Chapter 17
System Identification of Structures with Incomplete Modal Information

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Abstract The topic of this paper is system identification of structures conducted with consideration of incomplete modal information. A continuum structure has infinite number of degrees of freedom and an infinite number of modes. Modal estimation method is therefore not employed to identify all modes of systems. In addition, incomplete modal information may be led from insufficient response data in practical vibration measurement. In this paper, by using channel-expansion technique, modal estimation from response data of insufficient channel can be performed through the Ibrahim time-domain method. Applicability and effectiveness of the proposed method is demonstrated by numerical simulation of a chain model. Identification of the mode shapes, however, is still a challenging problem to be resolve in the proposed method from incomplete modal information data.

Keywords Incomplete modal information · Modal estimation · Channel-expansion technique · Ibrahim time-domain method

17.1 Introduction

When performing modal analysis of structures based on the theory of system identification, the finite element model is usually constructed to demonstrate a realistic structural system. Due to the accuracy requirements of a practical complex structure itself, the high order (degree of freedom, DOF) of the finite element model is usually produced accordingly. However, in the practical vibration measurement, due to the economic restrictions and structural unavailability, i.e., measurement location within the structure, and measurement technology is immature, then the sensors placed on overall DOFs of a structure to measure response data is not available. Therefore, the number of degrees of freedom corresponding to a mode shape vector to be identified and the order of finite element models are inconsistent, so that it causes the problems of incomplete number of modes and incomplete degree of freedom of mode shape vector.

During the 1970s, Ibrahim proposed a method developed in the time domain, which is usually referred to as the Ibrahim time domain method (ITD Method) [1], and is applied to problems involving the free-decay response data of structures, and is employed to perform modal identification via an eigenvalue analysis. Based on the Prony's theory, Brown et al. developed the least square complex exponential algorithm (LSCE) [2] using a squared output matrix constructed by multichannel impulse response functions. The pseudo-inverse technique is employed to estimate the coefficients of the Prony's polynomials and then extract the modal parameters of a system through the Prony's technique. In 1982, Vold et al. further proposed poly reference complex exponential method (PRCE) [3] to perform modal identification for the case that one of the modes may not be present in the response data. In 1985, among follow-up developments on minimal realization algorithm and singular value decomposition (SVD) [4], Juang and Pappa [5] proposed the Eigensystem Realization Algorithm (ERA) using the impulse response or the free-vibration response of the system to construct the Hankel matrix, which is an augmented matrix containing Markov parameters, for reducing the effect of noise, and making the parameters estimation more accurate.

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A. Baldi et al. (eds.), *Residual Stress, Thermomechanics & Infrared Imaging, Hybrid Techniques and Inverse Problems, Volume 7*,
Conference Proceedings of the Society for Experimental Mechanics Series, https://doi.org/10.1007/978-3-319-95074-7_17

17.2 ITD Method and Channel Expansion Technique

The ITD method is based on the state equations of a dynamic system. A simplified version of the method is presented here. For simplicity of explanation, assume that the structure is measured in n places (this assumption can be relaxed) at 2n times, where n is the number of degrees of freedom exhibited by the test structure. From the measured free-decay responses at n stations on a structure under test, each with q sampling points, we define a system matrix [**A**], which is an $n \times n$ matrix, such that

$$[\mathbf{A}]\,[\mathbf{X}] = [\mathbf{Y}] \tag{17.1}$$

where [**X**] and [**Y**] are, respectively, $n \times q$ data-expansion matrices of the free decay and its time-shifted response. The number q is generally chosen to be larger than the number of measurement channels n, the system matrix [**A**] can be therefore estimated from [**X**] and [**Y**] through the least-squares method. Once the system matrix [**A**] is obtained via least-squares analysis from measured data, the modal parameters of the structural system can be determined by solving the eigenvalue problem associated with the system matrix [**A**]. Brief speaking, ITD method, which is the same as the most time domain approaches, constructs a matrix from the time response and numerically computes the modal data by solving an eigenvalue problem.

In reality, we do not know in advance how many modes are required to describe the dynamic behavior of the observed structural system. The number of (real) modes m involved in the response determines the number of measurement channels, which is chosen to be at least twice of the number of modes of interest to appropriately identify the $2m$ complex modes. If the number of measurement channels does not actually reach $2m$, we may employ the technique of channel expansion [3] with sampling time shifted to reach the total available number of measurement channels. It should be noted that, however, the identified mode shapes are composed of the components corresponding only to those physically measured response channels. In addition, due to the fact that the results of modal identification may be poor from the noise effect, through the channel-expansion technique in ITD method, which uses time-delayed sampling points from the original response to increase the total numbers of sampling points and measurement channels, we can therefore reduce the effect of noise to improve the accuracy of modal estimation based on the property of consistency in the theory of system identification.

17.3 Numerical Simulations

Consider a 6-dof system of the chain model with viscous damping, whose mass matrix $\boldsymbol{M}$, stiffness matrix $\boldsymbol{K}$, and damping matrix $\boldsymbol{C}$ of the system are given as follows:

$$\boldsymbol{M} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix} N \cdot \mathrm{s}^2/m \quad \boldsymbol{K} = 600 \cdot \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 3 & -2 \\ 0 & 0 & 0 & 0 & -2 & 5 \end{bmatrix} N/m$$

$$\boldsymbol{C} = 0.05\boldsymbol{M} + 0.001\boldsymbol{K}\; N \cdot \mathrm{s}/m$$

Note that the system has proportional damping, because the damping matrix $\boldsymbol{C}$ can be expressed as a linear combination of $\boldsymbol{M}$ and $\boldsymbol{K}$. The simulated impulse function serves as the excitation input acting on the sixth mass point of the system. Assume the system is initially at rest, and the displacement responses of the system can be obtained using Newmark's method. The results of modal identification are summarized in Tables 17.1, 17.2, 17.3 and 17.4, which shows that the errors in both natural frequencies and damping ratios are less than 1%, whether using the complete and incomplete response data from the corresponding overall or part of degree of freedom of this system, respectively. However, it is good agreement between the identified and exact mode shapes only from the complete response data only from the corresponding overall degree of freedom of this system. It may be because that the identified mode shapes are composed of the components corresponding only to those physically measured response channels, the pseudo response data through the channel-expansion technique is not available in identification of mode shape.

Table 17.1 Results of modal identification of a 6-DOF system from complete impulse response of overall degree of freedom

	Natural frequency (rad/s)			Damping ratio (%)		
Mode	Exact	ITD	Error (%)	Exact	ITD	Error (%)
1	5.03	5.03	0.00	1.25	1.25	0.00
2	13.44	13.44	0.00	1.04	1.04	0.00
3	19.79	19.79	0.00	1.24	1.24	0.00
4	26.68	26.67	0.00	1.52	1.52	0.00
5	31.65	31.64	0.00	1.74	1.74	0.00
6	33.72	33.71	0.00	1.83	1.83	0.00

Table 17.2 Results of modal identification of a 6-DOF system from incomplete impulse response of 2nd～6th degree of freedom

	Natural frequency (rad/s)			Damping ratio (%)		
Mode	Exact	ITD	Exact	Exact	ITD	Error (%)
1	5.03	5.033	0.00000	1.25	1.245	0.00006
2	13.44	13.45	0.00001	1.04	1.044	0.00032
3	19.79	19.80	0.00001	1.24	1.242	0.00058
4	26.68	26.68	0.00001	1.52	1.521	0.00095
5	31.65	31.65	0.00018	1.74	1.741	0.00363
6	33.72	33.72	0.00003	1.83	1.834	0.00060

Table 17.3 Results of modal identification of a 6-DOF system from incomplete impulse response of 1st～3rd degree of freedom

	Natural frequency (rad/s)			Damping ratio (%)		
Mode	Exact	ITD	Exact	Exact	ITD	Error (%)
1	5.03	5.03	0.00	1.25	1.25	0.00
2	13.44	13.45	0.00	1.04	1.04	0.00
3	19.79	19.80	0.00	1.24	1.24	0.01
4	26.68	26.68	0.01	1.52	1.52	0.01
5	31.65	31.66	0.01	1.74	1.74	0.02
6	33.72	33.72	0.01	1.83	1.83	0.02

Table 17.4 Results of modal identification of a 6-DOF system from incomplete impulse response of 4th～6th degree of freedom

	Natural frequency (rad/s)			Damping ratio (%)		
Mode	Exact	ITD	Exact	Exact	ITD	ITD
1	5.03	5.03	0.00	1.25	1.25	0.102
2	13.44	13.45	0.00	1.04	1.04	0.002
3	19.79	19.80	0.00	1.24	1.24	0.002
4	26.68	26.68	0.01	1.52	1.52	0.002
5	31.65	31.66	0.01	1.74	1.74	0.003
6	33.72	33.72	0.01	1.83	1.83	0.006

17.4 Conclusions

This paper mainly investigates the influence of incomplete vibration measurement information on modal estimation of structures. Through the channel- expansion technique using the time-delayed sampling response data, the Ibraham time domain method is used for modal estimation from response data with incomplete measurement information of structures. Numerical simulation shows that the proposed method can be employed to effectively identify the natural frequency and damping ratio of a chain model system, but identification of mode shapes is still a challenging problem to be resolved in the proposed method.

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