



Reverse Engineering Techniques for Investigating the Vibro-Acoustics of Historical Bells

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Abstract. In this paper, we present an effective methodology for assessing the vibrational properties of real-life bells, using reverse engineering techniques. When struck by a clapper, bells vibrate in rather complicated ways, which result in complex sounds. Typically, to obtain pleasant sounds, bell founders tune the first five partials (vibration modes) according to specific frequency ratios, while also trying to control the amount of beats, which also affect the musical quality. In practice, many musically important aspects are strongly related to fine details of the bell geometry. In this work, we use scanning imaging technology to obtain precise 3D geometry bell data, and then assess the bell tuning features by combining the acquired 3D geometrical data with Finite Element modal computations. Our numerical results are compared with experimentally identified bell modes, attesting the feasibility and effectiveness of the proposed approach. This analysis strategy is particularly suited in the context of cultural heritage preservation, by providing new and comprehensive ways to characterize and describe historical bells. Moreover, it can also shed light when addressing bell casting and tuning techniques throughout times.

Keywords: Vibro-acoustics · Reverse engineering · 3D scanning
Modal analysis · Bells

1 Introduction

This work was motivated by the need to enrich the musicological knowledge on a unique collection of 102 historical cast bronze bells, stemming from the largest surviving 18th century carillons in Europe. Although originally concerned with the assessment of the bell geometry using 3D scan imaging technology, the objective of

this paper centers on the development of an effective framework for assessing the bell vibrational features, using reverse engineering techniques. Enabling easy, accurate and fast measurements of objects with complex shapes, surface scanning technology is becoming widely used for many applications, ranging from quality control in the automotive industry (Kuş 2009) to healthcare applications (Treleaven and Wells 2007), or to support the safeguarding of cultural heritage (Levoy et al. 2000). It has benefited from the increasing computational capabilities to manipulate large dataset alongside with the advances in computer graphics and, now 3D scanning can provide detailed models of intricate objects ready to use for analysis purposes. In this work, the reverse engineering process involves a 3D geometrical capture of the bell geometry by manual scanning, the construction of a structural model by means of Finite Elements, and finally modal computations in order to estimate the modal frequencies and mode shapes of the bell. In spite of the specificity of the mechanical system of interest here, the developed approach is definitely very attractive for dynamical assessment of existing structural components in industry.

It is well known that, when struck by a clapper, bells vibrate in rather complicated ways, which result in complex sounds (Fletcher and Rossing 1998). Moreover, due to the bells axi-symmetry, their so-called “partials” appear in modal pairs, with identical natural frequencies but different orientations (thus their orthogonality). However, due to some unavoidable slight deviations from the perfect axi-symmetry, this modal degeneracy disappears and leads to some audible beats in the radiated sound, which affect our perception of bell quality.

Typically, to obtain a pleasant sound, bell founders tune the first five partials according to the specific frequency ratios 0.5, 1.0, 1.2, 1.5 and 2.0 (referred to the second mode pair, conventionally designated as “fundamental”), while also trying to control the amount of beating. In practice, this is achieved in two main steps: (1) through the design of the bell moulds, which allows a rough pre-tuning, and (2) by removing some of the bell material after casting, using a tool-shop lathe in order to correct the slight tuning errors. According to the amount and location of the removed metal, the bell modal frequencies may increase or decrease. One then understands that all the musically important aspects are strongly related to fine details of the bell geometry.

In this work, we demonstrate the efficiency of combining precise 3D geometrical data obtained using scanning imaging technology with modal computations, in order to assess the vibrational behaviour of three-dimensional shell-like structures. The feasibility of the methodology is illustrated on an historical bell of the 18th century, from the Witlockx carillon of the Mafra National Palace, near Lisbon, Portugal (Fig. 1), for which a full vibratory assessment was achieved from a sophisticated experimental modal identification technique previously developed (Debut et al. 2016a). The comparison of the results attest the overall feasibility and effectiveness of the reverse engineering approach developed in this paper, in particular for the analysis and subsequent re-tuning of ill-tuned bells.



Fig. 1. Photograph of the historical bell from the Witlockx carillon of the Mafra National Palace – approximately 20 cm high.

2 3D Bell Scanning

The precise metrology of structures with complex shapes typically poses considerable problems. Modern 3D scan imaging technology, combining sophisticated techniques of computationally intensive image acquisition with processing algorithms, enables a comparatively fast and very precise geometrical assessment of intricate objects. Figure 2 shows one such commercial device, used in the present work, the ARTEC EVA 3D handheld scanner. The assembly consists of one projector and two cameras, and uses structured light as sensing technology to capture the fine details of the objects. It captures shapes with an overall dimensional tolerance of 0.5 mm, as well as image colours, which are used to generate a texture map on the 3D rendering model. Basically, the scanner takes a sequence of images with different patterns of LED light projected onto the surface of the object. Once acquired the raw scan data for the entire object, a series of different post-processes, namely scans alignments, registration, surface reconstruction and rendering, is run either automatically or interactively, in order to create a complete 3D final model from the raw point data.

Figure 3 illustrates three aspects of the scanned bell geometry: the 3D mesh, which results from the transformation of the point data into polygons, the rendering model, and after mapping colour data onto it. These figures show the level of detail achievable.

Using such a device, when compared with conventional laser point-metrology, an order of magnitude improvement can typically be achieved in dimensional precision, while spatial coverage resolution is improved by several orders of magnitude. Moreover, the geometry data can be obtained in a fraction of the time, but post-processing for high-quality full rendering can be lengthy depending on the object size and the model complexity.



Fig. 2. 3D image scanning of the historical bell with the ARTEC EVA 3D handled scanner

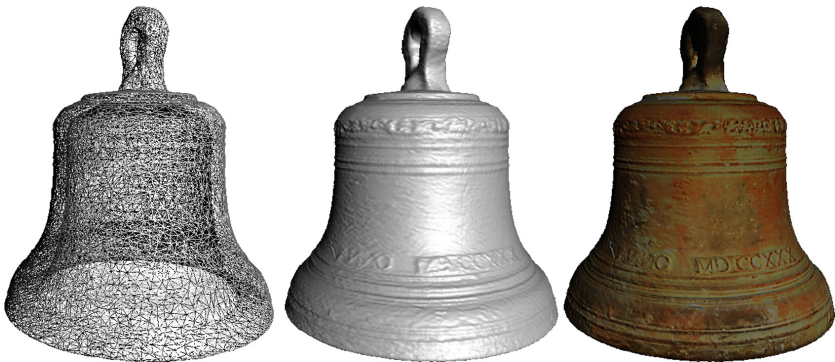


Fig. 3. 3D digital models of the historical bell. Polygons mesh (left), full-rendering model with texture (middle) and with colour (right)

3 Finite Element Modal Computations

The scanned bell geometry can be readily organized in a specific file form to input most finite element computer programs for subsequent structural analysis. Here, the Finite-Element package Cast3M (2017) developed by CEA (France) has been used to generate the structural model and carry out the modal computation as in our previous work (Debut et al. 2016b).

The bell bronze alloy was modelled with parameters $E = 100.3$ GPa, $\rho = 8542$ kg m⁻³ and $\nu = 0.34$. The generated mesh, displayed in Fig. 4, consists of 30,000

solid elements (4-node tetrahedral), which appears fine enough for accurate modal computations based on preliminary convergence tests. Modal computations were performed assuming the bell in free conditions, and ignoring damping, which is a legitimate assumption for this system. Energy dissipation, which occurs as a consequence of both internal losses and acoustic radiation, is actually very low for bells, typically in the range 0.01–0.1% (Debut et al. 2016a), so that the eigenfrequencies remain almost unchanged compared to the conservative case, and the mode shapes approximate closely real modes. Since losses by sound radiation remain the primary cause of dissipation for bells (Rossing 2000), damping strongly affect the sound radiated. The weakly damped bell vibration results in a slow sound decay, where the amplitude of each radiated partial decreases at a specific rate due to the frequency dependence of the sound radiation phenomena.

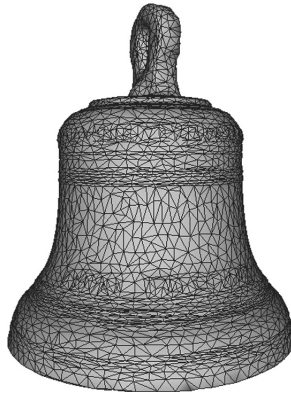


Fig. 4. Finite element mesh used for modal computations

The mode shapes ϕ_n and natural frequencies $f_n = \omega_n/(2\pi)$ were computed from the assembled mass and stiffness matrices \mathbf{M} and \mathbf{K} , which stem from the standard eigen-computation $(\mathbf{K} - \omega_n^2 \mathbf{M})\phi_n = \mathbf{0}$. The computed vibrational patterns and frequencies of the first five musically relevant modes are presented in Fig. 5. Note that although computed, rigid body modes, which correspond to translation and rotation motions of the undeformed structure, are not relevant here. As expected for typical axisymmetric structure, normal modes appear in degenerate pairs, so that one bell partial is constituted by a pair of modes. However, because slight asymmetries are present in the bell shape, one notices small differences in frequency between the two modal components of each partial, which lead to more or less intense beating when the bell is struck.

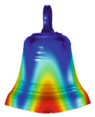


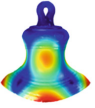


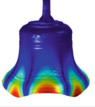


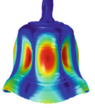


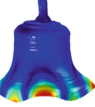


MODE PAIR	General XY view	ZX view	
		1 st family frequency [Hz]	2 nd family frequency [Hz]
1 (Hum)		1487.6 	1489.0 
2 (Fundamental)		2895.2 	2901.6 
3 (Tierce)		3597.6 	3599.1 
4 (Quint)		4831.1 	4841.4 
5 (Nominal)		6057.6 	6060.4 

Fig. 5. Mode shapes and modal frequencies of the relevant five mode pairs from FE modal computation of the scanned bell geometry

4 Experimental Modal Identification

For validation purposes, a full experimental modal identification of the bell, based on impact testing, was performed. The vibrational radial responses of the bell were recorded with three piezoelectric accelerometers glued on the outer rim of the bell, in the same horizontal plane, while impact excitations were performed on 32 locations regularly spaced near the rim – see Fig. 6. From the set of acquired impulse responses, modal identification was then performed using a custom developed program (Debut et al. 2016b) based on the Eigensystem Realization Algorithm (ERA) (Juang 1994), leading to estimate of the modal frequencies and modal damping values, as well as of the mode shapes.

The comparison of the FEM-computed and identified modal frequencies, reported in Table 1, confirms the effectiveness of the proposed approach. Once matching the modal frequency for the first mode, the errors of the modal frequencies predicted on the basis of the 3D scanned geometry and the finite element model proved consistently lower than 0.5%.



Fig. 6. Experimental test-rig for modal identification. Picture showing the bell suspended, with three accelerometers at the rim and the impact hammer

For the application of interest here, the internal tuning of the studied bell can be easily assessed, by calculating the frequency ratios of every partial relative to an arbitrary partial frequency (here the Fundamental). From the reported frequency ratios in Table 1, it appears that the founder achieved a satisfactorily tuning of the first two modes, while the tuning of the high-order modes remains less precise. One plausible speculation to explain this result is that Witlockx only attempted rigorous tuning of the Hum and Fundamental, being somehow aware of the decreasing sound intensity sensitivity of the ear toward very high frequencies (Campbell and Greated 1988).

Table 1. Computed and experimental modal frequencies of the bell

MODE PAIR	Modal frequencies (Hz) - Ratio f_n/f_2				Relative difference (%)
	FEM computations		Experimentally identified		
1 (Hum)	1487.6	0.51	1487.6	0.51	0
	1489.0	0.51	1490.8	0.52	0.12
2 (Fundamental)	2895.2	1.0	2891.8	1.0	0.12
	2901.6	1.0	2898.1	1.0	0.12
3 (Tierce)	3597.6	1.24	3593.8	1.24	0.11
	3599.1	1.24	3594.0	1.24	0.14
4 (Quint)	4831.1	1.67	4854.4	1.67	0.48
	4841.4	1.67	4855.9	1.68	0.30
5 (Nominal)	6057.6	2.09	6048.2	2.09	0.16
	6060.4	2.09	6060.8	2.09	0.01

Interestingly, this also reflects the difficulty of bell founders of casting and tuning small bells, which is an aspect of bell founding which is still challenging today (Lehr 2000). Finally, by comparing the modal frequencies between the doublet pair, one can notice that the beating frequency is rather large for the Fundamental, of approximately 6 Hz, which certainly has an important negative effect in terms of pitch perception and sound quality.

5 Conclusions

This paper presented a consistent and effective methodology for inferring the dynamical features of complex structures, based on scan data coupled to finite element computations. Application of the technique was presented for an historical bell from the 18th century, and convincing illustrative results were produced. The reverse engineering process involves the 3D geometrical capture of the bell geometry by manual scanning, the construction of a rendering model from the raw scan data, and finally, Finite Element modal computations in order to estimate its modal frequencies and mode shapes. As a result, an objective assessment of the tuning of this rare 18th century bell has been successfully achieved.

This analysis strategy, which can be applied in many different fields, seems particularly suited to support the preservation of cultural heritage. Besides the tuning assessment presented here, the high-resolution of the scans enables a precise, detailed and textured 3D image of the bell structure, for which no substantial information is today available. Besides documenting such a rare artefact, a careful analysis of the rendering model can also explain subtle details for the sound radiated, and might point toward other information to appreciate the practices and skills of the bell founder.

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