perspectively, all kinds of regular objects, from the simplest to the most complex; also, how far it is applicable to irregular Objects. Comprized in twelve Sections, on various Subjects, and adapted to various Professions.
BOOK IV.
Treats on shadows in general, in Theory and Practice, projected by the Sun, also by a Torch or Candle; of reflected Light on Objects, and the reflected Images of Objects, on the Surface of Water, and polished, plane Surfaces, of Aireal Perspective, or the effect of Distance, &c. In six Sections, containing nine Plates, which illustrate the whole.
By THOMAS MALTON.
LONDON:
Printed for the Author; and sold by Messrs. Royston, in Ludgate-street; Dodsley, Pall-Mall; Brewer, Adelphi-Strand; Taylor, near Great Turn-Stile, Holborn; Richardson and Urquhart, Royal Exchange; and by the Author, No. 3, Poland-street, Oxford Road, near the Pantheon.
MDCCLXXV.

Fig. 5. Malton 1775. Frontispiece and plate XXIV (photos by the author)

New sections on different topics were added to the 1778 edition: (1) History of Perspective; (2) Military Perspective; (3) Depictions of Vision and Imagination; Application of Perspective: (4) in Scenery, (5) in Ship, (6) in Landscape; (7) Methods of delineating curved surfaces; (8) The Globe.

This is the perspective treatise that uses the greatest number of moveable elements, in keeping with its title (…*with various moveable schemes* ...). They consist of actual pop-ups with moveable elements that create three-dimensional configurations.

The first pop-up in the treatise is in plate IV of the second book (*Theory of Perspective*), section III (*Elements of Perspective*). The figure is used to illustrate the relationships between the four main planes involved in the perspective construction (Malton 1775, 52–53) (Fig. 6): Original Plane (*DHAB*, the Ground Plane), Picture Plane (*ABLM*, the Projecting Plane), Vanishing plane (*KIML*, passing through the Eye, parallel to the Original Plane), Directing Plane (*KIHG*, passing through the Eye, parallel to the Picture).

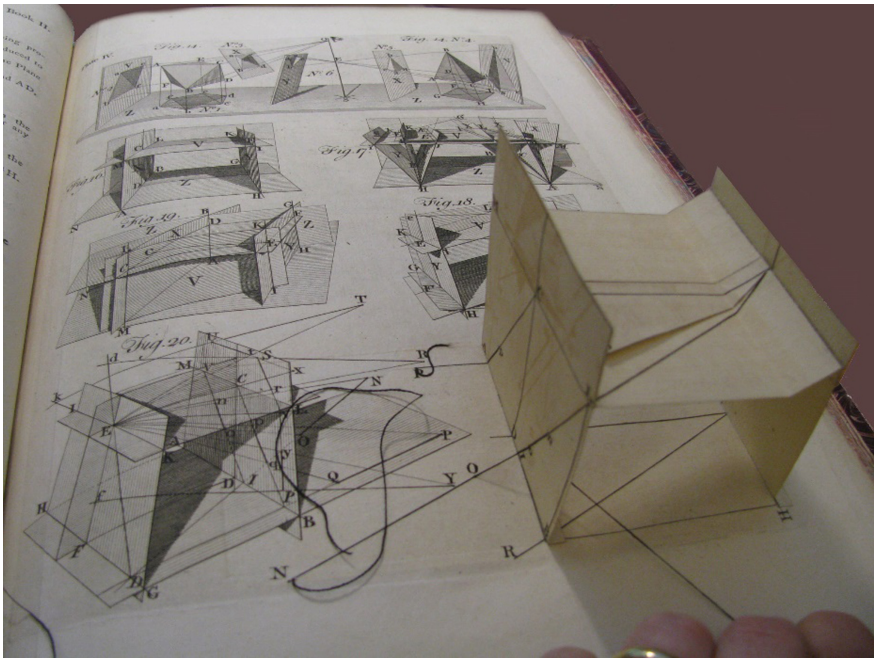

Fig. 6. Malton 1775. L II, after page 70: plate IV, Fig. 21 with threads (photo by the author)

Malton underlined that generally Original and Vanishing Planes are horizontal and Picture and Directing Planes are vertical, but they can assume different positions. Picture 21, with moveable planes, let the reader see this change of mutual position of the planes involved in perspective construction, without losing the parallelism. Pop up elements can be moved with the help of strings. Malton use the same picture to define other elements, like reference lines and points, demonstrating the extreme flexibility of using such kind of models in teaching perspective.

Figure 15 in plate V (book II, Section IV: *Theory of Rectilinear Perspective*) is used to prove the theory of vanishing points but also to show the change of perspective with rotation of the picture plane (Fig. 7).

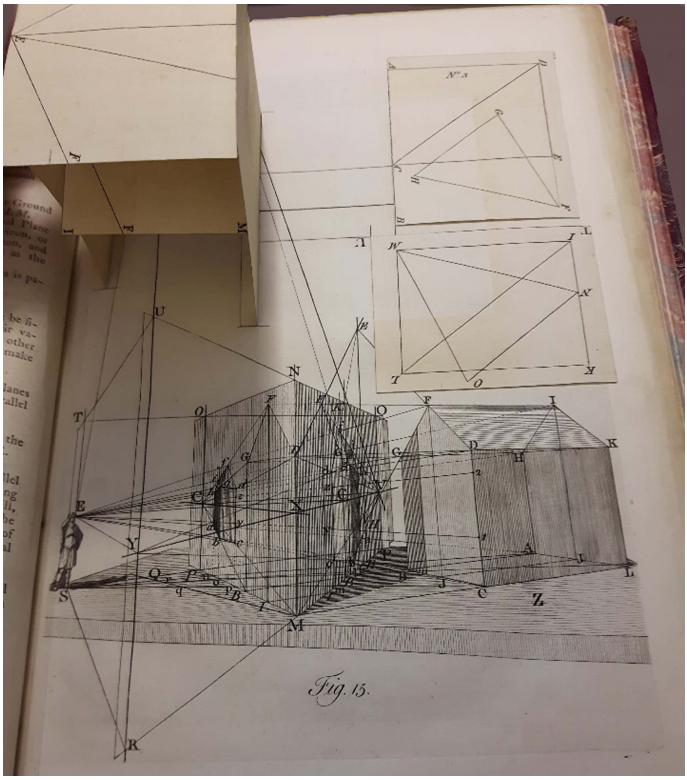

Fig. 7. Malton 1775. Plate V, Fig. 15. Above is a partly developed pop-up (photo by the author)

We can also find Fig. 37 in the plate VIII (Book III) with moveable elements used to explain "*How to prepare the picture for practice*" that also show the rabattement of the Vanishing Plane moving the Eye in the picture plane (Fig. 8), as we have seen in Brook Taylor's treatise and in Kirby's moveable image.

Other interesting pop ups contained in Book III are Figs. 136–140 in plate XXXV (about "Inclined planes"), and Fig. 12 in plate XXXIX in Book IV on Shadows.

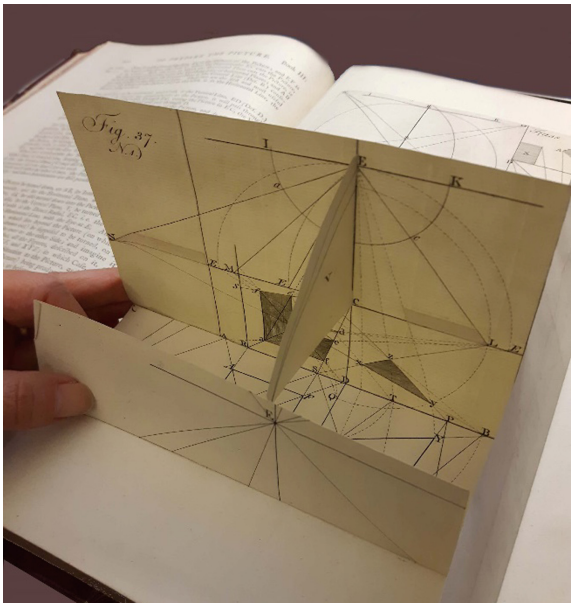

Fig. 8. Malton 1775: Plate VIII, Fig. 37 (photo by the author)

3 Some Possible Origins

These textbooks seem to reveal the presence of moveable elements as an innovative feature found in English treatises on perspective. However, there are some moveable elements in Abraham Bosse's treatise (1611–1676) (Bosse 1665), in which the author brings together the topics of his geometry and perspective courses held at the Royal Academy of Paris and inspired, as is known, by Girard Desargues's theories (1591–1661). In some copies, the book contains a moveable image with pop-up elements of the Picture Plan, with a perspective drawing of three elements (two geometric solids and a male figure) and the viewer. Along with a theoretical explanation, instructions are given to create a model for perspective construction, lifting both elements up and placing them perpendicular to the sheet (Bosse 1665, p. 100) (Fig. 9). The structure looks like the one created by Moxon five years later, but the Picture Plane already had a perspective drawing and only partially revealed its three-dimensionality, which was better illustrated by the transparency of the same element in Moxon.

It can't be ruled out that Moxon knew about Bosse's publication with moveable elements, but perhaps a more direct source of inspiration can be found in English publications. In the first English edition of Euclid's Geometry (Billingsley and Dee 1570) there are several drawings that include elements to create three-dimensional models. For example, book XI, fol. 339 shows the construction of a solid corner starting from a given line in a given point (problem 4, proposition 26). Next to the drawing of two triangles, the text describes the application of additional triangles that use the sides of the original triangles as their bases and a point A in the space as the third vertex (Fig. 10).

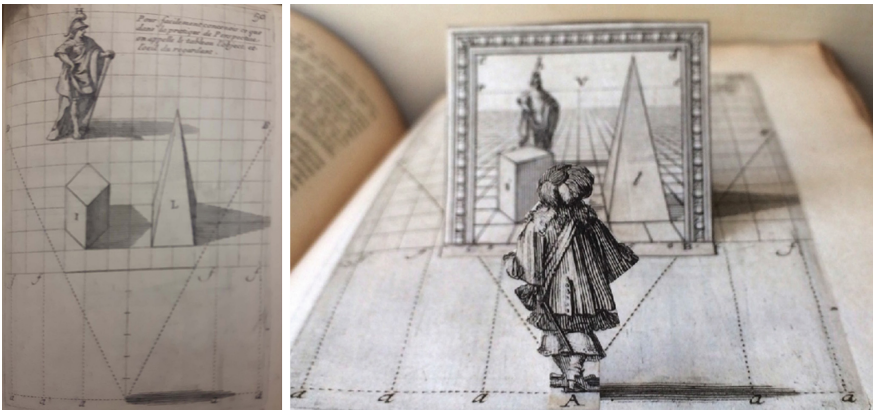

Fig. 9. Bosse 1665, plate 50 (photo by the author). Left: a scanned page of an edition without pop-up elements. Right: the same plate with raised pop-up elements (http://www.bldgblog.com/2015/07/pop-up/)

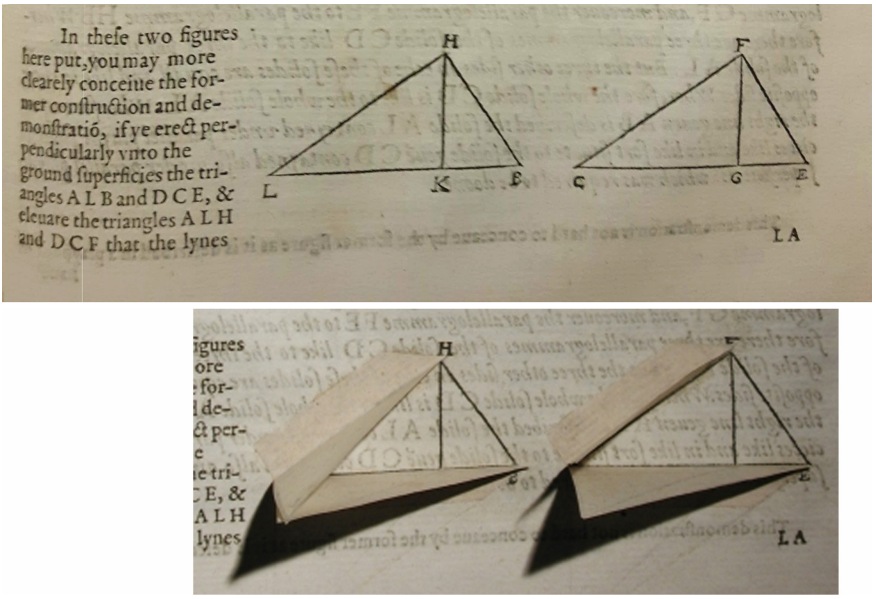

Fig. 10. Euclid, Day 1570: book XI, fol 339 (photo by the author and https://www.rcplondon.ac.uk/news/goodbye-john-dee)

The Preface was written by John Dee (1527–1609), a mathematician, astronomer and alchemist at the court of Elizabeth I, and probably suggests the use of a system of folding paper elements (Lindberg 1979, 71). This book was kept in John Dee's personal library, where there is also a text about cryptography, the 1561 edition of the *Polygraphiae* by Johannes Trithemius (original edition 1518), where we find the use of

volvelles. The copy in Dee's library may have been a source of inspiration for the addition of moveable elements in the geometry book, which does not necessarily reflect only the author's choice, but may come from other persons involved in the publication of the book.

Thomas Malton himself authored the aforementioned treatise on geometry of 1774, again based on the Euclidean book, with several pop-up elements that precede their use in the treatise on perspective, as in Book VII, theorem II (pp. 340, figure p. 341) about the perpendicularity between a straight line and a plane. The demonstration is found in the possibility of moving the pop-up element, which brings the geometry problem back to its spatial condition (Fig. 11).

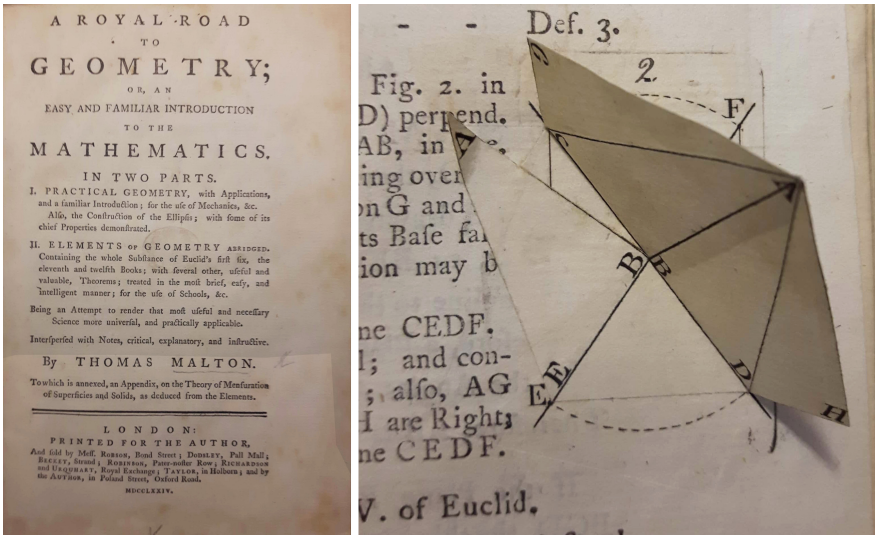

Fig. 11. Malton 1774: frontispiece and figure p. 341 (photos by the author)

4 Conclusion

The identification of moveable elements in some English treatises on perspective has led to deeper exploration of the topic, which has a vast literature in other scientific fields. Research has been conducted to find the origins of this peculiar feature and, even if they are to be considered partial, some possible traces have been identified. This was difficult because, in the past, moveable elements were not necessarily reported in the descriptions of books and studies on the topic are only recent and cannot be considered exhaustive, but the result of accidental discoveries while exploring ancient texts, albeit by groups of collectors and scholars with a strong commitment to the topic. Moreover, all previous research is centered on anatomy and astronomy textbooks.

In the field of perspective, Abraham Bosse's textbook (1665) was reported, as it contains moveable elements that create a three-dimensional model, used to explain the genesis of perspective projection. A similar pop up is found in Moxon's treatise (1670)

but here we can also have a view of the real object through the picture plane materialized only by a frame.

The same stratagems, however, can be found in the first edition of Euclid (1570) who may have been inspired to use moveable elements from cryptographic literature due to a specific interest by one of the book's editors, John Dee, who had a 1561 edition of the *Polygraphiae* by Trithemius. Tradition follows the aforementioned textbooks by Fontana and Alberti, also accompanied by volvelles, which also recall the illustrative and literary combinations obtained, using flaps, by Sayer (1771), almost contemporary of Malton treatises, and, more recently, by Queneau (1961).

Malton himself published a geometry textbook with pop-up elements, and the moveable elements in his book on perspective do not seem to derive from those previously adopted by Moxon or Kirby– in terms of number and complexity –but they tend to play more on transformations of elements, such as those found in geometry textbook.

There was also the English edition of the Cortés navigational textbook (Eden 1630), which includes volvelles and models and precedes Moxon's book by a few decades, in a line that follows the locations of the editions rather than the disciplines, as we tried to summarize in Table 1.

Table 1. Chronology and typology of some moveable elements

Year		Author	Title	Subject	Moveable elements
1565 (1518)	D	Tritemius	*Polygraphiae*	Cryptography	Volvelle
1570	GB	Billingsley and Dee	*The Elements ... Euclid*	Geometry	Pop up
1630	GB	Eden (Cortés)	*The art of navigation ...*	Navigation	Pop up/Volvelle
1662	F	Descartes	*De Homine*	Anatomy	Pop up
1665	F	Bosse	*Traité des pratiques ...*	Perspective	Pop up
1670	GB	Moxon	*Practical Perspective ...*	Perspective	Pop up
1751	GB	Kirby	*Dr Brook Taylor's Method ...*	Perspective	Pop up
1774		Malton	*A Royal Road to...*	Geometry	Pop up
1775	GB	Malton	*A compleat treatise...*	Perspective	Pop Up

While the techniques used have been updated over time, moveable elements and their ability to transform two-dimensional drawings into three-dimensional models and vice versa have proved to be effective tools for learning Euclidean or projective geometry, in the past as in the present.

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