Chapter 1
Cosmology

And that inverted Bowl we call the Sky,
Whereunder crawling coop't we live and die,
Lift not thy hands to It for help—for It,
Rolls impotently on as Thou or I

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In this Chapter we present an overview of cosmology, addressing its most important aspects and presenting some observational experiments and open problems.

1.1 The Expanding Universe and its Content

The starting point of our study of cosmology is the extraordinary evidence that we live in an expanding universe. This was a landmark discovery made in the XX century, usually attributed to Edwin Hubble (1929), but certainly resulting from the joint efforts of astronomers such as Vesto Slipher (1917) and cosmologists such as George Lemaître (1927). We do not enter here the debate about who is deserving more credit for the discovery of the expansion of the universe. The interested reader might want to read e.g. Way and Nussbaumer (2011); van den Bergh (2011).

Hubble discovered that the farther a galaxy is the faster it recedes from us. See Fig. 1.1. This is the famous **Hubble's law**:

$$v = H_0 r \;, \tag{1.1}$$

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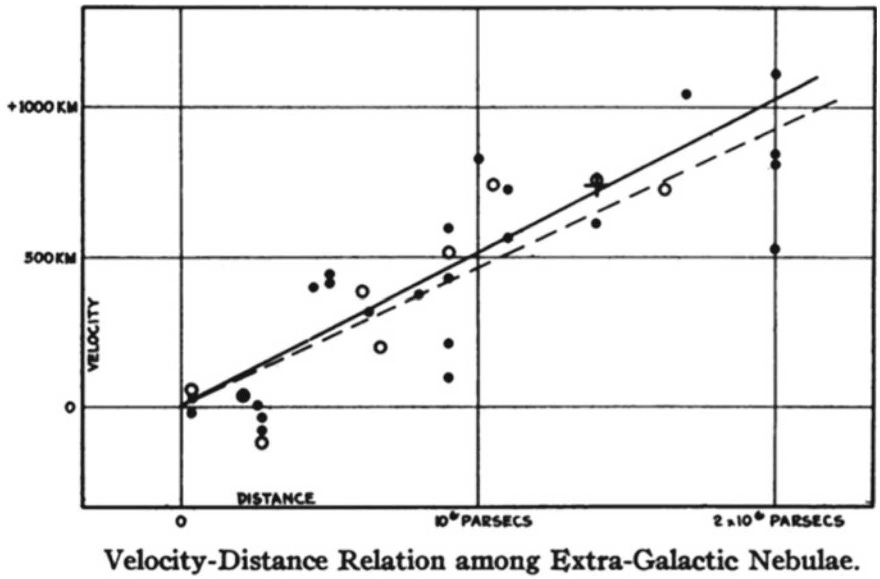

Fig. 1.1 Figure 1 of Hubble's original paper (Hubble 1929)

where v is the recessional velocity, r is the distance and H_0 (usually pronounced "H-naught") is a constant named after Hubble. The value of H_0 determined by Hubble himself was:

$$H_0 = 500\,\text{km s}^{-1}\,\text{Mpc}^{-1}\ , \tag{1.2}$$

with huge error, as can be understood from Fig. 1.1 by observing how much the data points are scattered. A more precise estimate was made by Sandage (1958) in 1958:

$$H_0 = 75\,\text{km s}^{-1}\,\text{Mpc}^{-1}\ . \tag{1.3}$$

A recent measurement done by the BOSS collaboration (Grieb et al. 2017) gives

$$H_0 = 67.6^{+0.7}_{-0.6}\,\text{km s}^{-1}\,\text{Mpc}^{-1}\ . \tag{1.4}$$

Roughly speaking, this number means that for each Mpc away a source recedes 67.6 km/s faster. At a certain radius, the receding velocity attains the velocity of light and therefore we are unable to see farther objects. This radius is called **Hubble radius**.

The Hubble constant can be measured also with fair precision by using the time-delay among variable signals coming from lensed distant sources (Bonvin et al. 2017) and via gravitational waves (Abbott et al. 2017b).

1.1.1 *Olbers's Paradox*

The expansion of the universe could have been predicted a century before Hubble by solving Olbers's paradox (Olbers 1826). See e.g. Dennis Sciama's book (Sciama 2012) for a very nice account of the paradox, which is also called **the dark night sky paradox** and goes as follows: if the universe is static, infinitely large and old and with an infinite number of stars distributed uniformly, then the night sky should be bright.

Let us try to understand why. First of all, the stars are distributed uniformly, which means that their number density say n is a constant. Consider a spherical shell of thickness dR and radius R centred on the Earth. The number of stars inside this spherical shell is:

$$dN = 4\pi n R^2 dR \; . \tag{1.5}$$

The total luminosity of this spherical shell is dN multiplied by the luminosity say L of a single star, and we assume L to be the same for all the stars. Only a fraction f of the radiation produced by the star reaches Earth, but this fraction is the same for all the stars (because we assume them to be identical). Therefore, the total luminosity of a spherical shell is $dNfL$. By the inverse-square law, the total flux received on Earth is:

$$dF_{\rm tot} = \frac{dNfL}{4\pi R^2} = nfLdR \; . \tag{1.6}$$

It does not depend on R and thus it diverges when integrated over R from zero to infinity. This means not only that the night sky should not be dark but also infinitely bright!

We can solve the problem of having an infinitely bright night sky by considering the fact that stars are not points and do eclipse each other, so that we do not really see all of them. Suppose that each star shows us a surface dA. Therefore, if a star lies at a distance R, we receive from it the flux $dF = dA\mathcal{L}/(4\pi R^2)$, where $\mathcal{L}$ is the luminosity per unit area. But $dA/R^2 = d\Omega$ is the solid angle spanned by the star in the sky. Therefore:

$$dF = \frac{\mathcal{L}}{4\pi} d\Omega \; . \tag{1.7}$$

Once again, this does not depend on R! When we integrate it over the whole solid angle, we obtain that $F_{\rm tot} = \mathcal{L}$, i.e. the whole sky is as luminous as a star! In other words: it is true that the farther a star is the fainter it appears, but we can pack more of them in the same patch of sky.

In order to solve Olbers's paradox, we can drop one or more of the initial assumptions. For example:

- The universe is not eternal so the light of some stars has not yet arrived to us. This is plausible, but even so we could expect a bright night sky and also to see some new star to pop out from time to time, without being a transient phenomenon such as a supernova explosion. There is no record of this.

- Maybe there is not an infinite number of stars. But we have showed that taking into account their dimension we do not need an infinite number and yet the paradox still exists.
- Are stars distributed not uniformly? Even so, we would expect still a bright night sky, even if not uniformly bright.

At the end, we must do something in order for the light of some stars not to reach us. A possibility is to drop the staticity assumption. The farther a spherical shell is, the faster it recedes from us (this is Hubble's law). In this way, beyond a certain distance (the Hubble radius) light from stars cannot reach us and the paradox is solved.

1.1.2 The Accelerated Expansion of the Universe and Dark Energy

The discovery of the type Ia supernova 1997ff (Williams et al. 1996) marked the beginning of a new era in cosmology and physics. The analysis of the emission of this type of supernovae led to the discovery that our universe is in a state of **accelerated expansion**. See e.g. Perlmutter et al. (1999); Riess et al. (1998).

This is somewhat problematic because gravity as we know it should attract matter, thereby causing the expansion to decelerate. So, what does cause the acceleration in the expansion? A possibility is that there exists a new form of matter, or rather energy, which acts as anti-gravity. This is widely known today as **Dark Energy** (DE) and its nature is still a mystery to us. The most simple and successful candidate for DE is the cosmological constant Λ.

1.1.3 Dark Matter

DE is not the only dark part of our universe. Many observations of different nature and from different sources at different distance scales point out the existence of another dark component, called Dark Matter (DM). In particular, these observations are:

- **The dynamics of galaxies in clusters**. The pioneering applications of the virial theorem to the Coma cluster by Zwicky (1933) resulted in a virial mass 500 times the observed one (which can be estimated by the light emission).

- **Rotation curves of spiral galaxies**. This is the famous problem of the flattish velocity curves of stars in the outer parts of spiral galaxies. See e.g. Sofue et al. (1999); Sofue and Rubin (2001).

The surprising fact of these flattish curves is that there is no visible matter to justify them and one would then expect a Keplerian fall $V \sim 1/\sqrt{R}$, where R is the distance from the galactic centre. In order to derive the Keplerian fall, simply assume a circular orbit and use Newtonian gravity (which seems to be fine for galaxies). Then, the centrifugal force is canceled by the gravitational attraction of the galaxy as follows:

$$\frac{V^2}{R} = \frac{GM(R)}{R^2} , \tag{1.8}$$

where we assume also a spherical distribution of matter in the galaxy and therefore use Gauss' theorem. Inside the bulge of the galaxy, the visible mass goes as $M \propto R^3$, and therefore $V \propto R$, which represents the initial part of the velocity curve. However, outside the bulge of the galaxy, the mass becomes a constant, thus the Keplerian fall $V \propto 1/\sqrt{R}$ follows.

- **The brightness in X-rays of galaxy clusters**. This depends on the gravitational potential well of the cluster, which is deduced to be much deeper than the one that would be generated by visible matter only. See e.g. Weinberg (2008).

- **The formation of structures in the universe**. In other words, the fact that the density contrast of the usual matter, called baryonic in cosmology, δ_b is today highly non-linear, i.e. $\delta_b \gg 1$. As we shall see in Chap. 9, relativistic cosmology predicts that δ_b grows by a factor of 10^3 between recombination and today and observations of CMB show that the value of δ_b at recombination is $\delta_b \sim 10^{-5}$. This means that today $\delta_b \sim 10^{-2}$ which is in stark contrast with the huge number of structures that we observe in our universe. So, there must be something else which catalyses structure formation.

- **The structure of the Cosmic Microwave Background peaks**. The temperature-temperature correlation spectrum in the CMB sky is characterised by the so-called **acoustic peaks**. The absence of DM would not allow to reproduce the structure shown in Fig. 1.2. We shall study this structure in detail in Chap. 10.

- **Weak Lensing**. The bending of light is a method for measuring the mass of the lens, and it is a classical test of General Relativity (GR) (Weinberg 1972). When the background source is distorted by the foreground lens, one has the so-called weak lensing. Analysing the emission of the distorted source allows to map the gravitational potential of the lens and therefore its matter distribution. See e.g. Dodelson (2017) for a recent textbook reference on gravitational lensing. Weak gravitational lensing is a powerful tool for the study of the geometry of the universe and its observation is one of the primary targets of forthcoming surveys such as the European Space Agency (ESA) satellite *Euclid* and the *Large Synoptic Survey Telescope* (LSST).

A remarkable combination of X-ray and weak lensing observational techniques made the **Bullet Cluster** famous (Clowe et al. 2006). Indeed X-ray maps show the result of a merging between the hot gases of two galaxy clusters which gravitational lensing maps reveal to be lagging behind their respective centres of mass. Therefore, most parts of the clusters simply went through one another, leaving behind a smaller fraction of hot gas. This is considered a direct empirical proof of the existence of DM forming a massive halo and a gravitational potential well in which gas and galaxies lie.

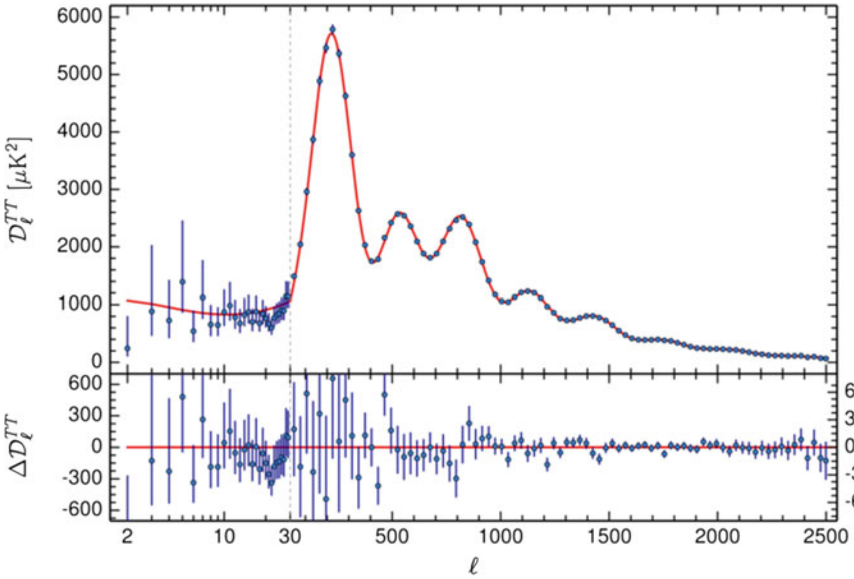

Fig. 1.2 CMB TT spectrum. Figure taken from Ade et al. (2016a). The red solid line is the best fit ΛCDM model

Popular candidates to the role of DM are particles beyond the standard model (Silk et al. 2010). Among these, the most famous are the *Sterile Neutrino* (Dodelson and Widrow 1994), the *Axion*, which is related to the process of violation of CP symmetry (Peccei and Quinn 1977), and *Weakly Interacting Massive Particles* (WIMP) which count among them the lightest supersymmetric neutral stable particle, the*Neutralino*. See Bertone and Hooper (2016) for a historical account of DM and (Profumo 2017) for a textbook on particle DM.

The observational evidences of DM that we have seen earlier do not only point to the existence of DM but also on the necessity of it being **cold**, i.e. with negligible pressure or, equivalently, with a small velocity (much less than that of light) of its particles (admitting that DM is made of particles, which is the common understanding to which we adhere in these notes). Hence **Cold Dark Matter** (CDM) shall be our DM paradigm.

As we shall see in Chap. 3 if DM was in thermal equilibrium with the rest of the known particles in the primordial plasma, i.e. it was **thermally produced**, then its being cold amounts to say that its particles have a sufficiently large mass, e.g. about 100 GeV for WIMP's. Decreasing the mass, we have DM candidates characterised by increasing velocity dispersions and thus with different impacts on the process of structure formation. Typically one refers to **Warm Dark Matter** (WDM) as a thermally produced DM with mass of the order of some keV and **Hot Dark Matter** (HDM) as thermally produced particles with small masses, e.g. of the order of the eV, or even massless. In fact neutrinos can be considered as a HDM candidate. Note that the axion has a mass of about 10^{-5} eV but nonetheless is CDM because it was not thermally produced, i.e. it never was in thermal equilibrium with the primordial plasma.

The combined observational successes of Λ and CDM form the so-called Λ**CDM** model, which is the standard model of cosmology.

1.2 Cosmological Observations

We dedicate this section to the most important cosmological observations which are ongoing or ended recently, or are planned.

1.2.1 The Cosmic Microwave Background

The cosmic microwave background (CMB) radiation provides a window onto the early universe, revealing its composition and structure. It is a relic, thermal radiation from a hot dense phase in the early evolution of our universe which has now been cooled by the cosmic expansion to just three degrees above absolute zero. Its existence had been predicted in the 1940s by Alpher and Gamow (1948) and its discovery by Penzias and Wilson at Bell Labs in New Jersey, announced in 1965 (Penzias and Wilson 1965) was convincing evidence for most astronomers that the cosmos we see today emerged from a **Hot Big Bang** more than 10 billion years ago.

Since its discovery, many experiments have been performed to observe the CMB radiation at different frequencies, directions and polarisations, mostly with ground- and balloon-based detectors. These have established the remarkable uniformity of the CMB radiation, at a temperature of 2.7 Kelvin in all directions, with a small ± 3.3 mK dipole due to the Doppler shift from our local motion (at 1 million kilometres per hour) with respect to this cosmic background.

However, the study of the CMB has been transformed over the last twenty years by three pivotal satellite experiments. The first of these was the *Cosmic Background Explorer* (CoBE, https://lambda.gsfc.nasa.gov/product/cobe/), launched by NASA in 1990. In 1992 CoBE reported the detection of statistically significant temperature anisotropies in the CMB, at the level of $\pm 30\,\mu$K on 10 degree scales (Smoot et al. 1992) and it confirmed the black body spectrum with an astonishing precision, with deviations less than 50 parts per million (Smoot et al. 1992). CoBE was succeeded by the *Wilkinson Microwave Anisotropy Probe* (WMAP, https://map.gsfc.nasa.gov/) satellite, launched by NASA in 2001, which produced full sky maps in five frequencies (from 23 to 94 GHz) mapping the temperature anisotropies to sub-degree scales and determining the CMB polarisation on large angular scales for the first time.

The *Planck* satellite (http://sci.esa.int/planck/), launched by ESA in 2009, sets the current state of the art with nine separate frequency channels, measuring temperature fluctuations to a millionth of a degree at an angular resolution down to 5 arc-minutes.

Planck's mission ended in 2013 and the full-mission data were released in 2015 in Adam et al. (2016) and in many companion papers. A fourth generation of full-sky, microwave-band satellite recently proposed to ESA within Cosmic Vision 2015-2025 is the *Cosmic Origins Explorer* (COrE, http://www.core-mission.org/) (Bouchet et al. 2011).

At the moment, a great effort is being devoted to the detection of the B-mode of CMB polarization because it is the one related to the primordial gravitational waves background, as we shall see in Chap. 10. Located near the South Pole, BICEP3 (https://www.cfa.harvard.edu/CMB/bicep3/) and the *Keck Array* are telescopes devoted to this purpose.

Among the non-satellite CMB experiments we must mention the *Balloon Observations Of Millimetric Extragalactic Radiation ANd Geophysics* (BOOMERanG) which was a balloon-based mission which flew in 1998 and in 2003 and measured CMB anisotropies with great precision (higher than CoBE). From these data the Boomerang collaboration first determined that the universe is spatially flat (de Bernardis et al. 2000).

1.2.2 Redshift Surveys

Redshift surveys are observations of certain patches of sky at certain wavelengths with the aim of determining mainly the angular positions (declination and right ascension) redshifts and spectra of galaxies.

The *Sloan Digital Sky Survey* (SDSS, http://www.sdss.org/) is a massive spectroscopic redshift survey which is ongoing since the year 2000 and it is now in its stage IV with 14 data releases available. It is ground-based and uses a telescope located in New Mexico (USA). The SDSS-IV is formed by three sub-experiment:

- *The Extended Baryon Oscillation Spectroscopic Survey* (eBOSS), focusing on redshifts $0.6 < z < 2.5$ and on the Baryon Acoustic Oscillations (BAO) phenomenon[1];
- The *Apache Point Observatory Galaxy Evolution Experiment* (APOGEE-2) is dedicated to the study of our Milky Way;
- The *Mapping Nearby Galaxies at Apache Point Observatory* (MaNGA) study instead nearby galaxies by measuring their spectrum along their extension and not only at the centre.

The V generation of the SDSS will start in 2020, consisting of three surveys: the *Milky Way Mapper*, the *Black Hole Mapper* and the *Local Volume Mapper*. See Kollmeier et al. (2017).

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[1] We shall not address BAO extensively in these notes, but only mention them in Chap. 10. Together with weak lensing, BAO are another powerful observable upon which present and future missions are planned.

The *Dark Energy Survey* (DES, https://www.darkenergysurvey.org/) measures redshifts photometrically using a telescope situated in Chile and looking for Type Ia supernovae, BAO and weak lensing signals.

Planned surveys are the already mentioned satellite *Euclid* (http://sci.esa.int/euclid/), whose launch is due possibly in 2021 and the telescope LSST, which is being built in Chile and whose first light is due in 2019. We also cite the NASA satellite *Wide Field Infrared Survey Telescope* (WFIRST, https://www.nasa.gov/wfirst) and the *Javalambre Physics of the accelerating universe Astronomical Survey* (J-PAS, http://j-pas.org/). The main cosmological goals of these experiments relay on the detection of weak lensing, BAO and type Ia supernovae signals with high precision.

1.2.3 Gravitational Waves Observatories

The recent direct detection of gravitational waves (GW) by the *LIGO-Virgo collaboration* (https://www.ligo.org/) (Abbott et al. 2016a, b) has opened a new observational window on the universe. In particular, GW are relevant in cosmology because they could be a relic from inflation containing invaluable informations on the very early universe. As already mentioned, they are being searched via the detection of the B-mode polarisation of the CMB.[2]

There are now three functioning ground-based GW observatories: *LIGO* (Hanford and Livingstone, USA) and *Virgo* (near Pisa, Italy). *KAGRA*, in Japan, is under construction and another one in India, *INDIGO*, is planned. The space-based *LISA* GW observer is still in a preliminary phase (*LISA* pathfinder).

1.2.4 Neutrino Observation

Neutrinos are relevant in cosmology, as we shall see throughout these notes, because they should form a cosmological background as CMB photons do. The great problem is that it is incredibly difficult to detect them and even more if they have low energy, as we expect to be the case for neutrinos in the cosmological background.

The most important neutrino observatory is *IceCube* (http://icecube.wisc.edu/), operating since 2005 (its construction was completed in 2010) and located near the South Pole. It detects neutrinos indirectly, via their emission of Cherenkov light.

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[2] The events detected by the LIGO-Virgo collaboration originated from merging of black holes or neutron stars. Thus are not part of the primordial GW background.

1.2.5 Dark Matter Searches

The search for DM particles counts on many observatories and the *Large Hadron Collider* (LHC). See Gaskins (2016); Liu et al. (2017) for the status of indirect and direct DM searches which, unfortunately, have not been successful until now.

1.3 Redshift

Redshift is a fundamental observable of cosmology. Its definition is the following:

$$\boxed{z = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} - 1} \tag{1.9}$$

It is always positive, i.e. observed radiation is redder than the emitted one, because the universe is in expansion. For the closest sources, such as Andromeda, it is negative, i.e. the observed radiation is bluer than the emitted one, because the Hubble flow is overcome by the peculiar motion due to local gravitational effects.

For the moment we can think of the redshift as a Doppler effect due to the relative motion of the sources. In Chap. 2 we will relate it to spacetime geometry with GR.

Redshift is measured in two ways: spectroscopically or photometrically. For the former one needs to do spectroscopy, i.e. detecting known emission or absorption lines from a source and comparing their wavelengths with the ones measured in a laboratory on Earth. Hence one uses Eq. (1.9) and thus calculate z.

Photometric redshifts are calculated by assuming certain spectral features for the sources and measuring their relative brightness in certain wavebands, using filters.

A simple example is the following. Sun's spectrum is almost a blackbody one with temperature of about 6000 K and then, by using Wien's displacement law, it has a peak emission at a wavelength of 500 nm. Therefore, if a star similar to the Sun had a peak emission of say 600 nm, then using Eq. (1.9) one would calculate $z = 0.2$.

The reason for using photometry instead of spectroscopy is that it is less time-consuming and allows to obtain redshifts of very far sources, for which it is difficult to do spectroscopy. On the other hand, photometric redshifts are less precise.

1.4 Open Problems in Cosmology

The fundamental issue in cosmology is to understand what are DM and DE. The effort of answering this question makes cosmology, particle physics and quantum field theory (QFT) to merge. The ways adopted in order to tackle these problems are essentially the search for particles beyond the standard model and the investigation of new theories of gravity, which in most of the cases are extensions of GR.

1.4.1 Cosmological Constant and Dark Energy

Pure geometrical Λ and vacuum energy have the same dynamical behaviour in GR. Estimating the latter via QFT calculations and comparing the result with the observed value leads to the famous **fine-tuning** problem of the cosmological constant. See e.g. Weinberg (1989). This roughly goes as follows: the observed value of ρ_Λ is about 10^{-47} GeV^4 (Ade et al. 2016a). The natural scale for the vacuum energy density is the Planck scale, i.e. 10^{76} GeV^4. There are 123 orders of magnitude of difference! Even postulating a false vacuum state after the electro-weak phase transition at 10^8 GeV^4, the difference is 55 orders of magnitude. See Martin (2012) for a comprehensive account of Λ and the issues related to it.

Another problem with Λ is the so-called **cosmic coincidence** (Zlatev et al. 1999). This problem stems from the fact that the density of matter decreases with the inverse of the cube scale factor, whereas the energy density of the cosmological constant is, as its name indicates, constant. However, these two densities are approximately equal at the present time. This coincidence becomes all the more intriguing when we consider that if the cosmological constant had dominated the energy content of the universe earlier, galaxies would not have had time to form; on the other hand, had the cosmological constant dominated later, then the universe would still be in a decelerated phase of expansion or younger than some of its oldest structures, such as clusters of stars (Velten et al. 2014).

The cosmic coincidence problem can also be seen as a fine-tuning problem in the initial conditions of our universe. Indeed, consider the ratio $\rho_\Lambda/\rho_\mathrm{m}$, of the cosmological constant to the matter content. This ratio goes as a^3. Suppose that we could extrapolate our classical theory (GR) up to the Planck scale, for which $a \approx 10^{-32}$. Then, at the Planck scale we have $\rho_\Lambda/\rho_\mathrm{m} \approx 10^{-96}$. This means that, at trans-Planckian energies, possibly in the quantum universe, there must be a mechanism which establishes the ratio $\rho_\Lambda/\rho_\mathrm{m}$ with a precision of 96 significant digits! Not a digit can be missed, otherwise we would have today 10 times more cosmological constant than matter, or vice-versa, thereby being in strong disagreement with observation.

So, we find ourselves in a situation of *impasse*. On one hand, Λ is the simplest and most successful DE candidate. On the other hand it suffers from the above-mentioned issues. What do we do? Much of today research in cosmology addresses this question. Answers are looked for mostly via investigation of new theories of gravity, extensions or modifications of GR, of which DE would be a manifestation. There are so many papers addressing extended theories of gravity that it is quite difficult to choose representatives. Probably the best option is to start with a textbook, e.g. Amendola and Tsujikawa (2010).

A different approach is to accept that Λ has the value it has by chance, and it turns out to be just the right value for structures to form and for us to be here doing cosmology. This also known as **Anthropic Principle** and exists in many forms, some stronger than others. It is also possible that ours is one universe out of an infinite number of realisations, called **Multiverse**, with different values of the fundamental constants. Life as we know it then develops only in those universes where the conditions are favourable. Again, it is difficult to cite papers on these topics (which are more about metaphysics rather than physics, since there is no possibility of performing experimental tests) but a nice reading is e.g. Weinberg (1992).

1.4.2 Dark Matter and Small-Scale Anomalies

On sub-galactic scales, of about 1 kpc, the CDM paradigm displays some difficulties (Warren et al. 2006). These are called **CDM small-scales anomalies**. See e.g. Bullock and Boylan-Kolchin (2017) for a recent account. They are essentially three and stem from the results of numerical simulations of the formation of structures:

1. The *Core/Cusp* problem (Moore 1994). The CDM distribution in the centre of the halo has a cusp profile, whereas observation suggests a core one;
2. The *Missing satellites problem* (Klypin et al. 1999). Numerical simulations predict a large number of satellite structures, which are not observed;
3. The *Too big to fail* problem (Boylan-Kolchin et al. 2011). The sub-structures predicted by the simulations are too big not to be seen.

Possible solutions to these small-scale anomalies are the following:

Baryon feedback. The cross section for DM particles and the standard model particles interaction must be very small, i.e. $\sim 10^{-39}\,\mathrm{cm}^2$, but in environments of high concentration, such as in the centre of galaxies, such interactions may become important and may provide an explanation for the anomalies. The problem is that the models of baryon feedback are difficult to be simulated and they seem not to be enough to resolve the anomalies (Kirby et al. 2014).

Warm dark matter. As anticipated, WDM are particles with mass around the keV which decouple from the primordial plasma when relativistic. They are subject to free streaming that greatly cut the power of fluctuations on small scales, thereby possibly solving the anomalies of the CDM. The problem with WDM is that different observations indicate mass limits which are inconsistent among them. In particular:

- To solve the *Core/Cusp* problem is necessary a mass of $\sim 0.1\,\mathrm{keV}$ (Macciò et al. 2012).
- To solve the *Too big to fail* problem is necessary a mass of $\sim 2\,\mathrm{keV}$ (Lovell et al. 2012).
- Constraints from the Lyman-α observation require $m_{\mathrm{WDM}} > 3.3\,\mathrm{keV}$ (Viel et al. 2013).

These observational tensions disfavour WDM. In addition, it was shown in Schneider et al. (2014) that for $m_{\mathrm{WDM}} > 3.3$ keV WDM does not provide a real advantage over CDM.

Interacting Dark Matter. CDM anomalies could perhaps be understood by admitting the existence of self-interactions between dark matter particles (Spergel and

Steinhardt 2000). It has been shown that there are indications that interaction models can alleviate the *Core/Cusp* and the *Too big to fail* problems (Vogelsberger et al. 2014). Recently, Macciò et al. (2015) pointed out that a certain type of interaction and mixing between CDM and WDM particles is very satisfactory from the point of view of the resolution of the anomalies.

1.4.3 Other Problems

Understanding the nature of DE and DM is the main open question of today cosmology but here follows a small list of other open problems:

- The problem of the initial singularity, the so-called **Big Bang**. This issue is related to a quantum formulation of gravity.
- There exists a couple of more technical, but nevertheless very important, issues which are called **tensions**. These happen when observations of different phenomena provide constraints on some parameters which are different up to 68% or 95% confidence level. There is now tension between the determination of H_0 via low-redshift probes and high-redshift ones (i.e. CMB). See e.g. Marra et al. (2013); Verde et al. (2013). Moreover, there is also a tension on the determination of σ_8 (Battye et al. 2015), recently corroborated by the analysis of the first year of the DES survey data collection (Abbott et al. 2017d).
- Testing the cosmological principle and the copernican principle. See e.g. Valkenburg et al. (2014).
- The CMB anomalies (Schwarz et al. 2016). These are unexpected (in the sense of statistically relevant) features of the CMB sky.
- The Lithium problem (Coc 2016). The predicted Lithium abundance is much larger than the observed one.

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