



Small World Networks in Education Sciences

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Abstract. Networks with a small world topology are distinguished by the characteristics of their connections, allowing two nodes, distant from each other, to be linked by a shorter path. This work relates the concept of small world network in the area of Education Sciences in particular in the integration of teaching cloister in the world system of higher education. A relation is established with micro-world and nanoworld concepts defined in previous works and the term of constellations of small worlds is defined, which allows to group subsets of the network that we analyze. It is concluded that the concept of small world network can be understood in the integration of teaching cloisters to the world system of higher education, which would allow to increase the quality of teaching and research work in Latin America.

Keywords: Small worlds networks · Micro-worlds · Nanoworlds
Higher education · Small worlds constellations

1 Introduction

In our environment, we coexist with several systems whose elements are related to each other, for example, the transportation system of the city in which we live, the banking system in which we conduct our financial transactions, the website where we make personal purchases, among others. These systems have become part of our daily life, in such a way that it is likely that in some cases we will interact with them without thinking about the structure of the relationship maintained by its components. This relationship can be represented by graphs in which, in some way, it can be seen that its structure can be understood as a type of network. In this great diversity of network types that exist in our environment exists one called *small world network* introduced by Watts y Strogatz [1]. This concept has been widely used to study many systems in which it can be concluded that its structure has a small world topology, studies have been deployed in areas such as: social networks [2–4], electrical networks [5, 6], pandemics [7, 8], neurosciences [9, 10], etc.

The Salesian Polytechnic University (UPS), in the search of its academic excellence, where the teaching task must be dynamic and be strengthened by the cooperative

work between peers, has managed to develop connections that favor the constitution of *teaching cloisters*, which they consist of the grouping of professors in scientific domains, thus ensuring coherence between related subjects and the academic profile of the teacher. The members of each cloister, plan the development of the subject, develop support material that will be used in the teaching-learning process, design cooperative virtual environments, all from the synergy of information and experiences; This process is intended to establish a common didactic model, with homogeneous methodologies and consensus evaluations. Each cloister of the UPS represents a node of the network, however, they are not isolated nodes, since the teachers of each cloister maintain permanent contact with other cloister and in turn with professors or teaching groups of universities from different parts of the world; these contacts correspond, in the majority of cases, to doctoral thesis tutors, therefore, it is normal that the UPS teaching cloisters create links or short routes, with teaching and research groups from other universities physically located at a greater distance; through these links, forming a network with a small world typology, all the teachers of the cloister, and the teachers of the other cloisters, can access and enrich themselves with the experience, not only of their local colleagues, but also with the experience of the foreign teachers and researchers with whom they are related; in the same way, foreign teachers can exchange knowledge, material, information and experiences with UPS teachers. This work also relates the analogy posed with concepts of micro-worlds and nanoworlds defined in previous works by researchers from the UPS. Finally, the concept of constellations of small worlds is proposed, which allows to group nodes of the network that we analyze and that have great affinity in the work they do.

2 Small World Networks: Basic Definitions and Notation

The following concepts and definitions are important for the understanding of this work.

A *network* is a finite set of nodes and edges. An *edge* represents a relationship between a pair of nodes of the network. Figure 1 shows a graph that could be an illustration of a train transport system in a city, in which each point named with a letter can represent a stop station, and each line joining a pair of points can represent the path of a train line from a source station to a destination station, for example, there is a path without intermediate stops between station *A* and station *H*, therefore, in this graph there is an edge between node *A* and node *H* and we denote it as *AH*, we also say that node *A* is neighbor of node *H*.

A *path* between a source node v_1 and a destination node v_2 is the sequence of nodes that must be visited within the graph to move from v_1 to v_2 , it is denoted as v_1v_2 -path. For example, in Fig. 1 exists a path from source node *A* to the destination node *V*. where $AV\text{-path}_1 = A, H, C, V$, however, this is not the unique way between these nodes, since there is also the path $AV\text{-path}_2 = A, H, I, J, K, L, S, U, R, D, E, Z, W, V$, there are also other paths between *A* and *V*. The *distance* or *path length* is the number of “jumps” required to travel from a source node to a destination node, for example, the distance between *A* and *V* in $AV\text{-path}_1$ is 4, while the distance in $AV\text{-path}_2$ is 13.

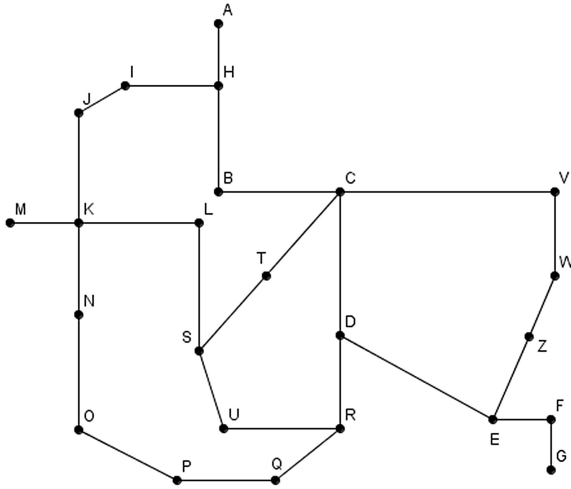


Fig. 1. Illustration of a network.

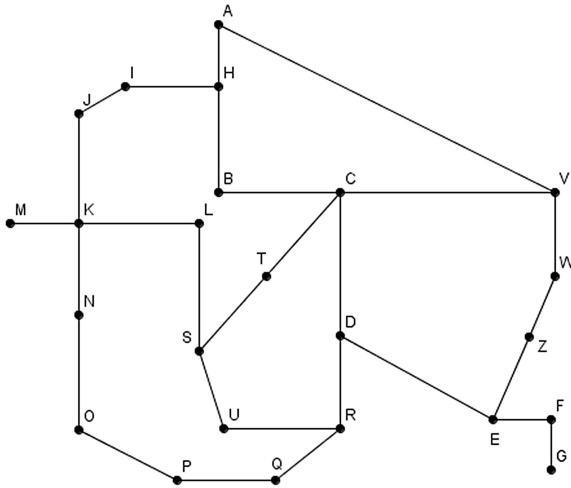


Fig. 2. Illustration of a network where the edge AV has been created.

A *small world network* is a network type introduced by Watts and Strogatz [1], the topology of this network has the characteristic that given a network R , in which there exist nodes v_1 and v_2 such that $v_1 \neq v_2$, related by a path v_1v_2 - $path_1$ with length $d > 1$, it is possible to establish an edge v_1v_2 in R such that it defines a network R' that contains a path v_1v_2 - $path_2$ whose length is equal to 1 [11, 12]. Therefore, in R' will exist a path with the minimum length between two different nodes. A network in which these paths may occur is called *small world network*. For example, in Fig. 1 the minimum distance between A and V is 4, but Fig. 2 shows a network in which an edge AV has been created, therefore, there exists a AV - $path$ whose length is 1.

The *degree of a node* is the number of edges that a node v_1 has in a network, and we denote it as $g(v_1)$, for example, in Fig. 1, $g(A) = 1$ while $g(K) = 4$. The higher the degree of a higher node will be its importance in the network, because it will be connected with a greater number of neighboring nodes. For example, if in Fig. 1 we eliminate the node K then the edges KJ , KL , KM and KN also disappear, in addition the node M would be isolated from the other nodes.

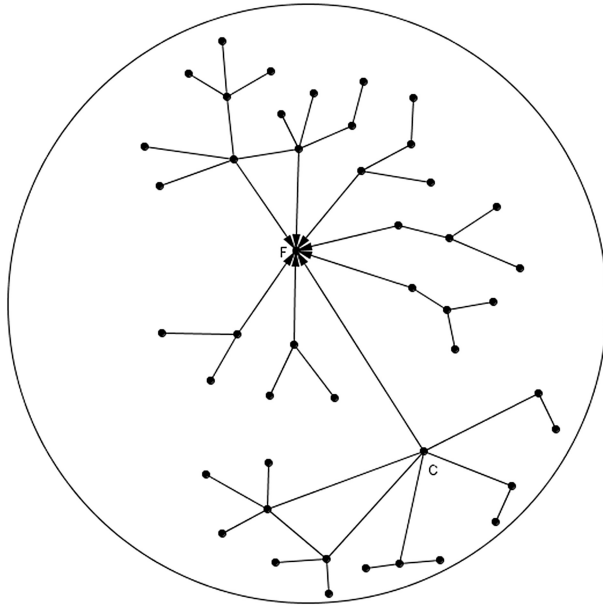


Fig. 3. Illustration of a social network with topology of small world network.

2.1 Importance of a Node According to the Degree of Its Neighbors

The importance of a node in a network can be measured according to its degree, however, within a network with small world topology, the importance of a node can also be measured according to the degree of its neighbors. In the graph of Fig. 1 we can see that degree $grado(M) = 1$, however, its only neighbor node K has the highest degree in the network ($grado(K) = grado(C) = 4$), then, node M acquires, in some way, a certain level of importance within the graph thanks to its neighbor.

We can imagine an ordinary person who, in general, will not have much chance of approaching a celebrity like a famous football player, a movie star or the president of a nation; however, in the environment of a social network this reality can change drastically; Normally this common person will not have a high number of followers, but it is very likely that he is a follower of a world famous celebrity; as well as it is probable that this celebrity has a quite high number of followers. Figure 3 shows an illustration of a scheme of a social network; in it the node C represents an ordinary person with a low number of followers, however it follows a possibly famous person (node F), hen it is said that node C has approached node F .

Another way of conceiving a small world network is through the relationship that may exist between the subsets of a network. We can imagine a finite set of “farms” in which the members of each one maintain a relationship with each other except for some that may be connected to members of another farm. In Fig. 4 we can see a set of nine farms in which the farm f_0 is directly related to five “smaller” farms, and indirectly with three others, for example, the farm f_7 is related to f_0 through the path $f_7f_0\text{-path} = f_7, f_8, f_0$. It is notorious then the importance of the farm f_0 in this graph, then, the more members of a farm have relationship with members of another farm, the more important the farm will be in the network.

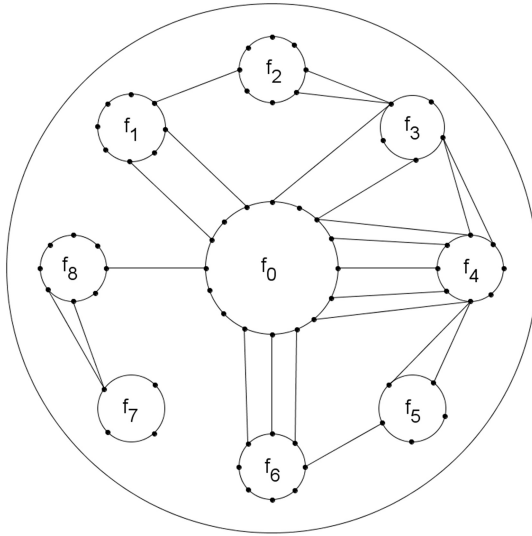


Fig. 4. Illustration of a small world network formed by farms.

3 Teachers’ Cloisters, like Microworlds, Nanoworlds and Constellations of Small Worlds

On the one hand, social reality is a complex system of interrelations. The educational spectrum does not escape this consideration, so that in order to understand it, administer it and model it, man needs to break it down into smaller but equally complex parts.

Taking the education system of a nation as an example, these segments can be the educational levels that make it up. At each level, it is also possible to decompose it from the functions that articulate it; in all cases, their structures maintain characteristics of complexity.

On the other hand, for each human being their conception of education as well as other concepts such as culture and reality, are different, in all cases they respond to their own experiences, knowledge and abilities, and they are also directly dependent on the context in which they develop, in that sense we would say that each person has the opportunity to model their world and that this is finally the sum of smaller but equally

complex segments. These segments have a fractal logic, that is, they reproduce their structure and complexity, for Pesántez [13], these segments are called micro and nanoworlds. For this work this concept is adapted to small worlds.

From the previous considerations, educational constellations mediating small worlds are understood as multiple-scale learning support ecosystems, which use microworlds and interactive nanoworlds [14], which through different processes strengthen at the level of conglomerates -teaching cloisters- educational competences to achieve. Thus, the teaching-learning exercise constitutes the nanoworlds of each teacher; the university functions of teaching, research, linking with society and administrative management would represent the microworlds, while the projects that these interactions give off would be the constellations of small worlds.

3.1 The Teaching Cloisters as Network Nodes

A nanoworld, as an individual teaching-learning exercise for each teacher where the task must be dynamic and strengthened by cooperative work among peers, must lead to the development of connections that favor the formation of teaching cloisters, which consist of the grouping of professors in scientific domains. This ensures coherence between related subjects and the academic profile of the teaching staff, as well as enhancing their research and specialization skills.

The members of each cloister, plan the development of subjects, develop support material that will be used in the teaching-learning process, design cooperative virtual environments, all from the synergy of information and experiences; This process is intended to establish a common didactic model, with homogeneous methodologies and consensus evaluations.

Each cloister of the UPS represents a node of the network, however, they are not isolated nodes, since the teachers of each cloister maintain permanent contact with other cloisters and in turn with professors or teaching groups of universities from different parts of the world; these contacts correspond, in the majority of cases, to professors with doctoral degrees, therefore, it is normal that the teaching cloisters of the UPS create links or short routes, with teaching and research groups from other universities physically located at a greater distance.

Through these links, they form a network with small world topology, all the teachers of the cloister, and the teachers of the other cloisters, can access and enrich themselves with the experience, not only of their local colleagues, but also with the experience that the foreign teachers and researchers with whom they relate have; in the same way, foreign teachers can exchange knowledge, material, information and experiences with UPS teachers.

From this explanation, we assert that the personal reality is also a micro or nanoworld that converge at some point in educational constellations, insofar as they constitute “communities of practice” Wenger [15]. Where learning maintains meaning within a process of collective and community commitment, in this case participation is about the social practice of education, in other words, social learning is generated that generates resources thanks to the interaction of the members of the local and global university community.

The 4 premises of Wegner regarding the social theory of learning are fulfilled in the constellations of small worlds, thus: (a) each member is conceived a social being; (b) knowledge is a matter of competence; (c) there is an active commitment to be valued as members of a university community; (d) the meaning of their identity is what motivates their exercise. From what has been described, micro and nanoworlds as learning development elements are designed, at an individual level as a member of a higher education institution, to reinforce the practices of university functions, reaffirming them; and at the organizational level to keep their conglomerates integrated.

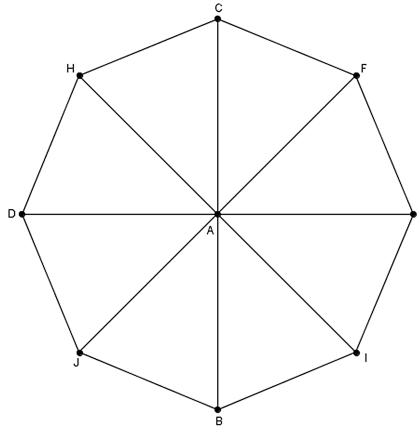


Fig. 5. Example of the structure of a teaching cloister.

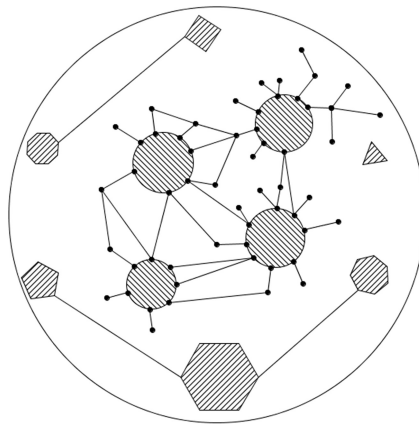


Fig. 6. Example of relationship between teaching cloisters.

4 Small World Networks Conformed by Teaching Cloisters

In the previous section, we can see that a teaching cloister works if there is a group of professors that interact with each other in order to obtain material and define common methodologies that benefit the development of their teaching activities. It can be said

then that a teaching cloister can be represented with a graph whose network structure is strongly connected [16], that is, each cloister member is “connected” or maintains a relationship with all other professors, Fig. 5 shows an example of the structure of a teaching cloister.

Now, consider all the groups of teaching cloisters that exist in the UPS. In view of the fact that each cloister specializes in a very specific area of knowledge or subject, then there are very few relationships between teachers of different cloisters, this case would be given, for example, if a teacher member of a cloister advises or shares material with a teacher from another cloister, in this sense, there will not be many teachers who have relations with teachers from other cloisters. In Fig. 6 the existence of six teaching cloisters (represented by regular striped polygons) and the scarce relationship between them are illustrated, in addition, the resulting graph could be non-connected. The striped circles represent teachers or groups of teachers from other universities.

However, the teachers of each cloister, in the course of their profession and their fourth level academic training, have had the opportunity to establish strong links with teachers from other prestigious universities, for example, if a teacher is studying at Harvard University, he will most likely maintain a link with at least the tutor of his thesis; considering that the UPS teacher needs to specialize in the academic area than his subject, then all cloister members of the cloister are probably interested in the subject. In this way, a teacher from the UPS, and all the teachers of his cloister, maintain a “*short path*” with a Harvard University professor located geographically far away and at a much higher ranking, which benefits his work since have the opportunity to enrich themselves with experiences raised in other places of the world. If we establish these relationships in the graph of Fig. 6 the result would be a graph with a structure of small world network (Fig. 7).

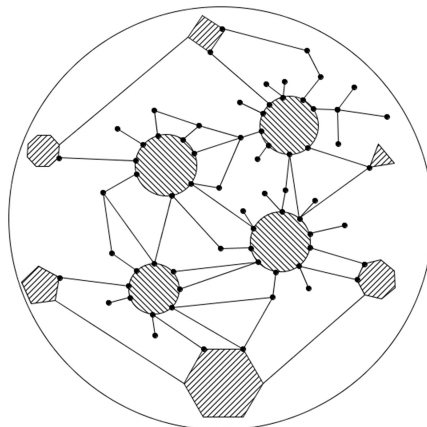


Fig. 7. Illustration of a small world network built through the relationship of teachers’ cloisters with other universities.

5 Conclusions

The concept of small world networks allows to understand the interaction of two elements of a network in a direct way when they are physically or logically located at great distances. The teaching cloisters allow the UPS to improve the teaching-learning process because it brings together teachers with common scientific domains to plan the development of the subject, develop support material, design cooperative virtual environments, with the intention of establishing a model didactic, with homogeneous methodologies and consensual evaluations, with this the UPS manages to have a coherence between related subjects and the academic profile of the teacher. The integration of teaching cloisters into the world higher education system allows for direct links with prestigious professionals and research groups, which helps to enrich their experience and resources. The direct relationship between UPS teaching cloisters and the important elements of the world education system can be understood as a small world network.

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