Chapter 13
Radial Inertia Effect on Dynamic Compressive Response of Polymeric Foam Materials

Bo Song, Brett Sanborn, and Wei-Yang Lu

Abstract Polymeric foams have been extensively used in shock isolation applications because of their superior shock or impact energy absorption capability. In order to meet the shock isolation requirements, the polymeric foams need to be experimentally characterized and numerically modeled in terms of material response under shock/impact loading and then evaluated with experimental, analytical, and/or numerical efforts. Measurement of the dynamic compressive stress-strain response of polymeric foams has become fundamental to the shock isolation performance. However, radial inertia has become a severe issue when characterizing soft materials. It is even much more complicated and difficult to address the radial inertia effect in soft polymeric foams. In this study, we developed an analytical method to calculate the additional stress induced by radial inertia in a polymeric foam specimen. The effect of changing profile of Poisson's ratio during deformation on radial inertia was investigated. The analytical results were also compared with experimental results obtained from Kolsky compression bar tests on a silicone foam.

Keywords Poisson's ratio · Radial inertia · Polymeric foam · Dynamic compression · Kolsky compression bar

13.1 Introduction

Polymeric foams have been extensively used in shock isolation applications because of their superior shock or impact energy absorption capability. Experimentally, it is challenging when characterizing very soft polymeric foams with a Kolsky compression bar because radial-inertia-induced stress can overshadow the intrinsic material response. Currently existing analyses on radial inertia effect are based on the assumption of constant Poisson's ratio [1–5], which is not applicable to foam materials. Foam materials usually possess very small Poisson's ratio before densification and can approach a nearly rubbery state after densification. This drastic change in Poisson's ratio may result in a sudden radial confinement in the specimen during densification process. Therefore, it is desirable to understand the radial inertia effect of polymeric foams that are subjected to large deformation (passing densification) at high strain rates.

In this paper, we develop an analytical method to calculate the additional stress induced by radial inertia in a polymeric foam specimen. The effect of changing profile of Poisson's ratio during deformation on radial inertia is investigated. The analytical results are also compared with experimental results obtained from Kolsky compression bar tests on a silicone foam.

13.2 Radial Inertia in a Compressible Cylindrical Solid

In this study, a cylindrical specimen configuration, shown in Fig. 13.1, with initial dimensions of radius, a_0, and length, l_0, for radial inertia analysis. An axial compression at velocity $V_x = V_x(t)$ generates a radial expansion with the velocity, $V_r = V_r(r, t)$. From mass and momentum conservations, the additional axial stress induced by radial inertia is calculated as

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J. Kimberley et al. (eds.), *Dynamic Behavior of Materials, Volume 1*, Conference Proceedings of the Society for Experimental Mechanics Series, https://doi.org/10.1007/978-3-319-95089-1_13

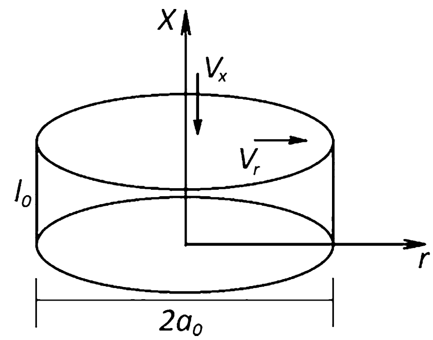

Fig. 13.1 Cylindrical specimen configuration

$$\sigma_r\left(r,t\right)=\rho(t)\cdot\frac{a^2(t)-r^2(t)}{2\left(1-e_x(t)\right)}\cdot\left[\nu(t)\ddot{e}_x(t)+\nu(t)\cdot(\nu(t)+1)\frac{\dot{e}_x^2(t)}{1-e_x(t)}+\frac{d\nu\left(e_x\right)}{de_x}\dot{e}_x^2(t)\right]\tag{13.1}$$

where ρ and ν are instantaneous density and Poisson's ratio of the specimen; e_x is engineering axial strain subjected to the specimen. Equation (13.1) indicates that the radial inertia effect includes three parts induced by (1) strain acceleration (the first term in the bracket in Eq. (13.1)); (2) large strain and high strain rate (the second term in the bracket in Eq. (13.1)); and (3) change of Poisson's ratio and high strain rate (the third term in the bracket in Eq. (13.1). The first two parts are also dependent on instantaneous Poisson's ratio.

For a hyperelastic foam, i.e. silicone foam, the Poisson's ratio follows a Boltzmann sigmoidal function of engineering axial strain,

$$\nu\left(e_x\right)=\frac{\nu_1-\nu_2}{1+\exp\left(\frac{e_x-e_{x0}}{\delta}\right)}+\nu_2\tag{13.2}$$

where ν_1, ν_2, e_{x0}, and δ are constants. Applying Eq. (13.2) into Eq. (13.1) yields an average radial-inertia-induced stress along axial direction,

$$\overline{\sigma}_r(t)=\frac{\rho_0\cdot a_0^2}{4(1-e_x(t))^2}\cdot\left[\begin{array}{l}\ddot{e}_x(t)\cdot\left(\nu_2+\frac{\nu_1-\nu_2}{1+\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)}\right)\\+\left(\nu_2+\frac{\nu_1-\nu_2}{1+\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\left(1+\nu_2+\frac{\nu_1-\nu_2}{1+\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)}\right)\cdot\frac{\dot{e}_x^2(t)}{1-e_x(t)}\\+\dot{e}_x^2(t)\cdot\frac{\nu_2-\nu_1}{\delta}\cdot\frac{\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)}{\left[1+\exp\left(\frac{e_x(t)-e_{x0}}{\delta}\right)\right]^2}\end{array}\right]\tag{13.3}$$

For a silicone foam specimen with a density of $\rho_0 = 600$ kg/m^3, a porosity of 50%, a diameter of 15 mm ($a_0 = 7.5$ mm), the constants of Poisson's ratio are $\nu_1 = 0.21$, $\nu_2 = 0.43$, $\varepsilon_{x0} = 0.525$, $\delta = 0.01$. The average radial-inertia-induced stress of a silicone foam specimen subjected to a constant strain rate of 4000 s^{-1} with a linear ramping time of 40 µs was calculated with Eq. (13.3) and is plotted in Fig. 13.2.

13.3 Conclusions

A general radial inertia analysis is conducted in this study to better understand the dynamic compressive response of compressible materials, particularly polymeric foam materials with drastic change of Poisson's ratio before and after densification. This analysis is applicable to any compressible or incompressible material, with a constant or varied Poisson's ratio, subjected to high-strain-rate compression. Strain acceleration, large strain, high strain rate, and the change of Poisson's ratio all contribute, but not necessarily equally, to radial inertia, which results in additional axial stress in dynamic specimen stress measurements.

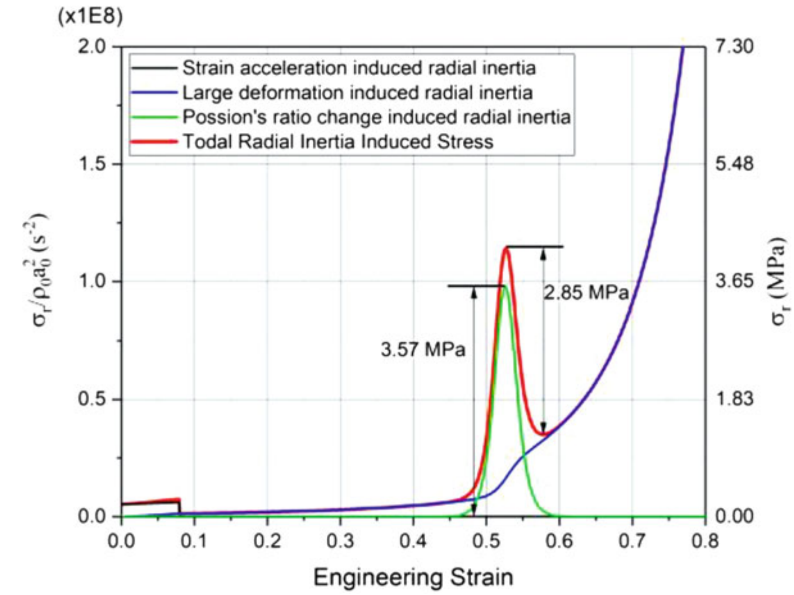

Fig. 13.2 Additional axial stress induced by radial inertia

Acknowledgement Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525. The views expressed in this article do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

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