

A Mathematical Model for Customer Lifetime Value Based Offer Management

Ahmet Şahin^{1(\boxtimes)}, Zehra Can^{1,2(\boxtimes)}, and Erinc Albey^{1(\boxtimes)}

¹ Department of Industrial Engineering, Ozyegin University, Istanbul, Turkey {ahmet.sahin,zehra.can}@ozu.edu.tr, erinc.albey@ozyegin.edu.tr
² Turkcell Technology Research and Development Inc., Istanbul, Turkey

Abstract. Customers with prepaid lines possess higher attrition risk compared to postpaid customers, since prepaid customers do not sign long-term obligatory contracts and may churn anytime. For this reason, mobile operators have to offer engaging benefits to keep prepaid subscribers with the company. Since all such offers incur additional cost, mobile operators face an optimization problem while selecting the most suitable offers for customers at risk. In this study, an offer management framework targeting prepaid customers of a telecommunication company is developed. Proposed framework chooses the most suitable offer for each customer through a mathematical model, which utilizes customer lifetime value and churn risk. Lifetime values are estimated using logistic regression and Pareto/NBD models, and several variants of these models are used to predict churn risks using a large number of customer specific features.

Keywords: Customer lifetime value · Offer management · RFM
Pareto/NBD · Logistic regression · Mobile subscribers · Prepaid Pareto/NBD · Logistic regression · Mobile subscribers · Prepaid subscribers
Telecommunication Telecommunication

1 Introduction

Mobile operators have to adapt to digital evolution of the customer services. Smartphones has changed the market conditions. Customers can experience hundreds of digital services and applications, easily, with their smartphones. Therefore, operators cannot be solely seen as communication providers anymore. They have to provide profitable services to survive in the business and to improve the customer experience to prevent customer churn.

In fact, mobile operators would prefer postpaid subscribers over prepaid subscribers, since it is easier to manage the postpaid subscriber's behavior and the revenue stream generated by a postpaid subscriber is more predictable (and most of the time higher) than that of a prepaid subscriber. But in today's digital marketing ecosystem, telecommunication operators have to act not only a call service provider (CSP), but also act like a full service provider ($\text{OTT} - \text{Over}$ the top provider) who offers many interactive social service products for their customers. Thus, keeping a customer is more valuable than before which means more users for the OTT services. These services are also an important means for revenue generation from the prepaid customers. Thus analyzing the churn behavior of prepaid customers as wells as their lifetime values has higher importance more than ever in the telecommunications sector.

The service and products of the operator should comply with the customer's needs. If this is not the case, it is highly probable that customers may leave and never come back or may never subscribe for a service of the operator. Customer experience management starts with potential customers' perception about the company brand. Starting with first impression, customer experience management aims to understand the desires of customers and to provide them with easy, simple and seamless experience with the offered products and services. The higher the quality in customer experience, the more customer will be attracted to and stay with the company.

Retaining customers has always been a costly challenge. Companies have developed different strategies to fight against customer attrition. Most of these efforts include proposing offers in varying forms, such as discounts, extra benefits etc. Almost in all of these efforts, increasing the amount of customer specific information pays off. In the digitalization era, the amount of customer specific data is more than ever, which yields numerous opportunities for developing granular and customized prediction models. When customer retention is considered, the challenge is turning this massive amount of data (and models built on the data) into business insights and eventually generate actions that decrease the retention cost, while increasing profit.

Churn analysis is one of the most fundamental customer behavior analysis in identifying silent, unhappy customers. The main output of churn analysis is a risk score, which quantifies the likelihood of customer churn. The output of churn analysis is best utilized when it is supported with extra customer specific features such as customer lifetime value (CLV). The relationship between churn and CLV is depicted in Fig. 1 which is based on the idea in the [\[1](#page-18-0)].

Fig. 1. Churn and CLV relationship.

As can be seen in Fig. 1, the ideal position of customers for a company is the bottom right, where customers produce the highest profit with lowest churn risk. It is recommended that companies should give special attention (provide offers or new

exciting services) to the customers falling in the upper right region. Since, if retained in the company, these are the "high profit promising" customers. Bottom left and upper left regions are the regions, where less profitable customers reside. If possible, companies should try to increase profit made out of these customers. On the other hand, if the customer is not promising higher profit and possessing high churn risk, companies may simply let these customers go and try to acquire new customers, who fall into high CLV, low churn risk region.

Companies usually identify risky customers according to the lift of their favorite "churn-prediction" model. In a typical setting, they try to give the best offer to the customers possessing highest risk. In this scenario, managing the campaign is simple but its efficiency (in terms of cost) is questionable. Because, in such an execution, decision maker does not know whether offer receiving customers are worth the effort or not. On the other hand, integrating CLVs into the offer management process helps to increase return of the campaign by targeting, and possibly retaining, customers with higher CLV.

Fig. 2. 3-level offer management process.

Figure 2 shows our proposed 3-level offer management framework. The first step calculates churn probability of customers, whereas the second step estimates CLVs. In the third step, an offer management optimization model is solved using the estimated churn and CLVs from the first two steps.

In our previous study [\[2](#page-18-0)] we focused on churn prediction models for prepaid subscribers of a mobile operator. This study can be seen as an extension, where we improve the performance of our churn prediction model, and include a CLV prediction approach. The major contribution of this paper is the proposed offer management model, which considers the predicted churn and CLV values for each subscriber and decide the best offer for each customer such that the expected overall CLV of the entire subscriber set is maximized where offers are addressed to subscribers considering budget and several offer allocation limitations.

The rest of the paper is organized as follows. Section 2 provides some background information on CLV and offer management (specifically in telecommunication sector). Section [3](#page-6-0) overviews the literature. Section [4](#page-7-0) introduces the data set used in this study. Section [5](#page-8-0) presents performance comparison several logistic regression models for churn prediction, whereas Sect. [6](#page-10-0) presents performance of Pareto/NBD model on churn and CLV prediction. Section [7](#page-13-0) presents the proposed offer management model and discuss solutions of the optimization model for varying scenarios.

2 Background

2.1 Customer Lifetime Value (CLV)

CLV can be simply described as net the present value of the future cash flows associated with a customer [[3\]](#page-18-0). If a mobile operator wants a profitable customer management they have to calculate the CLV of a subscriber [[1\]](#page-18-0) and a profitable customer means loyalty. Loyal customers spread positive word of mouth about the brand if they feel satisfied with the product and/or services. The studies on customer life time value in literature can be classified in four main categories [\[4](#page-18-0)] as shown in Table 1:

Focus	Category	Model			
Structural model	Customer unit	Individual model			
		Segment (Customer base) model			
	Prediction data	Retrospective model			
		Prospective model			
	Transaction	Contractual model			
		Non-contractual model			
	Purchase cycle	Discrete model			
		Continuous model			
Strategic model	Strategic use of CLV in management				
Normative model	Relationship between duration and cost				
Analytic model	Resource allocation (Budget allocation)/Pricing				

Table 1. CLV models [[4](#page-18-0)].

In this study we focus on both "Non-Contractual model" and "Discrete model" settings. In [\[5](#page-18-0)], the assumption monetary value is independent of the underlying transaction process, helps to decompose the CLV estimation into two sub-problems, namely transaction flow determination and monetary value estimation for transactions. A sub-model for the transaction flow (recency, frequency); discounted expected transaction is calculated based on the Pareto/NBD parameters, the confluent hypergeometric function of the second kind, and the Pareto/NBD likelihood function. The second sub-model for revenue per transaction (expected average transaction value); is calculated as a weighted average of the population mean and the observed average transaction value, which is the average value across transactions.

2.2 Offer Management

In the first years of mobile communication, a very limited number of customer services (only voice and short message services) were available. As mobile phone technology evolved, GSM service providers improve the quality and variety of services they provide and adapted to the rapidly changing technology faster than other industries. If subscribers are not satisfied with the service they receive or they think that the value proposed by the operator is minimal, they can easily switch their mobile service providers. Therefore, during the years, mobile operators generated various products and services for their subscribers to retain them with the company. Mobile operators created their own product lines in a very broad perspective and each product can easily be seen as an offer to the customer. In addition to this, offers also include marketing activities to attract and acquire customers over multiple channels. All these activities and offers are analyzed under the name campaign management (or alternatively offer management), which mainly aims to contact with the right customers at the right time over the right channel with right offer. Figure 3 overviews the general campaign management cycle.

Fig. 3. Campaign/Offer management cycle.

A sample offer management framework for a mobile operator is presented in Fig. [4](#page-5-0). The data is collected in "Enterprise Data Warehouse" (EDW) and also the mining models are fed from the EDW. Both the score from the mining models and the EDW data are used to calculate the best offer for each subscriber. The offer management system controls the calculated offers to propose to the subscribers. Integrated Voice Response (IVR), Short Message Service (SMS) and Web channels are different channels used in the offer management systems.

The Average Revenue per User in Turkish Lira (TL) (ARPU-TL) and subscriber trend is shown in Figs. [5](#page-5-0) and [6,](#page-6-0) respectively [[2\]](#page-18-0). This market data is published

Fig. 4. Offer management framework.

Fig. 5. ARPU trend for postpaid & prepaid subscribers.

quarterly to report the market trends in mobile sector in Turkey by "Bilgi Teknolojileri Kurumu" (BTK) which is the Governmental Organization of Information Technologies. It can be seen that ARPU of the postpaid subscribers is higher than the prepaid subscribers and the time shows that the number of the postpaid and prepaid subscribers converge to each other. It is also known that keeping a subscriber is cheaper than acquiring a new customer. If a mobile operator calculates the CLV of the prepaid subscriber and propose them the right offer than it is obvious that the operator can increase their revenue.

Fig. 6. Postpaid & prepaid subscribers trends.

3 Related Works

Recency-frequency-monetary (RFM) data is considered as one of the major set of features in segmenting customers. Our study uses refill history of prepaid subscribers which can be seen as a purchase order thus can be easily converted to *recency*frequency-monetary (RFM) data. Details RFM applications in telecommunication sector can be found in [[6\]](#page-18-0), which emphasizes that although using RFM for customer analytics is a simple yet a strong approach, there are also some disadvantage, such as:

- RFM only focuses on best customers. If customer do not buy often or spend little or do not generate any transaction lately, it provides little information,
- RFM only focuses on limited variables. It is better to take into account other customer relation variables along with RFM data.

Although it has some disadvantages, RFM data still being frequently used by analysts due to its simplicity. Besides analytics activities for telecommunication sector, RFM has been applied to many other areas and activities [[7\]](#page-18-0). One specific example is using RFM data for calculating CLVs. The calculation of customer based CLV is not a very frequently studied topic for mobile operator companies. However, there are a lot of churn analysis studies in the telecommunication analytics literature.

A churn analysis that is not backed up by a CLV analysis may not always provide the correct insights about customers. Although there are only a few studies which cover CLV calculation for telecommunications industry, these studies do not consider prepaid mobile subscribers. The study [[4\]](#page-18-0) deals with wireless telecommunication subscribers and calculate the CLV based on a new approach of Markov chain model. In [[8](#page-18-0)], authors conduct a churn prediction study for land line subscribers, whereas [[9\]](#page-18-0) is focused on contracted telecommunication services (land-line or mobile phone or internet line). In [\[9](#page-18-0)], several hypotheses are defined in terms of validation and calculation of the CLV and customer equity values.

CLV calculation cannot be ignored in todays' challenging marketing. In the big data era, making descriptive analysis on historical data is not enough for improving customer loyalty. The competition conditions in any market compelled the companies to analyze the future behavior of their customer to improve the customer experience with their product. Thus, the CLV calculation is a "must" in todays' market. Especially, like mobile market where the prepaid subscribers almost constitute the 50% of the customer base (see Fig. [6\)](#page-6-0), the mobile companies have to focus analyzing their prepaid subscribers, who have more complicated behaviors (harder to predict) than that of postpaid subscribers. As mentioned previously we utilize CLV information to allocate the best possible offer to subscribers in order to retain them with the company. Especially in the context of offer management, inclusion of CLVs play a critical role [[10\]](#page-18-0), hence the developed mathematical model is designed to reflect the effect of proposed offers on the CLVs.

The RFM data was extended by [\[5](#page-18-0)] for the CLV calculation. As stated in the paper, the study focused on using only the RFM data for the CLV calculation in customer base. The customer base data is the transactions for the non-contractual setting. This CLV calculation based on RFM data can be easily adopted to prepaid subscribers in mobile sector. However, there is no application of this method for mobile prepaid subscribers, we can give some other applications of CLV calculation based on RFM data. This method is used widely in many industries, there are many case studies like retail in these papers [[11,](#page-18-0) [12\]](#page-18-0).

In [\[4](#page-18-0)], the Pareto/NBD model is used for conducting customer base analysis. Pareto/NBD model is used to define the non-contractual purchase transactions for customers, for whom the actual churn time is never know with 100% certainty. There are many other models that can be used instead of Pareto/NBD model. In [\[12](#page-18-0)] a comparison for different models are provided.

Offer management is also another aspect of controlling churn of subscribers. A mobile operator has to manage their cost for customer loyalty. In [[1\]](#page-18-0), it is emphasized that companies cannot just only focus on profitable customer but also finding the loyal customers is more important than profitable customers. It is also important that companies have to solve the optimization problems for the promotions and calculate the maximum profit from their offer campaigns [[13\]](#page-18-0).

4 Data Set

The lifecycle period for the prepaid subscriber is 270-days. If they do not do any refill action, after 270-days the subscription will be ended by the mobile operator. But this period will be reset by refill transaction and another 270-days new period starts.

The total subscriber used in this study is 6480 which were activated two-year period starting from 1.2.2015 to 31.1.2017. Refill transactions are also for the same period for the activated subscribers. The subscribers are chosen among prepaid subscribers who did not do any charging method change and remains prepaid subscriber for their lifetime.

In this study we consider a three mutually exclusive segments, which are named as youth, mass, and other. Youth segment contains subscribers under age 26, and other segment is composed of subscribers for whom company has very limited information, and *mass* segment contains all other subscribers who are neither in *youth* nor in *other* segment. The distribution of subscribers over segments can be seen in Table 2.

For the RFM data, the refill transactions belong to again the same two-year period with the subscribers. For recency parameter we have taken the refill date, for the monetary value we have taken the refill amount in TL. The number of rows is 53.249. The # of distributions can be seen in Table 2 below.

Segment information		Other Youth Mass Total	
No. of subscriber		$\vert 1.063 \vert 1.352 \vert 4.065 \vert 6.480$	
No. of refill transactions 6.290 12.620 34.338 53.248			

Table 2. The distribution of subscribers over segments.

5 Churn Prediction Using Logistic Regression

In this section we present a comparative analysis for the performances of several Logistic Regression models for churn prediction. The base data set is assumed as the RFM data. In addition to this data set, we consider additional 395 features, which are collected by the company. The additional data set includes different types of features such as:

- call or usage behavior, includes incoming or outgoing calls and the types of communication like voice, data or short message service (SMS),
- network metrics, include call drop rates,
- refill metrics, include the behavior of the refill transactions of the subscriber like refill amount and also usage type based refill utilization like if the subscriber uses mostly SMS, data or voice,
- ARPU, includes the average revenue of per user in TL amount,
- number of lines, shows whether the subscriber has other lines, or has one or more lines.
- age,
- value-added-services (VAS) usage/subscription, indicates the subscriber's usage behavior of value added service or if the subscriber has any subscription for the value added services,
- pre-calculated social network analytics (SNA) metrics,
- network type (2G, 3G, 4G), includes the network type of the subscriber,
- equipment type, give if the subscriber has a smartphone or other basic phones.

Feature set selection is executed as follows:

- 395 features are analyzed with statistical methods,
- the features whose minimum and maximum values are equal are eliminated. 383 features are left,
- correlation analysis is run and 308 features are left,

Fig. 7. Flow of benchmarking.

- with domain knowledge and expertise, the total number of features is reduced to 50,
- Stepwise regression with backward elimination is run over the 50 variables and 10 features are selected.

Logistic Regression is applied in the following order (also shown in Fig. 7):

- Model 1: Logistic Regression with RFM
- Model 2: Logistic Regression with 10 features
- Model 3: Logistic Regression with 10 features and RFM
- Model 4: Logistic Regression with 50 features
- Model 5: Logistic Regression with 50 features and RFM

The results for all combinations are shared in Table [3.](#page-10-0) Table [3](#page-10-0) contains the following columns:

- cutoff value, which is used as a threshold to decide whether customer considered as a churner or non-churner combination,
- result, indicates the combination name
- set name, shows the details of the combination (selected data set),
- accuracy, true positive (TP), false positive (FP) rates and precision are the classical performance indicators for binomial classifiers, such as logistic regression.

The results presented in Table [3](#page-10-0) reveals that base logistic regression model using only RFM data performs very well (in terms of accuracy and prediction). However, inclusion of extra customer specific variables results in a performance increase. Overall, the cutoff rate 0.9 yields the best result and the top performing models at this cutoff level are Model 3 and Model 5. However, if one model is to be selected, it should be Model 3 due to the rule of parsimony.

Cutoff	Model	Set name	Accuracy	TP rate	FP rate	Precision
0.5	Model 1	RFM	95%	98%	24%	96%
	Model 2	Var 10	91%	100%	64%	90%
	Model 3	Var $10 + RFM$	95%	100%	31%	95%
	Model 4	Var 50	92%	100%	56%	91%
	Model 5	Var $50 + RFM$	96%	100%	28%	95%
0.6	Model 1	RFM	94%	97%	22%	96%
	Model 2	Var 10	92%	100%	50%	92%
	Model 3	Var $10 + RFM$	96%	100%	24%	96%
	Model 4	Var 50	93%	99%	41\%	93%
	Model 5	Var $50 + RFM$	96%	100%	23%	96%
0.7	Model 1	RFM	93%	95%	20%	96%
	Model 2	Var 10	94%	99%	34%	94%
	Model 3	$Var 10 + RFM$	97%	100%	20%	97%
	Model 4	Var 50	95%	99%	30%	95%
	Model 5	$Var 50 + RFM$	97%	100%	18%	97%
0.8	Model 1	RFM	92%	93%	17%	97%
	Model 2	Var 10	94%	98%	29%	95%
	Model 3	Var $10 + RFM$	97%	99%	15%	97%
	Model 4	Var 50	95%	98%	24%	96%
	Model 5	$Var 50 + RFM$	97%	99%	14%	98%
0.9	Model 1	RFM	86%	86%	13%	97%
	Model 2	Var_10	95%	98%	24%	96%
	Model 3	Var $10 + RFM$	98%	99%	12%	98%
	Model 4	Var 50	95%	98%	20%	97%
	Model 5	$Var 50 + RFM$	98%	99%	11%	98%

Table 3. Benchmark of data with logistic regression.

6 Churn and CLV Prediction Using Pareto/NBD Model

In preceding study [\[2](#page-18-0)], we focused on subscriber refill transactions and did not include any tenure or segment information of the subscriber in the transaction data. We found out that without any tenure or segment information the Pareto/NBD model fails in the test period while showing a good performance in the training period.

Our segment information includes youth, mass, and other (not segmented); definitions of which were provided in Sect. [4.](#page-7-0) First we generated the results based on the segment information then included 8 tenure groups which were based on the 3-months sub period information. The tenure is calculated according to the first refill action date. Segmentation based on tenure idea is illustrated in Fig. [8.](#page-11-0)

The Pareto/NBD model is executed based on the data sets, details of which is described above. The results are shared in Table [4.](#page-11-0) The Pareto/NBD model results is labeled as Model 6, Model 7, Model 8 and details of labeling is as follows:

Fig. 8. Tenure groups.

Cutoff	Model	Set name	Accuracy	TP Rate	FP Rate	Precision
0.5	Model 6	w/o Segment + Tenure	88%	98%	69%	89%
	Model 7	Segment		99%	29%	95%
	Model 8	Segment + Tenure	94%	98%	29%	95%
0.6	Model 6	w/o Segment + Tenure	88%	97%	64%	90%
	Model 7	Segment	96%	99%	18%	97%
	Model 8	Segment + Tenure	96%	98%	18%	97%
0.7	Model 6	w/o Segment + Tenure	89%	97%	58%	91%
	Model 7	Segment	96%	98%	13%	98%
	Model 8	Segment + Tenure	96%	97%	13%	98%
0.8	Model 6	w/o Segment + Tenure	90%	96%	46%	92%
Model 7		Segment	97%	98%	10%	98%
	Model 8	Segment + Tenure	96%	97%	10%	98%
0.9	Model 6	w/o Segment + Tenure	91%	93%	18%	97%
	Model 7	Segment	97%	97%	6%	99%
	Model 8	Segment + Tenure	96%	96%	6%	99%

Table 4. Pareto/NBD model results.

- Model 6: Pareto/NBD model is run with RFM without segment and tenure information,
- Model 7: Pareto/NBD model is run with RFM with segment information,
- Model 8: Pareto/NBD model is run with RFM with segment and tenure information.

The Pareto/NBD model's performance in predicting expected number of transactions is given in Fig. [9](#page-12-0)a–f for the all set, mass, youth and not segmented sets. For each segment, the prediction power is tested under two scenarios. In the first set of scenarios the customers are segmented further with respect to their tenure; where as in the second case customers are further sub-segmented using their tenure information. The segment & tenure based data sets have the best performance for the Pareto/NBD model.

Fig. 9. Pareto/NBD model prediction performance for varying segment and tenure combinations.

In this study by adding the tenure and segment information we have got a good performance for the model in test period for that the expected number of transactions and probability of alive (one minus probability of churn) of each subscriber are calculated and used in the proposed offer management model.

7 Proposed Mathematical Model for Offer Management

After identifying valuable customers to prevent them from churn, a best offer selection mathematical model is proposed. The model calculates output for each customer an eligible offer. Furthermore, the model maximizes the total CLV by proposing the customers with the eligible offers. Base CLV is calculated by the Pareto/NBD model which is based on the RFM data.

The notation used in the mathematical model, sets, parameters, decision variables, and the proposed mathematical model are provided below:

Sets;

- $i =$ Customer index, $i \in I$
- $i =$ Offer index, $i \in J$

Parameters;

- $clv_base_i =$ Base lifetime value of customer *i*. This amount should be interpreted as the expected present net worth of customer i for the following cases: (i) customer receives no offer or (ii) customer refuses the proposed offer. The base CLVs are estimated using Pareto/NBD model, Model 7 (see Table [4](#page-11-0)).
- p_i = Churn probability of customer i to company before receiving the offer. Churn probabilities are estimated using the Pareto/NBD model, Model 7 (see Table [4\)](#page-11-0).
- o_i = Upper bound of discount for customer *i*,
- w_i = Upper bound of offer j,
- c_i = Discount rate of offer j,
- f_i = Monthly payment of customer *i* to company before receiving the offer,
- f_{avg} = Monthly average payment of customers,
- β_{ij} = Probability of accepting offer *j* by customer *i*, $\beta_{ij} = p_i^{c_j \frac{c_j}{f_i}}$,
- ρ_{ij} = Churn probability of customer *i* after taking offer $\rho_{ij} = p_i \left(1 p_i^{\frac{C_i \log n}{f_i}}\right)$,
- clv_{ij} = Modified Customer Lifetime Value (CLV New in Fig. [10](#page-14-0)) of customer i, upon accepting of offer j.
- $t =$ Monthly budget for discount,
- $M = \text{Large positive number.}$

Decision Variables;

$$
x_{ij} = \begin{cases} 1, & \text{if customer } i \text{ gets offer } j \\ 0, & \text{o/w} \end{cases}
$$

Fig. 10. Tree demonstrating the relation between (i) offer proposal (ii) offer acceptance/rejection (iii) CLV to be used for each case.

Mathematical model;

$$
\max z = \sum_{i} \left[\sum_{j} x_{ij} \left[\beta_{ij} c l v_{ij} + (1 - \beta_{ij}) c l v_base_i \right] + \left(1 - \sum_{j} x_{ij} \right) c l v_base_i \right] \tag{1}
$$

$$
\sum_{i} \sum_{j} f_i c_j x_{ij} \le t \tag{2}
$$

$$
\sum_{i} f_i c_j x_{ij} \leq o_i, \quad \forall i \tag{3}
$$

$$
\sum_{i} x_{ij} \le w_j, \quad \forall j \tag{4}
$$

$$
x_{ij} \in \{0,1\} \forall i,j \tag{5}
$$

The objective function of the offer management mathematical model aims to maximize expected customer lifetime value while increasing retention of customers by giving the right offer. The expected CLV of each customer depends on (i) whether the customer receives an offer or not, and (ii) if the received offer is accepted or rejected by the customer. The expected CLV calculation idea used in the objective function is illustrated in Fig. 10. The constraints can be interpreted as follows: Constraint 2 states that offered discounts cannot exceed the total budget. Constraint 3 indicates the given offer cannot be greater than predetermined discount limit for each customer. Constraint 4 ensures that number of total proposed offers for each offer type cannot exceed the specified limit.

We made several assumptions in constructing links between offer acceptance/ rejection decision and its effect on updated CLV and churn probabilities. First of all it is assumed that the churn probabilities, p_i , are assumed to take values strictly greater than 0 and strictly less than one, in other words it is assumed that $p_i \in (0,1)$.

This assumption is required as for the boundary values (that is $p_i = 0$ and $p_i = 1$) calculating p_{ij} values becomes problematic (see (7)). It is assumed that the tendency of accepting an offer can be estimated using (6). It is assumed that the probability of accepting an offer by a customer (β_{ij}) is calculated depending on the customer's package refill payment (f_i) , churn probability (p_i) and average package refill payment of all customers (f_{avg}) . Having lower churn probability, higher package refill payment from average and higher discount rate yield higher probability of accepting an offer, as shown in (6) :

$$
\beta_{ij} = p_i^{c_j^{favg}} \forall i, j
$$
\n(6)

In the same way, new churn probabilities (ρ_{ii}) are calculated based on probabilities of accepting the offer (β_{ii}) and churn probabilities (p_i) . Having higher churn probability (p_i) and higher probability of accepting an offer (β_{ii}) gives lower new churn probability after receiving an offer, as shown in (7):

$$
\rho_{ij} = p_i \left(1 - p_i^{c_j \frac{f_{avg}}{f_i}} \right) \quad \forall i, j \tag{7}
$$

New churn probabilities (ρ_{ij}) are then used to calculate modified CLVs (clv_{ij}) . Modified CLV is the expected present net worth of customer i assuming customer accepts offer *j* as shown in (8) :

$$
clv_{ij} = \sum_{n=1}^{n=\infty} \left(1 - p_{ij}\right)^n \left[\left(1 - c_j\right) clv_base_i \right] \quad \forall i, j \tag{8}
$$

In order to propose predetermined set of offers to a set of selected customers aforementioned mixed integer mathematical model is developed, and solved by using IBM CPLEX Solver.

We have solved problem with 6480 customers and 5 offers in Table 5. Offers are generated solely in the form of discounts.

Offer	Discount %
Offer 1	10
Offer 2	20
Offer 3	30
Offer 4	40
Offer 5	50

Table 5. Offers.

Different scenarios are run with the 6480 customers. Below are the details of the scenarios:

- Scenario1, the budget was set to 10.000 and offer count was set to 300. There is no cut off value for the customer churn probability.
- Scenario2, the budget was set to 10,000 and no offer limit was applied. There is no cut off value for the customer churn probability.
- Scenario3, the budget was set to 5.000 and offer count was set to 300. There is no cut off value for the customer churn probability.
- Scenario4, the budget was set to 5,000 and no offer limit was applied. There is no cut off value for the customer churn probability.
- Scenario5, the budget was set to 1.000 and offer count was set to 300. There is no cut off value for the customer churn probability.
- Scenario6, the budget was set to 1.000 and no offer limit was applied. There is no cut off value for the customer churn probability.

Scenarios are generated by changing overall budget limit and offer limits. Results for all six scenarios are presented in Tables 6, 7 and [8.](#page-17-0) Table 6 summarizes the number of customers who gets an offer. Table 7 presents the breakdown of the offer receiving customers over churn probabilities. For instance, in Scenario1, out of 741 offer receiving customers, 294 have churn probability less than 0.5 and the remaining 447 have churn probability greater than or equal to 0.5. Table [8](#page-17-0) lists the distribution of offer suggestions over the possible list of offers for each scenario.

	Budget	Offer	Cutoff	Offers	% increase in CLV
		limit		received	(compared to base CLV)
Scenario1	10.000	300	0	741	3%
Scenario ₂	10.000	inf	Ω	3.398	3%
Scenario3	5.000	300	Ω	532	3%
Scenario ₄	5.000	inf	Ω	1.498	5%
Scenario ₅	1.000	300	Ω	168	4%
Scenario6	1.000	inf	Ω	168	2%

Table 6. Offer model scenario results.

Table 7. Churn rate breakdown.

	Churn rates	Total subs	
		$p < 0.5$ $p > = 0.5$	
Scenario1	294	447	741
Scenario 2 3.331		67	3.398
Scenario ₃	294	238	532
Scenario4	1.485	13	1.498
Scenario ₅	168		168
Scenario6	168		168

	Discount offers					
		Offer 1 % 10 Offer 2 % 20 Offer 3 % 30 Offer 4 % 40 Offer 5 % 50 Total subs				
Scenario 11300		57	39	60	285	741
Scenario $2 \mid 3.352$		10	2	11	23	3.398
Scenario $3 \mid 300$		22	13	35	162	532
Scenario 4 1.495		3				1.498
Scenario $5 \mid 168$						168
Scenario 6 168						168
Scenario $7 \mid 121$		57	39	60	285	562

Table 8. Suggested offer distribution.

8 Conclusion

A significant percent of the mobile subscribers are still prepaid subscribers (see Fig. [6\)](#page-6-0). In years, both the number of postpaid and prepaid subscriber has converged to each other as a result of increased data usage due to 4G network upgrade by the mobile operators. In this study, we concentrated on developing an offer management framework, which includes a state of the art mathematical model that is based on important estimated parameters for customer relations management, namely churn probabilities and CLVs. It is seen that, instead of proposing an offer to every churner, the mathematical model gives decision makers the flexibility of addressing offers to the most valuable subscribers who have higher potential to accept offers.

CLV estimation is seen as an important part of the offer management framework, and without CLV integration, the mobile operators should accept the burden of high rate of false positives in their offer proposals. Another important finding is, in estimating CLVs, segment and tenure information turns out critical. It can be seen from the (Fig. [9a](#page-12-0)–f) that Pareto/NBD model yields good results for the youth and mass segments in the test period only when tenure information is incorporated. It is also found that for the non-segmented customers (other segment) the Pareto/NBD model fails, which seconds the claim that both segment and tenure play critical role in order to obtain reasonable predictions from Pareto/NBD model.

In general, it should be said that there is still room for improving the parameter estimation for Pareto/NBD model. There are some different studies about the parameter estimation [\[13](#page-18-0)–[15](#page-18-0)], which may be followed to come up with a better estimation technique.

Lastly, it is planned to conduct an A/B testing in a real world setting, where responses of the subscribers will also be collected and the actual performance of the proposed framework will be revealed.

References

- 1. Kumar, V., Rajan, B.: Profitable customer management: measuring and maximizing customer lifetime value. Manag. Acc. Q. 10(3), 1 (2009). Spring 2009
- 2. Can, Z., Albey, E.: Churn prediction for mobile prepaid subscribers. In: Proceedings of the 6th International Conference on Data Science, Technology and Applications Volume 1, DATA 2017, ISBN 978-989-758-255-4, pp. 67–74 (2017)
- 3. Pfeifer, P., Haskins, M., Conroy, R.: Customer lifetime value, customer profitability, and the treatment of acquisition spending. J. Manag. Issues 17(Spring), 11–25 (2005)
- 4. Hwang, H.: A dynamic model for valuing customers: a case study. Adv. Sci. Technol. Lett. 120, 56–61 (2015)
- 5. Fader, P., Bruce, H., Ka, L.: RFM and CLV: using iso-value curves for customer base analysis. J. Mark. Res. 42, 415–430 (2005)
- 6. Wei, J., Lin, S., Wu, H.: A review of the application of RFM model. Afr. J. Bus. Manag. 4(19), 4199–4206 (2010). December Special Review
- 7. Zabkowski, T.: RFM approach for telecom insolvency modeling. Kybernetes 45(5), 815– 827 (2016)
- 8. Huang, B., Kechadi, M., Buckley, B.: Customer churn prediction in telecommunications. Expert Syst. Appl. 39(1), 1414–1425 (2012)
- 9. Segarra-Moliner, J., Moliner-Tena, M.: Customer equity and CLV in Spanish telecommunication services. J. Bus. Res. 69(10), 4694–4705 (2016)
- 10. Bahnsen, A., Aouada, D., Ottersten, B.: A novel cost-sensitive framework for customer churn predictive modeling. Decis. Anal. 2, 5 (2015)
- 11. Khajvand, M., Zolfaghar, K., Ashoori, S., Alizadeh, S.: Estimating customer lifetime value based on RFM analysis of customer purchase behavior: case study. Procedia Comput. Sci. 3, 57–63 (2011)
- 12. Platzer, M.: Stochastic models of noncontractual consumer relationships. Master thesis at the Vienna University of Economics and Business Adminstration under the Supervision of Dr. Thomas Reutterer (2008)
- 13. Homburg, C., Koschate, N., Hoyer, W.D.: Do satisfied customers really pay more? A study of the relationship between customer satisfaction and willingness to pay. J. Mark. 2(69), 84–86 (2005)
- 14. Ma, S., Liu, J.: The MCMC approach for solving the Pareto/NBD model and possible extensions. In: Third International Conference on Natural Computation, ICNC 2007, vol. 2. IEEE (2007)
- 15. Abe, M.: Counting your customers one by one: a hierarchical Bayes extension to the Pareto/NBD model. Mark. Sci. 28(3), 541–553 (2009)
- 16. Platzer, M., Reutterer, T.: Ticking away the moments: timing regularity helps to better predict customer activity. Mark. Sci. 35(5), 779–799 (2016)