Chapter 5
Content Sponsoring with Inter-ISP Transit Cost

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5.1 Introduction

As demand for mobile data increases, Internet service providers (ISPs) are turning to new types of smart data pricing to bring in additional revenue and to expand the capacity of their current network [7]. One way to keep up funding such investment is content sponsorship. Content providers (CPs) split the cost of transferring mobile data traffic, and sponsor the user's access to the content by making direct payment to the ISPs. For example, GS Shop, a Korea TV home shopping company, has partnered with SK Telecom to sponsor data incurred from its application, so consumers are incentivized to continue browsing and making purchases from their mobile devices without ringing up data charges [1]. Content sponsoring may benefit all players in the market: the ISPs can generate more revenue with CP's subsidies, and users can enjoy free or low-cost access to certain services, which in turn increases the demand and attracts more traffic, resulting in higher revenue of the CP.

There are several studies on content sponsoring despite a short history. Most of the works either focus on a simple model with a single ISP and a single CP interacting in a game theoretic setting or consider Quality-of-Service (QoS) prioritization and its implications for net neutrality [4, 5]. In a two-sided market with a single ISP providing connection between CPs and EUs, profit maximization of the players under sponsoring mobile data has been studied in [2, 8]. In [2], single monopolistic ISP determines optimal price to charge the CPs and the EUs, while the authors in [8] study the contractual relationship between the CPs and the ISP under a similar model. Nevertheless, none of them consider the interaction between multiple ISPs. Although the authors in [9] propose a model with a transit

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ISP and a user-facing ISP, their understanding of the interaction between these non-cooperative ISPs is limited to the environments without content sponsoring. Other works, e.g. [3, 10], have analyzed content sponsorship from the economic point of view. They examine the implications of sponsored data on the CPs and the EUs, and identify how sponsored data influence the CP inequality.

In many Internet markets, there are multiple ISPs that cooperate to provide end-to-end connectivity service between the CPs and the EUs, in which case the assumption of a single representative ISP no longer holds. Since each ISP aims to maximize its own profit, the establishment of interconnection among multiple ISPs is a thorough process that depends on specific profit sharing/inter-charging arrangements.

As the most commercial traffic originates from the CPs and terminates at the EUs, some ISPs positioned on the middle of the traffic delivery chain will have more power and request a transit-price. An ISP serving a large population of users might have a dominant influence in determining the transit price paid by other relatively weak ISPs for traffic delivery. For example, a large entertainment company Netflix directly uses the service provided by ISPs such as Level 3, which is connected with residential broadband ISPs like Comcast to get access to the customers [6]. Level 3 charges Netflix and Comcast charges the users. Netflix may partially or fully sponsor its traffic, which is likely to increase the amount of traffic through both ISPs. Due to high traffic volume, the access ISP (Comcast) may require additional transit price for traffic delivery, which will impact on the pricing decision at Level 3 and subsequently on the sponsoring decision at Netflix. In this work, we are interested in the dynamics between the players with focus on content sponsoring and transit pricing. To this end, we study the interplay among two ISPs, CP, and EU, where each player selfishly maximizes its own profit. We model this non-cooperative interaction between ISP$_1$, ISP$_2$, CP, and EU as a four-stage Stackelberg game. Specifically, in our model, we assume that the EU-facing ISP has a dominant power and can be considered as the game leader who decides the transit cost preceding the choice of the follower ISP. We aim to understand the behaviors of the players in non-cooperative equilibrium and their decisions to maximize their own utility.

The rest of the paper is organized as follows. We present the basic system model in Sect. 5.2, and investigate the strategies of the CP, the EU, and the ISPs to maximize their utility in Sect. 5.3. Numerical results are presented in Sect. 5.4, followed by the conclusion and future work in Sect. 5.5.

5.2 Two-ISP Pricing Model

We consider an Internet market model with one CP and two ISPs as shown in Fig. 5.1. Two interconnected ISPs have their own cost structures and each provides connectivity to either the CP or the EU. The CP-facing ISP (ISP_1) obtains its profits by directly charging the CP (CP) by p_{cp} for per unit traffic while the EU-facing ISP (ISP_2) charges the EU (EU) by p_{eu} for per unit traffic. Further ISP_2 charges ISP_1

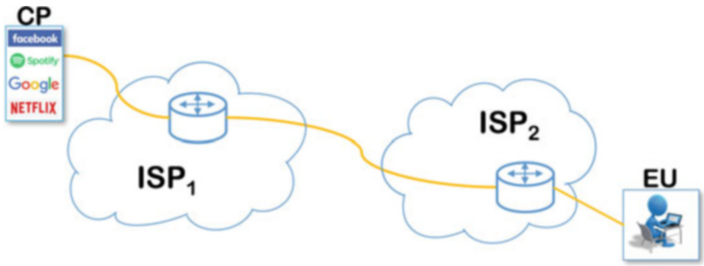


Fig. 5.1 Two-sided Internet market

with transit-price p_{tr} for traffic delivery. Let m_1 and m_2 denote the marginal costs of traffic delivery for ISP_1 and ISP_2, respectively. We denote x as the traffic amount of flow between CP and EU.

We assume that the players in this non-cooperative game make decisions in four stages as follows:

1. ISP_2 sets prices p_{eu} and p_{tr} to charge EU and ISP_1, respectively.
2. ISP_1 determines the optimal value of p_{cp} to charge CP.
3. CP decides how much content to sponsor, i.e., the value of s.
4. The traffic volume is decided by both EU and CP.

Each player selfishly maximizes its own profit subject to the others' decisions. We model this non-cooperative interaction as a four-stage Stackelberg game and use the backward induction method to find optimal strategy of each player.

Let us define the utility of EU by the multiplication of a scaling factor $\sigma_{eu} \geq 0$ and a utility-level function. The utility represents user's desire to obtain traffic. We assume a concave and non-decreasing function $u_{eu}(x)$ with decreasing marginal satisfaction, i.e., $u_{eu}(x) = \frac{x^{1-\alpha_{eu}}}{1-\alpha_{eu}}$ with parameter $\alpha_{eu} \in (0, 1)$. Given unit price p_{eu} that ISP_2 charges user, EU will maximize its utility minus the payment by solving

$$\textbf{(EU} - \textbf{P)} \quad \max_{x} \quad \sigma_{eu} \cdot u_{eu}(x) - (1-s) \cdot x \cdot p_{eu},$$
$$s.t. \quad x \geq 0, \tag{5.1}$$

where $s \in [0, 1]$ denotes the sponsored percentage, and $(1-s) \cdot x \cdot p_{eu}$ denotes the payment of EU to ISP_2. The solution x^*_{eu} to (5.1) can be obtained as $x^*_{eu}(s, p_{eu}) = \left(\frac{\sigma_{eu}}{(1-s)p_{eu}}\right)^{\frac{1}{\alpha_{eu}}}$.

Similarly, we model the behavior of CP. The utility of CP is given by $\sigma_{cp} u_{cp}(x)$, where $\sigma_{cp} \geq 0$ is a scaling factor (e.g., the popularity of the content) and $u_{cp}(x)$ is a concave utility-level function $u_{cp}(x) = \frac{x^{1-\alpha_{cp}}}{1-\alpha_{cp}}$ with parameter $\alpha_{cp} \in (0, 1)$. CP will maximize its payoff by solving

$$\textbf{(}\boldsymbol{CP-P}\textbf{)} \quad \max_{x,s} \quad \sigma_{cp} \cdot u_{cp}(x) - s \cdot x \cdot p_{eu} - x \cdot p_{cp},$$
$$s.t. \quad x \geq 0 \quad and \quad 0 \leq s \leq 1. \tag{5.2}$$

In the objective, the first term denotes its utility, the second term denotes the cost due to sponsorship, and the third term is from the network usage cost to ISP_1. Given s, p_{cp}, and p_{eu}, it can be easily shown that the optimal amount of traffic for CP is $x^*_{cp}(s, p_{cp}, p_{eu}) = \left(\frac{\sigma_{cp}}{sp_{eu}+p_{cp}}\right)^{\frac{1}{\alpha_{cp}}}$.

Since ISP_1 obtains its revenue from charging CP, it decides the optimal value of p_{cp} to maximize its total profit as

$$\textbf{(}\boldsymbol{ISP_1-P}\textbf{)} \quad \max_{p_{cp}} \quad (p_{cp} + s^* \cdot p_{eu} - p_{tr} - m_1) \cdot x^*(p_{cp}, p_{eu}),$$
$$s.t. \qquad p_{cp} \geq 0, \tag{5.3}$$

where m_1 is the marginal cost for traffic delivery and thus $p_{cp} + s^* \cdot p_{eu} - p_{tr} - m_1$ is the net-gain of ISP_1 per unit traffic.

ISP_2 obtains its revenue from charging ISP_1 with transit-price p_{tr} and charging EU with traffic-price p_{eu}. Therefore, in order to maximize its total profit, it will solve

$$\textbf{(}\boldsymbol{ISP_2-P}\textbf{)} \quad \max_{p_{eu},p_{tr}} \quad ((1-s^*) \cdot p_{eu} + p_{tr} - m_2) \cdot x^*(p_{cp}, p_{eu}),$$
$$s.t. \quad p_{eu} \geq 0 \quad and \quad p_{tr} \geq 0, \tag{5.4}$$

where m_2 is the marginal cost for traffic delivery.

Through the sequential decision, we investigate the interactions of the players described in (5.1), (5.2), (5.3), (5.4), and find the optimal strategies for pricing and sponsoring.

5.3 Strategies for Utility Maximization

In this section, we sequentially find the optimal strategies of CP, ISP_1, and ISP_2 by exploiting the backward induction.

5.3.1 Sponsoring of Content Provider

Note that each solution to (5.1) and (5.2) results in user-side traffic demand x^*_{eu} and CP-side traffic amount x^*_{cp}, respectively, and the actual traffic amount x^* between

CP and EU will be determined by their minimum, i.e., $x^* = \min\{x^*_{cp}, x^*_{eu}\}$. In general $x^*_{eu} \neq x^*_{cp}$. For instance, a certain website may restrict the number of simultaneous on-line clients, which implies $x^*_{cp} \leq x^*_{eu}$.

Suppose that p_{eu} and p_{cp} are given. The actual traffic $x^*(s)$ will be determined by the sponsoring rate s, and CP will decide its optimal sponsored percentage s^* by solving the following problem:

$$\begin{aligned} &\boldsymbol{(CP-P)} \quad \max_{s} \quad \sigma_{cp} \cdot u_{cp}(x^*(s)) - s \cdot x^*(s) \cdot p_{eu} - x^*(s) \cdot p_{cp}, \\ &\qquad\qquad\quad s.t. \quad 0 \leq s \leq 1. \end{aligned} \tag{5.5}$$

We assume $\alpha_{eu} = \alpha_{cp} = \alpha \in (0, 1)$, i.e., EU and CP utility components have the same utility shape. This assumption is reasonable in the scenarios where CP makes its pricing decision according to the user response. On the other hand, the scaling factors σ_{eu} and σ_{cp} of EU and CP can be quite different. The sponsoring behavior will be affected by whether the traffic volume is constrained by EU or CP. If $x^*_{eu} \leq x^*_{cp}$, we have $s \leq \frac{\sigma_{cp} p_{eu} - \sigma_{eu} p_{cp}}{(\sigma_{eu}+\sigma_{cp}) p_{eu}}$ and $x^* = x^*_{eu}$. Similarly, if $x^*_{eu} \geq x^*_{cp}$, we have $s \geq \max\left(\frac{\sigma_{cp} p_{eu} - \sigma_{eu} p_{cp}}{(\sigma_{eu}+\sigma_{cp}) p_{eu}}, 0\right)$ and $x^* = x^*_{cp}$. We consider each case.

Case (i) When $x^* = x^*_{cp}$. The profit of the CP can be written as

$$V(s) = \sigma_{cp} \cdot u_{cp}(x^*_{cp}(s)) - s \cdot x^*_{cp}(s) \cdot p_{eu} - x^*_{cp}(s) \cdot p_{cp}. \tag{5.6}$$

By substituting $x^*_{cp}(s, p_{cp}, p_{eu}) = \left(\frac{\sigma_{cp}}{s p_{eu} + p_{cp}}\right)^{\frac{1}{\alpha}}$ into (5.6), it can be easily shown that $V(s)$ is a decreasing function of s, and we have the optimal value $s^* = \max\left(\frac{\sigma_{cp} p_{eu} - \sigma_{eu} p_{cp}}{(\sigma_{eu}+\sigma_{cp}) p_{eu}}, 0\right)$. Thus, the traffic amount and the sponsoring rate will be

$$(x^*, s^*) = (x^*_{cp}, s^*) = \begin{cases} \left(\left(\frac{\sigma_{cp}}{p_{cp}}\right)^{\frac{1}{\alpha}}, \quad 0\right), & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} \leq \frac{p_{cp}}{p_{eu}}, \\ \left(\left(\frac{\sigma_{cp}+\sigma_{eu}}{p_{cp}+p_{eu}}\right)^{\frac{1}{\alpha}}, \quad \frac{\sigma_{cp} p_{eu} - \sigma_{eu} p_{cp}}{(\sigma_{eu}+\sigma_{cp}) p_{eu}}\right), & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}. \end{cases} \tag{5.7}$$

The maximum profit of CP is given as

$$V^*(x^*_{cp}, s^*) = \begin{cases} \frac{\alpha(\sigma_{cp})^{\frac{1}{\alpha}}}{1-\alpha}(p_{cp})^{1-\frac{1}{\alpha}}, & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} \leq \frac{p_{cp}}{p_{eu}}, \\ \frac{\alpha\sigma_{cp}}{1-\alpha}\left(\frac{p_{eu}+p_{cp}}{\sigma_{eu}+\sigma_{cp}}\right)^{1-\frac{1}{\alpha}}, & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}. \end{cases} \tag{5.8}$$

Case (ii) When $x^* = x^*_{eu}$. In this case, we have $s \leq \frac{\sigma_{cp} p_{eu} - \sigma_{eu} p_{cp}}{(\sigma_{eu}+\sigma_{cp}) p_{eu}}$, $x^*_{eu}(s, p_{eu}) = \left(\frac{\sigma_{eu}}{(1-s) p_{eu}}\right)^{\frac{1}{\alpha}}$ and $\frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}$. CP will optimize its sponsorship percentage by solving

$$\max \quad \frac{\sigma_{cp}\left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}-1}}{1-\alpha}(1-s)^{1-\frac{1}{\alpha}} - \frac{(sp_{eu}+p_{cp})\left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}}}{(1-s)^{\frac{1}{\alpha}}},$$
$$s.t. \quad 0 \le s \le \frac{\sigma_{cp}p_{eu}-\sigma_{eu}p_{cp}}{(\sigma_{eu}+\sigma_{cp})p_{eu}}, \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}. \tag{5.9}$$

From the first order condition, the optimal data rate x^* and the optimal sponsoring rate s^* can be obtained as

$$(x^*_{eu}, s^*) = \begin{cases} \left(\left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}}, \quad 0\right), & if \quad \frac{p_{cp}}{p_{eu}} < \frac{\sigma_{cp}}{\sigma_{eu}} \le \alpha + \frac{p_{cp}}{p_{eu}}, \\ \left(\left(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}}\right)^{\frac{1}{\alpha}}, \quad \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}\right), & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}, \end{cases} \tag{5.10}$$

and the maximum profit of CP is

$$V^*(x^*_{eu}, s^*) = \begin{cases} \left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}}\left[\frac{\sigma_{cp}p_{eu}}{(1-\alpha)\sigma_{eu}} - p_{cp}\right] & if \quad \frac{p_{cp}}{p_{eu}} < \frac{\sigma_{cp}}{\sigma_{eu}} \le \alpha + \frac{p_{cp}}{p_{eu}}, \\ \frac{\alpha(p_{cp}+p_{eu})}{1-\alpha}\left(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}}\right)^{\frac{1}{\alpha}} & if \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}. \end{cases} \tag{5.11}$$

From the two-case response of CP, we can obtain the following Proposition.

Proposition 1 *Given prices p_{cp} and p_{eu}, the optimal sponsorship rate s^* of the CP is*

$$\begin{aligned} &\textit{case (1) if} \quad \frac{\sigma_{cp}}{\sigma_{eu}} \le \frac{p_{cp}}{p_{eu}}, && s^* = 0, \\ &\textit{case (2) if} \quad \frac{p_{cp}}{p_{eu}} < \frac{\sigma_{cp}}{\sigma_{eu}} \le \alpha + \frac{p_{cp}}{p_{eu}}, && s^* = 0, \\ &\textit{case (3) if} \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}, && s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}. \end{aligned} \tag{5.12}$$

Proof For case 1, the maximum available profit of CP can be easily obtained as $V^*(x^*_{cp}, s^*) = \frac{\alpha(\sigma_{cp})^{\frac{1}{\alpha}}}{1-\alpha}(p_{cp})^{1-\frac{1}{\alpha}}$ from (5.8).

For $\frac{\sigma_{cp}}{\sigma_{eu}} > \frac{p_{cp}}{p_{eu}}$, the CP will choose the largest one among available profits of $V^*(x^*_{cp}, s^*)$ and $V^*(x^*_{eu}, s^*)$, given in (5.8) and (5.11), respectively. Let $\sigma = \frac{\sigma_{cp}}{\sigma_{eu}}$ and $p = \frac{p_{cp}}{p_{eu}}$. We decompose it into two subcases as below.

(1) When $p < \sigma \le \alpha + p$, each profit function can be written as

$$V^*(x^*_{cp}, s^*) = \frac{(\sigma_{eu})^{\frac{1}{\alpha}}(p_{eu})^{1-\frac{1}{\alpha}}}{(1-\alpha)}\left(\frac{1+p}{1+\sigma}\right)\left(\frac{1+p}{1+\sigma}\right)^{-\frac{1}{\alpha}}\alpha\sigma,$$

$$V^*(x^*_{eu}, s^*) = \frac{(\sigma_{eu})^{\frac{1}{\alpha}}(p_{eu})^{1-\frac{1}{\alpha}}}{(1-\alpha)}(\sigma-(1-\alpha)p).$$

Consider the ratio $\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)}$. By using the generalized form of Bernoulli's inequality $(1+x)^r \geq 1+rx$ for $r \leq 0$ or $r \geq 1$ and $x > -1$, we can obtain

$$\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)} \geq \left(\frac{\sigma-(1-\alpha)p}{\alpha\sigma}\right)\left(\frac{1+\sigma}{1+p}\right)\left(1+\frac{p-\sigma}{(1+\sigma)\alpha}\right) = 1+\frac{(1-\alpha)(\sigma-p)(p+\alpha-\sigma)}{\sigma\alpha^2(1+p)}.$$

Hence, if $p < \sigma \leq \alpha + p$, we have $\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)} \geq 1$, implying $x^* = x^*_{eu}$ and $s^* = 0$ from (5.10)

(2) When $\sigma > \alpha + p$, we have

$$V^*(x^*_{cp}, s^*) = \left(\frac{\alpha}{1-\alpha}\right)(p_{eu}+p_{cp})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}(\sigma)(1+\sigma)^{\frac{1}{\alpha}-1},$$

$$V^*(x^*_{eu}, s^*) = \left(\frac{\alpha}{1-\alpha}\right)(p_{eu}+p_{cp})^{1-\frac{1}{\alpha}}(\sigma_{eu})^{\frac{1}{\alpha}}(1+\sigma-\alpha)^{\frac{1}{\alpha}}.$$

Again we consider the ratio $\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)} = \frac{1+\sigma}{\sigma}\left(1-\frac{\alpha}{1+\sigma}\right)^{\frac{1}{\alpha}}$. Applying the generalized form of Bernoulli's inequality, we have $\frac{V^*(x^*_{eu},s^*)}{V^*(x^*_{cp},s^*)} \geq \frac{1+\sigma}{\sigma}\left(1-\frac{1}{1+\sigma}\right) = 1$, and thus we have $x^* = x^*_{eu}$ and $s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}$ from (5.10).

According to Proposition 1, CP has no incentive to invest in sponsored data plan when $\frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}$. On the other hand, when $\frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}$, CP will invest in sponsoring as in (5.10). The data rate under sponsoring will be

$$\begin{aligned}
&\textit{case (1) if} \quad \frac{\sigma_{cp}}{\sigma_{eu}} \leq \frac{p_{cp}}{p_{eu}}, && x^*(p_{cp}, p_{eu}) = \left(\frac{\sigma_{cp}}{p_{cp}}\right)^{\frac{1}{\alpha}},\\
&\textit{case (2) if} \quad \frac{p_{cp}}{p_{eu}} < \frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}, && x^*(p_{cp}, p_{eu}) = \left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}},\\
&\textit{case (3) if} \quad \frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}, && x^*(p_{cp}, p_{eu}) = \left(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}}\right)^{\frac{1}{\alpha}}.
\end{aligned} \tag{5.13}$$

5.3.2 *Utility Maximization of ISP_1*

ISP_1 also tries to maximize its total profit in each region specified in (5.13). We obtain the optimal response of ISP_1 in each case.

Case (1) When $x^* = \left(\frac{\sigma_{cp}}{p_{cp}}\right)^{\frac{1}{\alpha}}$ and $s^* = 0$. From (5.3), ISP_1 maximizes $(p_{cp} - p_{tr} - m_1)\cdot\left(\frac{\sigma_{cp}}{p_{cp}}\right)^{\frac{1}{\alpha}}$ subject to $\frac{\sigma_{cp}}{\sigma_{eu}}\cdot p_{eu} \leq p_{cp}$. The best response p^*_{cp} of ISP_1 can be easily obtained as $p^*_{cp} = \frac{p_{tr}+m_1}{1-\alpha}$.

Case (2) When $x^* = \left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}}$ and $s^* = 0$. From (5.3), ISP_1 has the objective of $\max_{p_{cp}\geq 0} \quad (p_{cp} - p_{tr} - m_1) \cdot \left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}}$ subject to $\frac{p_{cp}}{p_{eu}} - \frac{\sigma_{cp}}{\sigma_{eu}} \leq 0$ and $\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}}{p_{eu}} \leq 0$. From the constraints, we have $p_{cp} \in \left[\left(\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha\right) p_{eu}, \frac{\sigma_{cp}}{\sigma_{eu}} p_{eu}\right]$. Note that since the objective is an increasing function of p_{cp}, we set the largest $p_{cp} = \frac{\sigma_{cp}}{\sigma_{eu}} \cdot p_{eu}$ for the optimal solution, which gives us maximum utility $P^* = \left(\frac{\sigma_{cp}}{\sigma_{eu}} \cdot p_{eu} - p_{tr} - m_1\right) \cdot \left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}}$. By differentiating it with respect to p_{eu}, we can find $p^*_{eu} = \frac{\sigma_{eu}}{\sigma_{cp}} \cdot \left(\frac{p_{tr}+m_1}{1-\alpha}\right)$ that maximizes P^*, which results in the optimal $p^*_{cp} = \frac{p_{tr}+m_1}{1-\alpha}$.

Case (3) When $x^* = \left(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}}\right)^{\frac{1}{\alpha}}$ and $s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}$. The problem can be rewritten as $\max_{p_{cp}\geq 0} (p_{cp} + s^* p_{eu} - p_{tr} - m_1) \cdot \left(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}}\right)^{\frac{1}{\alpha}}$, subject to $p_{cp} \leq \left(\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha\right) p_{eu}$. From the first order condition, we can obtain the optimal price $p^*_{cp} = \frac{(k+1)(p_{tr}+m_1)}{k(1-\alpha)} - p_{eu}$, where $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$.

5.3.3 Utility Maximization of ISP_2

For the behaviors of ISP_2, we also consider the three cases of (5.13) and find the best strategy of ISP_2 for each case.

Case (1) When $x^*(p^*_{cp}, p_{eu}) = \left(\frac{\sigma_{cp}}{p^*_{cp}}\right)^{\frac{1}{\alpha}}$ and $s^* = 0$. We already have $p^*_{cp} = \frac{p_{tr}+m_1}{1-\alpha}$. From (5.4) and (5.13), the ISP_2 determines its prices p_{eu} and p_{tr} by solving $\max_{p_{eu}\geq 0, p_{tr}\geq 0} ((1 - s^*) \cdot p_{eu} + p_{tr} - m_2) \cdot \left(\frac{\sigma_{cp}}{p^*_{cp}}\right)^{\frac{1}{\alpha}}$, subject to $\frac{\sigma_{cp}}{\sigma_{eu}} - \frac{p^*_{cp}}{p_{eu}} \leq 0$.

Let P denote the objective function. From the Karush-Kuhn-Tucker (KKT) conditions, we have $\frac{\partial P}{\partial p_{eu}} = 0$, $\frac{\partial P}{\partial p_{tr}} = 0$, and $\lambda \cdot \left[\frac{\sigma_{cp}}{\sigma_{eu}} - \frac{p^*_{cp}}{p_{eu}}\right] = 0$. By solving these equations, we have the optimal prices

$$p^*_{eu} = \frac{(m_1+m_2)}{(1-\alpha)(1+(k+\alpha)(1-\alpha))} \quad \textit{and} \quad p^*_{tr} = \frac{(k+\alpha)(m_1+m_2)}{(1+(k+\alpha)(1-\alpha))} - m_1, \tag{5.14}$$

at which the maximum profit P^* is $\left[\frac{\alpha(m_1+m_2)}{(1-\alpha)}\right]\left(\frac{\sigma_{cp}(1-\alpha)(1+(k+\alpha)(1-\alpha))}{(k+\alpha)(m_1+m_2)}\right)^{\frac{1}{\alpha}}$, where $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$.

Case (2) When $x^*(p_{cp}^*, p_{eu}) = \left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}}$ and $s^* = 0$. In this case, we have $p_{cp}^* = \frac{p_{tr}+m_1}{1-\alpha}$. From (5.4) and (5.13), the ISP_2 determines its prices by solving the following problem.

$$\textbf{(}\boldsymbol{ISP_2 - P}\textbf{)} \quad \max_{p_{eu}\geq 0, p_{tr}\geq 0} \quad ((1-s^*)\cdot p_{eu} + p_{tr} - m_2)\cdot\left(\frac{\sigma_{eu}}{p_{eu}}\right)^{\frac{1}{\alpha}},$$

$$s.t. \quad \frac{p_{cp}^*}{p_{eu}} - \frac{\sigma_{cp}}{\sigma_{eu}} \leq 0 \quad and \quad \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}^*}{p_{eu}} \leq 0. \tag{5.15}$$

From the KKT conditions, we have $\frac{\partial P}{\partial p_{eu}} = 0$, $\frac{\partial P}{\partial p_{tr}} = 0$, $\lambda_1 \cdot \left(\frac{p_{cp}^*}{p_{eu}} - \frac{\sigma_{cp}}{\sigma_{eu}}\right) = 0$ and $\lambda_2 \cdot \left(\frac{\sigma_{cp}}{\sigma_{eu}} - \alpha - \frac{p_{cp}^*}{p_{eu}}\right) = 0$, where $\lambda_i \geq 0$, $p_{cp} \geq 0$, and $p_{eu} \geq 0$. There are three possible subcases: (i) $\lambda_1 = 0, \lambda_2 \neq 0$, (ii) $\lambda_1 \neq 0, \lambda_2 = 0$, (iii) $\lambda_1 = 0$ and $\lambda_2 = 0$.

(i) When $\lambda_1 = 0$ and $\lambda_2 \neq 0$, the optimal prices will be

$$p_{eu}^* = \frac{m_1+m_2}{(1-\alpha)(1+k(1-\alpha))} \quad and \quad p_{tr}^* = \frac{k(m_1+m_2)}{1+k(1-\alpha)} - m_1, \tag{5.16}$$

where $k = \frac{\sigma_{cp}}{\sigma_{eu}} - \alpha$, and we have the maximum profit $P_{\lambda_1}^* = \left[\frac{\alpha(m_1+m_2)}{(1-\alpha)}\right]\left(\frac{(\sigma_{cp}-\sigma_{eu}\alpha)(1-\alpha)^2+\sigma_{eu}(1-\alpha)}{m_1+m_2}\right)^{\frac{1}{\alpha}}$.

(ii) When $\lambda_1 \neq 0$ and $\lambda_2 = 0$, the optimal prices will be

$$p_{eu}^* = \frac{(m_1+m_2)}{(1-\alpha)(1+(k+\alpha)(1-\alpha))} \quad and \quad p_{tr}^* = \frac{(k+\alpha)(m_1+m_2)}{(1+(k+\alpha)(1-\alpha))} - m_1, \tag{5.17}$$

and the maximum profit $P_{\lambda_2}^* = \left[\frac{\alpha(m_1+m_2)}{(1-\alpha)}\right]\left(\frac{\sigma_{cp}(1-\alpha)^2+\sigma_{eu}(1-\alpha)}{m_1+m_2}\right)^{\frac{1}{\alpha}}$.

(iii) When $\lambda_1 = 0$ and $\lambda_2 = 0$, the two inequality constraints of (5.15) should be an active constraint (i.e., the equalities hold). However, it is not possible to satisfy both equalities, and hence, this case is infeasible.

From $P_{\lambda_2}^* > P_{\lambda_1}^*$, we should have $\lambda_2 = 0$ and the best response of the ISP_2 is (5.17), which equals the result of case 1 in (5.14).

Case (3) In this case, we have the optimal sponsoring rate $s^* = \frac{\frac{\sigma_{cp}}{\sigma_{eu}}-\alpha-\frac{p_{cp}}{p_{eu}}}{\frac{\sigma_{cp}}{\sigma_{eu}}+1-\alpha}$ and the traffic demand is $x^*(p_{cp}^*, p_{eu}) = \left(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p_{cp}+p_{eu}}\right)^{\frac{1}{\alpha}}$.

As shown in Sect. 5.3.2, the best-response p_{cp}^* of ISP_1 is $\frac{(k+1)(p_{tr}+m_1)}{k(1-\alpha)} - p_{eu}$. From (5.4) and (5.13), ISP_2 determines its prices by solving $\max_{p_{eu}\geq 0, p_{tr}\geq 0} ((1-s^*)\cdot$

$p_{eu} + p_{tr} - m_2) \cdot \left(\frac{\sigma_{cp}+(1-\alpha)\sigma_{eu}}{p^*_{cp}+p_{eu}}\right)^{\frac{1}{\alpha}}$, subject to $\frac{p^*_{cp}}{p_{eu}} + \alpha - \frac{\sigma_{cp}}{\sigma_{eu}} \leq 0$. From the KKT conditions, we have $\frac{\partial P}{\partial p_{eu}} = 0$, $\frac{\partial P}{\partial p_{tr}} = 0$, and $\lambda \cdot \left[\frac{p^*_{cp}}{p_{eu}} + \alpha - \frac{\sigma_{cp}}{\sigma_{eu}}\right] = 0$.

By solving the equations, we can obtain without difficulty that

$$p^*_{eu} = \frac{(m_1+m_2)}{(1-\alpha)(1+k(1-\alpha))} \quad and \quad p^*_{tr} = \frac{k(m_1+m_2)}{(1+k(1-\alpha))} - m_1. \tag{5.18}$$

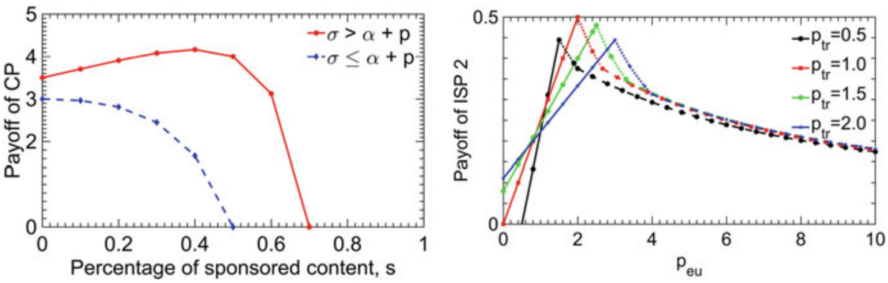
The maximum profit P^* will be $\left[\frac{\alpha(m_1+m_2)}{1-\alpha}\right]\left(\frac{(\sigma_{cp}(1-\alpha)+\sigma_{eu}(1-\alpha)^2)(1+k(1-\alpha))}{(k+1)(m_1+m_2)}\right)^{\frac{1}{\alpha}}$.

We have shown the optimal responses of the EU, the CP, and two ISPs in a non-cooperative equilibrium. They describe the sponsoring rate s^* and the pricing of p^*_{cp}, p^*_{eu}, and p^*_{tr} when each player maximizes its own utility in a greedy manner.

5.4 Numerical Simulations

We verify our analytical results through numerical simulations. We consider one CP, one EU, and two ISPs, where the CP and the EU share the same utility-level function $\alpha_{eu} = \alpha_{cp} = \alpha \in (0, 1)$. Figure 5.2a shows that CP has incentive to invest in sponsored data plan if $\frac{\sigma_{cp}}{\sigma_{eu}} > \alpha + \frac{p_{cp}}{p_{eu}}$. It means that as CP has higher utility level and EU consuming the content has relatively lower utility level (or similarly, the price charged to CP is relatively lower than the price charged to EU), CP tries to provide a higher sponsorship rate. In contrast, when $\frac{\sigma_{cp}}{\sigma_{eu}} \leq \alpha + \frac{p_{cp}}{p_{eu}}$, CP best strategy is not sponsoring the user access, i.e., $s^* = 0$.

Next we observe the payoff of ISP_2 as we change the price per unit traffic p_{eu} that charges to the user. Figure 5.2b illustrates the results and show that the payoff of ISP_2 linearly rises till some point, and then declines exponentially, which is due to the fact that the demand of users is inversely proportional to p_{eu}. Although ISP_2 obtains its revenue from charging ISP_1 with transit-price p_{tr}, the results show



(a) (b)

Fig. 5.2 Payoff changes of CP and ISP_2 when $\alpha = 0.5$. (**a**) CP. (**b**) ISP_2 with $\sigma_{cp} = 2, \sigma_{eu} = 1$

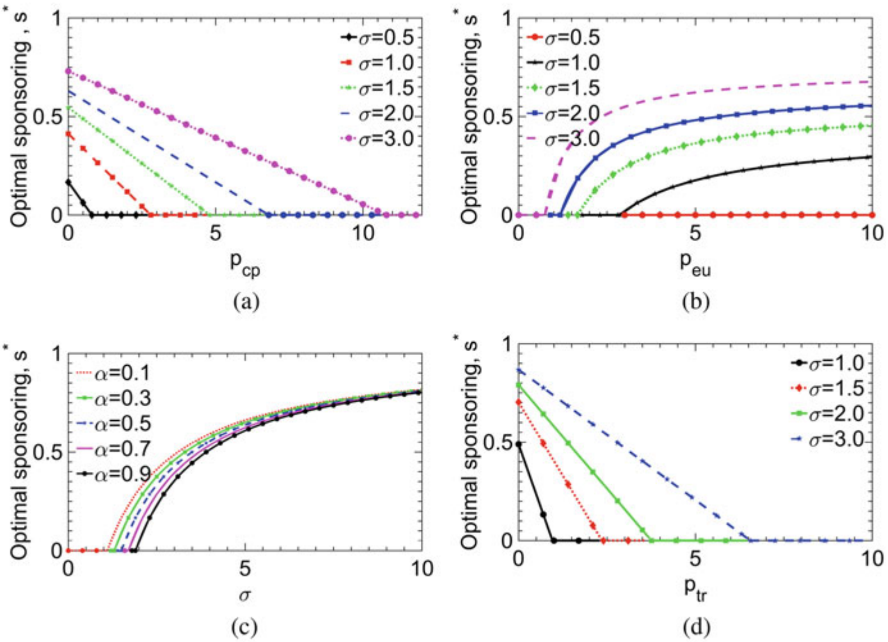


Fig. 5.3 The optimal sponsoring rate with respect to p_{cp}, p_{eu}, σ, and p_{tr}. (**a**) $p_{eu} = 4$, $\alpha = 0.3$. (**b**) $p_{cp} = 2$, $\alpha = 0.3$. (**c**) $p_{cp} = 2$, $p_{eu} = 2$. (**d**) $p_{eu} = 4$, $\alpha = 0.3$

that increasing the p_{tr} does not necessarily increase the payoff of ISP_2. As the transit price becomes higher, CP is forced to increase p_{cp} which in turn results in a decline of the traffic demand. Hence, the maximum point is achieved at $p_{tr} = 1$ and $p_{eu} = 2$.

We now examine the impact of ISP prices (p_{cp}, p_{eu}, and p_{tr}) and σ on the optimal sponsoring rate with different parameter sets. Figure 5.3a shows that as p_{cp} increases, the sponsoring rate drops sharply. The decreasing rate can be mitigated with higher σ. Figure 5.3b shows that with the increase of the p_{eu}, the marginal increase of the sponsoring rate is decreasing. Moreover, a larger σ value indicates a higher and rapidly growing sponsorship rate. Figure 5.3c demonstrates the change of the optimal sponsoring rate with respect to σ under different α values. The sponsorship rate logarithmically increases as σ increases. It can be explained from the fact that the CP with higher revenue level can afford more investment on the sponsoring content. We can also observe that the variation in α has a little impact on the traffic demand. Figure 5.3d will help us to understand the effect of the transit cost p_{tr} to the optimal sponsoring rate s^*. We can observe that the increase of the transit cost results in a sharp drop of s^*. The rise of transit cost will incur significant loss in ISP_1's revenue, which forces ISP_1 to increase its charge to CP, resulting in a rapid drop of the sponsoring rate.

5.5 Conclusion

In this work, we studied the sponsored data and non-cooperative inter-pricing among ISPs that jointly deliver traffic from CPs to EUs. We derived the best response of the EU, the CP, and the ISPs, and analyzed their implications for the sponsoring strategy of the CP. We investigate the interactions between strategic EU, CP, and two interconnected ISPs through a sequential Stackelberg game, and verify our results through numerical simulations. Our results clarify the high impact of the transit price of intermediate ISP on the sponsoring strategies of the CP, and demonstrate in what scenarios sponsoring helps. There are a couple of interesting direction to extend our results. A cooperation between the two ISPs will change the system dynamics and bring a different structure of pricing and sponsoring, and may improve the total payoff of the ISPs at the cost of the EU and the CP. On the other hand, multiple ISPs for the service to the EU or the CP may result in competition between the ISPs and can lead to a higher social welfare.

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