Chapter 4
Black Holes Sourced by a Massless Scalar

M. Cadoni and E. Franzin

Abstract We construct asymptotically flat black hole solutions of Einstein-scalar gravity sourced by a nontrivial scalar field with $1/r$ asymptotic behaviour. Near the singularity the black hole behaves as the Janis-Newmann-Winicour-Wyman solution. The hairy black hole solutions allow for consistent thermodynamical description. At large mass they have the same thermodynamical behaviour of the Schwarzschild black hole, whereas for small masses they differ substantially from the latter.

4.1 Introduction and Motivations

Static, spherically symmetric solutions of Einstein gravity sourced by scalar fields have played an important role for the development of black hole physics. The simplest solution of this kind, describing a asymptotically flat (AF) spherically symmetric solution with no horizon, sourced by a scalar with vanishing potential are known since a long time [1, 2]. They are called the Janis-Newmann-Winicour or Wyman (JNWW) solutions. Initially, the search for AF black holes (BHs) with scalar hair was motivated by the issue of the uniqueness of the Schwarzschild solution and related "old" no-hair theorems [3, 4], which forbid the existence of BHs if the scalar potential V is convex or semipositive definite.

In the early nineties it was discovered that low-energy string models may allow for black hole solutions with scalar hair [5–8]. But, in this case non-minimal couplings between the scalar field and the electromagnetic field.

In recent times, the quest for hairy black hole and black brane solutions has been motivated by the application of the AdS/CFT correspondence to condensed matter

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systems [9–17]. In holographic applications the scalar field has a nice interpretation as an order parameter triggering symmetry breaking/phase transitions in the dual field theory.

Several numerical and analytical, black hole and black brane with AdS asymptotics solutions with scalar hair have been found in this context [9–13, 16, 18, 19].

Shifting from AF to anti de Sitter (AdS) black holes allows to circumvent standard no-hair theorems because in AdS the scalar field may have tachyonic excitations without destabilizing the vacuum [20]. This led to the formulation of "new" no-hair theorems [21]. The violation of the positivity energy theorem (PET) [22]. being identified as a necessary condition for the existence of BH with scalar hair.

In this note, which is based on [23], we will show as the expertise achieved in the holographic context can be successfully used to find AF BH solutions with scalar hair. Extension to asymptotically flat BF is an important issue because we know that scalar fields play a crucial role in gravitational and particle physics. Experimental discovery of the Higgs particle at LHC has confirmed that there is a fundamental scalar particle [24]. Observation of the Plank 2013–2015 satellite gives striking confirmation of cosmological inflation driven by scalar field coupled to gravity [25]. Moreover, scalar field give a way to describe dark energy.

The main result presented here is that the solution generating techniques developed in the holographic context in [16] can be also successfully used to construct AF BH solutions sourced by a scalar behaving at $r = \infty$ as an harmonic function, $\phi = 1/r$.

The structure of the paper is as follows. In Sect. 4.2 we present the review the solution-generating technique of [16]. In Sect. 4.3 we rederive the JNWW solutions and discuss their main features. The boundary conditions on the scalar field and the corresponding asymptotic behavior for $V(\phi)$ are discussed in Sect. 4.4. In Sect. 4.5 we present our hairy BH solutions. The thermodynamical behaviour of our solutions is discussed in Sect. 4.6. Finally, in Sect. 4.7 we present our conclusions.

4.2 The Solution-Generating Technique

We consider Einstein gravity in four spacetime dimensions minimally coupled to a scalar ϕ ($\mathscr{R}$ is the scalar curvature),

$$A = \int \mathrm{d}^4x\sqrt{-g}\left(\mathscr{R} - 2(\partial\phi)^2 - V(\phi)\right) \tag{4.1}$$

and static, spherically symmetric solutions of the field equations,

$$\mathrm{d}s^2 = -U(r)\mathrm{d}t^2 + U^{-1}(r)\mathrm{d}r^2 + R^2(r)\mathrm{d}\Omega^2, \tag{4.2}$$

where $\mathrm{d}\Omega^2$ is the metric element of the two-sphere S^2.

Finding exact solutions of the field equations stemming from the action (4.1) is a very difficult task even for simple forms of the potential V. To solve the fields equation (FE) we use the solution generating technique developed in [16] to find asymptotically AdS solutions once the scalar field profile $\phi = \phi(r)$ is given. Using the variables introduced in [16]

$$R = e^{\int Y}, \quad u = UR^2, \tag{4.3}$$

the field equations take the simple form.

$$\begin{aligned} Y' + Y^2 &= -(\phi')^2, &(4.4)\\ (u\phi')' &= \frac{1}{4}\frac{\partial V}{\partial \phi}e^{2\int Y}, &(4.5)\\ u'' - 4(uY)' &= -2, &(4.6)\\ u'' &= 2 - 2Ve^{2\int Y}. &(4.7) \end{aligned}$$

Equations (4.4) for Y (Riccati equation) and (4.6) for u are universal, they do not depend on the potential. One starts from a given scalar field profile $\phi(r)$ and solves the Riccati equation for Y. Once Y is known can easily integrate the linear equation for u, (4.6) to obtain

$$u = R^4\left[-\int \mathrm{d}r\left(\frac{2r + C_1}{R^4}\right) + C_2\right], \tag{4.8}$$

where $C_{1,2}$ are integration constants.

The last step is to determinate the potential using (4.7)

$$V = \frac{1}{R^2}\left(1 - \frac{u''}{2}\right). \tag{4.9}$$

This is a very efficient solving method, very useful in the holographic context, allowing to find exact solutions of Einstein-scalar gravity in which the potential is not an input but an output of the theory.

4.3 The JNWW Solutions

The parametrization (4.3), allows a simple (re)derivation of solutions for $V = 0$ (the JNWW solutions). Equation (4.7) gives u as a quadratic function of r, (4.5) and (4.6) give $\phi(r)$ and $R(r)$, whereas the Riccati equation simply constrains the parameters,

$$U = \left(1 - \frac{r_0}{r}\right)^{2w-1}, \; R^2 = r^2\left(1 - \frac{r_0}{r}\right)^{2(1-w)}, \; \phi = -\gamma \ln\left(1 - \frac{r_0}{r}\right) + \phi_0, \; w - w^2 = \gamma^2. \tag{4.10}$$

According to old no-hair theorems, for $0 < w < 1$ the solution is not a BH (V=0) but interpolates between Minkowski space at $r = \infty$ and a naked singularity at $r = r_0$ (or $r = 0$). Nevertheless the solution is of interest for several reasons. The BH mass is $M = 8\pi(2w - 1)r_0$. We can have a solution with zero or positive mass even in the presence of a naked singularity. In particular for $w = 1/2$ we have $M = 0$, a degeneracy of the Minkowski vacuum. The JNWW appears as the zero charge limit of charged dilatonic black holes. Near to the singularity the solution has a scaling behavior typical of hyperscaling violation [17].

4.4 Asymptotic Behavior of the Scalar Field and of the Potential

We are looking for AF BH solutions sourced by scalar field, which decays as $1/r$. We also assume that the Minkowki vacuum is at $\phi = 0$ and that it is an extremum of the potential with zero mass: $V(0) = V'(0) = V''(0) = 0$. These conditions imply that near $\phi = 0$ the potential behaves as $V(\phi) = \mu\phi^n$ with $n \geq 3$. The corresponding asymptotic behavior for the scalar is determined by using the boundary conditions at $r = \infty$ $u = r^2$, $R = r$ in the FE. For $n = 5$ we get $\phi = \frac{\beta}{r} + \mathscr{O}(1/r^2)$. Hence, an harmonic decay of the scalar field requires a quintic behavior for the potential V.

4.5 Black Hole Solutions

Let us now use the solution-generating method of Sect. 4.2. We need an ansatz for the scalar. We use the JNWW scalar profile (also previously used to in the literature to derive AdS BHs): $\phi = -\gamma \ln(1 - r_0/r)$. The Riccati equation gives the form of the metric function R: $R^2 = r^2(1 - r_0/r)^{2(1-w)}$, $\quad w - w^2 = \gamma^2$, $1/2 \leq w < 1$. We get three different class of solutions ($X = 1 - r_0/r$),

$$U(r) = X^{2w-1}\left[1 - \Lambda\left(r^2 + (4w-3)rr_0 + (2w-1)(4w-3)r_0^2\right)\right] + \frac{\Lambda r^2}{X^{2(w-1)}} \tag{4.11}$$

$$U(r) = \frac{r^2}{r_0^2} X\left[\left(1 + r_0^2\Lambda\right)X - 2r_0^2\Lambda \ln X + \left(1 - r_0^2\Lambda\right)X^{-1} - 2\right], \tag{4.12}$$

$$U(r) = \frac{r^2}{r_0^2} X^{1/2}\left[\left(1 + \frac{r_0^2\Lambda}{2}\right)X^2 - 2\left(1 + r_0^2\Lambda\right)X + r_0^2\Lambda \ln X + 1 + \frac{3r_0^2\Lambda}{2}\right] \tag{4.13}$$

respectively for $1/2 < w < 1$, ($w \neq 3/4$), $w = 1/2$ and $w = 3/4$. The corresponding potentials are given by,

$$V(\phi) = 4\Lambda\left[w(4w-1)\sinh\frac{(2w-2)\phi}{\gamma} + 8\gamma^2\sinh\frac{(2w-1)\phi}{\gamma} + \right.$$
$$\left. +(1-w)(3-4w)\,\sinh\frac{2w\phi}{\gamma}\right], \tag{4.14}$$
$$V(\phi) = 4\Lambda\left[3\sinh 2\phi - 2\phi\,(\cosh 2\phi + 2)\right], \tag{4.15}$$
$$V(\phi) = \Lambda\left(8\sqrt{3}\phi\cosh\frac{2\phi}{\sqrt{3}} - 9\sinh\frac{2\phi}{\sqrt{3}} - \sinh 2\sqrt{3}\phi\right) \tag{4.16}$$

The previous solutions describe a one parameter family of AF black holes sourced by a scalar field behaving asymptotically as $1/r$ and with a curvature singularity at $r = r_0$ (or $r = 0$) and a regular event horizon at $r = r_h$. The scalar charge is not independent. Near to the singularity the solution have the same scaling behavior of the JNWW solutions. As expected near $\phi = 0$ the potential has always a quintic behavior. The existence of these BH solution represent a way to circumvent old and new no-hair theorems. In fact the potential V is not semipositive definite, it has an inflection point at $\phi = 0$ and is unlimited from below. The ADM mass is not semipositive definite (the PET is violated).

4.6 Black Hole Thermodynamics

Scalar charge σ is not independent from the mass but determined by the BH mass M, implying the absence of an associate thermodynamical potential. The First principle has therefore the form $dM = TdS$, where the temperature T and the entropy S are given by the usual forms $T = \left.\frac{U'}{4\pi}\right|_{r=r_h}$, $S = 16\pi^2 R^2|_{r=r_h}$. For $w = 1/2$ we have

$$T(\omega) = \frac{\sqrt{\Lambda}}{4\pi\sqrt{l}}\left[2\left(1-\frac{2}{\omega}\right)\ln(1-\omega) - 4\right], \quad S(\omega) = \frac{16\pi^2}{\Lambda l}\left(\frac{1}{\omega^2} - \frac{1}{\omega}\right) \tag{4.17}$$

where l is a function of ω defined implicitly by $2(1-\omega)\ln(1-\omega) - \omega^2(1+l) + 2\omega = 0$. We have an extremal low-mass state with non vanishing mass $M_m in$, zero entropy and infinite temperature. In the large mass (small temperature) limit we get the Schwarzschild behavior for the thermodynamical potentials: $M = 2/T,\ S = 1/T^2,\ F = M - TS = 1/T$. For $w = 3/4$ we have for T and S a different behaviour (see [23]). Both the low and large mass regimes have the Schwarzschild behavior. The extremal state has $M = S = 0$ and $T = \infty$. The thermodynamical behaviour of the solutions with $1/2 < w < 3/4$ and $3/4 < w < 1$ are similar respectively to the cases $w = 1/2$ and $w = 3/4$.

4.7 Concluding Remarks

AF BH solution sourced by a scalar field with $1/r$ fall-off do exist but require a potential unlimited from below. Because $\phi = 0$ is an inflection point for V, the $\phi = 0$ Schwarzschild black hole is unstable. For $3/4 \leq w < 1$ BH thermodynamics is similar to Schwarzschild. For $1/2 \leq w < 3/4$ the low-mass regime drastically different. Near to the $\phi = 0$ Minkowski vacuum V has a quintic behaviour. The corresponding Field theory is not renormalizable. It cannot be fundamental. However it could represent an effective description arising from renormalization group flow.

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