A Description Logic Based Knowledge Representation Model for Concept Understanding

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Abstract. This research employs Description Logics in order to focus on logical description and analysis of the phenomenon of 'concept understanding'. The article will deal with a formal-semantic model for figuring out the underlying logical assumptions of 'concept understanding' in knowledge representation systems. In other words, it attempts to describe a theoretical model for concept understanding and to reflect the phenomenon of 'concept understanding' in terminological knowledge representation systems. Finally, it will design an ontology that schemes the structure of concept understanding based on the proposed semantic model.

Keywords: Concept understanding · Conceptualisation
Terminological knowledge · Interpretation · Formal semantics
Description logics · Ontology

1 Introduction and Motivation

My point of departure is the special focus on the fact that there has always been a general problem concerning the notion of 'concept', in linguistics, psychology, philosophy, and computer science. In Kant's opinion, a concept is the "unity of the act of bringing various representations under one common representation". In addition, Kant believed that "no concept is related to an object immediately, but only to some other representation of it", see [16]. However, it has never been transparent (i) if concepts are some mental representations as well as mental images of various phenomena, or (ii) whether concepts always have to be bound up (and thus, be labelled) with some linguistic expressions.

Concepts are the main building blocks of this research. Note that this article, as an extended version of [4], aims at providing a logical analysis of a specific use of the phenomenon of 'concept' in terminological knowledge representation systems. In order to see and conceptualise concepts from the perspective of logics, we need to interpret them some logical-assessable phenomena. For example, we can interpret a concept a set (class) like the set of *Tree*s. Consequently, a set, that is an understandable and assessable mathematical phenomenon, can be applied to different contexts. Accordingly, a concept might be considered a conceptual entity and, in fact, could be

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correlated with a distinct 'entity' or with its essential attributes, characteristics, and properties. Note that a conceptual entity's properties express its relationships with itself and with other conceptual entities (e.g., reflexive, irreflexive, symmetrical, anti-symmetrical, transitive, anti-transitive relationships).

Through the lens of First-Order Predicate Logic, an entity is assessable a [unary] predicate. Accordingly, such an equivalence between a [conceptual] entity and a unary predicate can support terminological descriptions as well as logical representations of that conceptual entity. The main task of predicates is 'assigning'. In fact, predicates make mental mappings from the attributes and properties of conceptual entities into subjects, see 'predicate' in [7]. Furthermore, predicates can express the conditions that the conceptual entities referred to may satisfy. So, the most central logical premise is that predicates describe conceptual entities in order to determine the applications of logical descriptions and, accordingly, to play fundamental roles in reasoning processes and in giving satisfying conditions for definitions of truth. Consequently, predicates, as the outcomes of predications, express meanings and produce formal semantics. Subsequently, a formal semantics could focus on multiple conditions through definitions of truth (and falsity). Note that any formal semantics deals with the interrelationships between the signifiers of a description and what the signifiers do [or have been designed to do], see [5, 12, 15, 20]. This could be interpreted the most significant essence of any formal semantics.

In this research, concepts (conceptual entities) and their interrelationships will be employed to establish the basic terminology adopted in the modelled domain regarding the hierarchical structures. Relying on such a hierarchical structure, this research focuses on logical description as well as logical analysis of the phenomenon of 'concept understanding'. The desired logical descriptions will have a special focus on my methodological assumption that expresses that "one can find out that an individual thing/phenomenon is an instance of a concept (conceptual entity) and, thus, his[1] individual grasp of that concept (in the form of his conceptions) provide supportive foundations for producing his own conceptualisations. Accordingly, he restricts his produced conceptualisation to his concept understanding". This article will focus on describing and characterising humans' concept understandings and will deal with a formal-semantic model for uncovering the underlying logical assumptions of 'concept understanding' in knowledge representation systems. In other words, it attempts to describe a theoretical model for concept understanding and to reflect it in terminological knowledge representation systems. In this research, the phenomenon of 'concept understanding' will be seen from multiple perspectives. Subsequently, the expressiveness (as well as complexity) of the desired semantic model's descriptions will be improved (and increased).

In this research, the formal semantic analysis of concept understanding is based on Description Logics (DLs). DLs can support me in proposing a comprehensible logical description for clarifying the phenomenon of 'concept understanding'. DLs are, as the most well-known knowledge representation formalisms, used for representing predicates and for formal reasoning over them. They mainly focus on terminological

[1] For brevity, I use 'he' and 'his' whenever 'he or she' and 'his or her' are meant.

knowledge. It is of a terminological system's particular importance in providing a logical formalism for knowledge representation systems and, also, for ontology representations. In information and computer sciences, ontologies are formal and explicit specification of a shared conceptualisation, see [9, 23]. This research, thus, will focus on building up as well as formalising an ontology for 'concept understanding'. The desired ontology provides a structural representation of concept understanding based on the analysed semantic model.

2 The Phenomenon of 'Concept Understanding'

The term 'understanding' is very complicated and sensitive in psychology, neuroscience, cognitive science, philosophy, and epistemology. It shall be emphasised that there has not been any adequate model for understanding. More specifically, there has not been any complete, deterministic, and unexceptionable model for describing the phenomenon of 'understanding'. Anyhow, there have been some proper models for:

1. understanding of understanding (e.g., [11]),
2. understanding representation (e.g., [19, 26]), and
3. specification of the components of understanding (i.e., from the cognition's as well as desires' and emotions' perspectives), e.g., [8, 10, 17, 18, 24, 25, 27].

The first category of models can describe how the phenomenon of 'understanding' could be realised and figured out in different contexts. The second category focuses on epitomising, designing, visualising, and illustrating the phenomenon of 'understanding' in different contexts. And finally, the third category focuses on recognising and specifying the most significant ingredients and constructors of the phenomenon of 'understanding' (mostly from the cognitive perspectives).

Taking into consideration the phenomenon of 'understanding', the following assumptions can describe my main conceptions of the phenomenon of 'concept understanding':

1. I shall assume that if one is going to produce his understanding based on a concept (conceptual entity), then, he will be, either directly or indirectly, become concerned with the ontology as well as the existence of that concept. For instance, producing understanding based on the entity '*Tree*' makes one concerned with the existence and nature of the concept '*Tree*'. Therefore, it must be accepted that the phenomenon of 'concept understanding' relates a human being with the existence of a conceptual entity.
2. There is a strong interconnection and dependency between the phenomenon of 'concept understanding' and the phenomenon of 'explanation'. In addition, it shall be taken into account that the conceptual relationships between the explanans (that which does the explaining) and the explanandum (that which is to be explained) support the phenomenon of 'concept understanding'. Consequently, the phenomenon of 'explanation', as the outcome of the logical relationships between the explanans and the explanandum, attempts to shed light on the concept understanding's targets. For example, explaining *Tree* and understanding *Tree* are strongly interrelated to each other.

3. In order to describe the phenomenon of ‘concept understanding’, we can see it from the structuralist point of view. Such an overview can support us in explaining different facts and procedures about ‘concept understanding’. It is undeniable that understanding *Tree* and understanding *Bough* are tied to each other. Obviously, observing ‘concept understanding’ from the structuralist point of view can connect us to the dependencies between the phenomenon of ‘concept understanding’ and the phenomenon of ‘explanation’. Accordingly, we may become concerned with two issues.
   a. With the taxonomies and hierarchies of concept understanding. This means that seeing concept understanding from the structuralist point of view can link us to the existence of the phenomenon of ‘concept understanding’ and, subsequently, to concept understanding’s different levels of complexity. For example, understanding *Tree* can, inductively, connect us with understanding ‘young *Tree*’ and understanding ‘flowers of a young *Tree*’.
   b. With multiple aspects/criteria of concept understanding (Specially from the perspective of Cognition) that can relate various components of concept understanding to each other. Note that we may also apply different tools (e.g., graphical tools, statistical tools, concept maps) in order to, inductively, relate different components of a concept understanding to each other.

In Sect. 6, you will see that my semantic model focuses on the junctions between ‘understanding of concept understanding’ and ‘concept understanding representation’.

3 Description Logics

My main reference to Description Logics is [1]. Description Logics (DLs) represent knowledge in terms of (i) *individuals* (objects, things), (ii) *concepts* (classes of individuals), and (iii) *roles* (relationships between individuals). Individuals correspond to constant symbols, concepts to unary predicates, and roles to binary (or any other n-ary) predicates in Predicate Logic. A predicate in Predicate Logic can have a [possibly specified] equivalent concept in DLs. There are two kinds of atomic symbols, which are called *atomic concepts* and *atomic roles*. These symbols are the elementary descriptions from which we can inductively (by employing concept constructors and role constructors) construct the specified descriptions. Considering N_C, N_R, and N_O the sets of atomic concepts, atomic roles, and individuals, respectively the ordered triple (N_C, N_R, N_O) denotes a signature. The set of main logical symbols in ALC (Attributive Concept Language with Complements) that is the Prototypical DL (see [21]) is:

$$\{\text{Conjunction}(\sqcap : \text{And}), \text{Disjunction}(\sqcup : \text{Or}), \text{Negation}(\neg : \text{Not}),$$
$$\text{Existential Restriction}(\exists : \text{There exists}\ldots), \text{Universal Quantification}(\forall : \text{For all}\ldots)\}.$$

In addition, ALC contains:

$$\{\text{Atomic Concepts}(A),$$
$$\text{Top Concept}(\top : \text{Tautology}), \text{Bottom Concept}(\bot : \text{Contradiction})\}$$

In order to define a formal semantics, we need to apply terminological interpretations over our signatures. More particularly, any [terminological] interpretation consists of:

(a) a non-empty set Δ that is the interpretation domain and consists of any variable that occurs in any possible concept description, and
(b) an interpretation function $(.^I)$. I prefer to name it 'interpreter'.

The interpreter assigns m^I to every individual m. Note that m^I is in Δ^I (i.e., $m^I \in \Delta^I$). Also, it assigns to every atomic concept A, a set $A^I \subseteq \Delta^I$. In addition, it assigns to every atomic role P (between two individuals), a binary relation like P^I, such that: $P^I \subseteq \Delta^I \times \Delta^I$. This relationship is inductively extendable for any n-ary relationship over the interpretation domain. Table 1 reports the syntax and the semantics of ALC.

Table 1. The prototypical description logic

Syntax	Semantics
A	$A^I \subseteq \Delta^I$
P	$P^I \subseteq \Delta^I \times \Delta^I$
$\top$	Δ^I
$\bot$	$\varnothing$
$C \sqcap D$	$(C \sqcap D)^I = C^I \cap D^I$
$C \sqcup D$	$(C \sqcup D)^I = C^I \cup D^I$
$\neg C$	$(\neg C)^I = \Delta^I \setminus C^I$
$\exists R.\ C$	$\{a \mid \exists b.(a,b) \in R^I \wedge b \in C^I\}$
$\forall R.\ C$	$\{a \mid \forall b.(a,b) \in R^I \supset b \in C^I\}$

A knowledge base in DLs usually consists of a number of terminological axioms and world descriptions (so-called 'assertions'), see Table 2.

Table 2. Axioms and world descriptions in DLs

Name	Syntax	Semantics
Concept inclusion axiom	$C \sqsubseteq D$	$C^I \subseteq D^I$
Role inclusion axiom	$R \sqsubseteq S$	$R^I \subseteq S^I$
Concept equality axiom	$C \equiv D$	$C^I = D^I$
Role equality axiom	$R \equiv S$	$R^I = S^I$
Concept assertion	$C(a)$	$a^I \in C^I$
Role assertion	$R(a, b)$	$(a^I, b^I) \in R^I$

In DLs, in case a given terminological interpretation could assign the value True to a concept description, that interpretation is called a 'model' of that description. Consequently, a terminological interpretation (like I) can be a model of a terminological and, respectively, of an assertional description if and only if it can satisfy them semantically, see Tables 2 and 3. In these Tables P is an atomic role, R and S are role descriptions, A is an atomic concept, and C and D are concept descriptions.

Table 3. Inductive concept descriptions.

Over concept	Over role
$A^I \subseteq \Delta^I$	$P^I \subseteq \Delta^I \times \Delta^I$
$\bot^I = \varnothing$	$\bot^I = \varnothing$
$(\neg C)^I = \Delta^I \setminus C^I$	$(\neg R)^I = (\Delta^I \times \Delta^I) \setminus R^I$
$(C \sqcap D)^I = C^I \cap D^I$	$(R \sqcap S)^I = R^I \cap S^I$

4 Logical Characterisation of Terminological Knowledge

Description Logics (DLs) are a family of semi-formal descriptive languages. Any DL (like ALC) represents concepts and their interrelationships in order to represent terminological knowledge. Subsequently, it provides a logical backbone for concept-based reasoning processes. In this section, I do focus on logical analysis of the terminological background of concept-based reasoning.

Suppose that the function $M_K(C)$ denotes that machine (M) has—on a basis supported by its terminological knowledge (K)—focused on the concept C. Let T and W stand for a terminology and a world description, respectively. Thus, the terminological knowledge is equal to (T, W). More specifically, considering E_C as a set of examples of the concept C,

$$E_C = \{E_C^+, E_C^-\}$$

where, E_C^+ and E_C^- stand for positive and negative examples of C, respectively. Consequently:

- If a concept assertion (like $D(a)$) is satisfied by K (and, in fact, by (T, W)), then:

$$\forall a \in E_C^+(W), K \vDash D(a).$$

- If a concept assertion (like $D(b)$) is not satisfied by K, then:

$$\forall b \in E_C^-(W), K \nvDash D(b).$$

The logical symbol '$\vDash$' in the term '$K \vDash D(a)$' denotes that knowledge K has been supportive (and satisfactory) for satisfying the concept description $D(a)$. In fact, K (as a collection of terminology T and world description W) can satisfy the expressed concept description. Also, the symbol '$\nvDash$' in the term '$K \nvDash D(b)$' describes that K has not been supportive (and satisfactory) for satisfying the concept description $D(b)$.

These conclusions are also valid in the case of roles. More specifically, $M_K(R)$ denotes that machine M has—on a basis supported by its terminological knowledge (K)—focused on the n-ary role (R). Consequently:

$$E_R = \{E_R^+, E_R^-\}.$$

Therefore:

- If the role assertion $R(a_1, a_2, \ldots, a_n)$ is satisfied by K (and, in fact, by (T, W)), then:

$$\forall a_1, a_2, \ldots, a_n \in E_R^+(W), K \models R(a_1, a_2, \ldots, a_n).$$

- If the role assertion $R(b_1, b_2, \ldots, b_n)$ is not satisfied by K, then:

$$\forall b_1, b_2, \ldots, b_n \in E_R^-(W), K \not\models R(b_1, b_2, \ldots, b_n).$$

5 Logical Clarification of Concept Understanding

This section offers two examples in order to deal with a logical clarification of the phenomenon of 'concept understanding'.

5.1 Example I

Mary thinks that the term 'there is a young student' and the term 'there is a non-old student' are equivalent to each other. Mary's verification between these two propositions is expressible in DLs by:

$$\exists \text{hasStudent.Young} \equiv \exists \text{hasStudent.} \neg \text{Old}.$$

We can figure out that Mary has, mentally, assumed the axiom stating that Young and Old are two disjoint (= distinct) concepts. In fact, the logical description 'Young $\sqcap$ Old $\sqsubseteq \bot$' has formed a presupposition (in the form of a terminological axiom) in Mary's mind. It's obvious that Mary's interpretation has played crucial roles here. More specifically, her terminological interpretation has been carried out based on the following fundamental logical descriptions:

i. Young $\sqcap$ Old $\sqsubseteq \bot$. This fundamental description expresses that Young and Old are two disjoint concepts in Mary's mind.
ii. Person $\sqsubseteq$ Young $\sqcup$ Old. This fundamental description means that every person is either young or old in Mary's mind. Equivalently, any person could be described (and predicated) either by the predicate Young or by the predicate Old.

Mary has interpreted and, respectively, has understood that the proposition 'there is a young student' and the proposition 'there is a non-old student' have the same meanings. More specifically, Mary's terminological interpretations (over 'Young $\sqcap$ Old $\sqsubseteq \bot$' and 'Person $\sqsubseteq$ Young $\sqcup$ Old') have produced her understanding of the equivalence ($\equiv$) between the concept descriptions '$\exists$hasStudent.Young' and '$\exists$hasStudent.$\neg$Old'. We can see that Mary's interpretation has been restricted (limited) to her understanding of the disjointness of the concept descriptions '$\exists$hasStudent.Young' and

'∃hasStudent.¬Old'. Note that two concepts (concept descriptions) like C and D are logically and semantically equivalent when, 'for all' possible terminological interpretations like I, we have: $C^I = D^I$.

If one other person, say John, does not assume the axioms stating that 'Young and Old are two disjoint concepts' and 'every person is either young or old', then there will not be an equivalence relation between ∃hasStudent.Young and ∃hasStudent.¬Old. Let me conclude that Mary's and John's concept understandings are dissimilar, because they have had different terminological interpretations in their minds. Such a difference is caused by their different conceptions of the world. For example, John may—regarding his terminological interpretation—believe that the proposition 'there is a middle-aged student' comes next to 'there is a young student' and 'there is a non-old student'. In fact, John keeps in mind the axiom 'Person ⊑ Young ⊔ MiddleAged ⊔ Old'. It means that every person is young or middle-aged or old. Consequently, John by taking this axiom (based on his own conception) into consideration doesn't understand '∃hasStudent.Young' and '∃hasStudent.¬Old' as equivalent concept descriptions.

5.2 Example II

Mary believes that the propositions 'Anna has a child who is a philosopher' and 'Anna has a child who is a painter' could be jointly expressed by the proposition 'Anna has a child who is a philosopher and painter'. Translated into DLs we have her description as:

$$(\exists\, \text{hasChild.Philosopher} \sqcap \exists\, \text{hasChild.Painter}) \equiv \\ \exists \text{hasChild.}(\text{Philosopher} \sqcap \text{Painter}).$$

Suppose that Anna has two children and one is a philosopher and the other one is a painter. Then, ∃hasChild.(Philosopher ⊓ Painter) is not equivalent to ∃hasChild. Philosopher ⊓ ∃hasChild.Painter, because the one who is a philosopher, is not a painter, and vice-versa.

Actually, Mary has not proposed a correct description. Her non-correct description is caused by her inappropriate terminological interpretation. Accordingly, her concept understanding has followed her inappropriate interpretation. In fact, she has incorrectly understood that the proposition 'Anna has a child who is a philosopher and painter' is equivalent to the collection of the propositions 'Anna has a child who is a philosopher' and 'Anna has a child who is a painter'. Reconsidering the proposed formalism, the concept descriptions:

1. '∃hasChild.Philosopher ⊓ ∃hasChild.Painter', and
2. '∃hasChild.(Philosopher ⊓ Painter)'

are not, semantically, the same. In fact, there should not be an equivalence symbol between them. Thus, Mary's interpretation has not been satisfactory. Subsequently, her concept understanding is not satisfactory and appropriate.

6 A Semantic Model for Concept Understanding

Relying on Sect. 4, this section focuses on logical analysis of concept understanding and its terminological representation. More specifically, this section by taking into account my logical conceptions of ‘concept understanding’ (offered in Sect. 2) analyses a formal semantics and, subsequently, focuses on the junctions between ‘understanding of concept understanding’ and ‘concept understanding representation’ in terminological systems.

6.1 Concept Understanding as a Relation (and Function)

I shall claim that ‘concept understanding’ expresses a relationship. This relationship relates ‘the characteristics and attributes of a concept’ to ‘a description’. More specifically, *understanding* is a function (mapping) from a concept (conceptual entity) as well as its attributes into a statement. In fact, one could, based on his personal concept understanding, propose his personal concept descriptions. Therefore:

$$Concept\,Understanding : Concept \rightarrow Concept\,Description.$$

Let me be more specific:

A. A human being—by concept understanding—attempts to map the significant characteristics of concepts into some concept descriptions. For example, ‘breathing’, as a biological and psychological process, is a characteristic and trait of all animals. Then, breathing (that is a role) is the characteristic of the concept Animal. Therefore, (i) knowing the fact that the individual ‘horse’ is an instance of the concept ‘Animal’ (Formally: Animal(horse)), and (ii) drawing the [concept subsumption] inference ‘Horse ⊑ Animal’, collectively lead us to knowing and, subsequently, to understanding that ‘horses breathe’ (or equivalently: ‘horses do breathing’). Note that the role ‘breathing’ could be manifested in the concept ‘Breath’. Therefore, (i) and (ii) collectively lead us to expressing the concept description ‘Animal(horse) ⊓ ∃hasTrait.Breath’ for the individual ‘horse’ (as an instance of the concept ‘Animal’) and, respectively, for the concept ‘Horse’ (as a sub-concept of ‘Animal’).
B. A human being—by concept understanding—attempts to map the concepts’ reflexive properties (concepts’ interrelationships with themselves) into some concept descriptions. For example, one who knows that ‘male horses breathe’, by taking into consideration the terminological and assertional axioms:

   {
   Animal(horse),
   Horse ⊑ Animal,
   MaleHorse ⊑ Horse,
   FemaleHorse ⊑ Horse
   },

   can figure out and, accordingly, can understand that ‘female horses breathe’ as well.

C. A human being—by concept understanding—attempts to map the concepts' properties (and their relationships with other concepts) into some concept descriptions. For example, one who knows that 'horses breathe' (and as described: Animal(horse) $\sqcap$ $\exists$hasTrait.Breath), could, respectively figure out and understand that the individual 'rabbit' (that is an Animal) breathes as well. So, he could express that 'rabbits breathe' and, in fact, Animal(rabbit) $\sqcap$ $\exists$hasTrait.Breath.

Conclusion. Relying on Predicate Logic (and on DLs), the phenomenon of 'concept understanding' could be interpreted a 'binary predicate' (and a 'role' of human beings on expressing concept descriptions). This role will be represented by 'understanding' in my formalism.

6.2 Concept Understanding as a Conceptualisation

Concept understanding could be interpreted the limit/type of conceptualisation. Accordingly, humans need to conceptualise concepts in order to understand them. In [2, 3], I have interpreted a 'concept understanding' a local manifestation of a global 'conceptualisation'. Then, I acknowledge one's 'concept understanding' as a limited type of his own conceptualising. Note that 'conceptualising' could be recognised his role. This conclusion, relying on DLs, could be represented by the role inclusion (or role sub-sumption):

$$\text{understanding} \sqsubseteq \text{conceptualising}.$$

In fact, considering C a concept,

$$understanding\, C \subseteq conceptualising\, C.$$

On the other hand, 'it is not the case that all conceptualisations are concept understandings'. In fact, all the conceptualised concepts are not necessarily understood.

6.3 Concept Understanding as an Interpretation-Based Model

Generally, an interpretation is the act of elucidation, explication, and explanation, see [22]. According to [14] and through the lens of philosophy, "…in existential and hermeneutic philosophy, interpretation becomes the most essential moment of human life: The human being is characterized by having an 'understanding' of itself, the world, and others. This understanding, to be sure, does not consist—as in classical ontology or epistemology—in universal features of universe or mind, but in subjective–relative and historically situated interpretations of the social. …". Regarding [7] and through the lens of logic, an 'interpretation' of a logical system assigns meanings as well as semantic values to the formulae and their elements. At this point I shall emphasise that formal languages may see the phenomenon of 'interpretation' terminologically. In fact, a logical-terminological system can restrict the phenomenon of 'interpretation' to the phenomenon of 'terminological interpretation' in order to assess and apply it in logical as well as terminological contexts. More specifically, from the perspective of a logical-terminological system, one who has engaged his interpretations to explicate [and justify] what [and why] he means by classifying a thing/phenomenon as an instance of a concept, needs to interpret the non-logical signifiers of various concept descriptions within his linguistic expressions.

Considering any set of non-logical symbols (that have no logical consequences) in a terminology, a terminological interpretation of humans' languages could be described to be constructed based on the tuple:

$$(Interpretation\,Domain, Interpretation\,Function).$$

The first component (the interpretation domain) expresses the universe of the interpretation. Note that in linguistic and philosophical approaches it might be called 'universe of discourse'.

As mentioned above, an interpretation domain (like D) must be non-empty (i.e.., $D \neq \varnothing$). This non-empty set supports the range of any variable that occurs in any of the concept descriptions within logical descriptions of linguistic expressions. It is a fact that the collection of the rules and the processes that manage different terms and logical descriptions in linguistic expressions, cannot have any meaning until the non-logical signifiers and constructors are given terminological interpretations. The interpretations prepare human beings for producing their personal meaningful as well as understandable concept descriptions. Hence, I believe that all 'concept understandings' are 'concept interpretations'. According to [4], this conclusion could be represented by the role inclusion:

$$\text{understanding} \sqsubseteq \text{interpreting}.$$

In fact, considering C a concept,

$$understanding\,C \subseteq interpreting\,C.$$

But, on the other hand, all interpretations (over concepts) do not imply understandings (of concepts). Equivalently, 'it is not the case that all concept interpretations are concept understandings'. In other words, all the interpreted concepts may not be understood. Accordingly, considering any interpretation a function, 'concept understanding' is recognised an 'interpretation function'.

From this point I apply the function UND (as a limit of the interpretation function I) in my formalism. Then, considering C a concept,

$$C^{UND} = Understanding\,C.$$

Consequently, considering UND a kind of interpretation, there exists a tuple like (D_U, $C_{understood}$), where:

i. D_U represents the understanding domain (that consists of the variables that occur in any of the concept descriptions that are going to be understood), and
ii. $C_{understood}$ is the understood concept.

$C_{understood}$ is achievable based on the understanding function $-^{UND}$. Relying on the function $-^{UND}$,

$$C^{UND} \subseteq C^{I} \subseteq \Delta^{I} \\ \& \\ D_{U}^{UND} \subseteq \Delta^{I}.$$

D_{U}^{UND} means 'understanding all concepts belonging to the understanding domain'. Note that $-^{UND}$ (that is a function) can provide a model for terminological and, respectively, for assertional axioms. Therefore, the desired model:

- is a restricted form of a terminological-interpretation-based model, and
- can satisfy the semantics of the terminological and assertional axioms ('$UND \vDash$ Axiom' expresses that *UND* satisfies the axiom).

See Table 4. Consequently:

Table 4. *Concept Understanding*: terminologies and world descriptions.

Name	Description and semantics
Understanding a concept inclusion	$[UND \vDash (C \sqsubseteq D)] \Rightarrow [C^{UND} \subseteq D^{UND}]$
Understanding a role inclusion	$[UND \vDash (R \sqsubseteq S)] \Rightarrow [R^{UND} \subseteq S^{UND}]$
Understanding a concept equality	$[UND \vDash (C \equiv D)] \Rightarrow [C^{UND} = D^{UND}]$
Understanding a role equality	$[UND \vDash (R \equiv S)] \Rightarrow [R^{UND} = S^{UND}]$
Understanding a concept assertion	$[UND \vDash C(a)] \Rightarrow [a^{UND} \in C^{UND}]$
Understanding a role assertion	$[UND \vDash R(a_1, a_2, \ldots, a_n)] \Rightarrow$ $[(a_1^{UND}, a_2^{UND}, \ldots, a_n^{UND}) \in R^{UND}]$

$$C^{UND} \subseteq C^{I} \subseteq \Delta^{I} \\ \& \\ -^{UND} : C \to C^{UND} \\ \text{Where} : C^{UND} \subseteq D_{U}^{UND} \subseteq \Delta^{I}.$$

I shall emphasise that we are not able to conclude that $C^{I} \subseteq D_{U}^{UND}$. On the other hand, we certainly know that $C^{UND} \subseteq \Delta^{I}$ (because $C^{UND} \subseteq C^{I}$ and $C^{I} \subseteq \Delta^{I}$). According to the analysed characteristics, the *UND* understanding model in my terminology is constructed over the tuple:

$$(Understanding\ Domain, Understanding\ Function).$$

And, formally:

$$UND = (D_{U}^{UND}, -^{UND}).$$

Table 5 represents understanding inductive concept descriptions as the products of the proposed understanding model. This Table is logically supported by Table 4, see [4].

Table 5. Understanding inductive concept descriptions.

Model satisfies the vocabulary	Semantics
$UND \models \top$	$\top^{UND} = \top$
$UND \models \bot$	$\bot^{UND} = \varnothing$
$UND \models \neg R$	$(\neg R)^{UND} = \top \backslash R^{UND}$
$UND \models \neg C$	$(\neg C)^{UND} = D_U^{UND} \backslash C^{UND}$
$UND \models (R \sqcap S)$	$(R \sqcap S)^{UND} = R^{UND} \cap S^{UND}$
$UND \models (C \sqcap D)$	$(C \sqcap D)^{UND} = C^{UND} \cap D^{UND}$

6.4 Concept Understanding as a Consequence of Functional Roles

How could we employ DLs in order to describe a [concept] understanding function? In my opinion, an understanding function must be interpreted a functional role of human beings in order to be, logically, described. The functional roles (features) are the roles that are structurally as well as inherently functions and, hence, they can express *functional* actions, movements, procedures, and manners of human beings.

Let N_F be a set of functional roles and N_R be the set of roles (role descriptions). Obviously: $N_F \subseteq N_R$. Informally, functional roles are some kinds of roles.

Lemma. The *UND* understanding model is, semantically, structured based on:

a. the understanding domain (or D_U),
b. the understanding function (or $\cdot^{UND}$), and
c. the set D_U^{UND} (or equivalently, the effect of the understanding function $\cdot^{UND}$ on the Top concept) that represents understanding all atomic concepts (everything) in the understanding domain.

Analysis. The *UND* model associates with each atomic concept a subset of D_U^{UND} and with each ordinary atomic role a binary relation over $D_U^{UND} \times D_U^{UND}$. Assessed by Mathematics, any functional role can be seen as a partial function. More specifically, considering F as a chain of functional roles or, equivalently,

$$F = f_1 \circ \cdots \circ f_n,$$

the composition of n partial concept understanding functions can be represented by:

$$f_1^{UND} \circ \cdots \circ f_n^{UND}.$$

In fact, by employing *UND*, any f_i^{UND}—semantically—supports the overall functional role F^{UND}. Note that for all i in $(1,n)$, f_{i+1} produces the input of f_i. Therefore, understanding f_{i+1} (the output of f_{i+1}) provides the input of understanding f_i. In particular, any concept description could be understood over the subsets of D_U^{UND}. This characteristic is very useful in making a strong linkage between the terms 'concept understanding' and 'chain of functional roles'. It supports my semantic model in scheming and describing '*the concept understanding as the product of a chain of* *functional roles, where the functional roles are the partial understanding functions*'. You will see how it works.

6.5 Humans' Functional Roles Through SOLO's Levels

According to [6], the Structure of Observed Learning Outcomes (SOLO) taxonomy is a proper model that can provide an organised framework for representing different levels of humans' understandings. This model is concerned with various complexities of understanding on its different layers. According to SOLO taxonomy and taking into consideration humans' multiple layers of knowledge (based on concepts), we have:

- Pre-structured knowledge. Here, humans' knowledge of a concept is pre-structured. The pre-structured knowledge is the product of one's pre-conceptions of a concept.
- Uni-structured knowledge. Humans have a limited knowledge about a concept. Having a uni-structured knowledge is the outcome of knowing one or few isolated fact(s) about a concept.
- Multi-structured knowledge. Humans are getting to know a few facts relevant to a concept, but they are still unable to link and relate them together.
- Related Knowledge. Humans have started to move towards deeper levels of understanding of a concept. Here, they are able to explain their several conceptions of a concept. Also, they can link different facts (regarding their conceptions of a concept) to each other.
- Extended Abstracts. This is the most complicated level. Humans are not only able to link lots of related conceptions (of a concept) to each other, but they can also link them to other specified and complicated conceptions. Now, they are able to link multiple facts and explanations in order to produce more complicated extensions relevant to a concept.

Obviously, the extended abstracts are the products of deeper comprehensions of related structures. Related structures are the products of deeper comprehensions of multi-structures. The multi-structures are the products of deeper comprehensions of uni-structures, and the uni-structures are the products of deeper comprehensions of pre-structures.

Let me select a process as a sample of humans' functional roles from any of the SOLO's levels and formalise it. According to SOLO, (a) the phenomenon of '*creation*' (based on a concept) is an instance of the 'extended abstracts', (b) the phenomenon of '*justification*' (based on a concept) is an instance of the 'related structures', (c) the phenomenon of '*description*' (based on a concept) is an instance of the 'multi-structures', and (d) the phenomenon of '*identification*' (based on a concept) is an instance of the 'uni-structures'. Therefore, the phenomena of '*Creation*', '*Justification*', '*Description*', and '*Identification*' are four processes. These processes can be seen and interpreted functions in my semantic model. More specifically, any of these functions can support a functional role and, subsequently, can support a 'partial concept understanding function'. Actually,

i. *Creation* has interrelatedness with creatingOf that is a functional role and extends the humans' mental abstracts.

ii. *Justification* has interrelatedness with the functional role justifyingOf. This functional role relates the lower structures.
iii. *Description* has correlation with the functional role describingOf. This role produces the multi-structures.
iv. *Identification* has correlation with the functional role identifyingOf that generates the uni-structures.

It shall be emphasised that identifyingOf, describingOf, justifyingOf, and creatingOf are only four examples of functional roles within SOLO's categories and, in fact, the SOLO's levels are not limited to these functions. For example, followingOf and namingOf are two other instances of the uni-structures, combiningOf and enumeratingOf are two other instances of the multi-structures, analysingOf and arguingOf are two other instances of the related structures, and formulatingOf and theorisingOf are two other instances of the extended abstracts.

As mentioned, the functional roles creatingOf, justifyingOf, describingOf, and identifyingOf represent the equivalent roles of the *creation*, *justification*, *description*, and *identification* functions, respectively. Furthermore, these functions are the partial functions of the [concept] *understanding* function. Obviously, the concept understanding function (that is a process) could also be considered to be equivalent to a functional role like understandingOf. Employing the 'role inclusion' axiom we have:

(1) creatingOf $\sqsubseteq$ understandingOf,
(2) justifyingOf $\sqsubseteq$ understandingOf,
(3) describingOf $\sqsubseteq$ understandingOf, and
(4) identifyingOf $\sqsubseteq$ understandingOf.

Equivalently:

(1) *creation* $\subseteq$ *understanding*,
(2) *justification* $\subseteq$ *understanding*,
(3) *description* $\subseteq$ *understanding*, and
(4) *identification* $\subseteq$ *understanding*.

It shall be claimed that the role 'understandingOf', conceptually and logically, supports 'the [concept] *understanding* function based on the analysed [concept] understanding model (or *UND*)'. Similarly, we can define *CRN*, *JSN*, *DSN*, and *IDN* as sub-models of *UND* for representing *creation*, *justification*, *description*, and *identification*, respectively. Any of these models can, semantically, satisfy the terminologies and world descriptions in Table 4. Accordingly, relying on inductive rules, they can satisfy concept descriptions in Table 5.

Note that *CRN* (as a model) fulfils the desires of *UND* better (and more satisfying) than *JSN*, *DSN*, and *IDN*. Considering D_U as the understanding domain, we have:

$$D_U^{UND} \subseteq D_U^{CRN} \subseteq D_U^{JSN} \subseteq D_U^{DSN} \subseteq D_U^{IDN}.$$

More specifically:

- D_U^{CRN} represents the model of *creation* over the understanding domain. It consists of concepts which are (or could be) 'created' by human beings. Formally: $C^{CRN} \in D_U^{CRN}$.
- D_U^{JSN} represents the model of *justification* over the understanding domain. It consists of concepts which are (or could be) 'justified' by human beings. Formally: $C^{JSN} \in D_U^{JSN}$.
- D_U^{DSN} represents the model of *description* over the understanding domain. It consists of concepts which are (or could be) 'described' by human beings. Formally: $C^{DSN} \in D_U^{DSN}$.
- D_U^{IDN} represents the model of *Identification* over the understanding domain. It consists of concepts which are (or could be) 'identified' by human beings. Formally: $C^{IDN} \in D_U^{IDN}$.

Proposition. The terminological axioms and the world descriptions (in Table 4) and inductive concept descriptions (in Table 5) are all valid and meaningful for *CRN*, *JSN*, *DSN*, and *IDN*. Therefore, inductive concept descriptions are also valid and meaningful over the concatenation of the *creation*, *justification*, *description*, and *identification* functions that have supported these terminological models.

Proposition. All semantic satisfactions based on *IDN* are already satisfied by *DSN*, *JSN*, and *CRN* over D_U^{DSN}, D_U^{JSN}, and D_U^{CRN}, respectively. Informally, if one is able to focus on describing, justifying, and creating based on his conceptions of a concept, so, he is already capable of identifying that concept. Furthermore, he might be able to identify something else (some other phenomenon) with regard to his conception of that concept.

Formal Analysis. The formal semantics of the composite function '*creation* (*justification* (*description* (*identification* (*C*))))'—that is the product of the chain of functional roles—supports the proposed semantic model on D_U^{UND}, which is the central domain of concept understanding (central part of the understanding domain). Considering all the roles relevant for the concept *C*, we have:

$$(\forall R_1.C)^{CRN} = \{a \in D_U^{CRN} | \forall b.(a,b) \in R_1^{CRN} \rightarrow b \in C^{CRN}\}.$$

Therefore:

$$(\forall R_2.C)^{JSN} = \{a \in D^{JSN} | \forall b.(a,b) \in R_2^{JSN} \rightarrow b \in C^{JSN}\}.$$

Therefore:

$$(\forall R_3.C)^{DSN} = \{a \in D_U^{DSN} | \forall b.(a,b) \in R_3^{DSN} \rightarrow b \in C^{DSN}\}.$$

Therefore:

$$(\forall R_4.C)^{IDN} = \{a \in D_U^{IDN} | \forall b.(a,b) \in R_4^{IDN} \rightarrow b \in C^{IDN}\}.$$

In this formalism, R_1, R_2, R_3, and R_4 stand for creatingOf, justifyingOf, describingOf, and identifyingOf, respectively. Consequently, *CRN*, *JSN*, *DSN*, and *IDN* have been interpreted roles of human beings. Accordingly, it's possible to represent the chain of functional roles in the form of the collection of the following implications:

$$\begin{aligned}&(\forall R_1.C)^{CRN} \Rightarrow \\ &\quad(\forall R_2.C)^{JSN} \Rightarrow \\ &\qquad(\forall R_3.C)^{DSN} \Rightarrow \\ &\qquad\quad(\forall R_4.C)^{IDN}.\end{aligned}$$

It must be concluded that 'any role based on a conception of the concept *C*' to the left of any of implications ($\Rightarrow$) makes a logical premise for 'other roles based on the conceptions of the concept *C*' to the right of that implication. It shall be stressed that this is a very important terminological fact in semantic analysis of concept understanding. The deduced logical relationship represents a stream of concept understanding from deeper layers to shallower layers.

7 An Ontology for Concept Understanding

From the philosophical point of view, an ontology is described as studying the science of being and existence, see [13, 23]. Ontologies must be capable of demonstrating the structure of the reality of a thing/phenomenon. They check multiple attributes, particularities, and properties that belong to a thing/phenomenon because of its structural existence. From another perspective and through the lenses of information and computer sciences, an ontology is an explicit [and formal] specification of a shared conceptualisation.

However, in my opinion, there are some conceptual relationships between these two descriptions of ontologies. Actually, ontologies in information sciences attempt to mirror the phenomena's structures in virtual and artificial systems. In fact, they focus on conceptual descriptions of phenomena's structures in order to provide proper backgrounds for specifications of their conceptualisations. Hence, the ontological descriptions in information sciences (and in knowledge-based systems) tackle to provide appropriate logical and formal descriptions of a phenomenon's structure as well as its dependency to the other phenomena and to the environment. From this perspective, an ontology can be schemed and demonstrated by semantic networks and semantic representations. A semantic network is a graph whose nodes represent entities and whose arcs represent relationships between those entities.

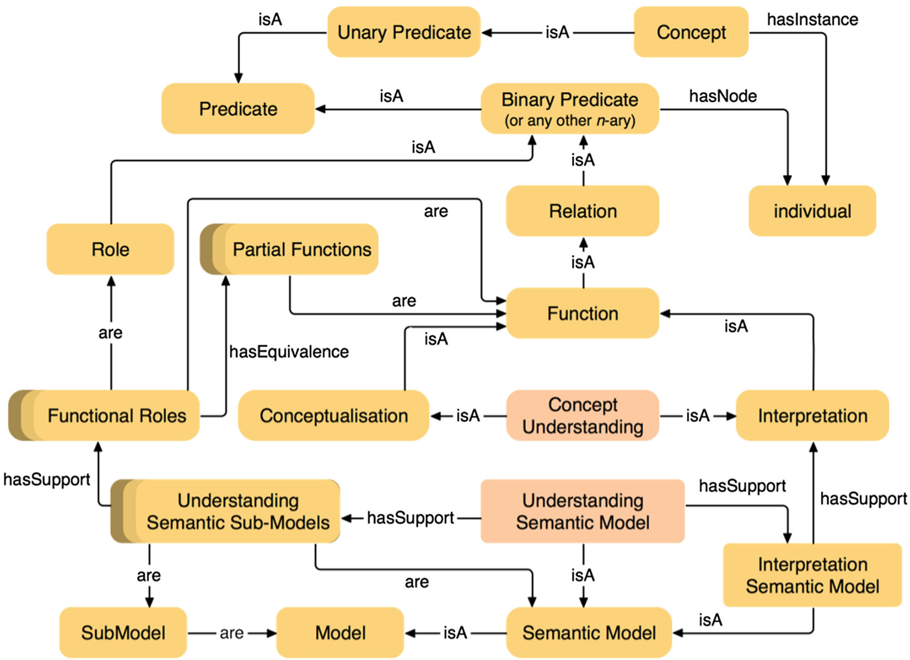

Fig. 1. An ontology for concept understanding.

According to [4], Fig. 1 represents a semantic network as an ontology for the phenomenon of 'concept understanding'. This hierarchical semantic representation:

- specifies the conceptual relationships between the most important ingredients of this research,
- demonstrates the logical representation of the phenomenon of 'concept understanding', and
- shows how the proposed semantic model attempts to represent concept understanding.

This semantic representation can be interpreted a specification of the shared conceptualisation of 'concept understanding' within terminological systems.

Note that the constructed ontology can be reformulated and formalised in ALC in the form of a collection of fundamental terminologies as following:

{
UnaryPredicate $\sqsubseteq$ Predicate,
BinaryPredicate $\sqsubseteq$ Predicate,
Concept $\sqsubseteq$ UnaryPredicate,
Concept $\sqsubseteq$ $\exists$hasInstance.Individual,
BinaryPredicate $\sqsubseteq$ ($\exists$hasNode.Individual $\sqcap$ $\exists$hasNode.Individual),
Role $\sqsubseteq$ BinaryPredicate,
Relation $\sqsubseteq$ BinaryPredicate,

Function $\sqsubseteq$ Relation,
Interpretation $\sqsubseteq$ Function,
Conceptualisation $\sqsubseteq$ Function,
ConceptUnderstanding $\sqsubseteq$ Interpretation,
ConceptUnderstanding $\sqsubseteq$ Conceptualisation,
PartialFunction $\sqsubseteq$ Function,
FunctionalRole $\sqsubseteq$ Role,
FunctionalRole $\sqsubseteq$ hasEquivalence.PartialFunction,
FunctionalRole $\sqsubseteq$ Function,
SubModel $\sqsubseteq$ Model,
SemanticModel $\sqsubseteq$ Model,
InterpretationSemanticModel $\sqsubseteq$ SemanticModel,
UnderstandingSemanticModel $\sqsubseteq$ SemanticModel,
UnderstandingSemanticSubModel $\sqsubseteq$ SubModel,
UnderstandingSemanticSubModel $\sqsubseteq$ SemanticModel,
InterpretationSemanticModel $\sqsubseteq$ $\exists$hasSupport.Interpretation,
UnderstandingSemanticModel $\sqsubseteq$ $\exists$hasSupport.InterpretationSemanticModel,
UnderstandingSemanticModel $\sqsubseteq$ $\exists$hasSupport.UnderstandingSemanticSubModel,
UnderstandingSemanticSubModel $\sqsubseteq$ $\exists$hasSupport.FunctionalRole
}

8 Concluding Remarks

Description Logics (DLs) attempt to provide descriptive knowledge representation formalisms to establish common grounds and interrelationships between human beings and machines. DLs have assisted me in revealing some hidden conceptual and logical assumptions about the phenomena of ‘concept’ and ‘concept understanding’. More specifically, these assumptions can produce a better conceptualisation (and respectively, understanding) of ‘concept understanding’. In this article, DLs have—by considering concepts as unary predicates and by applying terminological interpretations over them—proposed a realisable, as well as assessable, logical description for explaining the humans’ concept understanding. Relying on such a logical description a theoretical model for concept understanding has been offered. The proposed model attempts to reflect the phenomenon of ‘concept understanding’ in terminological knowledge representation systems. It shall be concluded that the most significant contribution of the article has been providing a formal semantics for logical analysis of concept understanding. According to the logical analysis, a logical background for terminological representation of concept understanding has been expressed. Consequently, an ontology for ‘concept understanding’ has been designed and formalised. The offered ontology specifies my conceptualisation of the phenomenon of ‘concept understanding’.

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