Chapter 6
History of Mathematics and Culture: Moments and Movements in Brazilian Mathematics Education

Cristiane Coppe de Oliveira, Cláudia Regina Flores, Daniel Clark Orey, and Maria Cristina Araújo de Oliveira

Abstract This chapter presents moments and movements discussed by the Working Group WG05-History of Mathematics and Culture of the Brazilian Society of Mathematics Education. Considering the proceedings and the books that have resulted from the six editions of the International Seminar of Research, SIPEM, we conducted a survey to map the current research and explore the discussions, drawing conclusions concerning the theoretical and methodological perspectives of the group. In the study, three axes have been characterized: ethnomathematics, history of mathematics, and history of mathematics education. We observed a variety of approaches that can be summarized around the area of multicultural education, the role of history in teaching and teacher education, and the importance of developing historical knowledge about the creation of knowledge, methodologies, and pedagogical practices. Here, the importance of research that incorporates diverse sociocultural methods to produce mathematical and historical knowledge in school mathematics and its pedagogical practices cannot be overstated.

6.1 Introduction

The recognition that many diverse cultures have developed a variety of techniques to work with calculations, measurements, comparisons, and volumes, among others throughout history has led researchers, nationally and internationally, to wonder

C. C. de Oliveira (✉)
Universidade Federal de Uberlândia, Uberlândia, MG, Brazil

C. R. Flores
Universidade Federal de Santa Catarina, Florianópolis, SC, Brazil

D. C. Orey
Universidade Federal de Ouro Preto, Ouro Preto, MG, Brazil

M. C. A. de Oliveira
Universidade Federal de Juiz de Fora, Juiz de Fora, MG, Brazil

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about a variety of ways in which mathematical knowledge is produced. According to Bishop (1994), this form of research is based on an anthropological-ethnographic perspective and seeks to understand mathematical knowledge from three major focuses.

The first, with an emphasis on traditional cultures, shows the existence of knowledge and practices experienced by members of different sociocultural groups. The second, centres on ancient documents of non-western societies, and is based on historical values. Finally, the third focuses on different groups in a society to enable investigations related to the production of mathematical knowledge in a socio-psychological way.

According to D'Ambrosio (2005), in the early 1970s, many Brazilian researchers, along with researchers from the United States, took an interest in those issues, bringing their perspectives and findings to the field of mathematics education. D'Ambrosio (1985) initiated a research line he named the "ethnomathematics program", which discusses the mathematics produced by a diversity of sociocultural groups, including the importance, appreciation, and awareness of those practices for mathematics education.

From that line of research, a working group called Mathematics Education, History and Culture was formed under the coordination of Prof. Ubiratan D'Ambrosio. In 2000, the group held its first meeting and developed several works, which were synthesized into a *Book of Abstracts* from SIPEM I held in Serra Negra, São Paulo, Brazil.

Since then, members of this group have continued their work around two key words or themes: history and culture. Thus, by including an ethnomathematics perspective, researchers started a dialogue between culture and the production, generation, institutionalization, and dissemination of knowledge related to diverse ways of counting, sorting, ordering, locating, modelling, explaining, and inferring the mathematics both found and used in cultural contexts that often break with classical education paradigms.

D'Ambrosio (1999b) stated that history shows that the evolution of academic mathematics reflects changes of cultural, linguistic, social, political, economic, ideological, and religious factors. Regarding this, many investigations have been conducted concerning the history of mathematics and mathematics education, where the researchers investigated interfaces between the history of mathematics and its contributions to mathematics education. This included work that sought to look at how the teaching and learning processes occur in the context of the historical understanding of school mathematics.

This chapter presents the ideas shared and movements discussed by the members of this group, as well as the analysis of proceedings and books from the former six editions of SIPEM. In the study, three axes have been characterized: ethnomathematics, history of mathematics, and history of mathematics education. We are pleased to see a variety of approaches articulated with multicultural education, with the role of history in teaching and teacher education, and the importance of historical knowledge on the constitution of knowledge and practices, methodologies, and pedagogical practices.

A remarkable trend has been growing steadily concerning research in the history of mathematics education that continues to engage the group in finding new boundaries between history and culture in mathematics education. Despite the many different approaches and problems, this WG focuses on history and culture specifically in the context of mathematics education.

6.2 History and Culture: Theory and Methodology

Since the research work of this group is concerned with both the history and culture of mathematics, it is necessary to understand how the researchers have used those terms in theoretical and methodological ways. According to the *Dictionary of Etymology*,[1] history comes from the Latin *historia* "narrative of past events, account, tale, story", from the Greek *historia* "a learning or knowing by inquiry; an account of one's inquiries, history, record, narrative", from *historein* "inquire", from *histor* "wise man, judge". From this, it follows that history as a scientific activity is the search for, and the study of traces of, past events, achievements, tragedies, and ideas that ensure the memory of a society and place humanity in a certain time. History, in this sense, is the methodical narration of facts that are important to a society.

During the 1960s and 1970s, a new way of conceiving history and historical research emerged, giving emphasis to studies through new approaches and objects that address new questions considered relevant to society. Within this new conception of history, De Certeau (2005) inspires researchers involved in this WG through the notions of deviations, ownership and the very question of the subject, i.e. understanding the speaker and to whom he speaks. So does the work of Chartier (1990), by a new theoretical horizon, which argues that cultural objects are produced "between practices and representations", and that the subjects that produce and receive culture correspond to "ways of doing" and "ways of seeing".

The notion of culture, in its broadest sense, goes along with the research in this WG. However, just like history, culture encompasses complex concepts that guide the way we do research. According to the *Dictionary of Etymology*, the word originates from the Latin *cultura* "cultivating, agriculture". However, since 1805, the word appears to mean "the intellectual side of civilization," and since 1867, the "collective customs and achievements of a people".

According to Laraia (1999, p. 69), "men of different cultures use various lenses and, therefore, have divergent views of things". The author also considers that the:

> way of seeing the world, the assessments of moral and value order, different social behaviours and even body postures are thus products of a cultural heritage, i.e., the result of the operation of a particular culture (1991, p. 70).

In this WG, culture has been, identified, on the one hand, by its:

[1] Retrieved from: http://www.etymonline.com. Access: October 21, 2016.

> systems of explanations, philosophies, theories, and actions and the daily behaviour. All of this is based on processes of communication, representations, classification, comparison, quantification, counting, measurement, inferences. These processes take place in differently in different cultures and transform over time. They always reveal the influences of the environment and are organized with an internal logic, they codify and formalize themselves. Thus, knowledge is born (D'Ambrosio, 2005, p. 101–102).

Certeau's discussions (2005), on the other hand, proposes that culture is a set of practices that establishes a field in which meanings are produced and shared through the processes of objectification, subjectivity, and identification. These processes generate representations that articulate a domain, a clash, a resistance, a speech, a value, and a rule. Hence, mathematical knowledge can be problematic in its practices, representations, and schooling. It is noticeable that researchers both connect favourably to, and allow for a certain focus on ethnomathematics, as well as seek to provide a connection between history and context as they are articulated by this notion of culture.

6.3 Ethnomathematics: Motivations

From the perspective of multicultural education and ethnomathematics existing in the proposal, and the work of WG5 in SIPEM I, Knijnik's research (2000) stands out in particular. The author discussed an ethnomathematics approach[2] from her seminal ethnographic research in relation to the landless movement in southern Brazil. The relations of ethnomathematics work with pedagogical implications arose from SIPEM II, held in the city of Santos, SP, with research that presents possibilities in different cultural contexts. Among the works, Monteiro (2003), for example, discussed components of mathematical modelling from the perspective of ethnomathematics, listing the possibilities and limits in school processes. Damasceno and Gomes (2003) focused on pedagogical possibilities that consider work with the cassava flour culture and found similarities between traditional knowledge and academic knowledge that helped them to identify mathematical thinking in this process.

Youth and Adult Education (EJA) is contemplated in Knijnik's work (2003) and continues to consider the relationship between culture, curriculum, and oral mathematics, in order to understand this connection as a sociocultural practice of the field. Indigenous knowledge, in turn, begins as a research strand by Mendes's work (2003), where the author analysed the number of meanings for the Kaiabi indigenous context and its consequences for the development of a mathematics textbook in their mother tongue.

[2] In 2000, the author conceptualized, as an ethnomathematics approach, the "research of mathematical traditions, practices and conceptions of a social group and the educational work that is developed, aiming for the group to interpret and decode its knowledge, acquire knowledge produced by academic mathematics, make comparisons between its knowledge and academic knowledge, analysing the power relations involved in the use of these two knowledges" (Knijnik, 2000, p. 178).

The third edition of SIPEM, held in the city of Curitiba, Paraná, Brazil, brought new kinds of research strands from an ethnomathematics perspective. Knijnik (2006) studied the dialogue between present-day ideas and Michel Foucault's discourse on peasant culture in EJA. Passos (2006) developed articulations between ethnomathematics and critical mathematics education, and Costa (2006) discussed the importance of considering the logical, mathematical, and mythological thinking in the cultural context of the African-Brazilians and indigenous peoples. Focus on research making a connection between teacher education and ethnomathematics was also emphasized. The work by Domingues (2006) discussed the issue of training indigenous teachers in the state of São Paulo, against the desire/need/complexity of acquisition of school mathematics. Domite (2006), in turn, moved from traditional teacher training paradigms as social and intellectual beings to the perspective of teachers' proximity to the culture that each student carries within him/herself. The legitimization of different forms of mathematical knowledge in teaching practices and in the mathematics curriculum was also highlighted, notably by the works presented by Mafra and Fossa (2006), Fantinato and Santos (2006), Gonçalvez and Monteiro (2006), and Santos (2006).

Pedagogical practices were discussed in the works of SIPEM IV, in the city of Brasília, Brazil. For example, Costa (2009) discussed the implementation of Federal Law 11645/08, including as mandatory African, African-Brazilian, and indigenous culture in the school curriculum. Mendonça and Pinto (2009) emphasized the practices of indigenous Xacriabá teachers in mathematics classes. The work conducted by Oliveira (2009a, 2009b) delineated new paths towards a dialogue between the Gilbert Durant's imaginary theory and ethnomathematics. The results of these studies reveal that the inclusion of cultural aspects in the mathematics curriculum has long-term benefits for students' mathematical achievements because these aspects contribute to their perception that mathematics is part of their daily lives. They also deepen the understanding of its nature by enhancing their ability to make meaningful connections. Thus, the pedagogical action of an ethnomathematics programme added value when compared to other editions of SIPEM because, generally speaking, indigenous training work gained greater prominence.

The main theme of SIPEM V, held in Petrópolis, Rio de Janeiro, Brazil, was the indigenous context. It brought the mathematical knowledge developed by members of the communities of Serra da Moça (Voltolini & Kaiber, 2012) and Paresí (Nascimento & Silva, 2012), while the research conducted by Bernardi, Caldeira, and Duarte (2012) presented contributions to the understanding of the continuous education process of indigenous Kaingang teachers in the city of Chapecó, in the state of Santa Catarina. Ethnomodelling also arose as an aspect of the ethnomathematics programme (Rosa & Orey, 2012), and Miarka (2012) brought important contributions to our thinking about the ethnographic possibilities in our work with ethnomathematics.

At SIPEM VI, the articles submitted and approved that had connection with ethnomathematics were classified in three categories: indigenous contexts and knowledge, countryside/rural contexts and knowledge, and theoretical research and reflections.

In the category of indigenous contexts and knowledge, Nascimento and Silva (2015) proposed strategies for the process of teaching and learning in geometry based on geometric patterns observed in the handicrafts developed by the Tikuna indigenous people of the Umariaçu community, in the state of Amazonas. The results of this study showed that the handicrafts, which play an integral part in the sociocultural context of the students, can help them understand academic curricular geometric concepts.

Similarly, within the school context, the results of the study conducted by Tomaz (2015) has exposed the tensions that emerge from activities related to notions of probability learned by indigenous students enrolled in an intercultural education training course for teachers at the Federal University of Minas Gerais. The analysis of the data collected in this research showed that the tensions involved in these activities occurred when these students did not recognize the application of probability in their cosmological beliefs. Tensions during the contact of two different cultural perspectives were softened when the power relationship between the teacher and the indigenous students became horizontal, allowing both indigenous and non-indigenous worldviews to be shared and incorporated.

Regarding the use of local knowledge developed by indigenous Brazilians in pedagogical actions of the ethnomathematics programme, the results of the study conducted by Costa and Mattos (2015) with members of the Karipuna ethnicity in the village of Manga, in Oiapoque, in the state of Amapá, showed that members of this cultural group developed specific community forms related to the productive process of cassava fields. From the cultural knowledge that these members collectively developed to organize the plantations, the researchers focused on the contributions of ethnomathematics regarding the teaching of mathematics. Thus, students were able to realize that this human and contextual knowledge helped them understand reality better.

Since the first Portuguese settlers arrived in Brazil, culture and customs, including mathematical knowledge and practices developed by Brazilian indigenous peoples, such as the Guaranis, were considered inferior and worthless. For example, considering the diverse multicultural characteristics of these indigenous peoples, Silva and Caldeira (2015) discussed the importance of the counting system and the graphic symbols developed by members of two Guarani groups: the *Itaty*, of Morro dos Cavalos, and the *M'Biguaçu*, located between the cities of Palhoça and Biguaçu, in the state of Santa Catarina. The results of this study show how the numbering and the graphic symbols systems the Guarani developed are intrinsically associated with the characteristics of their culture.

In the category of field/rural contexts and knowledge, the importance of establishing a connection between the school and practical knowledge that students acquire in educational institutions as well as in their own communities was verified. In this sense, Barbosa (2015) investigated how six rural education students confirmed the importance of bringing their reality to the mathematics classrooms. The results of this study showed that it was not possible to use the knowledge coming from a students' reality to establish a bridge between two different epistemologies, but it was

possible to verify the influence of this knowledge and the languages of the students to incorporate their reality to the school curriculum.

Regarding the use of local mathematical knowledge in a real context, the research conducted by Brito and Mattos (2015) analysed the mathematical knowledge produced by six farmers at Colônia Agrícola do Matapí, in Porto Grande, state of Amapá. The ethnomathematics perspective concerning the production, storage and commercialization of agricultural products and was used to analyse and interpret the results, and to establish their relationship with school knowledge.

With this in focus, Machado (2015) showed the contributions of rural producers to the processes of agricultural modernization and investigated the ways in which small producers deal with marketing their production. The results of this study showed that the mathematical knowledge used by these farmers enables them to play the role of traders in the agricultural activity. From this, a new agricultural training was proposed to adjust the model established by modernization from the very need of farmers to remain in rural areas. Other results showed that these producers were often not concerned with business accounting but used creative measures to mitigate the challenges posed by agricultural modernization.

The countryside and rural regions of Brazil present diverse and unique cultures with an increasing potential for the documentation of insights of the connections between mathematics, mathematics education, and culture. In this sense, Silva (2015) discussed the cultural approach of mathematization and measurement units that emerged from social practices developed in a rural settlement in the state of Maranhão. For that, they needed to verify possible historical convergences and cultural influences between Portugal and France in relation to the system of measures to identify the units used by rural workers in order to seek possible dialogues with school mathematics.

To discuss the principles of the ethnomathematics programme, it is necessary to propose a theoretical basis to discuss how the social, economic, and political contexts are important for the development of mathematical ideas. It is also important to know how and why different individuals reveal diverse interests, preferences, talents, and abilities, as well as specificities and strategies that we use to generate, organize, disseminate, and share mathematical knowledge to solve the problems we face in our daily lives.

In the area of research and theoretical reflections, Monteiro and Mendes (2015) investigated the ethnomathematics from studies that show of the variety of ways in which this programme in mathematical education is understood. Their goal was to analyse this movement from Foucault's perspective. The results of their work demonstrated the link between ethnomathematics and mathematics education based on the concept of a counter-conduct proposed by Foucault. Continuing the research on an ethnomathematics programme, Oliveira (2015) investigated emerging perspectives that arise in the discourse of a mathematics teacher in a school community, in a Costa da Caparica neighbourhood, in Portugal. This research, which is related to a project called Fronteiras Urbanas (Urban Frontiers—2012–2014), was used to develop an ethnomathematics perspective that involved the process of students' mathematics teaching and learning in this school community. This study used the

communicative, analytical, and technological instruments of D'Ambrosio's trivium curriculum for mathematics (1999a).

Regarding the theoretical reflections for this programme, Rosa and Orey (2015) argued that ethnomathematics shares several characteristics with Lakatos's methodological scientific research programmes, since its main components are a firm nucleus, positive and negative heuristics, and the protective belt of auxiliary hypotheses, which facilitate the analysis of empirical phenomena. Therefore, in the Lakatosian sense, the main objective of the ethnomathematics programme is to develop and strengthen the theories that make up its protective belt, expanding it and making it more precise in relation to the empirical predictions that aim to strengthen its nucleus. Consequently, ethnomathematics is a Lakatosian research programme composed of irrefutable theories that make possible the theoretical and methodological decisions that enable its progressiveness.

In another theoretical reflection, Marchon and Fantinato (2015) proposed the construction and development of a sociocultural basis for ethnomathematics as proposed by D'Ambrosio (1985) from his writings in mathematics education through a textual production that shows possible conceptual changes and transformations when adapting the theoretical base of this programme. These authors also presented some readings that contributed to the reflection on the sociocultural foundations of mathematics education and made possible to rethink fundamental theoretical, social, and philosophical aspects of ethnomathematics.

Continuing with the research that began in SIPEM V, in 2012, Rosa and Orey (2015) deepened the theoretical reflection on the application of particular techniques of ethnomathematics in conjunction with the tools used in modelling. Ethnomodelling, which is a tool that provides a holistic view of the nature of mathematical knowledge, connects the cultural and academic aspects of mathematics through a dialogical approach. In the ethnomodelling process, the emic and etic approaches to mathematical knowledge facilitate the translation of problem situations present in the systems drawn from the reality of members from distinct cultural groups. Emic knowledge is essential to understand ideas, procedures, and mathematical practices of the members of these groups intuitively, while etic knowledge is essential for a comparison. The dialogical perspective between the views uses the emic and etic approaches to obtain a more comprehensive understanding of the mathematical knowledge developed by members of diverse cultural groups.

The discussions presented above demonstrate that research in ethnomathematics appears in all editions of SIPEM as a kind of "firm nucleus", according to the Lakatosian conception of the programme. It is the motivating axis of the working group that deepens, but also renews itself, opening gaps for the exploration of other problems related to culture, and to history itself—of the characters, of the contents. This is the case, for example, of the research in history of mathematics that we now present.

6.4 History of Mathematics, History in Teaching: Configurations

According to D'Ambrósio (2000), the history of mathematics "aims to reflect on basic questions, leading to the construction of a historiographic project of the history of mathematics in Brazil" (p. 172). This would imply the investigation of ideas and achievements of renowned mathematicians, which were brought and disseminated in Brazil. Following this theoretical-methodological orientation, Sad and Dynnikov (2003) presented work that deals with the first scientific missions encouraged by agreements between Brazil and Europe, bringing Italian mathematicians to Brazil in the 1930s to teach at the University of São Paulo, in the city of São Paulo, in the state of São Paulo, Brazil. There was a concern about what and how to research in history of mathematics. Thus the work of Sad and Dynnikov (2003) presents questions pertaining to the delimitation of research, consideration of facts and sources and types of research.

However, the research in the area of the history of mathematics is shy in editions of SIPEM, despite the strong recognition of the need for a historical understanding of mathematics in Brazilian mathematics education. Two interest groups support this: one focuses on the historical study of influential characters in Brazilian education; and the other tries to understand mathematics contents historically.

In the first case, the work of Trentin (2009), which analyses Manoel Ferreira de Araújo Guimarães's translation of the work *Éléments de Géométrie*, by Adrien Marie Legendre, appears in SIPEM IV. Then, in SIPEM V, two works can be characterized in this axis, namely: Santos (2012) studies the historical and traditional story of Dedekind, who suggests the construction of real numbers from cuts, and Araújo (2012) presents an investigation about the life and works of Joaquim Gomes de Souza, a person known in the history of Brazilian mathematics as Souzinha (1829–1864).

The investigation of concepts and disciplines, from other perspectives, already appears in SIPEM I, as in Moura and Sousa's work (2000). The authors discuss historical elements that constitute the conceptual nexus of algebraic thought. Meneghetti (2003) investigated the intuitive and logical aspects in the development of calculation, and which were considered educational trends. Abdounur (2003) studied the mathematical concept of incommensurability through analogies with music and astronomy. However, considering new approaches in history that see the production of knowledge as the result of social practices, Lannes (2003), for example, dwells on the empirical construction of mathematical knowledge, discussing it as a network of meanings within a given collectivity. And Flores (2006a, 2006b) analyses social practices on how to draw and represent the space of the eighteenth century fortifications on Santa Catarina Island, in the city of Florianópolis, state of Santa Catarina, Brazil, using it to understand a form of knowledge that is delineated by geometry and perspective. From SIPEM IV, Silvai (2009) sought a historical (re) construction in the conceptual development of differential and integral calculus, looking at it as a construction of models.

Through two major centres of research, renowned mathematicians and mathematical concepts, researchers in mathematics history reflect on the development of a Brazilian mathematics education that started with foreign scholars and theories, but that was articulated within the specificities of Brazilian culture. Furthermore, the history of mathematics in mathematics teaching has become a subject of research in Brazil, mainly because it recognizes that it "provides a good opportunity to develop our view of 'what mathematics is' or that the history of mathematics allows us to have a better understanding of concepts and theories" (Baroni, Teixeira, & Nobre, 2004, p. 165).

In fact, we can say that there is a delimitation of internationally articulated trends. Heeffer (2006), for example, has raised three good reasons to argue that mathematics education benefits from the history of mathematics:

> The first is epistemological and addresses a contextual view on mathematical knowledge. The second concerns the phylogenic aspects of the development of mathematics. Conceptual difficulties with teaching children mathematics often correspond with historical periods of conceptual crisis in mathematics. A third, historical argument, draws on the vast repository of experience in mathematics education. We provide examples for each of these arguments from the history of algebra (Heeffer, 2006, p. 1).

Thus, from the third edition of SIPEM, the research focused on this relational aspect of the history of mathematics and mathematics education as presented. Batista (2003) discussed the inclusion of the history of mathematics in teaching, evidencing a study on the systems of equations from mathematicians Seki Kowa and Leibnz. Bayer (2006) deals with the use of the history of mathematics as a motivational and contributing factor to teaching and learning. Scheide (2006) also discussed the contribution of the history of mathematics to improve the teaching and learning process of the students in order to make them more critical and active citizens.

Maciel, Cardoso, and da Fonseca (2012) promoted meaningful learning of the concept and function through history. Roque (2012) identified pedagogical potentialities of the history of mathematics in activities related to integers. Dias and Saito (2009) justified and proposed an approach that favours the construction of an interface between the history and teaching of mathematics, based on new historiographical and methodological trends. Finally, Sá (2015), presented research on the use of history of mathematics in the classroom from an experience with teaching graph theory.

On the other hand, teacher education regarding the knowledge of mathematics history with the possibility of knowing methods and techniques that can aid in his/her practice (Baroni et al., 2004) has also been a research problem since the first edition of SIPEM. This is the case for Nobre (2000), who discusses the implications of the history of mathematics for the training of mathematics professionals. In practical situations, Brolezzi (2000), for example, proposed workshops with teachers to discuss the conceptual tension of the discrete/continuous pair, and to rethink the construction of the idea of number, and the birth of differential and integral calculus. Then, during SIPEM III, Motta and Brolezzi (2006) discussed the role of teachers in regard to the integration of history of mathematics in the process of teaching

and learning in practical situations. Similarly, Abdounur (2006) organized an exhibition to address historical-didactic aspects of the relationship between mathematics and music, which characterized a space for teachers to experience cultural and extension activities in their curricular tasks.

It is remarkable that a small research sample focusing on history allows for the circulation and delimitation of diversified research objects. This aspect led Miguel (2003), in SIPEM II, to propose a distinction between the investigative research fields of the history of mathematics, history of mathematics education, and history in mathematics education. Therefore, it is noteworthy that there is the configuration of three mixing strands that help to define their own ways to deal with history and culture, which allowed the dismemberment of these important investigation fields such as the case of the Brazilian history of mathematics education.

6.5 History of Mathematics Education: Dismemberment

During SIPEM II, held in 2003, the first works on what would be characterized later as the history of mathematics education started to appear. Here we comprise the works that are concerned with the historicity of the processes of teaching, learning, and teacher training in the context of mathematics. Therefore, historical studies on textbooks, teaching renewal movements, teacher training courses, legislation, pedagogical journals, notebooks, that is, all documents geared towards teaching or learning mathematics are included in this framework.

In 2003, four studies were approved, two of which used comparative perspectives in history. One of the authors, Valente (2003), sought to understand the appropriations of Brazilian and Argentinean proposals for the renewal of mathematics teaching from the first international movement that had the creation of ICMI, in 1908, as a milestone. Soares (2003) compared two national programmes for the evaluation of textbooks: The National Textbook Commission, created in 1938, during the management of Gustavo Capanema as a Minister of Education and Health during Getúlio Vargas's government, and the National Textbook Programme, created by the Ministry of Education, in 1985. Duarte (2003) investigated the mathematics proposed in the *Journal of Mathematics and Physics Notes*, published between 1953 and 1954, by students of the Mathematics and Physics Section of the Sciences and Letters Philosophy School of University of São Paulo that aimed at disseminating mathematical knowledge for students in college, teachers, and high school students. In a similar context, Ferreira (2003) analysed the mathematical content included in the school curricula of the teacher formation school, Escola Normal do Espírito Santo, from 1892 to 1971.

The third edition, held in 2006, contained works that explain the history of mathematical education as a keyword or major theme and other papers that, even without being explicit, could be considered in this framework, when identified by similarity of contexts.

Five studies were developed from textbooks as main sources. Carvalho (2006) compares editions of the work *Moyens d'apprendre à compter sûrementet avec facilité* by Condorcet in Brazil and Portugal. School mathematics in German-Brazilian schools founded in the late nineteenth and early twentieth centuries, in Rio Grande do Sul, is Mauro's object of research (2006). The author focuses on the process of producing textbooks and newspapers to serve that community. Another study analysed the teaching of mathematics in *General Geography*, by Varenius, published in 1650, highlighting the relations in knowledge in those times (Brito, 2006).

Two studies analysed specific topics in textbooks: Thales's theorem, in the collection of mathematics course *Matemática Ginasial—1ª série* written by Euclides Roxo, Cecil Thiré, Júlio César de Mello, and Souza,[3] published between 1940 and 1942 (Pereira, 2006); and the insertion of the decimal metric system in Brazil from 1856 (Zuin, 2006). Although both consider textbooks as sources for research, the way they are analysed is different. Whereas for the study on the Thales theorem the emphasis is on the mathematical treatment given in the work in which the segments are incommensurable, for the decimal metric system, the book is analysed as a vehicle for insertion of new contents that parameterize new practices in society.

The historical study of the education of teachers who teach mathematics was the subject of three studies dealing with different levels—elementary, middle and high school, and higher education. Regarding elementary education, the study by Soares (2006) analysed rules for recruiting and selecting teachers for the first schools in Rio de Janeiro, from the turn of the eighteenth century to the nineteenth century. The work by Baraldi (2006) investigated the Campaign for Improvement and Diffusion of Middle and High School Education (CADES) as a training course for middle and high school mathematics teachers based on oral history as a methodology. Oliveira (2006c) investigated the learning of mathematical analysis that Ubiratan D'Ambrosio had during his undergraduate time in mathematics in the 1950s, by associating his training to the activity of research in mathematics. The sources mobilized in the study ranged from sheets made by Ubiratan himself as a student, interviews with him and his teacher, Elza Gomide, and the books included in his course bibliography. The last two studies were both conducted in the 1950s and represent the contrasts in Brazilian educational reality. CADES course, the training offered aimed at preparing teachers in a short period, many times during vacations, to work in middle and high school. The mathematics course attended by Professor Ubiratan, on the other hand, aimed at formation for research in mathematics.

Two studies investigated knowledge and practices for mathematics teaching in Brazil's northern and northeastern states. Neto and Braga (2006) investigated mathematical practices and knowledge that guided the official curriculum for public schools in the state of Pará, between 1900 and 1920. The research, which uses

[3] The three authors taught at the secondary school Colégio Pedro II, an institution founded in 1837 in the colonial period, which, until the 1930s, served as an official reference for other Brazilian educational institutions, until the creation of the Ministry of Education, in the same decade.

written documents and image records, identifies the influence of Comte's positivist ideas in curricular organizations. Gutierre (2006) discussed mathematics education in middle and high school that was disseminated by *The Athenaeum* in the state of Rio Grande do Norte, Brazil, between the 1930s and 1960s, a period in which proposals for educational reform by Francisco Campos and Gustavo Capanema stabilized. The two works cover the first half of the twentieth century and are emblematic of the process of strengthening national references, particularly teachers and authors of mathematics textbooks. Two studies investigated specific knowledge areas from an ahistorical perspective. Miguel and Souza (2006) investigated the obsolescence process of the litmus test, an arithmetic method to confirm the result of the addition operation, in the Brazilian school context. Finally, Oliveira (2006a) investigated infinitesimal calculus taught at the polytechnic school, Escola Politécnica de São Paulo, in 1904. The studies have in common the way they analyse the specific contents inserted in the school/institutional, cultural, and social context that allow the maintenance or the rejection of certain knowledge in the scope of education.

The modern mathematics movement is also the subject of two studies. Duarte (2006) discussed the view of the mathematician, teacher, and author of textbooks Benedito Castrucci, on the movement in Brazil. Fischer and Carpes (2006) investigated the experience with—pilot and experimental—classes organized by the Mathematics Education Study Group of Porto Alegre (GEEMPA), starting in 1972. Using written documents and statements from teachers involved, they discuss how this experience is considered positive and marked by modern mathematics ideas as carried out at a time of criticism and decline of this movement.

Three studies analyse trajectories and productions of characters directly or indirectly related to Brazilian mathematics education and to the field of mathematics. A study by Oliveira (2006b) analysed proposals for the teaching and learning of mathematics published in the *Al-Karismi Magazine*, organized by MalbaTahan,[4] in the 1940s. Forner and Lopes (2006) investigate, through oral history, the interpretations and uses mathematics educators made of Paulo Freire's work, an important reference for Brazilian education. Using documents in archives in England and Berlin as sources, the process of historical recognition by English mathematician, Arthur Cayley, is the objective of the research by Mattos (2006a, 2006b).

In SIPEM IV, in 2009, the number of studies identified as history of mathematics education reduces to eight. The modern mathematics movement is a representative theme in this universe because it dealt with three of them. Oliveira (2009) discussed the growth of a modern mathematics programme for middle and high school education disseminated by G.E.E.M[5] (Mathematics Education Study Group) and outlined

[4] Pseudonym of the professor of mathematics, author of didactic textbooks, Júlio César de Mello e Souza.

[5] This group of mathematics teachers from the state of São Paulo, with different levels of education, played a prominent role in the dissemination of proposals of modern mathematics throughout Brazil. The group had the professor and consecrated author of mathematical textbooks, Osvaldo Sangiorgi, as a president, among other distinguished names of teachers and authors of books of

in pedagogical journals that circulated between the 1950s and 1960s in Brazil. Geometry in the perspective of this movement is analysed by Leme da Silva (2009), through didactic texts produced by the School Mathematics Study Group (SMSG), of the discussion in the Royaumont and Dubrovnik Seminars, promoted by the European Organization for Economic Cooperation (OECE) and the First Inter-American Conference on Mathematics Education (CIAEM). Batista et al. (2009) investigated the influences of this movement in the curricular organization of the Brazilian Federal District, inaugurated in the 1960s, at the same time as the ideas of modern mathematics were disseminated in the country.

The influence of the book *International Conferences on Public Instruction*, from 1934 to 1963, published by the Ministry of Education and Culture, National Institute of Pedagogical Studies, in 1965, on the elaboration, organization, curriculum constitution, teaching methods, and textbooks of mathematics in Brazilian schools for the middle and high school levels are the objective Ribeiro's study (2009).

The forms of mathematics taught at a middle and high school levels is the objective of the research carried out by Valente (2009), from the analysis of textbooks, indicating a differentiated process compared to that in middle school. The text conjectures the existence of two sequential school subjects—one for middle school and another for high school—with a certain autonomy. The process of professionalization of mathematics teachers is dealt with by Dias and Bertani (2009) in the context of the mathematics course in the state of Bahia, Brazil. The authors prioritized primary sources and notebooks between 1943 and 1965, and registers of actual practices.

Brito (2009) analysed the teaching of mathematics in the seventeenth century at the Akademisches Gymnasium in Hamburg and in the universities of Königsberg, through the analysis of the correspondence between Bernhard Varenius and his teacher, Joachim Jungius. Miorim, Brito, and Faria (2009) presented a survey of articles that studied mathematics history and education work published in journals, categorizing them according to the approach used.

In SIPEM V, held in 2012, five studies related to the history of mathematics education strand were presented. Three of them used a very similar theoretical-methodological contribution based on the history of the school subjects. Two were related to the teaching of mathematics in elementary school, one by Villela (2012), from the analysis of reports issued by the direction of a school in Vassouras, in the state of Rio de Janeiro, Brazil, in the late nineteenth century; and another by Borges and Duarte (2012), who investigated the magazine *A Escola*, published between 1923 and 1924. The third study, carried out by Ferreira (2012), investigated the historical trajectory of the mathematics teaching methodology disciplines in the teacher training courses in three universities in the state of São Paulo, Brazil.

The appropriation of innovation proposals for the teaching of mathematics disseminated from two international movements—the first, with the creation of ICMI in 1908, and the second, the movement of modern mathematics in the 1960s and 1970s—by two textbook authors representative of each of the periods is the theme

mathematics in Brazil, who integrated the group.

of the work done by Silva and Silva (2012). Mathematics education in colonial Brazil was addressed by Magalhães and Silva (2012).

In SIPEM VI, referenced researchers in the area of mathematics education began to appear with publications in the history of mathematics education, such as the work done by Ubiratan D'Ambrosio, Wagner Valente, Gert Schubring, and Maria Ângela Miorim.

During the SIPEM VI meeting, held in Pirenópolis, Goiás, in November 2015, many presentations dealt with studies in the history of mathematics education. Therefore, the coordination of the WG subdivided the works into axes, one of them on that topic. Six out of the 22 studies presented belonged to the history of mathematics education axis. Half of these studies drew on references, but discussed this knowledge from different perspectives. Two studies analysed the presence of the discipline and its purposes in different educational and historical contexts. The first context is the training of elementary school teachers during the first decades of the First Republic, between 1889 and 1930 (Oliveira, 2015), and the second is the context of elementary education at the end of the Empire (1883). It was institutionalized in the country at that time, in the framework of proposals by Rui Barbosa, an influential intellectual of the period (Guimarães, 2015). The third text discussed theoretical-methodological perspectives for the writing of a history of drawing, not getting restricted to the history of sciences or education, but crossing different fields of knowledge (Machado & Flores, 2015).

The other three works dealt with a wide scattering of themes comprising the historical study of mathematics education research in Brazil, from studies resulting from graduate courses in science and mathematics teaching between 1975 and 1984, which are currently considered milestones for the constitution of the mathematics education as a professional field in the country (Miranda, 2015); from the analysis of mathematics teachers' representations about the modern mathematics movement in a countryside city in the state of São Paulo (Rodrigues, 2015); and from the investigation of mathematics conceptions underlying professional and technological education between the 1940s and 1980s, in the federal technical institute, Instituto Técnico Federal do Espírito Santo (Pinto, 2015).

During SIPEM VI, the works used documentary and oral sources and, for the most part, used explicitly contributions of cultural history as a theoretical-methodological perspective, referenced in authors such as Peter Burke and Antoine Prost. Studies that include historical perspectives found in school subjects are also mobilized in most texts. In a specific way, one of the studies used the contributions of oral history, and another, Foucault's philosophical perspective.

From this perspective of research on the history of mathematics education, represented since SIPEM II, it was manifest that the research was concerned with a variety of ways of learning and teaching mathematics that occurred in the midst of changes and permanencies, at different levels of teaching and in different times and spaces in Brazilian education.

6.6 In Conclusion: Emerging Aspects, New Designs

At the beginning of the WG in 2000, D'Ambrosio's proposed delimitation brought in its essence the richness of many research perspectives, which, due to their very diversity, led to diverse and divergent theoretical and methodological frameworks. Throughout its trajectory, the group added more research concerned with themes involving history and culture in its wide and very diverse relations to mathematics, most notably: ethnomathematics, history of mathematics, history of mathematics education, and the history in mathematics education. The space has continued to prove to be quite fertile in terms of ongoing discussion, production, and dissemination of the topics, which can be observed through national and international scientific meetings and publications.

Particularly, the role of research in ethnomathematics can be affirmed by recent trends in education because it envisions hope for the transformation of the role of the school, specifically, for example, the school in rural and countryside contexts. In Brazil, there has been modest growth in relation to the discussion and the conduction of research related to these areas of study, due to the recognition of research contribution in the development of rural and countryside education.

In this sense, approximately two decades ago, Bishop (1994) argued that rural education can be considered as a potential construct to be applied in ethnomathematics research. In this case, the study of ethnomathematics helps educators to connect school mathematics with students and their communities. Extensive global work with ethnomathematics has been conducted in rural and countryside contexts; however, concepts related to rurality itself are rarely considered to have a relevant influence on mathematics education since there is a gap in the conduct of the investigations related to the problem of rural and countryside education.

In addition, an important aspect of the ethnomathematics research programme is the connection between indigenous knowledge and the curriculum of mathematics. In this case, it is considered necessary to explore the possibilities to use indigenous knowledge as a pedagogical action for mathematics education. Also required is the identification of issues and problems that occur in complex contexts of indigenous groups (such as natural, cultural, political, and social), so that different approaches to diverse educational situations can be discussed (D'Ambrosio & Rosa, 2008). In this sense, it is important to emphasize that Brazilian indigenous peoples have their rights guaranteed by the Federal Constitution, as well as by federal and state laws that govern indigenous school rights and education.

In this sense, it is important to promote discussion in relation to the dialogue between local (indigenous) and scientific (academic) knowledge, as well as the influence of mathematics on indigenous schools and teacher training. In initial indigenous teacher training courses, specifically in intercultural degrees, future teachers reflected on their own pedagogical practices and their actions, which enable them to understand various aspects of the teaching and learning process in the context of an indigenous school.

In order to promote intercultural and bilingual education in these schools, one of the main objectives of the discussions was to analyse some of the many challenges different training courses currently meet for indigenous teachers in the field of mathematics education. According to Rosa and Gavarrete (2016), these challenges involve the relationship between traditional and scientific knowledge during the development of pedagogical actions used in classrooms by indigenous teachers.

In order to debate the principles of an ethnomathematics programme, it is necessary to propose a theoretical basis to discuss how the social, economic, and political contexts are important for the development of mathematical ideas, a fact observed in the diversity of the work presented through all SIPEM's meetings. It is also important to know how and why different individuals have come to reveal their diverse interests, preferences, talents, and abilities, as well as specificities and strategies to generate, organize, disseminate, and share mathematical knowledge to solve the problems they face in their daily lives.

Therefore, when the focus of a study is on the nature of mathematics, attention should be centred on both the legitimization of the students' knowledge that develops from the experiences built up by their own experiences, and the study of pedagogical possibilities about how it is possible to work with the teaching and learning process that takes place inside and outside the school environment. For Shirley and Palhares (2016), discussions related to the educational aspects of ethnomathematics assist educators in the establishment of cultural models of beliefs, thoughts, and behaviours, to contemplate the pedagogical potential of the work that considers the prior knowledge of the students, as well as meaningful and empowering mathematical learning.

The pedagogical action of ethnomathematics can be considered as a contextualization of the ideas, procedures, and mathematical practices developed by the members of distinct cultural groups. However, how this pedagogical work can be done in classrooms is problematic (D'Ambrosio & Rosa, 2008). In this context, it is necessary to understand how ethnomathematics can contribute to the development of activities contextualized in multicultural classrooms.

In this regard, D'Ambrosio (1999b) stated that the growing trend towards multiculturalism recognizes ethnomathematics as a valid school practice that enhances creativity, reinforces cultural self-respect, and offers a broad view of humanity. In everyday life, an ethnomathematics perspective increasingly recognizes systems of knowledge, which offers the possibility of a more favourable and harmonious relation in human behaviour and between humans and nature.

The result of our discussions in these presentations continues to show that ethnomathematics is alive and embraces the ideas, thoughts, and mathematical practices developed by members of all cultures. In this perspective, an anthropological body of research focuses on diverse forms of intuitive mathematical thinking, as well as the development of equally diverse cognitive processes, which are widely developed by members of distinct cultural groups. Thus, ethnomathematics is a programme that seeks to study how students understand, articulate, process and use ideas, concepts, and mathematical practices that can solve problems related to their daily activities.

Seen in this context, the focus of ethnomathematics consists essentially of a critical analysis of the generation and production of the mathematical knowledge and intellectual processes, the social mechanisms in the institutionalization of knowledge; and the diffusion of that knowledge (Rosa & Orey, 2007). In the *holistic* context of mathematics that uses an anthropological perspective to include diverse perspectives, patterns of thought, and histories, the study of the *systems* taken from reality help students to reflect, and understand extant relations among all of the components of the system.

From this reflection, it is our opinion that these systems address the issues regarding mathematics education in non-western cultures by bringing the cultural background of students into the mathematics curriculum in order to connect it to the local-cultural aspects of the school community into the teaching and learning of mathematics. This alternative approach helps promote intellectually innovative ideas in mathematics education by both deepening and widening the western-academic understanding of mathematics (Rosa, 2015).

The unique cultural background of the students in Brazil represents a set of values and diverse worldviews that are diffused across generations. The studies presented in WG5 show that the ideas, procedures, and mathematical practices developed by members of different cultural groups favour our understanding of the internal logic and beliefs of members of these groups.

Regarding the history of mathematics, school knowledge, methodologies, and mathematical practices, it is considered that, in the same way as with ethnomathematics, researchers continue to contribute to mathematics education in its various social, political, and cultural dimensions, and is reflected by graduation courses and in the training and development of future mathematics teachers.

In particular, the development of mathematics education does not arise in isolation: it is part of an educational system, which, in turn, interacts with a certain sociopolitical system (Schubring, 2006). Understanding the reasons why certain mathematical contents become objects of teaching over others, or certain methodologies are more adequate than others, according to the research presented in this WG, seems to be fundamental for teacher education. In addition, this facet of the research was developed especially in the group requiring new outlines for the group itself from the researchers.

Although it is recognized, as Karp (2014) says, that "the history of mathematics education is a branch of research that is still only taking shape, and consequently its methodology, too, is still only in its formative stage" (p. 9), Brazilian researchers have been finding different ways to investigate historically mathematics education in Brazil, whether through documentary sources, textbooks, pedagogical journals, and even through what has been named as oral history.

Moreover, this research modality has attracted national and international attention of researchers in mathematics education, contributing to the rapid growth of the research. In addition, the creation of this WG brings together the research working specifically with aspects of the history of mathematics education in Brazil. Just at the international level, the 29 chapters in the *Handbook on the History of Mathematics*

Education (Karp & Schubring, 2014) demonstrate the amplification and need for this specific WG.

As far as the history of mathematics is concerned, from the discussion described above, research focuses on contents or characters that, in one way or another, are linked to either issues of ethnomathematics or the history of mathematics education. This does not mean that the history of mathematics in Brazil is no longer interesting, but that, because of the work involved by the group, it has grown enormously and has begun to answer questions emerging from education. It is true that research in education history has sparked investigations since the 2000s, but its production discussed in the group is still growing and should be done under the auspices of a separate strand.

In this way, by the very movement of research involved in it, WG5 has been characterized from emerging themes that deal with both history and culture of mathematics to address issues in Brazilian mathematics education, in a way that a new configuration was required. Just as history and culture are always in motion, the Group changes and transforms itself with every edition of SIPEM.

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