Application of Artificial Neural Network for Identification of Bearing Stiffness Characteristics in Rotor Dynamics Analysis

Ivan Pavlenko[1(✉)], Vitalii Simonovskiy[1], Vitalii Ivanov[1], Jozef Zajac[2], and Jan Pitel[2]

[1] Sumy State University, 2 Rymskogo-Korsakova Street, Sumy 40007, Ukraine
i.pavlenko@omdm.sumdu.edu.ua
[2] Technical University of Kosice, 1 Bayerova Street, 08001 Presov, Slovak Republic

Abstract. In this article the implementation of the mathematical model for rotor oscillations on non-linear bearing supports for the multistage centrifugal compressor is considered by using the computer program "Critical frequencies of the rotor". It realized the finite element mathematical model, which allows taking into account the non-linear dependence of bearing stiffness on the rotor speed, as well as gyroscopic moments of inertia of impellers and shell-type parts. The artificial neural network "Virtual Gene Developer" software is proposed for evaluating the operating parameters of the approximating curve "bearing stiffness – rotor speed" by the dataset of numerical simulation results in the abovementioned software. Actual parameters of non-linear bearing stiffness are obtained by the results of the experimental research of rotor critical frequencies for the multistage centrifugal compressor 295GC2-190/44-100M on the experimental accelerating-balancing stand "Schenck". The main advantages of the proposed approach and methodology for application of Artificial Neural Networks are stated.

Keywords: Rotor vibrations · Artificial intelligence · Non-linear support
Critical frequency · Mode shape

1 Introduction

Intensification of the development in the field of power engineering occurs through the usage of the modern energy-intensive equipment, an essential role of which is performed by multistage rotor machines. Permanently raising their parameters leads to increasingly significant problems of vibration reliability. Furthermore, the problem of investigating the dynamics of flexible rotors on the system of bearings is currently actual due to the impossibility of absolutely accurate dynamic balancing. It is complemented by an acute problem of evaluation of stiffness characteristics for rotor bearings in close connection with the dependence of critical frequencies of the rotor and corresponding mode shapes [1]. However, up-to-date methods for simulating the dynamics of rotor systems are predominantly based on the use of computer programs realizing the finite element method [2], but analytical studies generally use

V. Ivanov et al. (Eds.): DSMIE 2018, LNME, pp. 325–335, 2019.
https://doi.org/10.1007/978-3-319-93587-4_34

two-dimensional continuous mathematical models for beam elements taking into account the hypothesis of Kirhhoff [3] about the non-deformable cross-sections. Application of the three-dimensional finite element models, particularly within the use of "ANSYS" software requires a relatively large machine time, in contrast to the beam models. Moreover, available software by default cannot provide it, taking into account non-linearities, which may be inherent in rotors. Consequently, the attention of scientists is attached to the application of significant efforts to expand the capabilities of existing software products and create new ones, more often combining the fields of hard and soft modeling [4–7].

2 Literature Review

The recent achievements in the field of rotor dynamics can be marked. The research work [5] is aimed at the development of the computational techniques of rotor dynamics with the use of the finite element method. Using "Matlab" software to develop the simulation model for determining the eigenfrequencies of the elastic rotor based on the 2D beam bending model with testing by "SolidWorks" software is presented in the research [8]. The problem of determining eigenfrequencies of rotors on the basis of linear and non-linear mathematical models is well researched in the work [9]. The influence of the rotor speed on the bearing stiffness using numerical simulation with the use of "ANSYS" software is investigated in the work [10]. The approach for identification of the rotor unbalances is proposed within the work [11] for the case of a single-span rotor as an elastic Euler-Bernoulli beam with a single-disc. Techniques that allow taking into account the gyroscopic moments are considered in works [12], and the impact of the deformable parts on rotor dynamics is investigated in the article [13].

The work [14] is devoted to comprehending the inherent mechanism and the feature of the subharmonic resonance for the rotor system supported on the ball bearings using numerical analysis for the 6-degrees-of-freedom model with non-linearities, Hertzian contact forces and bearing clearance. Taking into account the random excitations for the analysis of gas turbine, the blade vibrations is presented in work [15]. The approach for experimental and numerical research of a new dynamic phenomenon for two-bladed wind turbines is stated in the article [16], and the novel methodology for the angular position identification of the unbalance force for the case of asymmetric rotors is proposed in the research paper [17]. Finally, a novel methodology for a stochastic presentation of non-linear dynamics problems is stated within the work [18]. The solution of the problem of evaluation of stiffness characteristics for the rotor bearings within the dependence of critical frequencies of the rotor has not been found. Due to the abovementioned, this paper is aimed at using the artificial neural network (ANN) for evaluating the parameters of the approximating curve "bearing stiffness – rotor speed" by the dataset of numerical simulation results in the proposed computer program. The results of the experimental research of critical frequencies allow obtaining actual parameters of bearing stiffness.

3 Research Methodology

3.1 Bearing Stiffness Characteristics

To develop a technique for determining eigenfrequencies and critical frequencies of rotor oscillations, an approximating curve for describing the dependence "rotor speed – bearing stiffness" from the recent experience in the design and calculation of compressor units can be used. It confirms the sufficiency of using quadratic polynomial to describe the dependence of the bearing stiffness c on the rotor speed ω:

$$c(\omega) = c_0 + \alpha\omega + \beta\omega^2, \tag{1}$$

where c_0 – bearing stiffness in case of the non-rotating rotor (N/m); α – initial slope (N·s/m); β – initial curvature of the curve (N·s^2/m). The following investigation is aimed at identifying parameters c_0, α, β by using ANN.

3.2 The Mathematical Model of Rotor Dynamics

The finite element mathematical model of rotor oscillations, which allows taking into consideration gyroscopic moments of inertia for impellers and shell-type parts, as well as the non-linear dependence of bearing stiffness on the rotor speed (1), is realized by the computer program [19].

Due to the finite element method, the mathematical model of rotor oscillations is described with the following equation [20, 21]:

$$\left([C(\omega)] - \omega^2[M]\right)\{Y\} = \{F\}, \tag{2}$$

where $\{F\}$, $\{Y\}$ – column vectors of amplitudes F_k and y_k of external mono-harmonic forces F_k sinωt and node displacements y_k sinωt respectively; k – node number (k = 1, 2, …, 2N – 1); N – total number of finite elements; $[C(\omega)]$, $[M]$ – global stiffness and inertia matrices formed from local ones by summarizing at corresponding nodes k:

$$[C(\omega)]_{ij} = \sum_{k=1}^{n}\left([C(\omega)]_e\right)_{ij}^{\langle k\rangle};\;\; [M]_{ij} = \sum_{k=1}^{n}\left([M]_e\right)_{ij}^{\langle k\rangle};\;\; (i,j = 1,\ 2,\ldots,2N-1). \tag{3}$$

In case of two-nodes beam finite elements with 4 degrees of freedom, the local matrices of stiffness $[C(\omega)]_e$ and inertia $[M]_e$ for the finite element $e = (i,\ j)$ are determined by the following expressions:

$$C_e = \frac{EI}{l^3}\begin{bmatrix} 12 + \frac{cl^3}{EI} & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix};$$

$$M_e = \frac{m}{420}\begin{bmatrix} 136 & 22l & 54 & -13l \\ 22l & \begin{matrix} -136l^2- \\ -105r^2/4+ \\ +420I_g/m \end{matrix} & 13l & -3l^2 \\ 54 & 13l & 136 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}, \tag{4}$$

where m – finite element mass (kg); l – finite element length (m); E – Young's modulus of the material (N/m^2); I – axial moment of inertia for the cross-section (kg·m^2); r – cross-sectional radius (m); I_g – gyroscopic moment of inertia for impellers and shell-type parts (kg·m^2).

In case of free oscillations $F_k = 0$ (k = 1, 2, … 2N – 1), and the condition of existence of non-trivial solutions of the system (2) is vanishing, the determinant

$$\det\left([C(\Omega)] - \Omega^2[M]\right) = 0, \tag{5}$$

which is the higher order non-linear algebraic equation with respect to critical frequency Ω.

As a result of the rotor dynamic modeling for the different values of parameters c_0, α, β, the dataset describing the matrix dependence $\{K\} = f\,\{\Omega\}$ between stiffness characteristic $\{K\} = \{c_0, \alpha, \beta\}^T$ and spectrum $\{\Omega\} = \{\Omega_1, \Omega_2, \Omega_3, \ldots\}^T$ of critical frequencies can be obtained.

The abovementioned technique is implemented on the example of the multistage centrifugal compressor 295GC2-190/44-10M on magnetic bearings. The compressor with a capacity of 16,85 megawatt is a part of the gas pumping unit GPU-C-16/102-3,32M produced by the Public Joint Stock Company "Sumy Machine-Building Science-and-Production Association", has the operating rotor speed in the range 3710…5565 rpm.

Obtained maximum parameters c_0^{max} = 2,5·10^7 N/m, α^{max} = 3·10^4 N·s/m, β^{max} = 3·10^7 N·s^2/m and Ω^{max} = 536 will be used for normalizing the dataset within the regression procedure and ANN implementation.

3.3 Regression Procedure

The dataset obtained as a result of numerical simulation of rotor dynamics by using the computer program "Critical frequencies of the rotor" can be used within the regression procedure [19, 20] for determining the linear dependence between bearing stiffness and critical frequencies in the following form:

$$\bar{\Omega}_i = a_i\bar{c}_0 + b_i\bar{\alpha} + c_i\bar{\beta}, \tag{6}$$

where i – number of critical frequency ($i = \{1, 2, 3\}$); a_i, b_i, c_i – unknown weight factors; $\bar{\Omega}_i$, $\bar{c}_0$, $\bar{\alpha}$, $\bar{\beta}$ – dimensionless normalized parameters in a range 0…1 determined by formulas:

$$\bar{\Omega}_i = \Omega_i/\Omega^{\max}; \ \ \bar{c}_0 = c_0/c^{\max}; \ \ \bar{\alpha} = \alpha/\alpha^{\max}; \ \bar{\beta} = \beta/\beta^{\max}. \tag{7}$$

Dependence (6) is equal to a set of three planes in the 4D hyperspace unit "$\bar{c}_0 - \bar{\alpha} - \bar{\beta} - \bar{\Omega}_i$" after normalizing (7).

Unknown parameters a_i, b_i, c_i as a components of the column vector of weight factors $\{A\}_i = \{a_i, b_i, c_i\}^{\mathrm{T}}$ can be obtained as a result of solving the system of non-homogeneous linear equations

$$[\overline{K}]\{A\}_i = \{\bar{\Omega}\}_i, \tag{8}$$

where $[\overline{K}]$ – rectangular matrix of bearing stiffness coefficients of size $n \times 3$ (n – number of rows of numerical simulation results dataset); $\{\bar{\Omega}\}_i$ – column vector of normalized critical frequencies of size $n \times 1$:

$$[\overline{K}] = \begin{bmatrix} c_0^{\langle 1\rangle} & \alpha^{\langle 1\rangle} & \beta^{\langle 1\rangle} \\ c_0^{\langle 2\rangle} & \alpha^{\langle 2\rangle} & \beta^{\langle 2\rangle} \\ \cdots & \cdots & \cdots \\ c_0^{\langle n\rangle} & \alpha^{\langle n\rangle} & \beta^{\langle n\rangle} \end{bmatrix}; \ \ \{\bar{\Omega}\}_i = \begin{Bmatrix} \bar{\Omega}_i^{\langle 1\rangle} \\ \bar{\Omega}_i^{\langle 2\rangle} \\ \cdots \\ \bar{\Omega}_i^{\langle n\rangle} \end{Bmatrix}. \tag{9}$$

Due to $n > 3$, column vectors, $\{A\}_i$ can be obtained by the formula of linear regression:

$$\{A\}_i = \underbrace{\left([\overline{K}]^T[\overline{K}]\right)^{-1}[\overline{K}]^T}_{n\times n}\underbrace{\{\bar{\Omega}\}_i}_{n\times 1}. \tag{10}$$

Critical frequencies can be determined by the linear regression model by the following formula:

$$\{\tilde{\Omega}\} = [A]\{\bar{K}\}, \tag{11}$$

where $\{\tilde{\Omega}\}$ – column vector of experimental values of critical frequencies $\tilde{\Omega}_1$, $\tilde{\Omega}_2$, $\tilde{\Omega}_3$; $\{\bar{K}\}$ – column vector of bearing stiffness parameters; $[A]$ – the rectangular matrix of weight factors determined by formula (10):

$$[A] = \begin{bmatrix} \{A\}_1 \\ \{A\}_2 \\ \{A\}_3 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}; \ \ \{\tilde{\Omega}\} = \begin{Bmatrix} \tilde{\Omega}_1 \\ \tilde{\Omega}_2 \\ \tilde{\Omega}_3 \end{Bmatrix}; \ \ \{\bar{K}\} = \begin{Bmatrix} \bar{c}_0 \\ \bar{\alpha} \\ \bar{\beta} \end{Bmatrix}. \tag{12}$$

Unknown weight factors $\{\bar{K}\}$ are determined in Eq. (11) by the inverse matrix formula:

$$\{\bar{K}\} = [A]^{-1}\{\tilde{\Omega}\}. \quad (13)$$

3.4 Using Artificial Neural Network

Using the continuum analytical models for investigating rotor dynamics is not possible in the general form. Therefore, the finite element method is mainly applied. However, solving the inverse problem related to the identification of bearing stiffness characteristic providing the actual operating parameters or critical frequencies and form shapes is the complicated research problem that can not be solved using traditional finite element analysis due to the initial nonlinearity of the mathematical model. In this case, artificial neural networks (ANN) as universal approximators can be implemented due to their ability to provide general mechanisms for creating models with highly nonlinear relationships between input and output parameters [21].

A variety of ANN is ensured due to the specific requirements of the problems by adopting a different degree of network complexity, types of inter-connections, transfer functions, training method, etc. In this work, ANN with input, output and system of hidden layers is used, that creates a correspondence between critical frequencies of the rotor and parameters of the bearing stiffness characteristic. The procedure for identification of rotor bearing stiffness characteristic by combined using finite element model of rotor dynamics and ANN is schematically presented in Fig. 1a.

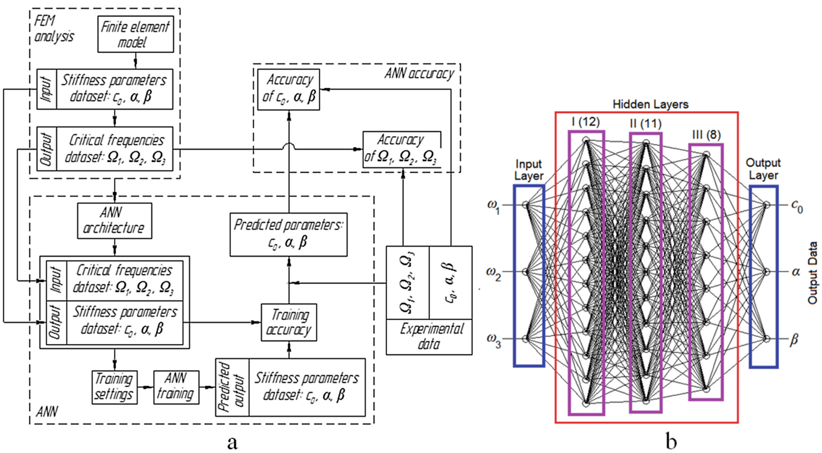


Fig. 1. The procedure of using ANN (a) and ANN architecture (b).

It should be noted that the output parameters (critical frequencies Ω_1, Ω_2, Ω_3) of numerical simulation in the computer program “Critical frequencies of the rotor” are input data for ANN training, and the output parameters of ANN (stiffness coefficients

c_0, α, β) must be compared with their corresponding actual values determining as a result of experimental research.

The configuration of ANN is presented in Fig. 1b. The number of layers and the distribution of neurons in layers are determined by the condition of full-time operating of all neurons. Decreasing the number of hidden layers and corresponding neurons leads to decreasing accuracy of the subsequent evaluation of rotor bearing stiffness characteristics, while an unreasonable increasing of neurons and layers leads to increasing the learning time and to non-involved neurons.

4 Results

4.1 Using Regression Procedure

Experimental research of the rotor dynamics for the multistage centrifugal compressor 295GC2-190/44-100M was provided on the accelerating-balancing stand "Schenck" for accelerating testing and dynamic balancing in vacuum of flexible rotors of centrifugal compressors with a mass up to 2500 kg. As a result, actual critical frequencies Ω_1 = 117 rad/s, Ω_2 = 256 rad/s, Ω_3 = 511 rad/s were determined, and corresponding normalized parameters $\bar{\Omega}_1 = 0.218$; $\bar{\Omega}_2 = 0.478$; $\bar{\Omega}_3 = 0.953$ were calculated. Thus, due to the formula (13), the column vector $\{\bar{K}\} = \{0.817, 0.204, 0.891\}^T$, and the evaluated bearing stiffness parameters are as follows: $c_0 = 0.817 \cdot 2.5 \cdot 10^7 = 2.179 \cdot 10^7$ (N/m); $\alpha = 0.204 \cdot (-3 \cdot 10^4) = -0.613 \cdot 10^4$ (N·s/m); $\beta = 0.891 \cdot 2 \cdot 10^2 = 1.782 \cdot 10^2$ (N·s^2/m).

All the abovementioned results are presented in Table 1.

Table 1. Comparison of achieved results.

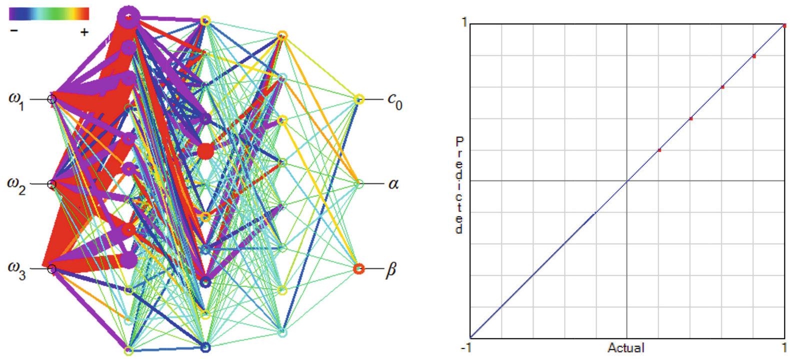
Methods and parameters	$c_0 \cdot 10^7$	$\alpha \cdot 10^4$	$\beta \cdot 10^2$	ω_1	ω_2	ω_3
Regression analysis	2.179	−0.613	1.782	117	264	513
Artificial Neural Network	2.455	−2.682	1.996	118	256	511
Actual parameters	2.450	−2.900	2.086	117	256	511
Error in regression analysis	11.1	78.9	14.6	0.0	3.1	0.4
Error for using ANN	0.2	7.6	4.4	0.9	0.0	0.0

The verification of the results is carried out by determining the critical frequencies as a result of numerical simulation in the program "Critical frequencies of the rotor". In case of the parameters $c_0 = 2.179 \cdot 10^7$ N/m; $\alpha = -0.613 \cdot 10^4$ N s/m; $\beta = 1.782 \cdot 10^2$ N s^2/m, the critical frequencies are obtained: Ω_1 = 117 rad/s, Ω_2 = 264 rad/s, Ω_1 = 513 rad/s, which corresponds to actual critical frequencies.

However, it should be noted that actual parameters $c_0 = 2.45 \cdot 10^7$ N/m; $\alpha = -2.9 \cdot 10^4$ N s/m; $\beta = 2.086 \cdot 10^2$ N s^2/m of bearing stiffness characteristic allow concluding that there is insufficient accuracy of the linear regression procedure due to relative errors 11.1%, 78.9% and 14.6% respectively. Thus, there is a need to use the method giving more accurate results.

4.2 Implementation of Artificial Neural Network

"Visual Gene Developer" software provides graphical visualization of ANN training procedure (Fig. 2). Lines present weight factors and nodes means threshold values. In the diagram, red color corresponds to the high positive number, and violet color means high negative number. Line width is proportional to the absolute number of weight factor or threshold value.



a b

Fig. 2. ANN map analysis (a) and results of regression analysis (b).

The following training settings were selected: Learning rate – 0.001; Transfer function – Hyperbolic tangent; Total number of training cycles – $1 \cdot 10^6$; Target error – $1 \cdot 10^{-5}$; Initialization method of threshold – Random; Initialization of weight factor – Random; Analysis update interval – 500 cycles.

ANN training results include initial parameters c_0, α, β, resulting frequencies Ω_1, Ω_2, Ω_3 and predicted critical frequencies ω_1, ω_2, ω_3.

Comparison of predicted critical frequencies with the corresponding resulting critical frequencies leads to the conclusion about high accuracy (up to the third decimal place) of ANN learning process. In addition, the following training results are obtained: Sum of error – $3.5 \cdot 10^{-4}$; Average error per output per dataset – $3.6 \cdot 10^{-6}$; Regression coefficient – 0.99996.

Evaluated bearing stiffness parameters are $\bar{c}_0 = 0.982$ ($c_0 = 2.455 \cdot 10^7$ N/m); $\bar{\alpha} = 0.894$ ($\alpha = -2.681 \cdot 10^4$ N s/m); $\bar{\beta} = 0.998$ ($\beta = 1.996 \cdot 10^2$ N s^2/m). All the abovementioned results are presented in Table 1. The verification of the results is carried out by determining the critical frequencies as a result of numerical simulation in the program "Critical frequencies of the rotor". In case of the parameters $c_0 = 2.455 \cdot 10^7$ N/m; $\alpha = -2.681 \cdot 10^4$ N s/m; $\beta = 1.996 \cdot 10^2$ N s^2/m, the critical frequencies are obtained: $\Omega_1 = 118$ rad/s, $\Omega_2 = 256$ rad/s, $\Omega_1 = 511$ rad/s, which corresponds to the actual critical frequencies. However, it should be noted that actual parameters $c_0 = 2.45 \cdot 10^7$ N/m; $\alpha = -2.9 \cdot 10^4$ N s/m; $\beta = 2.086 \cdot 10^2$ N s^2/m of bearing stiffness characteristic indicates high accuracy of results obtained by using ANN due to the relative errors less than 1%. The design scheme and mode shapes obtained using the computer program "Critical frequencies of the rotor" for bearing stiffness evaluated by the use of ANN are presented in Fig. 3.

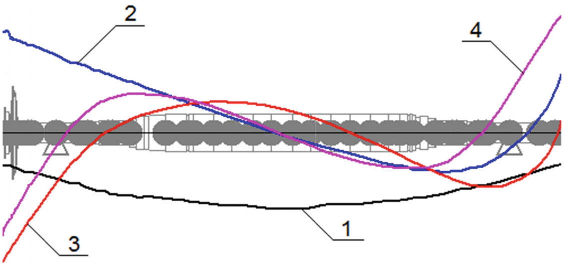


Fig. 3. Design model and mode shapes of the rotor.

The main advantages of using ANN are sufficient accuracy of the obtained results, and lack of need for the re-optimizing procedure when changing experimental values of critical frequencies within the same model of rotor dynamics.

5 Conclusions

Thus, the regression dependences for identification of bearing stiffness parameters by using numerical simulation and experimental data are proposed. The computer program "Critical oscillations of the rotor" realizes the mathematical model of rotor dynamics, which allows taking into consideration gyroscopic moments of inertia for impellers and shell-type parts, as well as the non-linear dependence of bearing stiffness on the rotor speed.

The implementation of ANN is verified on the example of the compressor 295GC2-190/44-10M on magnetic bearings with taking into account dependence of stiffness characteristics on the rotor speed. Comparison of the bearing stiffness and critical frequencies obtained with the use of ANN and physical experiment on the accelerating-balancing stand confirms the reliability of the proposed approach with sufficient accuracy for practical purposes.

There are different results of the evaluation of bearing stiffness characteristics as a result of the implementation of the linear regression procedure and artificial neural network. Moreover, sufficient accuracy of calculation of critical frequencies does not ensure a sufficient accuracy of regression procedure due to its original linearity. However, this problem is completely eliminated with the use of an artificial neural network. In addition, it should be noted that using artificial neural network significantly improves the accuracy of identification of the parameters for the mathematical model of bearing stiffness in comparison with the linear regression procedure. ANN has the advantage over traditional optimization methods due to the fact, that changing the initial data for the same model requires a new optimization procedure, while the previously trained ANN does not need this.

Achieved results allow setting directions of further research within the development an approach for implementation ANN for identification non-linear bearing stiffness characteristics by the results of numerical simulation and experimental research of forced oscillations, as well as providing virtual dynamic balancing of flexible rotor systems for multistage centrifugal machines.

Acknowledgements. This work was supported by the Slovak Research and Development Agency under the contract No. APVV-15-0602.

The main scientific results were achieved within The National Scholarship Programme of the Slovak Republic at the Faculty of Manufacturing Technologies with a seat in Prešov of Technical University of Košice (Prešov, Slovakia) and recent research at Sumy State University: No. 0117U004922 and No. 0117U003931.

References

1. Norfield, D.: Practical Balancing of Rotating Machinery. Elsevier, Amsterdam (2006)
2. Swanson, E., Powell, C.D., Weissman, S.: A practical review of rotating machinery critical speeds and modes. Sound Vib. **39**(5), 10–17 (2005)
3. Vance, J.M., Zeidan, F.Y., Murphy, B.: Machinery Vibration and Rotordynamics. Wiley, Hoboken (2010)
4. Sika, R., Rogalewicz, M., Popielarski, P., Wytyk, G.: Forecasting of steel consumption with use of nearest neighbors method. In: Proceedings of 13th International Conference on Modern Technologies in Manufacturing, Cluj-Napoca and AMaTUC, MATEC Web of Conference, vol. 137, p. 01010 (2017)
5. Sika, R., Rogalewicz, M.: Demerit Control Chart as a decision support tool in quality control of ductile cast-iron casting process. In: Proceedings of 8th International Conference on Manufacturing Science and Education (MSE 2017), Trends in New Industrial Revolution, Sibiu, MATEC Web of Conference, vol. 121, p. 05007 (2017)
6. Sika, R., Hajkowski, J.: Synergy of modeling processes in the area of soft and hard modeling. In: Proceedings of 8th International Conference on Manufacturing Science and Education (MSE 2017), Trends in New Industrial Revolution, Sibiu, MATEC Web of Conference, vol. 121, p. 04009 (2017)
7. Vollan, A., Komzsik, L.: Computational Techniques of Rotor Dynamics with the Finite Element Method. CRC Press, Boca Raton (2012)
8. Muminovic, A.J., Braut, S., Muminovic, A., Saric, I.: Numerical and analytical analysis of elastic rotor natural frequency. Technol. Educ. Manag. Inform. **3**(4), 323–328 (2014)
9. Yamamoto, T., Ishida, Y.: Linear and Nonlinear Rotordynamics: A Modern Treatment with Applications. Wiley, Hoboken (2013)
10. Jin, C., Xu, Y., Zhou, J., et al.: Active Magnetic Bearings Stiffness and Damping Identification from Frequency Characteristics of Control Systems, pp. 1–8. Hindawi Publishing Corporation, Cairo (2016)
11. Wang, A., Cheng, X., Meng, G., et al.: Dynamic analysis and numerical experiments for balancing of the continuous single-disc and single-span rotor-bearing system. Mech. Syst. Sig. Process. **86**, 151–176 (2017)
12. Wang, T., Wang, F., Bai, H., et al.: Stiffness and critical speed calculation of magnetic bearing-rotor system based on FEA. In: International Conference on Electrical Machines and Systems (**10458520**) (2008)

13. Villa, C., Sinou, J., Thouverez, F.: Stability and vibration analysis of a complex flexible rotor bearing system. Commun. Nonlinear Sci. Numer. Simul. **13**(4), 804–821 (2008)
14. Bai, C., Zhang, H., Xu, Q.: Subharmonic resonance of a symmetric ball bearing-rotor system. Int. J. Non Linear Mech. **50**, 1–10 (2013)
15. Ogbonnaya, E.A., Poku, R., Ugwu, H.U., et al.: Analysis of Gas Turbine Blade Vibration Due to Random Excitation. InTech, London (2015)
16. Larsen, T.J., Kim, T.: Experimental and numerical study of a new dynamic phenomenon for two-bladed wind turbines. In: Proceedings of the 25th International Ocean and Polar Engineering Conference, pp. 547–553 (2015)
17. Ocampo, J.C., Wing, E.S.G., Moroyoqui, F.J.R., et al.: A novel methodology for the angular position identification of the unbalance force on asymmetric rotors by response polar plot analysis. Mech. Syst. Signal Process. **95**, 172–186 (2017)
18. Sinou, J., Didier, J., Faverjon, B.: Stochastic non-linear response of a flexible rotor with local non-linearities. Int. J. Non Linear Mech. **74**, 92–99 (2015)
19. Pavlenko, I.V., Simonovskiy, V.I., Demianenko, M.M.: Dynamic analysis of centrifugal machine rotors supported on ball bearings by combined using 3D and beam finite element models. In: IOP Conference Series: Materials Science and Engineering, vol. 233, pp. 1–8 (2017)
20. Pavlenko, I.: Static and dynamic analysis of the closing rotor balancing device of the multistage centrifugal pump. Appl. Mech. Mater. **630**, 248–254 (2014)
21. Reddy, M.C.S., Sekhar, A.S.: Application of artificial neural networks for identification of unbalance and looseness in rotor bearing systems. Int. J. Appl. Sci. Eng. **11**(1), 69–84 (2013)