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The Early Oligopolistic Models: Market Power in the Paretian Tradition

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9.1 Introduction

In economics, the idea of market power is often associated with the functioning of non-competitive markets. In such markets, firms can set a price over marginal costs.[1] The study of non-competitive markets was a very active research field in the period between the two World Wars also called the years of high theory, to use George L.S. Shackle's (1967) useful expression. During that period, there was not only theoretical interest in this topic but also the aim to find explorations for great emerging transformations in the economic structure, chief among them the growing concentration of large industrial and financial firms. Heinrich von Stackelberg's book, *Market Structure and Equilibrium* (2011), vividly

[1] This difference can be considered a rough measure of market power as in the case of Lerner Index (Giocoli 2012).

Paper presented at the AISPE Conference – Lecce, 28–30 April 2016.

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M. Mosca (ed.), *Power in Economic Thought*, Palgrave Studies in the History of Economic Thought, https://doi.org/10.1007/978-3-319-94039-7_9

illustrates the complexity and richness of the debate among mathematical economists of the period.

The mathematical treatment of non-competitive markets was proposed by Francis Edgeworth in the seminal 1897 article, "La teoria pura del monopoli," published in Italian in *Giornale degli Economisti*. Maintaining a critical attitude toward Cournot, Edgeworth concluded that in a duopolistic market, it would be impossible to establish a stable equilibrium in production, and therefore in price, because of the interdependence of firms' behavior. This problem would later be taken up from the mathematical point of view by Pareto in the "Mathematical Appendix" of the French edition of his *Manuale* (1909). In the following decades, the mathematical economics literature grew owing to contributions by eminent scholars, such as Joseph Schumpeter (1908), Pareto (1909), Cecil Pigou (1924), Luigi Amoroso (1921, 1930), Arthur Bowley (1924), Edgeworth (1922, 1925), Knut Wicksell (1926), Harold Hotelling (1929), Edward Chamberlin (1929), and Wassily Leontief (1935). The Italian followers of Pareto participated actively in this debate, and indeed Amoroso played a prominent role. In Europe, the Italian economist had always been considered the most authoritative figure in the field and a fierce defender of Cournot's approach. Other Paretians who made important contributions were Arrigo Bordin (1934, 1936), Felice Vinci (1944), and especially Emilio Zaccagnini (1947), who made novel revisions to Pareto's ideas in the 1940s.

This chapter focuses on a particular strand of literature from the interwar period, that is, the analytical developments developed within the Paretian tradition, starting with the original position of Pareto. This historical retrospective is useful for assessing the main problems that emerged in the theory of non-competitive markets. First, the chapter discusses how economists anticipated important analytical results, whose developments only emerged to their full extent in the postwar period. Second, by considering the theory of non-competitive markets in relation to its historical perspective, rather than in relation to other aspects of economic theory, the chapter sheds light on the internal dialectic of economic reasoning between the search for analytical rigor and the necessity of recognizing its full interpretative value.

The chapter is structured as follows. The next section outlines the Paretian approach, and the third section is dedicated to the change brought about by Amoroso, the most brilliant exponent of the Paretian School. In the fourth section, Edgeworth's controversial response and Amoroso's subsequent reply are considered. The fifth section illustrates the different perspective that Amoroso adopted in the 1930s. The sixth section discusses the new mathematical tool developed by de Finetti, the theory of simultaneous maxima. The seventh section examines the renewal of Paretian theory by Emilio Zaccagnini, a mathematician and exponent of the third Paretian generation. The final section provides the concluding remarks.

9.2 Pareto's Ambiguous Position

This section provides a brief overview of Pareto's position. Pareto's "Mathematical Appendix" of the French edition of his *Manuale* (1909) deals with the issue of non-competitive markets in mathematical terms, focusing on the case of duopoly. Paragraph 69 of the text presents the case of two firms and one good, and the subsequent paragraph 70 deals with the case of two firms and two goods. For our purposes, the relevant case is the former. In the discussion, Pareto confronts Edgeworth, albeit without naming him. In a previous long essay published in *Giornale degli Economisti* (1897), Edgeworth concluded that in the case of duopoly, the solution presented an oscillatory character. The individualistic actions of the two firms had continuous fluctuations in quantities produced, and therefore in price. In mathematical terms, the solutions were indeterminate. Therefore, Pareto contests this conclusion, affirming that the problem is rather impossible, since the number of equations is greater than the number of unknowns.

As regards Pareto's reasoning (Stackelberg would in 1934 speak of Pareto's monopoly), in the duopolistic market, price depends on quantities produced together, $p = f(q_1 + q_2)$. Assuming for simplicity, like Pareto, that the costs are null, the profit will be $\pi_1 = q_1 f(q_1 + q_2)$ for one firm and $\pi_2 = q_2 f(q_1 + q_2)$ for the other. Pareto assumes that each firm behaves like a monopolist. This hypothesis, from a mathematical point of view, requires that we have four partial derivatives to consider in the process of optimization. One of the four conditions can be eliminated using the market demand function, but three equations remain in the two unknowns—that is, the quantities produced by the two firms. At the end of his brief discussion, Pareto (2014 [1909]) observes the following:

> From a mathematical point of view it is inaccurate to say, as is often the case, that in the case of two monopolists and one commodity, the equilibrium is indeterminate. On the contrary it is overdetermined since the conditions are imposed that are incompatible. (p. 348)[2]

Pareto therefore refutes Edgeworth's approach, and it is worth noting that he also completely ignores Cournot's approach. The reason for this unsatisfactory state of oligopoly theory at the beginning of the century is clearly stated by Pareto (2014 [1909]) in the following paragraph:

> § 73 From an economic point of view, [the duopoly] it may be observed in the case of the problem in §69 [the duopoly] that by assuming there to be a position in which one of the monopolists obtain s_1, and the other obtains s_2, it is sufficient for the first one to lower his price to increase his gain and reduce his competitor's share to zero; and vice versa. It is therefore impossible to obtain a solution of the problem we have posed, since no position s_1, s_2 is an equilibrium position. (p. 350)

To find a solution, Pareto proposes abandoning pure economics and viewing this problem from another angle, concluding as follows:

> § 75. It is idle to ask pure economics what will happen when two individuals who have the power to act as monopolists in the sale of one and the same commodity confront each other. Pure economics, by telling us that it is impossible for these two individuals to use their monopoly power in fact, for them both to behave according to type II, has answered as full as it can.
>
> Pure economy cannot even tell us that the two individuals will go back and forth infinitely between two extreme positions of equilibrium. This is no way results from the fact that equilibrium is determined by two incompatible equations. (p. 351)

[2] The translation of quotations is mine, except when indicated otherwise.

In the Paretian framework, the interdependence of oligopolistic behavior creates situations that are impossible to model in mathematical terms. In Pareto's view, only factual experience can determine what the final economic equilibrium will be. In this case, pure economics is overtaken by both applied economics and sociology. In general, for Pareto, in the oligopolistic market, the solution will not be unique; depending on the particular circumstances, there would be many solutions to verify on a case-by-case basis. Therefore, for Pareto, the theory of non-competitive markets remains an open theory depending on institutional and historical elements.

9.3 Amoroso's Turn in *Lezioni di Economia Matematica* (1921)

Pareto's view was immediately adopted by the young Amoroso, the main exponent of the Paretian School, whose early articles were devoted to disseminating and clarifying his mentor's ideas (Amoroso 1909). The theory of general economic equilibrium was seen with some suspicion in Italy, as it was judged to be too abstract, both for its unrealistic assumptions and for the wide use of mathematics. Amoroso's 1909 article, "La teoria dell'equilibrio economico del prof. Pareto,"[3] considers the case of duopoly and provides the following observation:

> Edgeworth's solution is incorrect: but Loria's article adds nothing. A rigorous solution is given to the problem in *Manuale*. There are two missing equations in system (C), ϕ_{1y} and ϕ_{1z}. All the unknowns can be expressed in function of two of them. These are determined by the two monopolists, who can impose on each one a single condition. However, it is not possible for both to achieve the maximum profit: these two conditions are incompatible. If they swing from one position to another, if they agree, if one dominates the other, or if there is something else which changes the framework of the problem, it is not a task of pure economics. It is a kind of research belonging to applied economics or rather to sociology. Pure economics tells us that the conditions set out in our problem are incompatible, and that is enough. (Amoroso 1909, p. 364)[4]

[3] English transl. "The theory of general equilibrium by prof. Pareto."

[4] The translation of quotations are mine.

The young Amoroso was in agreement with Pareto. He shared the idea that a mathematical treatment of this kind of markets was not possible for logical reasons and that economic theory should have been supplemented with sociological considerations. However, his initial position changed at the end of the subsequent decade. Before becoming an economist, Amoroso was a mathematician; thus, he could not be satisfied with the solution proposed by Pareto.

The 1921 book *Lezioni di Economia Matematica* constitutes Amoroso's first mature and very innovative contribution to pure theory. The book is a collection of his lectures on mathematical economics at the University of Bari (Italy). These lectures present deep mathematical analyses—the first in Europe—of the theory of oligopoly markets, or the theory of n monopolists, to use the term of that period. Seeking a coherent mathematical treatment of this topic, Amoroso chose to adopt Cournot's approach. In *Lezioni*, whose content would be strengthened and further developed in his 1930 article "La curva statica di offerta," Amoroso develops oligopoly theory following Cournot's suggestions. Consequently, he would go on to be considered in the interwar period as the main and most prestigious defender of Cournot's approach internationally (Edgeworth 1925).

Amoroso presents Cournot's approach as the natural extension of the monopolistic scheme in the case of n producers of the same good—that is, the oligopolistic market. In analytical terms, the system to be solved consists of $n+1$ equations (n profit equations of n monopolists and the demand function) that determine $n+1$ unknowns, the quantities produced by each firm, and the price, in the spirit of general equilibrium. The maximization of the profits of n firms, under the constraint of a demand curve, determines the necessary equations to obtain equilibrium in the market.

Amoroso's published lectures focus essentially on two problems. The first is the existence of a solution (§ 37) that is easily resolved by considering, similar to Cournot but unlike Pareto, that each oligopolistic firm considers as given the production of all the others. This individualistic behavior of the firm determines a very peculiar equilibrium defined by Amoroso in metaphorical terms as an *equilibrium of war* (*Lezioni*, p. 210), since it arises from the contrast of egoistic interests. Therefore, Amoroso was able to overcome the difficulties encountered by Pareto without ever naming him. Following the strategy proposed by Cournot, according to Amoroso, every firm maximizes its proper level of production, and it makes no sense to assume that one firm takes into consideration the production of other firms inside its profit function. The same criticism was vigorously expressed by Chamberlin in the 1929 article, "Value where sellers are few," containing a detailed review of the literature on oligopolistic models including the Amoroso's model.

In keeping with his usual style of reasoning, Amoroso proffers a numerical example. After making the necessary calculations, he notes that the equilibrium values found have a peculiar feature: neither of the two firms has the convenience of varying the quantity produced, given the production of the other. Thus, it is an equilibrium that is optimal in a very restricted meaning. From the historical perspective, we can say that we are faced with a rudimentary early definition of what would be the equilibrium of optimal response or the Nash equilibrium 30 years later. Amoroso does not capture all the potentialities of this new formulation of economic equilibrium, which would be possible only after the publication of *Theory of Games and Economic Behavior* (1944) by John Von Neumann and Oskar Morgenstern and the birth of game theory.

The second problem considered in *Lezioni* is the optimality of the equilibrium reached not only for the individual firm but also for the society. Amoroso observes that if the firms behave like a single monopolist and try to maximize joint profits, a different position of equilibrium will be achieved, which will be more convenient for each oligopolistic firm. In this case, production is lower and the price is higher. Amoroso demonstrates that whether the production in this collusion will be greater or lower depends on the structure of the marginal costs of the single firm compared with the marginal costs of the entire sector. The subsequent problem of the distribution of total profit among individual firms is considered by Amoroso as an institutionally relevant element, but one that is outside of economic theory. Amoroso's approach, which is based on Cournot's model, was followed in the Paretian tradition by Arrigo Bordin, a second-generation exponent of the Paretian School in Italy. The topic is developed in his book, *Lezioni di Economia Politica* (1936), by introducing a novel formal analysis of the optimal dimension of a cartel in the case of an oligopolistic market.

Owing to these works, Amoroso earned a solid reputation as a mathematical economist. In particular, he was considered to be the main exponent of Cournot's approach to the analysis of non-competitive markets. However, this assessment would prove not to be very rewarding, as the community of mathematical economists viewed Cournot's approach as highly unsatisfactory (Chamberlin 1929). The main criticism concerned Cournot's lack of realism. If the mathematical argument was strong and unquestionable, the relevant issue called into question the interpretative power of the model. It seemed to the small community of mathematical economists, from Edgeworth to Pigou and von Stackelberg, that Cournot's assumptions were erroneous because they were in stark contrast to the facts of the economic reality. The hidden contrast, always present in economics, between the rigor of mathematical models and interpretative realism, was very strong. In the interwar period, the pendulum swung in favor of the request for greater realism.

9.4 Edgeworth's Criticism and Amoroso's Response

The paragraphs on the theory of oligopoly contained in *Lezioni* had great resonance internationally, as they induced Edgeworth's criticism. In the following year, Edgeworth responded in *The Economic Journal* via an extensive review of Amoroso's book, with the main part dedicated to providing a critical interpretation of Amoroso's duopoly theory. Edgeworth was very critical of the Paretian scholar, almost to the point of being harsh. In his review, he observes the following:

> We believe that Professor Amoroso is alone among high authorities in siding with Cournot in this matter. The view that in monopolistic competition "the output is indeterminate" … "Is now commonly accepted," says Professor Pigou; And, he adds, "it seems to me to be the right one" (Wealth and Welfare, p. 193). Altogether our author's teaching about duopoly cannot be considered as part of accepted science. We should recommend the omission of this topic, if it was proposed to translate the work into English

with the view of supplying the much-felt need of an introduction to mathematical economics. (p. 405)

The critical argument was directed essentially toward Cournot's model, which was considered a misleading representation of the functioning of the oligopolistic market. Edgeworth's reasoning, however, proceeded in a indirect way. Since there were no formal or conceptual errors in Amoroso's analysis, Edgeworth chose to use the same numerical example employed by Amoroso adopting prices as strategic variables, as in Bertrand's model, instead of quantities. He demonstrated numerically that in this case, a position of stable equilibrium was impossible and that the price was ever oscillating. Ultimately, Edgeworth intervened to defend the model that he had advanced in 1897 rather than to contest that of Amoroso with purely external criticism.

Amoroso did not respond to Edgeworth's criticism. He revisited this topic only in the 1930 article, "La curva statica di offerta." This work represents a highly relevant contribution (Keppler 1994) in which Amoroso introduces new concepts for analyzing the construction of the firm supply function, such as the relevance of the minimum average cost or the concept of the monopoly power index, whose discovery would be attributed to Abba Lerner in an article printed in English in 1934 (Giocoli 2012). In the third part of "La curva statica di offerta," Amoroso returns to the problem of the duopolistic market and reaffirms the centrality of the Cournot model, but adopts a different approach, focusing on the problem of the stability of the equilibrium obtained. Through a numerical example, Amoroso shows that it is possible to determine a stable equilibrium position depending on the slope of the reaction curves. Thus, he rejects the cooperative solution and instead develops the case for using a nonlinear demand function.

For our purposes, it is relevant to consider the final part of the article, in which Amoroso distinguished his position from those of Bertrand–Edgeworth and Pareto. According to Amoroso, the Edgeworth–Bertrand approach was insufficient because it assumed that each of *n* monopolists tried to put the other firms out of the market. In his opinion, this way of thinking diverged widely from the methods of mathematical economics. He observed the following:

> It also means that Cournot is right and Bertrand and Edgeworth are wrong. In the mathematical theory of duopoly, it is not about predicting what one or the other will do in a certain factual situation, but it is interesting to look for what the two would find worth doing if either of them were pure homo oeconomicus. (Amoroso 1930, p. 17)

He also rejected Pareto's criticism of Edgeworth, since he thought it was based on a logical mistake; that is, it was not reasonable to assume that both duopolists could behave like monopolists. This strong hypothesis was considered incorrect because "It means that it is not possible for A to force B to do his will and even at the same time B forces A to do his" (Amoroso 1930, p. 19). Cournot's idea was different, according to Amoroso, because it involved the assumption that each duopolist was able to achieve the maximum profit within his sphere of action. At the thresholds of the theoretical revolution provoked by Joan Robinson and Edward Chamberlin with the appearance of the theory of imperfect competition, Amoroso remained one of the few within the community of mathematical economists to support the validity of the Cournot model.

9.5 The Partial Monopoly in *Meccanica Economica* (1942)

In the 1930s, Amoroso's research focused primarily on dynamic analysis, returning to the youth project to dynamize the theory of general economic equilibrium (Pomini and Tusset 2009). However, he also revisited the topic of non-competitive markets from a different perspective. The international debate, above all with Chamberlin's publication in 1933, had taken a very specific direction, and Amoroso was trying to adapt to the new context. His first contribution was "La produzione in regime di concentrazione industriale" (1935), followed by Chapter X of the textbook *I Principi di Economia Corporativa* (1938). He established his definitive position in "Lezione XI. Monopolio totale e parziale" included in a collection of lectures on mathematical economics delivered at Istituto di Alta Matematica in Rome entitled *Meccanica Economica* (1942)

Amoroso's new and different perspective took up the problem of analyzing a specific criterion of price formation in the oligopolistic market or in the context of a concentration of firms (Mistri 1970, Gaeta 1967). Since Cournot's scheme was inadequate in this case, it was necessary to change the research direction. Amoroso resumed the equation of monopolistic pricing put forward in the 1930 article and introduced some modifications to account for the relevance of the market share of each firm. The 1935 essay provides the following price equation:

$$\frac{p-m}{\eta p}=\frac{x}{x+y\left(1+\sigma\right)} \tag{9.1}$$

In Eq. (9.1), the term η indicates the elasticity of demand, x represents the production of a single firm, y indicates the production of the other firms, and the parameter σ represents the new element, with respect to the previous article, measuring the variation of the production of other firms when the production of a single firm changes. According to Amoroso (1935), this extension of the previous equation was necessary because "in an industrial concentration scheme, the choice of production of a single firm must take into account three things: its costs, what it does, what the competitors do" (p. 951). The new element that Amoroso tried to formalize using the traditional instruments was the interdependence of choices in the oligopolistic market. Equation (9.1) is generalized in the lectures from 1941 to 1942 and assumes the following definitive form:

$$\frac{p-m}{\eta p}=\frac{u}{D+\sigma E}=S \tag{9.2}$$

In the new Eq. (9.2), the term D represents the total demand, E the residual demand of the market, and parameter u the quantity produced by a single firm. In general terms, this equation can assume a value between zero and one. The case $S = 0$ is perfect competition, where the price coincides with the marginal cost. The case $S = 1$ is when the entire production is concentrated in a single firm and the market becomes monopolistic. As Amoroso (1942) observes, “The quantity *S* can therefore be taken as an index of the power of the firm in the market: the greater its value, the greater the possibility for the firm to influence the price, the more accentuated its monopolistic character” (p. 116). In light of Eq. (9.2), Amoroso can be said to have found the general expression for price determination in different market forms (Giocoli 2012).

In the Paretian tradition, the next step of the theory of the oligopolistic market is represented by the contribution of Zaccagnini, who tried to adopt the approach of Pareto in a different mathematical setting which was advanced by the Italian statistician de Finetti in the second half of the 1930s.

9.6 Some Mathematical Developments: de Finetti’s Theory of Simultaneous Maxima

In the second half of the 1930s, the young Bruno de Finetti was passionately engaged in the field of welfare economics. His seminal article, “Il tragico sofisma,”[5] was published in 1935 (de Finetti 1935a, b), and in the following years, he produced several related essays. His final contribution, published in 1943, was “La crisi dei principi e l’economia matematica”[6] (de Finetti 1943). The main target of de Finetti’s criticism was, in current terms, the first theorem of welfare economics. De Finetti’s criticism of Pareto’s general equilibrium theory led him to build the theory of simultaneous maxima, which is probably his most important contribution to the field of economic theory. The starting point was a problem that remained open in Pareto’s system. In fact, from a given initial allocation, Pareto’s optimum position could not be determined uniquely; however, there could be more than one optimum position, as in the well-known case of the Edgeworth box. Consequently, once again, there was a problem of choosing which allocation was preferable for the society as a whole. This problem had no solution within Pareto’s approach, as the utilities of each individual were not comparable like they were in traditional utilitarianism.

[5] English translation “The tragic sophism.”

[6] English translation “The crisis of principles and the mathematical economics.”

In his 1937 article, de Finetti dealt with a very general context, which he defined as simultaneous maximization. This type of optimization differed from traditional constraint optimization, which is the common case considered by economists, because the problem was to obtain the maximum values of many functions at the same time, in a particular way. These maximum results were simultaneous in the sense that it was not possible to increase the value of one function without decreasing that of another. The analogy with the case of the Pareto optimum—whose simultaneous maximization constituted a generalization—is evident. De Finetti offered an in-depth analytical discussion on this very peculiar situation. We consider the simplified case of two functions in two variables, $f(x, y)$, $g(x, y)$.[7]

De Finetti starts with the assumption that to obtain a solution of a simultaneous optimum, it is necessary for the total differentials of the two functions to cancel out; otherwise, it would be possible to increase the value of one of the two without decreasing that of the other. In our simplified case, the following expressions must not be greater than zero:

$$\begin{aligned} df &= f'_x\,dx + f'_y\,dy \\ dg &= g'_x\,dx + g'_y\,dy \end{aligned} \tag{9.3}$$

This was because, if all equations in system (9.3) were positive, no point of coordinates x, y could be the optimal one. Moreover, for both differentials of (9.3) to be null, it was necessary that the determinant of the coefficients dx, dy canceled out. Therefore, the following must occur:

$$\begin{vmatrix} f'_x & f'_y \\ g'_x & g'_y \end{vmatrix} = 0 \tag{9.4}$$

This equation represents the curve that contains the optimum points. De Finetti concludes this first step by showing how, in general, the points of the simultaneous maxima belong to a variety of dimension $n-1$, on

[7] In the article, de Finetti considers the general case with n functions and q variables. In the exposition, we follow Zaccagnini (1947).

which the determinant of the matrix of the partial derivatives of the functions to maximize is zero.

De Finetti then takes an additional original step with the aim of offering an operational version of the condition of simultaneous maxima. If the determinant of (9.4) is zero, it follows that the equations in (9.3) will be represented by a linear relationship. In this case, we can find two coefficients, λ_1, λ_2, connected by the following relationship:

$$\lambda_1 df + \lambda_2 dg = 0 \tag{9.5}$$

Substituting *df*, *dg* in (9.5) with their development given by (9.3), the system obtained is as follows:

$$\begin{aligned} \lambda_1 f'_x + \lambda_2 g'_x &= 0 \\ \lambda_1 f'_y + \lambda_2 g'_y &= 0 \end{aligned} \tag{9.6}$$

System (9.6) establishes parameters λ_1, λ_2. For these two parameters to be consistent with (9.4), it is necessary for them to have the same sign. These parameters are easy to determine because de Finetti shows that they are defined by the cofactors of the initial Jacobian matrix (9.4). In general, de Finetti shows that in order to verify if a specific arbitrary vector represents a position of simultaneous maxima for *n* functions, two conditions must be met: the determinant of the Jacobian matrix must be zero and its cofactors must all be of the same sign. These are necessary conditions that become sufficient to the extent that some restrictions are added, such as the concavity of the functions being considered. De Finetti states the following:

> The optimal point belongs to a variety of $n-1$ dimensions, for which the determinant of the partial derivatives cancels out. Knowing the value of the n cofactors, $\lambda_1, \lambda_2, \ldots, \lambda_n$ we can exclude that it is a point of optimum if two of them have opposite signs (de Finetti 1937a, p. 54).

Lastly, de Finetti identifies the entire set of optimal points, starting from the evident property that each point of the maximum of one of the *n* functions, given the value of the others, results in a point of optimum. Since this property is true for all points of optimum related to *n* functions, the conclusion is the topological hypothesis that the set of points of optimum form a simplex of $n-1$ dimension, whose faces are the points of optimum of a function with $n-1$ components, the $(n/2)$ corners for those of $n-2$ components, and so on, up to *n* vertexes, with each one representing the maximum of one of the *n* functions. This topological analysis of the positions of a simultaneous optimum occupies a relevant part of de Finetti's 1937 essay and demonstrates the high level of mathematics involved.

The question is what the implications of simultaneous maxima for economic theory are. They are undoubtedly deep, because the optimization process is at the basis of the economic agents' behavior. The fundamental implication for de Finetti is that, in this way, it is possible to prove formally that the optimal points are infinite, and therefore the identification between the Pareto optimum and free competition is purely arbitrary. For de Finetti, anarchic market forces reach only one of the many positions that have this property. Hence, it was de Finetti's belief that he had revealed the logical weakness of the sophism of economic liberalism. According to de Finetti (1937a),

> We demonstrate that normally in the case of n individuals, the points of optimum are∞^{n-1}. Suppose set the ophelimities $\Theta_1 = a_1$, $\Theta_2 = a_2$, …., $\Theta_{n-1} = a_{n-1}$ of $n-1$ individuals; on the variety so defined, the $\Theta_n = a_n$ will admit a maximum value, and therefore at least a point of optimum. Of such points there are at least ∞^{n-1}; they actually constitute a variety at $n-1$ dimensions. (p. 62)

Thus, de Finetti formally demonstrated a result that would become well known among Italian economists, especially those aligned with the Paretian School (Bordin 1948). Considering the problem from a different point of view, his result implied that the social optimum in Pareto's sense was always a relative optimum, depending on the initial distribution of resources for him. For him this fact was the essential point of the matter and not just a pessimistic assessment of perfect competition.

9.7 Zaccagnini and the Return to Pareto

The theory of simultaneous maxima was then developed by Emilio Zaccagnini.[8] As a graduate in mathematics, Zaccagnini can be considered as belonging to the third Paretian generation, along with Giuseppe Palomba[9] and Valentino Dominedò.[10] He followed the Paretian approach of his teacher Arrigo Bordin in Torino. His first contributions were wholly orthodox, dealing with barter theory and the problem of obtaining the demand function from the preference relation (Zaccagnini 1942). Subsequently, the main focus of his research was the application of de Finetti's simultaneous maxima theory to some topics of economic theory, in particular to oligopoly theory. In several articles (Zaccagnini 1947, 1953, 1958), he attempted to offer the theory of simultaneous maxima as a general methodology for studying many economic phenomena. The main contribution was the article "Massimi simultanei in economia pura" (1947) that was included in the first edition of the *International Economic Papers* (1951) with the title "Simultaneous Maxima in Pure Economics." For our purposes, we limit our attention to the theories of duopoly and oligopoly. Zaccagnini's project was to return to Pareto via the mathematical lens offered by de Finetti.

The starting point of the 1947 article is the traditional criticism of the lack of realism of Cournot's approach. This criticism, as we have seen, was widespread and had reached its peak in the 1930s. The problematic point was the fundamental assumption that each of the two firms considered as given, in its process of maximizing choice, the production of the other. In the essay, Zaccagnini (1947) observes the following:

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[8] Emilio Zaccagnini (1903–1979) held the chair of Economics at the Faculty of Law of Turin (Italy) from 1953 to 1974. He was also a Fellow of Econometric Society. Both his scientific training and his research career were centered on the theory of general equilibrium.

[9] Giuseppe Palomba (1908–1986), a second-generation member of the Italian Paretian School, devoted his scientific activity to devise an original axiomatic framework for economic dynamics, primarily designed to address economic change.

[10] Valentino Dominedò (1905–1985) did many contributions in the field of demand theory. He was the first economist at the international level to grasp the relevance of the Slutsky's article on the theory of consumers' choice.

> From this point of view, Cournot's hypothesis clashes with common sense and daily observations. It is evident that without experience it is not possible to conceive of any rational economic operation; how it is possible to admit that this experience teaches nothing to the two single firms in the market and it allows them to assume the Cournot hypothesis about the assumption of each operator regarding the quantity negotiated by the rival? (p. 262)

In addition, in Zaccagnini's view, Bowley's solution of conjectural variations could not be considered as an adequate answer, as it introduced a psychological element that was extraneous to economic reasoning. Zaccagnini recognized the need to return to Pareto's position and revise it in the light of the new mathematical contribution of de Finetti.

> To Cournot's solution and to these criticisms, Pareto opposed an important observation which in our opinion is decisive for the general approach to the problem. In fact, writes the ingenious scientist, the two quantities q_1 and q_2 are both variables and have to be considered in the two total profit functions to be maximized. The solution requires considering four partial derivatives. In this case, the number of variables is greater with respect to the number of equations and the problem is indeterminate. (Zaccagnini 1947, p. 264)

Moreover,

> The second solution originates from Paretian criticism and it is really general, as it excludes any subjective hypothesis of an operator on the behavior of the other, and only considers the hedonistic postulate in its more general expression and the simultaneity of the solutions. But it was the technical means adopted by Pareto that were unsuitable and that led the Author to an erroneous conclusion: it is not possible to resolve the two maxima separately when they behave simultaneously. By applying a proper mathematical technique, the problem does not seem determined, as was the case with Cournot, nor was it impossible, as Pareto stated. (Zaccagnini 1947, p. 266)

In Zaccagnini's view, the tool that could allow for overcoming the analytic impasse in Pareto's theory was the application of de Finetti's simultaneous maxima methodology. With this approach, it was possible, on the one hand, to maintain the Paretian assumptions and, on the other hand, to overcome the problem of the modest interpretative capacity of Cournot's perspective, which had never been quite convincing to economists. Zaccagnini considered numerous examples of applications of the theory of simultaneous maxima to economic theory with the aim of demonstrating the generality of de Finetti's analytical scheme. For our purposes, it is sufficient to consider the theories of duopoly and oligopoly.

In his 1947 article, Zaccagnini extensively treats the classic case of duopoly without cost functions and with a linear demand curve, comparing the solution of the simultaneous maximum with that of Cournot. Directly applying de Finetti's methodology (and hence Eq. (9.5)), Zaccagnini comes to the following expression for the price:

$$p = -p'\left(q_1 + q_2\right) \tag{9.7}$$

Equation (9.7), as Zaccagnini observes, is not new; it corresponds to the case in which the firms maximize their joint profits. The price depends on the derivative of the demand function and the total production, while the level of production of each firm remains indeterminate. Zaccagnini concludes as follows:

> To determine the equilibrium between q_1 and q_2 it is necessary to: (1) abandon the static hypothesis of simultaneity and thus admit a particular sequence of actions and reactions based on specific assumptions of each operator on the behavior of the rival; or (2) impose additional conditions that cancel the indeterminacy of the problem (arbitration, hedonistic strength, etc.). (Zaccagnini 1942, p. 268)

In Eq. (9.7), the old, non-uniqueness of the equilibrium solution reappears, but this fact does not seem to be a problem for Zaccagnini. In the case of n monopolists, Eq. (9.7) can easily be extended, and the following general expression is obtained:

$$p = -p'\left(\sum_{i=1}^{n} q_i\right) \tag{9.8}$$

Zaccagnini concludes his analysis of oligopoly theory with these words:

> We believe that the economic applications of the mathematical process of simultaneous maxima of different functions, show, in our opinion, how it is possible to obtain the equilibrium conditions of a whole class of economic problems quickly and rigorously. Such conditions are based on the economic explanation which is often confirmed by ordinary reasoning in a way that really fulfills our requirements of logical representation of reality. (Zaccagnini 1947, p. 292)

The final result is not a single equilibrium position but an entire path depending on the choice of firms, as in the Bertrand's case. From a geometric point of view, the equilibrium points identified by Eq. (9.7) are found on the tangency of the isoprofit curves of the duopolists. It is an equilibrium different from that of Cournot, with higher prices and less quantity produced. However, the problem of the distribution of production between the two firms remained unresolved, and, as in Pareto, it could not been solved using pure economic theory. In the following years, Zaccagnini attempted to extend his interpretation of the simultaneous maxima in various directions, especially in the case of a socialist economy and the labor market (Zaccagnini 1958).

The attempt to overcome the difficulties posed by Cournot's model with a different alternative derived from de Finetti's mathematical model did not enjoy success in the theory of non-competitive markets. The reasons for this failure can be noted immediately, considering Eqs. (9.7) and (9.8), which show a situation where the price is the same as that in the monopolistic market, while the quantities produced by n firms remain indeterminate. From the mathematical point of view, Cournot's model is a more powerful logical scheme, as it allows for determining the quantities produced, and thus the market price. It does not matter that for decades it has been considered an inappropriate framework for studying the market with few firms. The model of simultaneous maxima could certainly be more coherent with the logic of economic maximization, but this left open the economic discourse to sociologically or institutionally external factors. This perspective did not create any discomfort for a Paretian economist such as Zaccagnini. Pareto's scholars were well aware of the limitations of the applications of mathematics to economic theory. On the contrary, the request for mathematical rigor would become, after the Second World War, the main lens through which to judge the validity of economic reasoning. The indeterminacy of the equilibrium solution would be considered as a fatal mistake and the Cournot model would become a fundamental starting point for the study of non-competitive markets, mainly for its analytical virtues.

9.8 Conclusions

The period between the two World Wars was characterized by intense debate over the theory of oligopolistic markets. In 1934, von Stackelberg documented in detail the variety and complexity of the different positions emerging among mathematic economists. It was not a purely theoretical or academic debate. The economic transformations that were taking place, with the increase in the size of firms and industrial concentrations, required the economists to offer a proper theoretical interpretation. In particular, most economists critically considered the Cournot model for not being for a convincing interpretation of the economic reality, and new ways were sought. We saw how, in this context, the Paretian economists participated actively in this debate, openly defending the Cournot model. In particular, Amoroso became an authoritative voice in this field of economic research. Zaccagnini in the 1940s moved on a different path using the mathematical insights of De Finetti to amend Pareto's theory.

The analysis of this debate can help us understand the reasons why the Cournot model became the dominant approach of oligopoly theory rather late and with some difficulties. Weak from the interpretative point of view, the equilibrium of the best response approach became dominant only when it was included in the new paradigm of game theory. In this new theoretical setting, the model obtained centrality that had been long questioned. It should be noted, however, that the supremacy of the Cournot–Nash approach is not complete, as an alternative path of research is always present. Even the most advanced textbooks (Martin 2002) take into account other behavioral hypotheses, such as those incorporated in Bertrand's model, that are completely different and closer to reality. Ultimately, there is always doubt that the search for certain analytic properties, such as that of the uniqueness of equilibrium, can only be obtained by sacrificing the necessary realism of the theory. There are properties of the formal model that may be of interest to the mathematician, but certainly not to the economist. This epistemological contention may perhaps explain why Nash's equilibrium only slowly developed in the postwar period in the community of economists, mainly because of its fragility in terms of economic rationality, and only when the scientific climate profoundly changed (Hurwicz 1953).

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