Chapter 10
Cognitive and Linguistic Processes in Brazilian Mathematics Education: Theoretical Considerations and Educational Implications

Airton Carrião, Sintria Labres Lautert, and Alina Galvão Spinillo

Abstract The aim of this chapter is to contribute to the debate about the trends, objects of study, and theoretical and methodological assumptions that have marked and constituted the current identity of working group Cognitive and Linguistic Processes in Mathematics Education of the Brazilian Society of Mathematics Education. The considerations presented bring together a range of investigations that explore the cognitive and linguistic aspects involved in the teaching and learning of mathematics in different learning contexts and different levels of schooling. The work developed by the group has been characterised by investigations conducted on language and communication in the classroom and their sociocultural aspects, alongside studies into the cognitive processes involved in mathematical reasoning. The discussion about the theoretical-methodological questions underlying the reflections on the cognitive and linguistic processes in the Brazilian scenario has been divided into two parts. In the first, we present a historical review of the main trends considered during the first 10 years of working group's history. In the second, explore the more recent objects of study. We describe how this development indicates a convergence of Brazilian researchers with different theoretical and methodological affiliations, as they search for theoretical models that can explain the role of language, cognition and cultural aspects in the teaching and learning of mathematics. The advances made in terms of knowledge of cognitive and linguistic processes in mathematics education within Brazil are also presented.

A. Carrião (✉)
Universidade Federal de Minas Gerais, Belo Horizonte, Brazil

S. L. Lautert · A. G. Spinillo
Universidade Federal de Pernambuco, Recife, Brazil

A. J. Ribeiro et al. (eds.), *Mathematics Education in Brazil*,
https://doi.org/10.1007/978-3-319-93455-6_10

10.1 Introduction

This chapter presents and discusses the proposals, ideas, and theoretical assumptions that underlie the research conducted by the Working Group Cognitive and Linguistic Processes in Mathematics Education (Processos Cognitivos e Linguísticos em Educação Matemática), which has been part of the Brazilian Society of Mathematics Education (SBEM) for the last 16 years. The chapter also presents the changes that have taken place over this period. Before considering the issues that have generated reflections on the cognitive and linguistic processes in the area of mathematics education in Brazil, it is important to consider the objectives that led to the creation of the Working Group (WG09).

This working group aims to discuss and disseminate research on the cognitive and linguistic aspects involved in the teaching and learning of mathematics at different levels of schooling and in different contexts. Three factors have characterised these studies: (1) language as the means of formation and expression of mathematical reasoning; (2) communication in the classroom; and (3) the cognitive processes involved in the mathematical reasoning. Thus, both the role of language in the production of meaning and communication in the classroom, and the description of the reasoning processes (limits and possibilities) involved in mathematical knowledge are topics that have permeated the production of the working group.

In an attempt to give visibility to the diversity of proposals, objects of study and theoretical-methodological assumptions that mark and constitute the current identity of WG09, we have chosen to present the most frequent and relevant questions that can be seen in the work produced by the group. To do so, we have presented the investigations that demonstrate the extensiveness and variety of subjects considered, and the interdisciplinary nature of the work conducted by the group, as it brings together different areas of knowledge such as cognitive psychology and mathematics education. The studies of this working group have been arranged into two groups. The first offers a historical review of the first 10 years of the working group, presenting the main trends that characterised the first meetings of the group. The second discusses the research presented in the last two SIPEMs, which took place in Petrópolis (2012) and Pirenópolis (2015). These studies depict two thematic and methodological tendencies that remain the focus of the investigations, namely: (1) research which emphasises cognitive processes from a developmental perspective and (2) research emphasising the language and cultural aspects in the teaching and learning processes of mathematics. This organisation reflects the thematic and methodological trends that have characterised the work of WG09 since its establishment.

10.2 The Research Conducted in the First Years of WG09

The first meetings were marked by discussions about how students at different levels of education (elementary, middle and high school, university education and youth and adult education—EJA)[1] were taught mathematics and about their mathematical learning processes. The aim of these first meetings was to study teacher–student interactions, student–student interactions, as well as how students interacted with the mathematical knowledge in which argumentative and metacognitive strategies were identified as support for learning situations in the classroom context. The solving of problems involving various concepts (e.g. percentage, function, graphs, and fraction) and different fields of the mathematical knowledge (e.g. geometry, combinatorics, and arithmetic) were also investigated. These studies, according to Rabello, Lins, and Da Rocha Falcão (2000), focused on issues of great theoretical and practical interest in the field of mathematics education:

1. Language, cognition, and communication—emphasising the role of language, embodied language (according to Lakoff & Johnson, 1999), representation (according to Duval, 1993, 1995), and discourse genres (according to Bakhtin & Voloshinov, 1992), and highlighting the differences between natural language and the language of mathematics (Pires, Curi, Rabelllo, Pavanello, & Valente, 2003) and the boundaries between the use of rationality in daily life and formal logic;
2. Cognitive and linguistic processes in the classroom, focusing on the interface between theory and practice in relation to the development of cognitive and language skills considered relevant to the teaching and learning of mathematics.
3. Vygotsky's social-historical-cultural perspective and Piaget's constructivism, addressing some of the theoretical debates about conceptualisation in mathematics and processes such as internalisation, scaffolding, and interaction.

In summary, the results of the studies presented at the first meetings pointed to the following facts: (1) students used concepts acquired in extracurricular social contexts and different strategies to grasp the mathematical concepts presented in the classroom; (2) the mathematical language used in the classroom, particularly notation and the use of symbols, was of little significance for the students; and (3) the dialogues between teacher and student often inhibited the expression of the ways of reasoning of the students.

Some of the research studies presented have gone on to gain international recognition, especially those which focused on the study of cognitive and linguistic

[1] The educational system in Brazil is divided into basic education and higher education. Basic education corresponds to: nursery or early-years education (0–5 years old), (2) elementary school (elementary and middle school, from 6 to 14 years), and high school (15–17 years). Higher education corresponds to university education, undergraduate and postgraduate courses (Masters and PhD). This educational system also includes youth and adult education (EJA), distance learning and educational technologies, technological education and vocational training, special education and indigenous education.

aspects in the learning of mathematical concepts by blind students (Healy & Fernandes, 2008), and on the role of language, corporeality, and technology in supporting the analysis of the process of teaching and learning mathematics (Frant, 2009). Research on the tacit and explicit knowledge involved in the learning of mathematics in the school context (Frade & Borges, 2006); and discussions about the learning of mathematics in the context of youth and adult education (Fonseca, 2010) have also received international attention.

Another feature of the work conducted by WG09 during this period was the concern with how students with atypical profiles learn mathematics, such as blind learners and children with epilepsy. There was also a great interest in how the processes of teaching and learning take place in the classroom, in investigating how teachers understand the learning process and the exchanges between the students and the teacher. In other words, there was an interest in how the teacher's role was defined in relation to their teaching practice, the transfer of learning, and the classroom as a "community of practice" (Garnica, Soares, & Buriasco, 2006). These studies seek to construct a knowledge of flexible bases, one that no longer focuses on the idealised student. Instead, this knowledge brings into play a teacher who respects and integrates different ways of learning and of communicating in the classroom. The discussions derived from these studies later appeared in national and international publications (David & Watson, 2008; Frade & Falcão, 2007; Frade & Tatsis, 2009; Frade, Winbourne, & Braga, 2009; Hazin, Da Rocha Falcão, & Leitão, 2006).

Despite the thematic, theoretical, and methodological variations that characterised the research conducted by members of WG09 during this period, it is possible to see a general common ground in relation to the teaching and learning of mathematics. This involves cognitive and linguistic processes that, whether investigated separately or jointly, are interconnected and become inseparable in the classroom.

10.3 The Most Recent Trends in the Research Conducted by WG09

In this part, we consider the propositions and theoretical assumptions present in the theoretical-methodological production of the V and VI Brazilian Symposium on Mathematics Education, in which twelve and three papers were presented, respectively. First, we offer an overview of the investigations presented at these two seminars. Then we discuss a selected set of studies that show the heterogeneity of the research conducted by the group. These studies also illustrate two recurring themes of the investigations since the creation of the GW9, namely: research that emphasises cognitive processes from a development perspective, and research that emphasises the language and the cultural aspects in the teaching and learning processes of mathematics.

The fifth seminar illustrates the diversity of the objects of investigation. The vast majority of the investigations focused on language and cultural aspects of the processes of reasoning and of learning mathematics. These aspects were investigated both from the perspective of those performing the activity individually (questions, tasks, or problems) and in the interactions which occur during a mathematics class (e.g. Carrião, 2012; David & Tomaz, 2012a; Frant, 2012; Torisu, 2012). There were also three studies on the processes involved in the teaching and learning of specific contents: rational numbers in fractional form (Freire & Lima, 2012), division (Lautert & Spinillo, 2012), and geometry (Viana, 2012). In addition, we identified two studies which looked into students' ability to solve mathematical problems and their relationship with the activity of playing chess (Lopes & Magina, 2012) and creativity in mathematics (Oliveira, Albuquerque, & Gontijo, 2012). There was also a paper on the production of meanings and the decision-making ability of consumers (Kistemann Jr., 2012). Two studies drew attention to the education of blind and deaf students, with regard to the material and semiotic tools for the acquisition of mathematical knowledge (Fernandes, Healy, & Serino, 2012) and the configuration of the mathematics classroom when deaf students are present and interpreters are required (Borges & Nogueira, 2012).[2]

Although these investigations focus mainly on elementary school students, diversity is maintained by also conducting studies that consider high school and undergraduate students. The three studies presented at SIPEM VI focus on the solving of problems involving the multiplicative conceptual field in elementary school. Two of them focus on the teachers' conceptions (e.g. Merlini, Santos, Teixeira, & Magina, 2015; Spinillo, Lautert, Santos, & Silva, 2015) and one on the students' conceptions (e.g. Magina, Spinillo, & Melo, 2015).

In summary, in recent years, despite the thematic, theoretical, and methodological variability that has characterised the work developed by WG09 since its inception, the studies that have been carried out can be grouped together according to their main theme:

1. The cognitive aspects involved in solving problems relating to specific mathematical concepts (Freire & Lima, 2012; Lautert & Spinillo, 2012; Lopes & Magina, 2012; Magina et al., 2015; Oliveira et al., 2012; Viana, 2012) and the views of the teachers (e.g. Merlini et al., 2015; Spinillo et al., 2015);
2. Communication in the classroom which includes the participation of deaf and blind students (Borges & Nogueira, 2012; Fernandes et al., 2012) and the cultural aspects involved in the teaching and learning of mathematics (e.g. Carrião, 2012; David & Tomaz, 2012a, b; Frant, 2012; Kistemann Jr., 2012; Torisu, 2012).

In the next section, we discuss the advances made with regard to the knowledge about the cognitive and linguistic processes in mathematics education. We consider the studies that focus on the cognitive aspects in a developmental perspective, and the studies which emphasise the language and the cultural aspects involved in the reasoning processes. The division of the latest research by WG09 in two groups shows how Brazilian researchers from different theoretical and methodological

affiliations have come together in their search for theoretical models that can explain the role of language, cognition, and of cultural aspects in the teaching and learning of mathematics.

10.3.1 Studies That Highlight the Cognitive Processes in a Developmental Perspective

According to Spinillo and Lautert (2006), cognitive psychology has distanced itself from more general theories that explain cognitive development in a broad manner, such as the Piagetian theory. Instead, cognitive psychology has become more closely identified with more specific theories, such as the theory of conceptual fields, proposed by Vergnaud, which deals specifically with mathematical knowledge. This has created a scenario that has led to the understanding of the multiple connections between psychology and education, and between theory and school practice. In the last few years, much of the research on cognitive processes that has been presented in the Cognitive and Linguistic Processes in Mathematics Education Working Group has adopted a developmental perspective. Based mainly on the theory of conceptual fields (Vergnaud, 1983, 1988, 1990, 2003), these investigations have examined problem solving in the field of multiplicative structures. Problems of this nature have been widely documented, both in the national and international literature and are considered a challenge for both those who teach and for those who learn them. Following the international trend,[2] the issues addressed in WG09 emphasise problem solving, drawing attention to the importance of learning how to solve problems in order to learn mathematics, and to the importance of learning mathematics in order to solve problems (Charnay, 2001; Onuchic & Alevato, 2004).

Problem solving has been investigated, particularly, aiming at: (1) understanding how students conceive certain concepts considered to be complex, such as fractions, proportions, probability, divisions, multiplications, and the solving of problems involving combinatorial reasoning (e.g. Freire & Lima, 2012; Lautert & Spinillo, 2012; Lopes & Magina, 2012; Merlini et al., 2015; Spinillo et al., 2015); (2) analysing the nature of different types of problems since the way of reasoning adopted is related to the type of problem being solved (e.g. Magina et al., 2015).

The research conducted in the last few years has taken an interdisciplinary approach, involving psychology and mathematics education. To illustrate this interdisciplinary nature of the group, we will briefly present some studies that look into the solving of problems in the field of multiplicative structures. They are

[2] The solving of problems continues to be a relevant topic in the field of mathematics education. It has been the topic of numerous international conferences, such as the 13th International Congress on Mathematical Education (ICME) in 2016, the 40th Conference of the International Group for the Psychology of Mathematics Education (PME) also in 2016, and of many of the National Council of Teachers of Mathematics (NCTM) documents.

grouped as follows: (1) *diagnostic research*, which seeks to describe how reasoning is organised and how it develops in students from different age groups and at different levels of schooling, when these are working with a particular concept; analysing the strategies adopted; (2) *intervention research*, which seeks to examine ways to develop mathematical reasoning; and (3) research which looks into *the teacher's conception* when dealing with problem-situations in the field of multiplicative structures.

A study by Magina et al. (2015) is an example of *a diagnostic study*. The authors analysed the performance and the strategies adopted by 3rd, 4th, and 5th graders of elementary school in the resolution of Cartesian product problems. The results showed that the students used different strategies, expressing different levels of combinatorial reasoning, although their performance, in terms of the process employed, did not vary throughout the different school years. The most elementary of these levels corresponds to the inability to understand the relationships between the variables, and the use of inappropriate operations. The strategies at this elementary level were the ones adopted more frequently. The less elementary strategies employed by the students were characterised by the establishment of fixed combinations based on term-to-term correspondences. Slightly more sophisticated strategies, but still not yet entirely appropriate, were those in which a flexible combination of the elements was observed. Such strategies, however, did not exhaust all the possible combinations between the elements from the two elementary sets. Appropriate strategies were rare. These were considered to be the strategies that, as stated by Moro and Soares (2006), resulted in combinatorial solutions derived from one-to-many correspondences, and that made use of all possible combinations, involving all the elements from both elementary sets. These results confirm Vergnaud's (1983, 1988) claim that conceptualisation in mathematics develops gradually and over a long period since the most sophisticated solutions, although rare, were found among the 5th graders. Moreover, the results of this study point to both the limitations and to the potential of the reasoning ability of elementary school students. These results have educational implications that can be translated into didactic actions that can support the development of these concepts, as well as helping to overcome the difficulties identified.

As an instance of *intervention research,* we have selected the study conducted by Lautert and Spinillo (2012). In this study, the authors investigated the effect of a specific intervention on the understanding of division in children who had difficulties understanding this concept. The research involved an experimental group (EG) and a control group (CG), and all the participants in the groups were required to take a pre-test and a post-test. The intervention, which was offered to the EG only, consisted of problem-solving activities, and discussed the forms of resolution adopted by the children. The intervention also explained two invariant principles of division considered as determinants of the difficulties experienced by students during the early years of elementary school: co-variation relations between the terms of division when the dividend is kept constant, and how to deal with the remainder. The results showed that the EG students reached more advanced levels of understanding than those of the CG, overcoming the difficul-

ties they had before the intervention. According to the authors, this progress was due to the fact that the intervention allowed participants to perform a metacognitive activity—that is, to think about their ways of thinking when solving problems—and to reflect on the invariant principles of the concept of division.

Finally, as examples of studies that investigate *the teacher's conception* with regard to mathematical concepts, we focus on two studies that have analysed the notions held by teachers about the resolution of problems involving both division and multiplication. Most research is based on the assumption that the understanding of a concept manifests itself not only in problem-solving situations, but also in situations where the individual is asked to formulate problems. Research conducted in Brazil (Chica, 2001; Itacarambi, 2010), and abroad (English, 1997; Lowrie, 2002; Zunino, 1995) assume this perspective regarding the formulation of problems by students, showing that students' ability to formulate problems is related to their understanding of the concept involved in the solving of the problems they formulate. If we apply this reasoning to the teachers, we can see that the posing of problems by teachers needs further investigation. If problem solving is considered a didactic strategy, and if the problems presented relate to the content to be taught, we can conclude that knowledge about problem solving is a didactic knowledge of the content that needs to be mastered by those who teach mathematics.

As a result, research in Brazil (Cunha, 2015; Souza, 2015) and abroad (Crespo, 2003; Leung & Silver, 1997) have looked at how teachers of mathematics formulate problems. The results of these studies led Spinillo et al. (2015) to investigate the formulation of problems by elementary school teachers. Participants in this study were asked to formulate, in writing, problems whose resolution required the use of multiplication and/or division. It was observed that the vast majority of the problems involved simple proportion. With regard to problems involving division in particular, most of them involved the idea of partition. Thus, it was concluded that simple-proportion problems are considered prototypical, and that teachers seem to have a limited notion about problem-situations that involve multiplication. Such limitation may have repercussions on how teachers deal with the textbooks that they adopt, especially, on their ability to analyse critically the problems proposed in these books, and on their capacity to make adjustments that they consider necessary in order to develop the knowledge of mathematics of their students.

Another study also investigated elementary school teachers was conducted by Merlini et al. (2015). In this study, about the introductory teaching of multiplication, the authors analysed how the participants, during a teaching training, reflected about their own didactical actions when teaching multiplication to 4th graders. A semi-structured interview was held shortly after the researchers observed classes the participants ministered. The way the teachers reflected about their own didactical practices were stimulated according to Schön's model (2000): reflection-on-action (reflection on the actions carried out in the classroom), reflection-in-action (reflection made when the action is being carried out), and reflection-for-action (reflection on the outcomes of the actions performed). The main result was that after applying the three types of reflection, teachers tended to change their unsuccessful didactical actions and could adopt ones that were more

efficient. For the authors, teachers showed a progress in their understanding and evaluation of their own didactical actions, being able to improve their teaching practice. Probably, this was because the teachers were aware of the discontinuity between the addition and multiplication conceptual fields, and of the need to explore situations that require students to apply forms of reasoning that were appropriate to the field of multiplication.

Taken together, the investigations mentioned above have relevant educational implications. A first implication is that teachers need to be fully aware of the limitations and difficulties experienced by students in the field of multiplicative structures. Diagnostic studies contribute to this knowledge. Once these difficulties are known, it is possible to think of ways of teaching that will help students overcome these difficulties, and that will promote increasingly more sophisticated forms of reasoning. In this sense, intervention research can serve as a basis for the creation of more effective and appropriate didactic situations that will allow the development of the mathematical reasoning. It is necessary, however, to adapt these types of research to the school context. Such adaptation, although complex, is possible.

A second implication is that the teacher's knowledge and conceptions are equally crucial to creating an effective learning environment. Therefore, it is necessary to make teachers more knowledgeable, both in relation to the knowledge that the students possess, and in relation to methodological issues to be considered by the teacher, such as the ability to formulate problems and the ability to reflect about their actions in the classroom (Gonzales, 1994; Tardif, 2002).

It is important to mention that the studies carried out within this cognitive approach are characterised by continuity, complementarity, and depth. More than theoretical or methodological tensions and disagreements, the results of these investigations have gradually added new information to this field of knowledge, both with regard to the operative invariants mobilised by the students, as well as the teacher's conception of the conceptual field of multiplicative structures. Moreover, in spite of the theoretical cohesion between them, these studies present a wide diversity of methodological resources, and allow investigating different aspects of the same phenomenon. A final, but equally important, point about these cognitive-based studies is that they have implications for mathematics education, and belong therefore to a field of knowledge of international recognition: the psychology of mathematics education.

10.3.2 Studies Which Focus on the Linguistic and the Cultural Aspects Involved in the Processes of Teaching and Learning of Mathematics

To consider language in the classroom during a mathematics class means to discuss its different aspects and uses. These can range from the individual level (expressing the understanding of knowledge) to the social level (involving the different socially

constructed discourses and interactions, and how they affect the learning process). In this focus, the studies discussed in the Working Group Cognitive and Linguistic Processes in Mathematical Education also adopted different theoretical and methodological perspectives. Most frequently adopted is the social-historical-cultural perspective (Vygotsky and Leontiev), which calls attention to the fact that all human activity is *mediated* by semiotic instruments, and that these instruments are constructed during the interactions and discursive practices developed in the social context, in this case, the mathematics class. These issues have been widely discussed in the international literature, with reference to the role attributed to language and the cultural aspects in the practices involving the learning of mathematics.

The latest studies presented in the working group (Carrião, 2012; Frant, 2012; Kistemann Jr., 2012) draw attention to the fact that language in the investigation of learning processes not only has the role of *mediator* between the student and the mathematical knowledge, but it is also the sole form of access to students' way of thinking. In other words, according to the authors, it is through the language used by the teacher and by the students that we can obtain information about how they understand a particular topic.

Another language aspect considered in the investigations is the social norms of conduct that determine what can be said, how and where. This stems from the fact that every social group has its own repertoire of forms of discourse in communication (Carrião, 2012). Kistemann Jr. (2012) draws attention to the fact that we aspire to be socially accepted, and this will determine our discursive choices. This view of language as social practice means that the study of language cannot be restricted to the analysis of isolated phrases or words. For someone to understand the use of words in a given context, it is necessary to interact with the social group that makes use of those words. Thus, in order to analyse the discourse used in the classroom during a mathematics class, we must consider the various elements present in the interactions that take place, such as gestures, intonation, written material, and drawings. This is because the mental processes emerge, mainly, through the internalisation of the social discourse (Carrião, 2012; Frant, 2012).

In order to examine the role of language and the cultural aspects involved in the acquisition of mathematical knowledge, researchers have adopted some theoretical references, namely: (1) the semantic field theory and critical mathematics education (Lins, 1994; Skovsmose, 2005); (2) the historical-cultural perspective of the activity (Engeström, 1987; Leontiev, 1978); (3) the conceptual mapping theory (Lakoff & Johnson, 1999) and the idea of metaphor (Sfard, 2008); and (4) Vygostky's sociocultural perspective and Bakhtin and Voloshinov's (1992) discourse analysis.

Research that adopt these theoretical perspectives often involve the observation and systematic analysis of the classroom activities and of the teaching material used by the teacher. Our focus on this topic is in the discussion of three papers: Carrião (2012), David and Tomaz (2012a, b), and Frant (2012). These studies carried out in WG09 address recurrent questions regarding the role of language and the cultural aspects present in the processes of teaching and learning of mathematics.

The recourses of analysis used, as in any research, are determined by the research question and by the theoretical perspective adopted. Frant (2012) uses the idea of

conceptual mapping to verify the use of compression and metonymy in the student's text and in the textbook. David and Tomaz (2012a, b), on the other hand, focus on the rules that govern the activities and the actions of the individuals, analysing the way students participate in the classroom. Carrião (2012), also investigating classroom discourse, uses strategies of discourse analysis to look for language features brought from other fields, in this case nominalisations.

A number of studies in WG09 share the idea that social aspects are involved in the learning process and that the development of thought is determined by language, in a specific way and by the cultural context of the individual's social group.

David and Tomaz (2012a, b) and Frant (2012) contrast two different perspectives of learning. Frant (2012) stands in favour of the one that she calls participatory metaphor, and according to which mathematics is a discourse and learning mathematics is to participate in the development of this discourse. The author highlights the role of everyday language in the construction of knowledge. For her, from the moment that the cognoscente subject takes hold of the text, everything that is said by others acquires meaning, they create objects and produce knowledge. David and Tomaz (2012a, b), on the other hand, adopt the perspective of expansive learning. From this perspective, it is assumed that learners construct a new object and motive for its use, and that they put it into practice in a process of expansive transitions of the actions of an activity.

Both the studies by Frant (2012) and Carrião (2012) point to questions related to language elements (e.g. nominalisation, compression, and metonymy) in the classroom discourse. These studies show that students' ability to use such elements will determine how they participate in the classroom activities, as well as influence their understanding of the concepts presented. The authors also draw attention to the fact that these language elements are used in our daily life although we are not always aware of this fact. In order to understand the use of words in the classroom dialogue, participants need to agree on the meanings attributed to these words, in particular with regard to compression and metonymy. Metonymy occurs when one entity is used to refer to another, and its main function is referential. Compression is a process that shortens the distance between mental spaces, juxtaposing two spaces and creating a third. Without this agreement, the classroom dialogue can become difficult (Frant, 2012).

Nominalisation is the linguistic process that turns verbs into nouns, indicating the resulting action or event, instead of the verb. Carrião (2012) shows that nominalisation is a feature of the mathematics class discourse, being a direct reflection of the discourse of the field of academic mathematics, as shown by other studies (e.g. Burton & Morgan, 2000; Meaney, 2005). For Carrião (2012), a grasp of the mathematics class discourse, nominalisations in particular, can reconfigure the meaning-construction process. This will not only lead to better school performance, but also, and most importantly, to the development of a specific form of thought that is valued socially.

David and Tomaz (2012a, b), on the other hand, discuss the complexity of the activities performed during a mathematics class. By analysing the mobility of the components of these activities, they explain the complexity of their structure and the

superposition of the rules that govern the activities in the classroom and the actions of the subjects, identifying tensions and potential learning. The authors also point out that in order not to miss learning opportunities, we must focus on the main object of the activity. Moreover, they point out that the changes associated to momentary overlapping rules are an aspect that seems characteristic of mathematics activities in school. David and Tomaz (2012a, b) bring to the analysis of the micro level of the classroom, the perspective of Engeström's third generation activity theory (1987, 2001), which is interdisciplinary. This approach connects different realities. It analyses situations ranging from mathematical situations produced in the classroom to everyday situations, being therefore a methodology that can relate the micro and the macro context.

Research developed according to this approach has revealed important educational aspects. These aspects need to be problematised and discussed with the teachers when pondering the role they attribute to language, and to the cultural aspects present in the teaching and learning processes. Among such aspects, the following can be highlighted: (1) the need to develop pedagogical practices in which the tensions brought about by the different ways of interpreting mathematical records are confronted, rather than ignored; (2) the need for teachers and authors of textbooks and curricula to use an approach based on research on cognitive and linguistic processes, and not only on mathematical logic; (3) the need for students to have a good command of the mathematics class discourse, in particular nominalisations, and of the use of symbols, because not mastering these can result in exclusion, or at least limited classroom participation.

The studies that consider the role of language and of the cultural aspects in the teaching and learning processes, although presenting different theoretical-methodological approaches, share a common ground. For example, all such studies consider the student as a participant in a community, and analyse the learning process in a contextualised way, looking not only at the nearest environment, the classroom, but also at the wider social context. This contributes to the construction of a learning that is consistent and, to a certain extent, coherent. In addition, this diversity of research has contributed to the area of mathematics education, bringing different references from other areas and adapting them to the interpretation of several questions in this field, such as learning and communication in the school context.

10.4 Final Remarks

A retrospective look at the investigations about the cognitive and linguistic processes in mathematics education in Brazil shows that they reflect an ongoing search for possible answers about the role of language, of cognition, and of the cultural aspects present in the teaching and learning of mathematics. WG meetings represent an opportunity to open dialogues between different theoretical frameworks from different areas of knowledge and the use of a variety of methodological approaches. The diversity of the studies also contributes to establishing a dialogue between these

theoretical frameworks and methodologies and the cultural reality of Brazil, and, when necessary, adapt them to this reality.

Such diversity has also been recognised on the international scene (e.g. English & Sriraman, 2005; Hoffmann, 2006; Moschkovich, 2010; Sfard, 2012), among researchers interested in the role of language in teaching and learning in the area of mathematics education. For Hoffmann (2006), this diversity can be related to the fact that there is no unifying theory that can encompass the different ways in which semiotic questions have been addressed in research. This indicates the need for more investigations that can avoid an undue separation between the internal and external processes of learning mathematics of the individual, from its historical and semiotic aspects.

The theoretical choices made with regard to the linguistic, cognitive, and cultural aspects explored make it possible to organise different explanatory models about the cognitive functioning of individuals and its relation to the mathematical knowledge. The research presented by WG09, and which constitutes its identity, portrays the interests within this field in the Brazilian scenario.

It should be stressed that the research developed in Brazil provides data that support theoretical questions posed by different scholars (e.g. Bakhtin, Engeström, Lakoff, Leontiev, Nemirovsky, Vergnaud, Piaget, and Vygotsky). These investigations have also contributed to the discussion of various topics. Among the several contributions of the studies area presented in the group, we highlight the following:

1. The role of gestures in language with reference to the coordination of semiotic tools to create and communicate mathematical meanings, and to the coordination of material tools and semiotics in the construction of learning environments that can contribute to the acquisition of mathematical knowledge by students with special needs. Such discussions highlight the cognitive importance of the body in mathematical thinking, a topic explored by several researchers in the international arena. The studies with blind students developed in Brazil (Fernandes & Healy, 2010; Healy & Fernandes, 2008, 2011) have revealed the role that gestures can play in the construction of mathematical knowledge. For Healy and Fernandes (2008, p. 2), "[...] If we can identify the differences and similarities in the mathematical practices of those whose knowledge of the world is mediated through different channels, perhaps we gain more robust understanding of the relationships between experience and cognition more generally".
2. The resolution of mathematical problems as a way to access both students' and teachers' conceptions, and as a way to relate problem solving to specific mathematical concepts (e.g. Lautert, Spinillo, & Correa, 2012; Spinillo & Lautert, 2006; Spinillo & Silva, 2010; Spinillo, Silva, & Lautert, 2016).
3. The analysis of classroom practices, in particular the multiple perspectives of the different participants in this complex social context, has led Brazilian researchers to make use of different methodological perspectives. Some researchers have adopted the theory of activity, taking Leontiev and Engeström as reference. Tomaz and David (2011) bring to the analysis of the micro level of the mathematics

classroom Engeström's interdisciplinary perspective, in an attempt to understand the complexity of the structure of the activities in the classroom, identifying the tensions and potential learning.

4. The language currently used in mathematics classes and in mathematics schoolbooks is the result of several research works that seek to understand their role in the communication and learning processes. Research by Frant (2012) and Carrião (2012) brings this discussion to the daily activities of the classroom, analysing how elements such as metonymy and nominalisation, which characterise this language, affect communication, the participation in the activities, and student's learning. In these works, we can see this language being used in the social interactions of the classroom.

To conclude, we bring the statement made by Rabello et al. (2000, p. 346) at SIPEM I. With reference to two approaches relevant to the theory and practice that have accompanied all the reflections produced over the last 16 years, they said "[...] At the same time that theory makes practice meaningful, practice makes theory significant: practice reveals to what extent the theoretical assumptions that support it can provide answers to the questions that emerge in the classroom".

References

Bakhtin, M., & Voloshinov, V. (1992). *Marxismo e Filosofia da Linguagem* (6th ed., M. Lahud & Y. L. Vieira, Trans.). São Paulo: Ed. Hucitec. (Original work published 1929).

Borges, F. A., & Nogueira, C. M. I. (2012). Aulas de matemática para alunos surdos inclusos no ensino fundamental. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–20), Petrópolis, Rio de Janeiro.

Burton, L., & Morgan, C. (2000). Mathematicians writing. *Journal for Research in Mathematics Education, 31*(4), 429–453.

Carrião, A. (2012). A nominalização como marca do discurso na aula de matemática. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–18), Petrópolis, Rio de Janeiro.

Charnay, R. (2001). Aprendendo (com) a resolução de problemas. In C. Parra & I. Saiz (Orgs.), *Didática da matemática: reflexões pedagógicas* (pp. 36–47). Porto Alegre: Artmed.

Chica, C. (2001). Por que formular problemas? In K. Smole & M. Diniz (Orgs.), *Ler, escrever e resolver problemas: habilidades básicas para aprender matemática* (pp. 151–173). Porto Alegre: Artmed.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics, 52*, 243–270.

Cunha, M. J. G. (2015). *Elaboração de problemas combinatórios por professores de matemática do ensino médio.* Dissertação de Mestrado, Programa de Pós-graduação em Educação Matemática e Tecnológica da Universidade Federal de Pernambuco, Brazil.

David, M. M., & Tomaz, V. S. (2012a). Perspectiva de análise micro da estrutura da atividade matemática em sala de aula. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–20), Petrópolis, Rio de Janeiro.

David, M. M., & Tomaz, V. S. (2012b). The role of visual representations for structuring classroom mathematical activity. *Educational Studies in Mathematics, 80*, 413–431.

David, M. M. M. S., & Watson, A. (2008). Participating in what? Using situated cognition theory to illuminate differences in classroom practices. In A. Watson & P. Winbourne (Orgs.), *New directions for situated cognition in mathematics education* (pp. 31–57). New York: Springer.

Duval, R. (1993). Registres de représentation sémiotique et fonctionnement cognitif de la pensée. In *Annales de Didactique et de Sciences cognitives, IREM de Starsbourg* (Vol. 5, pp. 37–65).

Duval, R. (1995). *Sémiósis et pensée humaine:registres sémiotiques et apprentissages intellectuels*. Suisse: Peter Lang.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki: Orienta-Konsultit.

English, L. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics, 34*, 183–217.

English, L., & Sriraman, B. (2005). Theories of mathematics education. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 170–202). Melbourne: PME.

Fernandes, S. H. A. A., & Healy, L. (2010). Embodied Mathematics: Relationships between doing and imagining in the activities of a blind learner. In *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 345–352), Belo Horizonte, Brazil.

Fernandes, S. H. A. A., Healy, H., & Serino, A. P. (2012). Das relações entre figuras para relações em um espaço matematizável: as percepções de alunos cegos sobre transformações geométricas. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–19), Petrópolis, Rio de Janeiro.

Fonseca, M. C. F. R. (2010). Adult education and ethnomathematics: Appropriating results, methods, and principles. *ZDM Mathematics Education, 42*(3), 361–369.

Frade, C., & Borges, O. (2006). The tacit-explicit dimension of the learning of mathematics: An investigation report. *International Journal of Science and Mathematics Education, 4*, 293–317.

Frade, C., & Falcão, J. (2007). Exploring connections between tacit knowing and situated learning perspectives in the context of mathematics education. In A. Watson & P. Winbourne (Orgs.), *New directions for situated cognition in mathematics education* (Vol. 1, pp. 203–231). Norwell: Springer.

Frade, C., & Tatsis, K. (2009). Learning, participation and local school mathematics practice. *The Montana Mathematics Enthusiast, 6*, 96–112.

Frade, C., Winbourne, P., & Braga, S. A. M. (2009). A mathematics-science community of practice: Crossing boundaries. *For the Learning of Mathematics, 29*, 14–22.

Frant, J. B. (2009). As time goes by: Technology, embodiment and Cartesian graphics. In *Proceedings of the 33th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 343), Tessaloniki, Greece.

Frant, J. B. (2012). Linguagem, compressão e algumas implicações para a matemática escolar. Perspectiva de análise micro da estrutura da atividade matemática em sala de aula. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–10), Petrópolis, Rio de Janeiro.

Freire, P. C., & Lima, R. N. (2012). O subconstruto parte-todo: uma análise com os três mundos da matemática. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–20), Petrópolis, Rio de Janeiro.

Garnica, A. V. M., Soares, M. T. C., & Buriasco, R. L. C. (2006). *Anais do III Seminário Internacional de Pesquisa em Educação Matemática*. Águas de Lindoia: Sociedade Brasileira de Educação Matemática, São Paulo.

Gonzales, N. A. (1994). Problem posing: A neglected component in mathematics courses for prospective elementary and middle school teachers'. *School Science and Mathematics, 94*(2), 78–84.

Hazin, I., Da Rocha Falcão, J. T., & Leitão, S. (2006). Mathematical impairment among epileptic children. In *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 249–256), Prague, Czech Republic.

Healy, L., & Fernandes, S. H. A. A. (2008). The role of gestures in the mathematical practices of blind learners. In *Proceedings of the Joint Meeting of PME 32 and PME-NA XXX* (Vol. 3, pp. 137–144). Morelia, México: Cinvestav-UMSNH.

Healy, L., & Fernandes, S. H. A. A. (2011). The role of gestures in the mathematical practices of those who do not see with their eyes. *Educational Studies in Mathematics, 77*(2), 157–174.

Hoffmann, M. H. G. (2006). What is a "semiotic perspective", and what could it be? Some comments on the contributions to this special issue. *Educational Studies in Mathematics, 61*, 279–291.

Itacarambi, R. R. (2010). *Resolução de problemas nos anos iniciais do ensino fundamental*. São Paulo: Editora Livraria da Física.

Kistemann, M. A., Jr. (2012). A produção de significados e a tomada de decisão de indivíduos-consumidores. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–13), Petrópolis, Rio de Janeiro.

Lakoff, G., & Johnson, M. (1999). *Philosophy in the flesh: The embodied mind and its challenge to western thought*. New York: Basic Books.

Lautert, S. L., & Spinillo, A. G. (2012). Os princípios invariantes da divisão como foco de um estudo de intervenção com crianças. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–13), Petrópolis, Rio de Janeiro.

Lautert, S. L., Spinillo, A., & Correa, J. (2012). Children's difficulties with division: An intervention study. *Journal of Medicine and Medical Sciences, 1*, 447–456.

Leontiev, A. N. (1978). *Activity, consciousness, personality*. Englewood Cliffs, NJ: Prentice-Hall.

Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal, 9*(1), 5–24.

Lins, R. C. (1994). O modelo teórico dos campos semânticos: uma análise epistemológica da álgebra e do pensamento algébrico. *Revista Dynamis, 1*(7), 29–39.

Lopes, A. C., & Magina, S. (2012). O xadrez e o estudante: uma relação que pode dar certo na resolução de problema matemáticos. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–20), Petrópolis, Rio de Janeiro.

Lowrie, T. (2002). Young children posing problems: The influence of teacher intervention on the type of problems children pose. *Mathematics Education Research Journal, 14*(2), 87–98.

Magina, S. M. P., Spinillo, A. G., & Melo, L. M. (2015). As estratégias de estudantes dos anos iniciais na resolução de problema combinatório. In *Anais do VI Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–12), Pirenópolis, Goiás.

Meaney, T. (2005). Mathematics as text. In A. Chronaki & I. M. Christiansen (Orgs.) *Challenging perspectives on mathematics classroom communication.* Connecticut: Information Age Publishing.

Merlini, V., Santos, A., Teixeira, A. C., & Magina, S. M. P. (2015). Processo formativo centrado na escola: as reflexões da professora Maria. In *Anais do VI Seminário Internacional de Pesquisa em Educação Matemática* (p. 13), Pirenópolis, Goiás.

Moro, M. L. F., & Soares, M. T. C. (2006). Níveis de raciocínio combinatório e produto cartesiano na escola fundamental. *Revista Educação Matemática Pesquisa, 8*(1), 99–124.

Moschkovich, J. N. (2010). *Language and mathematics education: Multiple perspectives and directions for research*. Charlotte: Information Age Publishing.

Oliveira, D. L., Albuquerque, L. C., & Gontijo, C. H. (2012). Criatividade matemática: alguns elementos na divisão de quadrados. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–19), Petrópolis, Rio de Janeiro.

Onuchic, L. R., & Alevato, N. S. G. (2004). Novas reflexões sobre o ensino-aprendizagem de matemática através da resolução de problemas. In M. A. V. Bicudo & M. C. Borba (Orgs.), *Educação matemática: Pesquisa em movimento* (pp. 213–223). São Paulo: Cortez.

Pires, C. M. C., Curi, E., Rabelllo, M., Pavanello, R., & Valente, W. R. (2003). *Anais do II Seminário Internacional de Pesquisa em Educação Matemática.* Sociedade Brasileira de Educação Matemática. São Paulo: Santos.

Rabello, M., Lins, R., & Da Rocha Falcão. (2000). Processos cognitivos e linguísticos na Educação matemática. In T. M. Campos & C. M. C. Pires Anais do I Seminário Internacional de Pesquisa em Educação Matemática. *Sociedade Brasileira de Educação Matemática* (pp. 329–372). São Paulo: Serra Negra.

Schön, D. (2000). *Educando o Profissional Reflexivo: um novo design para o ensino e aprendizagem*. Porto Alegre: Artes Médicas Sul.

Sfard, A. (2008). *Thinking as communicating*. Cambridge University Press. Ebook.

Sfard, A. (2012). Introduction: Developing mathematical discourse—Some insights from communicational research. *International Journal of Educational Research, 51–52*, 1–9.

Skovsmose, O. (2005). Foregrounds and politics of learning obstacles. *For the Learning of Mathematics, 25*(1), 4–10.

Souza, E. R. R. de. (2015). *Estruturas multiplicativas: Concepção de professor de Ensino Fundamental*. Dissertação de Mestrado, Programa de Pós-Graduação em Educação Matemática, Universidade Estadual de Santa Cruz.

Spinillo, A. G., & Lautert, S. (2006). Exploring the role played by the remainder in the solution of division problems. In *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 153–160).

Spinillo, A. G., Lautert, S. L., Santos, E. M., & Silva, J. F. G. (2015). Uma análise de problemas do campo multiplicativo elaborados por professores do Ensino Fundamental I. In *Anais do VI Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–9), Pirenópolis, Goiás.

Spinillo, A. G., & Silva, J. F. G. (2010). Making explicit the principles governing combinatorial reasoning: Does it help children to solve Cartesian product problems? In *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 216–224).

Spinillo, A. G., Silva, J. F. G., & Lautert, S. L. (2016). Ensino e aprendizagem de conceitos matemáticos a partir da explicitação dos princípios invariantes. In J. A. de Castro Filho, M. Chagas, P. M. Barguil, D. L. Maia, & J. Lima (Orgs.), *Matemática, cultura e tecnologia: perspectivas internacionais* (pp. 35–48) Curitiba: CRV.

Tardif, M. (2002). *Saberes docentes e formação profissional*. Petrópolis, Rio de Janeiro: Editora Vozes.

Tomaz, V. S., & David, M. M. (2011). Classroom activity promoting students' learning about the use of drawings in geometry. In *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 257–264).

Torisu, E. M. (2012). Acompanhamento extraclasse e fortalecimento das crenças de autoeficácia matemática. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* (pp. 1–22), Petrópolis, Rio de Janeiro.

Vergnaud, G. (1983). Multiplicative structures. In R. Lesh & M. Landau (Eds.), *Acquisitions of mathematics concepts and procedures* (pp. 127–174). New York: Academic.

Vergnaud, G. (1988). Multiplicative structures. In H. Hiebert & M. Behr (Eds.), *Research agenda in mathematics education. Number concepts and operations in the middle grades* (pp. 141–161). Hillsdale, NJ: Lawrence Erlbaum.

Vergnaud, G. (1990). La théorie des champs conceptuels. *Recherches en Didactique des Mathématiques, Grenoble, 10*(23), 133–170.

Vergnaud, G. (2003). A gênese dos campos conceituais. In E. Grossi (Org.). *Por que ainda há quem não aprende? A teoria* (pp. 21–60). Ed. Vozes, RJ: Petrópolis.

Viana, O. A. (2012). A identificação de propriedades e a habilidade de planificação de figuras geométricas espaciais. In *Anais do V Seminário Internacional de Pesquisa em Educação Matemática* Petrópolis, Rio de Janeiro.

Zunino, D. L. (1995). *A matemática na escola: aqui e agora*. Porto Alegre: Artes Médicas.