Chapter 12
Aspects of Quantum Chaos Inside Black Holes

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12.1 Introduction and Conclusions

Theoretical physicists are all agreed that Semiclassical Black holes are paradoxical objects (as nicely reconfirmed by several discussions during the Karl Schwarzschild meeting 2015). However, a clear strategy in order to solve this problem is still unknown.

In this paper, we would like to suggest that infalling information could be chaotized inside a black hole. Our claim is related to a different picture about quantum black holes' nature: we retained unmotivated to think seriously about a quantum black hole as a conformal Penrose's diagram, i.e as a smoothed semiclassical geometry with a singularity in its center (eventually cutoff at the planck scale). In particular, one could expect that, in a "window" of length scales among the Schwarzschild radius and the Planck scale, there is a non-topologically trivial region of space-time rather than a smoothed one. A realistic black hole could be a superposition of different horizonless solutions, perhaps associated GR gravitational instantons or "exotic" gravitational instantons.[1] In this picture, a black hole' horizon is an approximated Chauchy null-like surface (for energy scales closed to an inverse Schwarzschild radius). However, for lenght scales L in the range $l_{Pl} << L << R$, geometrical deviations and asperities with respect to semiclassical smoothed geometries are reasonable expected. In this regime, gravitational interactions among horizonless geometries can be neglected as well as microscopical exchanges of matter and gauge fields among their surfaces. In

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[1]In string theory, the class of instantons is much larger than in field theories. Applications of a particular class of these solutions in particle physics were recently studied in [1–10].

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this sense, a black hole cannot be described by a single Penrose's diagram at all the lenght scales. In particular, in the "middle region" a black hole would be described by a superposition of a large number of Penrose's diagrams.

Such a black hole can be rigorously defined in an euclidean path integral formulation. It emits a thermal radiation like a semiclassical BH, with small corrections on Bekenstein–Hawking entropy [1].

At this point, a further question is the following: what happen to infalling informations in such a "scale variant" system? Let us consider the usual thought experiment of a infalling radiation in a quantum pure state, with a very small initial frequency $\omega \simeq R^{-1}$. Such a radiation will start to probe a smoothed semiclassical geometry of a black hole, near the horizon. However, radiation will be inevitably blueshifted inside the gravitational potential of a black hole, i.e its De Broglie wave length starts to be smaller than the Schwarzschild's radius. So that, infalling radiation will start to probe the middle region before than the full quantum quantum regime. In the middle region, radiation is scattered back and forth among asperities that usually are not present at all in semiclassical BH solutions. As a consequence, radiation will be chaotically diffracted inside this system. At that middle scales, a black hole is a sort of *space-temporal chaotic Sinai billiard* rather than a smoothed manifold. Usually, in simpler classical chaotic billiards than our one, chaotic zones of unstable orbits trapped forever in the system are formed. Simple examples of such a trapped paths: (i) an orbit trapped in back and forth scatterings among the asperity A and the asperity B (AB segments); (ii) one trapped among A, B, C asperities (triangular orbits); and so on. Considering quantum fields rather than classical trajectories, one has also to consider quantum transitions induced by inelastic scatterings on gravitational backgrounds $< g, \ldots, g >$ (thought as a vacuum expectation value of gravitons). $\phi + < g, \ldots, g > \rightarrow X + < g, \ldots, g >$ where ϕ is a generic gauge/matter field, and X is a collection of N fields. For example a process like a photon-background scattering

$$\gamma + < g, \ldots, g > \rightarrow q\bar{q} + < g, \ldots, g > \rightarrow \text{hadronization} + < g, \ldots, g >$$

will lead to a complicated hadronic cascade of entangled fields. As a consequence, such a system is even more chaotic than classical one. So that, a part of the initial infalling information is effectively fractioned in a "forever" (black hole lifetime or so) trapped part and another one, so that

$$|\text{IN}\rangle = a|\text{OUT}\rangle + b|\text{TRAPPED}\rangle$$

where a, b parametrize our ignorance about the space-time billiard, $|\text{OUT}\rangle$ is emitted as Bekestein–Hawking radiation. As a consequence, the in-going information is a linear combination of outgoing informations and trapped informations during $0 << t << t_{Evaporation}$. In this picture, information paradoxes are understood as an *apparent* losing of unitarity. In fact, $|\text{IN}\rangle \rightarrow |\text{OUT}\rangle$ is not allowed by quantum mechanics: $|\text{IN}\rangle$ is a pure state, while $|\text{OUT}\rangle$ is a mixed one. However, also $|\text{TRAPPED}\rangle$ is a mixed state, and a linear combination of two mixed states can

be a pure one. In this approach, a $|\text{IN}\rangle \rightarrow |\text{OUT}\rangle$ transition can be effectively described in a density matrix approach, with an effective non-unitary evolution. However, unitarity is not lost at fundamental level because of the real transition $|\text{IN}\rangle \rightarrow a|\text{OUT}\rangle + b|\text{TRAPPED}\rangle$ is not contradicting unitarity. Let us consider, for example, a (famous) Bekenstein–Hawking particle-antiparticle pair created nearby the black hole horizon. As usual, one of the two is captured inside the black hole space-like interior, while the second one can tunnel outside the horizon. As well known, the two particles are entangled, and this will lead to the undesired fire-wall paradox. However, in a frizzy black hole, the infalling particle will start to be blueshifted so that it will start to scatter back and forth inside the system, giving rise to an exponentially growing cascade of N particles continuing to scatter and to scatter in the billiard. The process will be even more chaotic in a realistic case in which a large number of infalling partners from a large number of Bekenstein–Hawking pairs have to be considered. As a consequence, P outgoing pairs will be entangled with a total number $N >> P$ of particles inside the system. This practically disentangles the P outgoing pairs from the original ones, as a quantum decoherence effect induced by the non-trivial space-time topology. In other words, the space-time topology is collapsing the entangled wave function as a quantum decoherence phenomena, as well as two entangled pairs are disentangled by a an experimental apparatus. The entanglement entropy is linearly growing with the number of back and forth scatterings n of a particle, because of the density matrix of the internal black states are exponentially growing with n:

$$S_{\text{interior}} = -Tr\rho_{\text{interior}}\log S_{\text{interior}} \sim n$$

so that is growing with time. On the other hand, for P Bekenstein–Hawking particles $S_{\text{int}} \sim n\log P$. Our model predicts $S_{B.H.} \sim P$ from entanglement entropy definition.

However, if a frizzy black hole emits a Bekenstein–Hawking radiation with small deviations from thermality, it cannot have an infinite life-time. On the other hand, the non-trivial topological space-time configuration of a frizzy black hole is sourced by the black hole mass. The final configuration after the complete black hole evaporation is a Minkowski space-time with a dilute residual radiation. As a consequence, a space-time phase transition from the "frizzy" topology to the Minkowski space-time is expected at the Page time or so. As a consequence, chaotic saddles of trapped information will be emitted in the environment as a *final information burst*. For this motivation, the S-matrix describing BH evolution from the initial collapse/formation to its complete evaporation is unitary:

$$\langle\text{COLLAPSE}|\text{S}|\text{EVAPORATION}\rangle$$

$$= \langle\text{TOTAL INFALLING}|\,\text{S}|\,(a|\text{TRAPPED}\rangle + b|\text{OUT}\rangle)$$

The trapped probability density $\rho(T)$ is approximately described by

$$\frac{\mathrm{d}\rho(T)}{\mathrm{d}T} \sim -\frac{1}{T^2}e^{-\Gamma T}$$

In fact, $\rho(T)$ is dependent by the number of asperities N_s as $\rho \sim N_s e^{-\Gamma T}$, where Γ is proportional to *effective average deepness* of asperities (trapping ρ). But the number of asperities is depending by the Black hole mass. In turn, the black hole mass decreases with the temperature as $\mathrm{d}M/\mathrm{d}T = -1/8\pi T^2$.

To conclude, chaotic aspects of quantum black holes could be relevantly connected to the information paradoxes. In particular, a semiclassical black hole could be reinterpreted as a superposition of horizonless geometries, chaotizing infalling informations. Such an approach could have surprising connections with recent results in contest of AdS/CFT correspondence [11–13].

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