Chapter 15
Bertrand Meets Ford: Benefits and Losses

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Abstract The paper carries out the detailed comparison of two types of imperfect competition in a general equilibrium model. The price-taking Bertrand competition assumes the myopic income-taking behavior of firms, another type of behavior, price competition under a Ford effect, implies that the firms' strategic choice takes into account their impact to consumers' income. Our findings suggest that firms under the Ford effect gather more market power (measured by Lerner index), than "myopic" firms, which is agreed with the folk wisdom "Knowledge is power." Another folk wisdom implies that increasing of the firms' market power leads to diminishing in consumers' well-being (measured by indirect utility.) We show that in general this is not true. We also obtain the sufficient conditions on the representative consumer preference providing the "intuitive" behavior of the indirect utility and show that this condition satisfy the classes of utility functions, which are commonly used as examples (e.g., CES, CARA and HARA.)

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15.1 Introduction

"The elegant fiction of competitive equilibrium" does not dominates now the frontier of theoretical microeconomics as stated by Marschak and Selten in [11] in early 1970s, being replaced by also elegant monopolistic competitive Dixit-Stiglitz "engine". The idea that firms are price-makers even if their number is "very large", e.g., continuum, is a common wisdom. But what if the monopolistic competitive equilibrium conception, where firms has zero impact to market statistics and, therefore, treat them as given, is just a brand new elegant fiction? When firms are sufficiently large, they face demands, which are influenced by the income level, depending in turn on their profits. As a result, firms must anticipate accurately what the total income will be. In addition, firms should be aware that they can manipulate the income level, whence their "true" demands, through their own strategies with the aim of maximizing profits [8]. This feedback effect is known as the *Ford effect*. In popular literature, this idea is usually attributed to Henry Ford, who raised wages at his auto plants to five dollars a day in January 1914. Ford wrote "our own sales depend on the wages we pay. If we can distribute high wages, then that money is going to be spent and it will serve to make... workers in other lines more prosperous and their prosperity is going to be reflected in our sales", see [7, p. 124–127]. To make things clear, we have to mention that the term "Ford effect" may be used in various specifications. As specified in [5], the Ford effect may have different scopes of consumers income, which is sum of wage and a share of the distributed profits. The first (extreme) specification is to take a whole income parametrically. This is one of solutions proposed by Marschak and Selten [11] and used, for instance, by Hart [9]. This case may be referred as "No Ford effect". Another specification (also proposed by Marschak and Selten [11] and used by d'Aspremont et al. [5]) is to suppose that firms take into account the effects of their decision on the total wage bill, but not on the distributed profits, which are still treated parametrically. This case may be referred as "Wage Ford effect" and it is exactly what Henry Ford meant in above citation. One more intermediate specification of The Ford effect is an opposite case to the previous one: firms take wage as given, but take into account the effects of their decisions on distributed profits. This case may be referred as "Profit Ford effect". Finally, the second extreme case, Full Ford effect, assumes that firms take into account total effect of their decisions, both on wages and on profits. These two cases are studied in newly published paper [4]. In what follows, we shall assume that wage is determined. This includes the way proposed by Hart [9], in which the worker fixed the nominal wage through their union. This assumption implies that only the Profit Ford effect is possible, moreover, firms maximize their profit anyway, thus being price-makers but not wage-makers, they have no additional powers at hand in comparison to No Ford case, with except the purely informational advantage—knowledge on consequences of their decisions. Nevertheless, as we show in this paper, this advantage allows firms to get more market power, which vindicate the wisdom "Knowledge is Power". As for welfare effect of this Knowledge, we show that it is ambiguous, but typically it is harmful for consumes. It should be mentioned also that being close in ideas with paper [4], we have no intersections in results, because the underlying economy model of this paper differers from our one, moreover, that research focuses on existence and uniqueness of equilibria with different specifications of Ford effect and does not concern the aspects of market power and welfare. We leave out of the scope of our research all consideration concerning Wage Ford effect, such as Big Push effect[1] and High Wage doctrine of stimulating consumer demand through wages. The idea that the firm could unilaterally use wages to increase demand for its own product enough to offset wage cost seems highly unlikely and was criticized by various reasons, including empirical evidences. For further discussions see [10, 15].

15.2 Model and Equilibrium in Closed Industry

15.2.1 Firms and Consumers

The economy involves one sector supplying a horizontally differentiated good and one production factor—labor. There is a continuum mass L of identical consumers endowed with one unit of labor. The labor market is perfectly competitive and labor is chosen as the numéraire. The differentiated good is made available under the form of a finite and discrete number $n \geq 2$ of varieties. Each variety is produced by a single firm and each firm produces a single variety. Thus, n is also the number of firms. To operate every firm needs a fixed requirement $f > 0$ and a marginal requirement $c > 0$ of labor. Without loss of generality we may normalize marginal requirement c to one. Since wage is also normalized to 1, the cost of producing q_i units of variety $i = 1, ..., n$ is equal to $f + 1 \cdot q_i$.

Consumers share the same additive preferences given by

$$U(\mathbf{x}) = \sum_{i=1}^{n} u(x_i), \tag{15.1}$$

where $u(x)$ is thrice continuously differentiable function, strictly increasing, strictly concave, and such that $u(0) = 0$. The strict concavity of u means that a consumer has a love for variety: when the consumer is allowed to consume X units of the differentiated good, she strictly prefers the consumption profile $x_i = X/n$ to any other profile $\mathbf{x} = (x_1, ..., x_n)$ such that $\sum_i x_i = X$. Because all consumers are identical, they consume the same quantity x_i of variety $i = 1, ..., n$.

[1] Suggesting that if firm profits are tied to local consumption, then firms create an externality by paying high wages: the size of the market for other firms increases with worker wages and wealth, see [12].

Following [17], we define the relative love for variety (RLV) as follows:

$$r_u(x) = -\frac{xu''(x)}{u'(x)}, \tag{15.2}$$

which is strictly positive for all $x > 0$. Technically RLV coincides with the Arrow-Pratt's relative risk-aversion concept, which we avoid to use due to possible misleading association in terms, because in our model there is no any uncertainty or risk considerations. Nevertheless, one can find some similarity in meaning of these concepts as the RLV measures the intensity of consumers' variety-seeking behavior. Under the CES, we have $u(x) = x^\rho$ where ρ is a constant such that $0 < \rho < 1$, thus implying a constant RLV given by $1 - \rho$. Another example of additive preferences is paper [2] where authors consider the CARA utility $u(x) = 1 - \exp(-\alpha x)$ with $\alpha > 0$ is the absolute love for variety (which is defined pretty much like the absolute risk aversion measure $-u''(x)/u'(x)$); the RLV is now given by αx.

A consumer's income is equal to her wage plus her share in total profits. Since we focus on symmetric equilibria, consumers must have the same income, which means that profits have to be uniformly distributed across consumers. In this case, a consumer's income y is given by

$$y = 1 + \frac{1}{L}\sum_{i=1}^{n} \Pi_i \geq 1,$$

where the profit made by the firm selling variety i is given by

$$\Pi_i = (p_i - 1)q_i - f, \tag{15.3}$$

p_i being the price of variety i. Evidently, the income level varies with firms' strategies.

A consumer's budget constraint is given by

$$\sum_{i=1}^{n} p_i x_i = y, \tag{15.4}$$

where x_i stands for the consumption of variety i.

The first-order condition for utility maximization yields

$$u'(x_i) = \lambda p_i, \tag{15.5}$$

where λ is the Lagrange multiplier of budget constraint. Conditions (15.4) and (15.5) imply that

$$\lambda = \frac{\sum_{j=1}^{n} u'(x_j)x_j}{y} > 0. \tag{15.6}$$

15.2.2 Market Equilibrium

The **market equilibrium** is defined by the following conditions:

1. each consumer maximizes her utility (15.1) subject to her budget constraint (15.4),
2. each firm i maximizes its profit (15.3) with respect to p_i ,
3. product market clearing: $Lx_i = q_i \quad \forall\, i = 1, ..., n$,
4. labor market clearing: $nf + \sum_{i=1}^{n} q_i = L$.

The last two equilibrium conditions imply that

$$\bar{x} \equiv \frac{1}{n} - \frac{f}{L} \tag{15.7}$$

is the only possible symmetric equilibrium demand, while the symmetric equilibrium output $\bar{q} = L\bar{x}$.

15.2.3 When Bertrand Meets Ford

As shown by (15.5) and (15.6), firms face demands, which are influenced by the income level, depending in turn on their profits. As a result, firms must anticipate accurately what the total income will be. In addition, firms should be aware that they can manipulate the income level, whence their "true" demands, through their own strategies with the aim of maximizing profits [8].

Let $\mathbf{p} = (p_1, ..., p_n)$ be a price profile. In this case, consumers' demand functions $x_i(\mathbf{p})$ are obtained by solving of consumer's problem—maximization of utility $U(\mathbf{x})$ subject to budget constraint (15.4)—with income y defined as

$$y(\mathbf{p}) = 1 + \sum_{j=1}^{n} (p_j - 1) x_j(\mathbf{p}).$$

It follows from (15.6) that the marginal utility of income λ is a market aggregate that depends on the price profile $\mathbf{p}$. Indeed, the budget constraint

$$\sum_{j=1}^{n} p_j x_j(\mathbf{p}) = y(\mathbf{p})$$

implies that

$$\lambda(\mathbf{p}) = \frac{1}{y(\mathbf{p})} \sum_{j=1}^{n} x_j(\mathbf{p}) u'\left(x_j(\mathbf{p})\right),$$

while the first-order condition (15.5) may be represented as $\lambda(\mathbf{p})p_i = u'(x_i(\mathbf{p}))$. Since $u'(x)$ is strictly decreasing, the demand function for variety i is thus given by

$$x_i(\mathbf{p}) = \xi(\lambda(\mathbf{p})p_i), \tag{15.8}$$

where ξ is the inverse function to $u'(x)$. Thus, firm i's profits can be rewritten as

$$\Pi_i(\mathbf{p}) = (p_i - 1)x_i(\mathbf{p}) - f = (p_i - 1)\xi(\lambda(\mathbf{p})p_i) - f. \tag{15.9}$$

Remark 15.1 The definition of ξ implies that the Relative Love for Variety (15.2) may be equivalently represented as follows

$$r_u(x_i(\mathbf{p})) \equiv -\frac{\xi(\lambda(\mathbf{p})p_i)}{\xi'(\lambda(\mathbf{p})p_i)\lambda(\mathbf{p})p_i}. \tag{15.10}$$

Indeed, differentiating ξ as inverse to u' function, we obtain $\xi' = 1/u''$, while $x_i(\mathbf{p}) = \xi(\lambda(\mathbf{p})p_i)$, $u'(x_i(\mathbf{p})) = \lambda(\mathbf{p})p_i$.

Definition 15.1 For any given $n \geq 2$, a *Bertrand equilibrium* is a vector $\mathbf{p}^* = (p_1^*, ..., p_n^*)$ such that p_i^* maximizes $\Pi_i(p_i, \mathbf{p}_{-i}^*)$ for all $i = 1, ..., n$. This equilibrium is symmetric if $p_i^* = p_j^*$ for all i, j.

Applying the first-order condition to the profit (15.9) maximization problem, yields that the firm's i relative markup

$$m_i \equiv \frac{p_i - 1}{p_i} = -\frac{\xi(\lambda p_i)}{\xi'(\lambda p_i)\lambda p_i \cdot \left(1 + \frac{p_i}{\lambda}\frac{\partial\lambda}{\partial p_i}\right)}, \tag{15.11}$$

which involves $\partial\lambda/\partial p_i$ because λ depends on $\mathbf{p}$. Unlike what is assumed in partial equilibrium models of oligopoly, λ is here a function of $\mathbf{p}$, so that the markup depends on $\partial\lambda/\partial p_i \neq 0$. But how does firm i determine $\partial\lambda/\partial p_i$?

Since firm i is aware that λ is endogenous and depends on $\mathbf{p}$, it understands that the demand functions (15.8) must satisfy the budget constant as an identity. The consumer budget constraint can be rewritten as follows:

$$\sum_{j=1}^{n} p_j\xi(\lambda(\mathbf{p})p_j) = 1 + \sum_{j=1}^{n}(p_j - 1)\xi(\lambda(\mathbf{p})p_j),$$

which boils down to

$$\sum_{j=1}^{n}\xi(\lambda(\mathbf{p})p_j) = 1. \tag{15.12}$$

Differentiating (15.12) with respect to p_i yields

$$\xi'(\lambda p_i)\lambda + \frac{\partial \lambda}{\partial p_i} \sum_{j=1}^{n} p_j \xi'(\lambda p_j) = 0$$

or, equivalently,

$$\frac{\partial \lambda}{\partial p_i} = -\frac{\xi'(\lambda p_i)\lambda}{\sum_{j=1}^{n} \xi'(\lambda p_j) p_j}. \tag{15.13}$$

Substituting (15.13) into (15.11) and symmetrizing the resulting expression yields the candidate equilibrium markup:

$$\bar{m}^F = -\frac{\xi(\lambda p)}{\xi'(\lambda p) \cdot \lambda p \cdot \dfrac{n-1}{n}} = \frac{n}{n-1} r_u\left(\bar{x}\right), \tag{15.14}$$

where we use the identity (15.10) and $\bar{x} = \frac{1}{n} - \frac{f}{L}$ due to (15.7).

Proposition 15.1 *Assume that firms account for the Ford effect and that a symmetric equilibrium exists under Bertrand competition. Then, the equilibrium markup is given by*

$$\bar{m}^F = \frac{n}{n-1} r_u \left(\frac{1}{n} - \frac{f}{L} \right).$$

Note that $r_u\left(\frac{1}{n} - \frac{f}{L}\right)$ must be smaller than 1 for $\bar{m}^F < 1$ to hold. Since $\frac{1}{n} - \frac{f}{L}$ can take on any positive value in interval (0, 1), it must be

$$r_u(x) < 1 \; \forall x \in (0, 1). \tag{15.15}$$

This condition means that the elasticity of a monopolist's inverse demand is smaller than 1 or, equivalently, the elasticity of the demand exceeds 1. In other words, the marginal revenue is positive. However, (15.15) is not sufficient for $\bar{m}^F$ to be smaller than 1. Here, a condition somewhat more demanding than (15.15) is required for the markup to be smaller than 1, that is, $r_u\left(\frac{1}{n} - \frac{f}{L}\right) < (n-1)/n$. Otherwise, there exists no symmetric price equilibrium. For example, in the CES case, $r_u(x) = 1 - \rho$ so that

$$\bar{m}^F = \frac{n}{n-1}(1 - \rho) < 1,$$

which means that ρ must be larger than $1/n$. This condition is likely to hold because econometric estimations of the elasticity of substitution $\sigma = 1/(1-\rho)$ exceeds 3, see [1].

15.2.4 Income-Taking Firms

Now assume that, although firms are aware that consumers' income is endogenous, firms treat this income as a parameter. In other words, firms behave like *income-takers*. This approach is in the spirit of Hart (see [9]), for whom firms should take into account only some effects of their policy on the whole economy. Note that the income-taking assumption does not mean that profits have no impact on the market outcome. It means only that no firm seeks to manipulate its own demand through the income level. Formally, firms are income-takers when $\dfrac{\partial y}{\partial p_i} = 0$ for all i. Hence, the following result holds true. For the proof see Proposition 1 in [13].

Proposition 15.2 *Assume that firms are income-takers. If (15.15) holds and if a symmetric equilibrium exists under Bertrand competition, then the equilibrium markup is given by*

$$\bar{m}(n) = \frac{n}{n-1+r_u\left(\frac{1}{n}-\frac{f}{L}\right)} r_u\left(\frac{1}{n}-\frac{f}{L}\right). \tag{15.16}$$

Obvious inequality

$$\frac{n}{n-1+r_u\left(\frac{1}{n}-\frac{f}{L}\right)} < \frac{n}{n-1}$$

implies the following

Corollary 15.1 *Let number of firms n be given, then the income-taking firms charge the lesser price (or, equivalently, lesser markup) than the "Ford-effecting" firms.*

In other words, Ford effect provides to firms more marker power than in case of their income-taking behavior.

15.3 Free Entry Equilibrium

In equilibrium, profits must be non-negative for firms to operate. Moreover, if profit is strictly positive, this causes new firms to enter, while in the opposite case, i.e., when profit is negative, firms leave industry. The simple calculation shows that symmetric Zero-profit condition $\Pi = 0$ holds if and only if the number of firms satisfies

$$n^* = \frac{L}{f}m. \tag{15.17}$$

Indeed, let $L(p-1)\bar{x} - f = 0$ holds, where the symmetric equilibrium demand $\bar{x}$ is determined by (15.7). On the other hand, budget constraint (15.4) in symmetric case boils down to $n \cdot p\bar{x} = 1 + \sum_{i=1}^{n} \Pi_i = 1$ due to Zero-profit condition. Combining these identities, we obtain (15.17).

Assuming number of firms n is integer, we obtain generically that for two adjacent numbers, say n and $n+1$ the corresponding profits will have the opposite signs, e.g., $\Pi(n) > 0$, $\Pi(n+1) < 0$, and there is no integer number n^* providing the Zero-Profit condition $\Pi(n^*) = 0$. On the other hand, both markup expressions, (15.14) and (15.16), allow to use the arbitrary positive real values of n. The only problem is how to interpret the non-integer number of firms.[2] To simplify considerations, we assume that the fractional part $0 < \delta < 1$ of non-integer number of firms n^*, is a marginal firm, which entered to industry as the last, and its production is a linear extrapolation of typical firm, i.e., its fixed labor cost is equal to $\delta f < f$, while the production output is δq. In other words, marginal firm may be considered as "part-time-working firm".

Therefore, the equilibrium number of firms increases with the market size and the degree of firms' market power, which is measured by the Lerner index, and decreases with the level of fixed cost. Note also that

$$\bar{x} = \frac{f(1-m)}{Lm} > 0, \tag{15.18}$$

provided that m satisfies $0 < m < 1$. Substituting (15.17) and (15.18) into (15.14) and (15.16), we obtain that the equilibrium markups under free-entry must solve the following equations:

$$\bar{m}^F = \frac{f}{L} + r_u\left(\frac{f}{L}\frac{1-\bar{m}^F}{\bar{m}^F}\right), \tag{15.19}$$

$$\bar{m} = \frac{f}{L} + \left(1 - \frac{f}{L}\right) r_u \left(\frac{f}{L}\frac{1-\bar{m}}{\bar{m}}\right). \tag{15.20}$$

Under the CES, $\bar{m}^F = f/L + 1 - \rho$, while $\bar{m} = \rho f/L + 1 - \rho < \bar{m}^F$. It then follows from (15.17) and (15.18) that the equilibrium masses of firms satisfy $\bar{n}^F > \bar{n}$, while $\bar{q}^F < \bar{q}$. This result may be expanded to the general case. To prove

[2] Note that interpretation of non-integer *finite* number of oligopolies is totally different from the case of monopolistic competition, where mass of firms is *continuum* $[0, n]$, thus it does not matter whether n is integer or not. For further interpretational considerations see [13, subsection 4.3].

this, we assume additionally that

$$r_u(0) \equiv \lim_{x\to 0} r_u(x) < 1,\ r_{u'}(0) \equiv \lim_{x\to 0} r_{u'}(x) < 2 \tag{15.21}$$

Proposition 15.3 *Let conditions (15.21) hold and L be sufficiently large, then the equilibrium markups, outputs, and masses of firms are such that*

$$\bar{m}^F(L) > \bar{m}(L),\ \bar{q}^F(L) < \bar{q}(L),\ \bar{n}^F(L) > \bar{n}(L)$$

Furthermore, we have:

$$\lim_{L\to\infty} \bar{m}^F(L) = \lim_{L\to\infty} \bar{m}(L) = r_u(0).$$

Proof Considerations are essentially similar to the proof of Proposition 2 in [13]. Let's denote $\varphi = f/L$, then $L \to \infty$ implies $\varphi \to 0$ and condition "sufficiently large L" is equivalent to "sufficiently small φ."

It is sufficient to verify that function

$$G(m) \equiv \varphi + r_u\left(\varphi\frac{1-m}{m}\right) - m$$

is strictly decreasing at any solution of $\hat{m}$ of equation

$$m = \varphi + r_u\left(\varphi\frac{1-m}{m}\right) \tag{15.22}$$

Indeed, direct calculation show that

$$G'(m) = -\frac{1}{m}\left[\frac{1}{1-m}\frac{\varphi(1-m)}{m}r'_u\left(\varphi\frac{1-m}{m}\right) + m\right]. \tag{15.23}$$

Differentiating $r_u(x)$ and rearranging terms yields

$$r'_u(x)x = (1 + r_u(x) - r_{u'}(x))r_u(x)$$

for all $x > 0$. Applying this identity to $\hat{x} = \varphi\frac{1-\hat{m}}{\hat{m}}$ and substituting (15.22) into (15.23), we obtain

$$G'(\hat{m}) = -\frac{1}{\hat{m}}\left[\frac{r_u(\hat{x})\left(2 - \varphi - r_{u'}\left(\hat{x}\right)\right)}{1-\hat{m}} + \varphi\right] < 0 \tag{15.24}$$

for all sufficiently small $\varphi = f/L$, or, equivalently, for all sufficiently large L. Moreover, inequality (15.24) implies, that there exists not more than one solution of Eq. (15.22), otherwise the sign of derivative $G'(m)$ must alternate for different roots.

An inequality $r_u(x) > 0$ for all x implies $G(0) \geq \varphi > 0$, while $G(1) = \varphi + r_u(0) - 1 < 0$, provided that $\varphi < 1 - r_u(0)$, therefore, for all sufficiently small φ there exists unique solution $\bar{m}^F(\varphi) \in (0, 1)$ of Eq. (15.22), which determines the symmetric Bertrand equilibrium under the Ford effect. In particular, inequality $m < \bar{m}^F(\varphi)$ holds if and only if $G(m) > 0$.

Existence an uniqueness of income-taking Bertrand equilibrium for all sufficiently small φ was proved in [13, Proposition 2]. By definition, the equilibrium markup $\bar{m}$ satisfies $F(\bar{m}(\varphi)) = 0$ for

$$F(m) \equiv \varphi + (1-\varphi) r_u \left(\frac{\varphi(1-m)}{m} \right) - m.$$

It is obvious that $G(m) > F(m)$ for all m and φ, therefore,

$$G(\bar{m}(\varphi)) > F(\bar{m}(\varphi)) = 0,$$

which implies $\bar{m}^F(\varphi) > \bar{m}(\varphi)$. The other inequalities follow from formulas (15.17) and (15.18).

The last statement of Proposition easily follows from the fact, that both equations $G(m) = 0$ and $F(m) = 0$ boil down to $m = r_u(0)$ when $\varphi \to 0$ (see proof of Proposition 2 in [13] for technical details.)

Whether the limit of competition is perfect competition (firms price at marginal cost) or monopolistic competition (firms price above marginal cost) when L is arbitrarily large depends on the value of $r_u(0)$. More precisely, when $r_u(0) > 0$, a very large number of firms whose size is small relative to the market size is consistent with a positive markup. This agrees with [3]. On the contrary, when $r_u(0) = 0$, a growing number of firms always leads to the perfectly competitive outcome, as maintained by Robinson [14]. To illustrate, consider the CARA utility given by $u(x) = 1 - \exp(-\alpha x)$. In this case, we have $r_u(0) = 0$, and thus the CARA model of monopolistic competition is not the limit of a large group of firms. By contrast, under CES preferences, $r_u(0) = 1 - \rho > 0$. Therefore, the CES model of monopolistic competition is the limit of a large group of firms.

15.4 Firms' Market Power vs. Consumers' Welfare

Proposition 15.3 also highlights the trade-off between *per variety* consumption and product diversity. To be precise, when free entry prevails, competition with Ford effect leads to a larger number of varieties, but to a lower consumption level per variety, than income-taking competition. Therefore, the relation between consumers' welfare values $\bar{V}^F = \bar{n}^F \cdot u(\bar{x}^F)$ and $\bar{V} = \bar{n} \cdot u(\bar{x})$ is a priori ambiguous.

In what follows we assume that the elemental utility satisfies $\lim_{x\to\infty} u'(x) = 0$, which is not too restrictive and typically holds for basic examples of utility functions. Consider the Social Planner's problem, who manipulates with masses of firms n trying to maximize consumers' utility $V(n) = n \cdot u(x)$ subject to the labor market clearing condition $(f + L \cdot x)n = L$, which is equivalent to maximization of

$$V(n) = n \cdot u\left(\frac{1}{n} - \varphi\right), \ n \in (0, \varphi^{-1}),$$

where $\varphi = f/L$.

It is easy to see that

$$V(0) \equiv \lim_{n\to 0} n \cdot u\left(\frac{1}{n} - \varphi\right) = \lim_{x\to\infty} \frac{u(x)}{x+\varphi} = \lim_{x\to\infty} u'(x) = 0 = V(\varphi^{-1}),$$

where $x \equiv 1/n - \varphi$. Moreover,

$$V''(n) = \frac{1}{n^3} \cdot u''\left(\frac{1}{n} - \varphi\right) < 0,$$

which implies that graph of $V(n)$ is bell-shaped and there exists unique social optimum $n^* \in (0, \varphi^{-1})$, and $V'(n) \leq 0$ (resp. $V'(n) \geq 0$) for all $n \geq n^*$ (resp. $n \leq n^*$.)

This implies the following statement holds

Proposition 15.4

1. *If equilibrium number of the income-taking firms $\bar{n} \geq n^*$, then $\bar{V}^F < \bar{V}$*
2. *If equilibrium number of the Ford-effecting firms $\bar{n}^F \leq n^*$, then $\bar{V}^F > \bar{V}$*
3. *In the intermediate case $\bar{n} < n^* < \bar{n}^F$ the relation between $\bar{V}^F$ and $\bar{V}$ is ambiguous.*

In what follows, the first case will be referred as the "bad Ford" case, the second one—as the "good Ford" case.

Let's determine the nested elasticity of the elementary utility function

$$\Delta_u(x) \equiv \frac{x\varepsilon_u'(x)}{\varepsilon_u(x)},$$

where

$$\varepsilon_u(x) \equiv \frac{xu'(x)}{u(x)}.$$

The direct calculation shows that this function can be represented in different form

$$\Delta_u(x) = [1 - \varepsilon_u(x)] - r_u(x),$$

where $r_u(x)$ is Relative Love for Variety defined by (15.2), while $1 - \varepsilon_u(x)$ is so called *social markup*. Vives in [16] pointed out that social markup is the degree of preference for a single variety as it measures the proportion of the utility gain from adding a variety, holding quantity per firm fixed, and argued that 'natural' consumers' behavior implies increasing of social markup, or, equivalently, *decreasing of elasticity* $\varepsilon_u(x)$. In particular, the 'natural' behavior implies $\Delta_u(x) \leq 0$.

Lemma 15.1 *Let* $r_u(0) < 1$ *holds, then* $\Delta_u(0) \equiv \lim_{x\to 0} \Delta_u(x) = 0$.

Proof Assumptions on utility $u(x)$ imply that function $xu'(x)$ is strictly positive and

$$(xu'(x))' = 2u'(x) + xu''(x) = u'(x) \cdot (2 - r_{u'}(x)) > 0$$

for all $x > 0$, therefore there exists limit $\lambda = \lim_{x\to 0} x \cdot u'(x) \geq 0$. Assume that $\lambda > 0$, this is possible only if $u'(0) = +\infty$, therefore using the L'Hospital rule we obtain

$$\lambda = \lim_{x\to 0} x \cdot u'(x) = \lim_{x\to 0} \frac{x}{(u'(x))^{-1}} = \lim_{x\to 0} -\frac{(u'(x))^2}{u''(x)} = \lim_{x\to 0} \frac{xu'(x)}{-\frac{xu''(x)}{u'(x)}} = \frac{\lambda}{r_u(0)} > \lambda$$

because $r_u(0) < 1$ by (15.21). This contradiction implies that $\lambda = 0$. Therefore, using the L'Hospital rule, we obtain

$$\lim_{x\to 0} (1 - \varepsilon_u(x)) = 1 - \lim_{x\to 0} \frac{xu'(x)}{u(x)} = 1 - \lim_{x\to 0} \frac{u'(x) + xu''(x)}{u'(x)} = \lim_{x\to 0} r_u(x),$$

which implies $\Delta_u(0) = 0$.

The CES case is characterized by identity $\Delta_u(x) = 0$ for all $x > 0$, while for the other cases the sign and magnitude of $\Delta_u(x)$ may vary, as well as the directions of change for terms $1 - \varepsilon_u(x)$ and $r_u(x)$ may be arbitrary, see [6] for details.

Let $\delta_u \equiv \lim_{x\to 0} \Delta'_u(x)$, which may be finite or infinite. The following theorem provides the sufficient conditions for both "bad" and "good" Ford cases, while the obvious gap between (a) and (b) corresponds to the ambiguous third case of Proposition 15.4.

Theorem 15.1

(a) Let $\delta_u < r_u(0)$, *then for all sufficiently small* $\varphi = f/L$ *the 'bad Ford' inequality* $\bar{V} > \bar{V}^F$ *holds.*
(b) Let $\delta_u > \frac{r_u(0)}{1-r_u(0)}$, *then for all sufficiently small* $\varphi = f/L$ *the 'good Ford' inequality* $\bar{V}^F > \bar{V}$ *holds.*

Proof See Appendix.

It is obvious that in CES case $u(x) = x^{\rho}$ we obtain that $\delta_{CES} = 0 < r_{CES}(0) = 1-\rho$, thus CES is "bad For" function. Considering the CARA $u(x) = 1-e^{-\alpha x}$, $\alpha > 0$, HARA $u(x) = (x+\alpha)^{\rho} - \alpha^{\rho}$, $\alpha > 0$, and Quadratic $u(x) = \alpha x - x^2/2$, $\alpha > 0$, functions, we obtain $r_u(0) = 0$ for all these functions, while $\delta_{CARA} = -\alpha/2 < 0$, $\delta_{HARA} = -(1-\rho)/2\alpha < 0$ and $\delta_{Quad} = -1/2\alpha < 0$. This implies that these widely used classes of utility functions also belong to the "bad Ford" case.

To illustrate the opposite, "good Ford" case, consider the following function $u(x) = \alpha x^{\rho_1} + x^{\rho_2}$. Without loss of generality we may assume that $\rho_1 < \rho_2$, then

$$1 - \varepsilon_u(x) = \frac{\alpha(1-\rho_1) + (1-\rho_2)x^{\rho_2-\rho_1}}{\alpha + x^{\rho_2-\rho_1}},$$

$$r_u(x) = \frac{\alpha\rho_1(1-\rho_1) + \rho_2(1-\rho_2)x^{\rho_2-\rho_1}}{\alpha\rho_1 + \rho_2 x^{\rho_2-\rho_1}},$$

Using the L'Hospital rule we obtain

$$\lim_{x\to 0} \Delta_u' = \lim_{x\to 0} \frac{\alpha(\rho_2-\rho_1)^2 \cdot x^{-\rho_1-(1-\rho_2)}}{(\alpha + x^{\rho_2-\rho_1})(\alpha\rho_1 + \rho_2 x^{\rho_2-\rho_1})} = +\infty > \frac{r_u(0)}{1-r_u(0)} = \frac{1-\rho_1}{\rho_1}.$$

Corollary 15.2 *Let* $\varepsilon_u'(0) < 0$, *then* $\bar{V} > \bar{V}^F$.

Proof Using L'Hospital rule we obtain that

$$\delta_u = \lim_{x\to 0} \Delta_u'(x) = \lim_{x\to 0} \frac{\Delta_u(x)}{x} = \lim_{x\to 0} \frac{\varepsilon_u'(x)}{\varepsilon_u(x)} = \frac{1}{\varepsilon_u(0)} \lim_{x\to 0} \varepsilon_u'(x) < 0 \le r_u(0),$$

where $\varepsilon_u(0) = 1 - r_u(0) > 0$ due to assumption (15.21).

Remark 15.2 The paper [13] studied comparison of the Cournot and Bertrand oligopolistic equilibria under assumption of the income-taking behavior of firms. One of results obtained in this paper is that under Cournot competition firms charge the larger markup and produce lesser quantity, than under Bertrand competition, $\bar{m}^C > \bar{m}^B$, $\bar{q}^C < \bar{q}^B$, while equilibrium masses of firms $\bar{n}^C > \bar{n}^B$. This also implies ambiguity in comparison of the equilibrium indirect utilities $\bar{V}^C$ and $\bar{V}^B$. It is easily to see, that all considerations for $\bar{V}^F$ and $\bar{V}$ may be applied to this case and Theorem 15.1 (a) provides sufficient conditions for pro-Bertrand result $\delta_u < r_u(0) \Rightarrow \bar{V}^B > \bar{V}^C$. Moreover, considerations similar to proof of Theorem 15.1 (b) imply that inequality $\bar{V}^C > \bar{V}^B$ holds, provided that $\delta_u > 1$.

15.5 Concluding Remarks

Additive preferences are widely used in theoretical and empirical applications of monopolistic competition. This is why we have chosen to compare the market outcomes under two different competitive regimes when consumers are endowed with such preferences. It is important to stress, that unlike the widely used comparison of Cournot (quantity) and Bertrand (price) competitions, which are we compare two similar price competition regimes with "information" difference only: firms ignore or take into account strategically their impact to consumers' income. Moreover, unlike most models of industrial organization which assume the existence of an outside good, we have used a limited labor constraint. This has allowed us to highlight the role of the marginal utility of income in firms' behavior.

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Appendix

Proof of Theorem 15.1

Combining Zero-profit condition (15.17) $m = \frac{f}{L}n = \varphi n$ with formula for symmetric equilibrium demand $x = n^{-1} - \varphi \iff n = (x + \varphi)^{-1}$ we can rewrite the equilibrium mark-up equation for income-taking firms (15.20) as follows

$$\frac{\varphi}{x + \varphi} = \varphi + (1 - \varphi) r_u(x).$$

Solving this equation with respect to x we obtain the symmetric equilibrium consumers' demand $x(\varphi)$, parametrized by $\varphi = f/L$, which cannot be represented in closed form for general utility $u(x)$, however, the inverse function $\varphi(x)$ has the closed-form solution

$$\varphi = \frac{1 - x}{2} - \sqrt{\left(\frac{1 - x}{2}\right)^2 - \frac{x r_u(x)}{1 - r_u(x)}}. \tag{15.25}$$

It was mentioned above that graph of indirect utility $V(n)$ is bell-shaped and equilibrium masses of firms satisfy $n^* \le \bar{n} \le \bar{n}^F$ if and only if $V'(\bar{n}) \le 0$. Calculating the first derivative $V'(n) = u(n^{-1} - \varphi) - n^{-1} \cdot u'(n^{-1} - \varphi)$ and substituting both $n = (x + \varphi)^{-1}$ and (15.25) we obtain that

$$n^* \le \bar{n} \le \bar{n}^F \iff u(x) \le \left(\frac{1 + x}{2} - \sqrt{\left(\frac{1 - x}{2}\right)^2 - \frac{x r_u(x)}{1 - r_u(x)}}\right) u'(x),$$

at $x = \bar{x}$—the equilibrium consumers demand in case of income-taking firms. The direct calculation shows that this inequality is equivalent to

$$\Delta_u(x) \leq (1 - r_u(x))\frac{1-x}{2}\left[1 - \sqrt{1 - \frac{4xr_u(x)}{(1-r_u(x))(1-x)^2}}\right]. \qquad (15.26)$$

We shall prove that this inequality holds for all sufficiently small $x > 0$, provided that $\Delta'(0) < r_u(0)$. To do this, consider the following function

$$A_u(x) = \frac{x \cdot r_u(x)}{1-x},$$

which satisfies $A_u(0) = 0 = \Delta_u(0),\ \Delta'_u(0) < A'_u(0) = r_u(0)$. This implies that inequality $\Delta_u(x) \leq A_u(x)$ holds for all sufficiently small $x > 0$.

Applying the obvious inequality $\sqrt{1-z} \leq 1 - z/2$ to

$$z = \frac{4xr_u(x)}{(1-r_u(x))(1-x)^2},$$

we obtain that the right-hand side of inequality (15.26)

$$(1 - r_u(x))\frac{1-x}{2}\left[1 - \sqrt{1 - \frac{4xr_u(x)}{(1-r_u(x))(1-x)^2}}\right] \geq A_u(x) \geq \Delta_u(x)$$

for all sufficiently small $x > 0$, which completes the proof of statement (a).

Applying the similar considerations to Eq. (15.19), which determines the equilibrium markup under a Ford effect, we obtain the following formula for inverse function $\varphi(x)$

$$\varphi = \frac{1 - r_u(x) - x}{2} - \sqrt{\left(\frac{1 - r_u(x) - x}{2}\right)^2 - xr_u(x)}$$

Using the similar considerations, we obtain that

$$\bar{n}^F \leq n^* \iff u(x) \geq \left(\frac{1 - r_u(x) + x}{2} - \sqrt{\left(\frac{1 - r_u(x) - x}{2}\right)^2 - xr_u(x)}\right) u'(x)$$

at $x = \bar{x}^F$—the equilibrium demand under Bertrand competition with Ford effect. The direct calculation shows that the last inequality is equivalent to

$$\Delta_u(x) \geq \frac{1 - r_u(x) - x}{2}\left[1 - \sqrt{1 - \frac{4xr_u(x)}{(1-r_u(x)-x)^2}}\right]. \qquad (15.27)$$

Now assume

$$\delta_u > \frac{r_u(0)}{1 - r_u(0)},$$

which implies that

$$\alpha \equiv \frac{r_u(0) + (1 - r_u(0))\delta_u}{2r_u(0)} > 1.$$

Let

$$B_u(x) \equiv \frac{\alpha x r_u(x)}{1 - r_u(x) - x},$$

it is obvious that $\Delta_u(0) = B_u(0) = 0$, and

$$B_u'(0) = \frac{\alpha r_u(0)}{1 - r_u(0)} = \frac{r_u(0) + (1 - r_u(0))\delta_u}{2(1 - r_u(0))} < \delta_u = \Delta_u'(0),$$

which implies that inequality $\Delta_u(x) \geq B_u(x)$ holds for all sufficiently small x.

On the other hand, the inequality $\sqrt{1 - z} \geq 1 - \alpha z/2$ obviously holds for any given $\alpha > 1$ and $z \in \left[0, \frac{4(\alpha-1)}{\alpha^2}\right]$. Applying this inequality to

$$z = \frac{4 x r_u(x)}{(1 - r_u(x) - x)^2}, \alpha = \frac{r_u(0) + (1 - r_u(0))\delta_u}{2r_u(0)},$$

we obtain that the right-hand side of (15.27) satisfies

$$\frac{1 - r_u(x) - x}{2}\left[1 - \sqrt{1 - \frac{4 x r_u(x)}{(1 - r_u(x) - x)^2}}\right] \leq B_u(x) \tag{15.28}$$

for all sufficiently small $x > 0$, because $x \to 0$ implies $z \to 0$. This completes the proof of Theorem 15.1.

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