A Multistage Risk Decision Making Method for Normal Cloud Model with Three Reference Points

Wen Song and Jianjun Zhu(✉)

College of Economics and Management,
Nanjing University of Aeronautics and Astronautics,
Nanjing 211106, People's Republic of China
{songwen, zhujianjun}@nuaa.edu.cn

Abstract. Decision making problems become more complicated due to the dynamically changing environment. Consequently, decision making methods with reference points are increasing. Reference points provide a good basis for decision makers. This paper proposes a multistage risk decision making method for normal cloud model considering three reference points. Firstly, the setting method of three reference points is proposed considering the dimensions of multistate, development and promotion. The value function is defined based on the characteristics of three reference points. Secondly, the aggregation methods for different prospect values are proposed with the preference coefficients, which are calculated by the synthetic degree of grey incidence. Thirdly, a two-stage weight optimization method is proposed to solve the attribute weights and stage weights based on the idea of minimax reference point optimization. Finally, a numerical example illustrates the feasibility and validity of the proposed method.

Keywords: Multistage risk decision making · Three reference points
Normal cloud model · Two-stage weight optimization method

1 Introduction

Multistage risk decision making (MSRDM) methods aim to rank alternatives or select the best alternative(s) by the aggregation of multistage risk decision-making (DM) information. MSRDM problems include risk, uncertainty and dynamics. Psychological factors of the decision-makers need to be taken into consideration to solve the risk DM problems. Decision-makers often consider the gain and loss under a reference point due to the bounded rationality. The fairness and satisfaction of DM is significantly influenced by a single reference point. In dynamic and uncertain conditions, using a single reference point will lead to the loss of some of the information about the distribution of the results. In the DM process with risk and dynamics, the psychological behavior of decision makers is inconsistent. In this context, the consideration of multiple reference points helps decision-makers to uncover the dynamic and risk characteristics of MSRDM problems, thus making a reasonable and comprehensive assessment of results.

Y. Chen et al. (Eds.): GDN 2018, LNBIP 315, pp. 14–32, 2018.
https://doi.org/10.1007/978-3-319-92874-6_2

Work on MSRDM methods have been increasing recently. In a multistage DM problem of finite-state automaton, a new optimization method stochastically develops a solution step-by-step in combination with a simulated annealing [1]. In an optimal investment problem with several projects, a new methodology is proposed based on experts' evaluations. It consists of three stages [2]. The multistage one-shot DM problems under uncertainty are studied based on scenario [3]. A multi-stage technical screening and evaluation tool is proposed to determine the optimal technique scheme under fuzzy environment [4]. A multistage assignment model is presented for rescue teams to dynamically respond to the disaster chain [5].

With the increasing complexity of DM problems, more effective methods are developed to support decision makers' judgments. DM methods considering reference points are one kind of resourceful methods. TOPSIS [6, 7] is widely used in MCDM problems. The idea of TOPSIS is to compare each solution with the positive ideal solution and the negative ideal solution, which are actually the two reference points. VIKOR methods [8] are also dependent on the positive and negative ideal solution, which are similar to the TOPSIS methods. Kahneman and Tversky [9, 10] presented the Prospect Theory to solve the risky DM problems.

The actual utility is obtained from comparison with a reference point. Due to the limited rationality of decision-makers' thinking, it is difficult to judge by the evaluation value. The consideration of reference points can provide the basis for decision makers, and lead to better informed and well-reasoned decisions. DM methods considering the reference point have been gradually enriched. A prospect theory-based interval dynamic reference point method has been proposed for emergency DM [11]. A risk DM method has been proposed considering the dynamic reference point, the external reference point and the internal reference point [12]. A new method based on the concept of ideal solution has been presented as a possible variant of TOPSIS and VIKOR methods [13]. The newsvendor's pricing and stocking decisions have been studied considering the impact of reference point effects [14].

Decision information often shows different forms, such as fuzzy numbers [2], interval numbers [12], linguistic sets [15], cloud models and so on. Due to the dynamic continuity of MSRDM process and the risky DM environment, information often shows fuzziness and randomness at the same time. The transformation between qualitative concepts and quantitative concepts is often needed to be dealt with. Linguistic set is usually used to express the decision maker's judgment. However, linguistic sets are often ambiguous and uncertain, and very difficult to form accurate information [15]. In this context, Li presented cloud models to propose conversion between qualitative concept and quantitative representation [16]. Many new approaches to cloud models have been proposed to solve existing problems. Cloud Hierarchical Analysis (CHA) is an extension of AHP [17]. The Cloud Delphi hierarchical analysis has been presented for practical multi-criteria group DM problems [18]. DM methods combining linguistic sets and cloud model have been investigated [19]. Cloud model has been widely used in many problems, such as water quality assessment [20], image segmentation [21], and clustering problems [22].

This paper makes contributes to the MSRDM problems for normal cloud model with three reference points.

(1) The new setting method for three reference points is proposed based on the data in different states and stages.
(2) The aggregation method for different prospect values is proposed based on the synthetic degree of grey incidence.
(3) A two-stage weight optimization method is proposed to solve the attribute weights and stage weights.

The rest of this paper is organized as follows. In Sect. 2, some related concepts and definitions are reviewed. Section 3 presents the MSRDM method for normal cloud model with three reference points. Section 4 provides a case followed by its analysis. Section 5 concludes the paper.

2 Preliminaries

This section briefly reviews the basic concepts and definitions associated with normal cloud model and prospect theory, and describes the problem addressed in this paper.

2.1 Basic Concepts and Definitions

Due to the complexity and uncertainty of DM problems, DM information often shows fuzziness and uncertainty. Normal cloud model provides an important background to represent fuzziness and randomness at the same time. Many scholars [18, 20, 23, 24] have carried out studies using normal cloud model.

Definition 1. [23] Let U be the universe of discourse and $\tilde{A}$ be a qualitative concept in U. If $x \in U$ is a random instantiation of concept $\tilde{A}$ that satisfies $x \sim N(Ex, En'^2)$, $En' \sim N(En, He^2)$, and the certainty degree of x belonging to $\tilde{A}$ satisfies

$$y = e^{-\frac{(x-Ex)^2}{2(En')^2}} \tag{1}$$

The distribution of x in the universe U is called the normal cloud model and x can be called a cloud drop. The normal cloud model can effectively integrate the randomness and fuzziness of a concept through three parameters: Expectation Ex, Entropy En and Hyper Entropy He. Expectation Ex is the mathematical expectation of the cloud drops belonging to a concept in the universe. It can best represent the qualitative concept. Entropy En represents the uncertainty measurement of a qualitative concept. It is the measurement of randomness and fuzziness of the concept. Hyper Entropy He is the uncertain degree of entropy En [23].

Given two normal cloud models $C_i(Ex_i, En_i, He_i)$ and $C_j(Ex_j, En_j, He_j)$. Certain operation rules between two normal cloud models have been included in [18, 20].

(1) $C_i + C_j = \left(Ex_i + Ex_j, \sqrt{En_i^2 + En_j^2}, \sqrt{He_i^2 + He^2}\right)$.
(2) $C_i - C_j = \left(Ex_i - Ex_j, \sqrt{En_i^2 + En_j^2}, \sqrt{He_i^2 + He^2}\right)$.

(3) $C_i \times C_j = \left(Ex_i \times Ex_j, |Ex_i Ex_j| \sqrt{\left(\frac{En_i}{Ex_i}\right)^2 + \left(\frac{En_j}{Ex_j}\right)^2}, |Ex_i Ex_j| \sqrt{\left(\frac{He_i}{Ex_i}\right)^2 + \left(\frac{He_j}{Ex_j}\right)^2} \right).$

(4) $C_i \div C_j = \left(\frac{Ex_i}{Ex_j}, \left|\frac{Ex_i}{Ex_j}\right| \sqrt{\left(\frac{En_i}{Ex_i}\right)^2 + \left(\frac{En_j}{Ex_j}\right)^2}, \left|\frac{Ex_i}{Ex_j}\right| \sqrt{\left(\frac{He_i}{Ex_i}\right)^2 + \left(\frac{He_j}{Ex_j}\right)^2} \right).$

(5) $\lambda C_i = \left(\lambda Ex_i, \sqrt{\lambda} En_i, \sqrt{\lambda} He_i \right).$

(6) $(C_i)^\lambda = \left(Ex_i^\lambda, \sqrt{\lambda} Ex_i^{\lambda-1} En_i, \sqrt{\lambda} Ex_i^{\lambda-1} He_i \right).$

When applying normal cloud model, the similarity between two normal cloud models is commonly used.

Definition 2. [24] Let $C_i(Ex_i, En_i, He_i)$ and $C_j(Ex_j, En_j, He_j)$ be two normal cloud models. The similarity between two normal cloud models based on shape and distance is defined as:

$$sim_c(C_i, C_j) = sim_d(C_i, C_j) \times sim_s(C_i, C_j) \tag{2}$$

Where $sim_d(C_i, C_j)$ represents the similarity between two normal cloud models based on distance, $sim_s(C_i, C_j)$ represents the similarity between two normal cloud models based on shape.

In prospect theory, alternatives are selected based on the prospect value.

Definition 3. [9] The prospect value is defined by the value function and the probability weight function:

$$V(x) = \sum_k \pi(p_k) \cdot v(x_k) \tag{3}$$

Definition 4. [10] The value function is expressed in the form of a power law according to the following formula:

$$v(x_k) = \begin{cases} (x_k)^\alpha, & x_k \geq 0 \\ -\theta(-x_k)^\beta & x_k < 0 \end{cases} \tag{4}$$

Where x_k denotes the gain or loss of the value when comparing an alternative to its reference point. When $x_k \geq 0$, it represents a gain. When $x_k < 0$, it represents a loss. α and β represent the concave and convex degree of the value power function $v(x_k)$ in the region of gain and loss respectively. θ indicates the loss-averse coefficient.

Definition 5. [10] The probability weight function is defined as

$$\pi(p_k) = \begin{cases} \frac{(p_k)^\gamma}{((p_k)^\gamma + (1-p_k)^\gamma)^{1/\gamma}} & x_k \geq 0 \\ \frac{(p_k)^\delta}{\left((p_k)^\delta + (1-p_k)^\delta\right)^{1/\delta}} & x_k < 0 \end{cases} \tag{5}$$

where γ and δ are the risk gain and loss attitude coefficients respectively.

Tversky and Kahneman [10] found that when $\alpha = \beta = 0.88, \theta = 2.25, \gamma = 0.61$, $\delta = 0.72$, the experimental results are more consistent with the empirical results. To simplify calculation, we also take the above values in the paper.

2.2 Problem Description

The MSRDM process involves multiple stages and multiple states, which is usually risk, uncertainty and dynamic. Considering multiple reference points is helpful to make decisions results from multiple perspectives. This paper aims to select the desirable alternative(s) from a set of feasible alternatives according to the MSRDM problems.

In the MSRDM problem, let $A = \{a_i | i = 1, 2, \cdots, I\}$ be the set of I alternatives. Let $C = \{c_j | j = 1, 2, \ldots, J\}$ be the set of J attributes. Let $M = \{m^t | t = 1, 2, \ldots, T\}$ be the set of T stages. Let $S = \{s^n | n = 1, 2, \ldots, N\}$ be the set of N natural states. Let $WC = \{wc_1, wc_2, \ldots, wc_J\}$ be the weighting set of J attributes $\left(wc_j \in [0, 1], \sum_{j=1}^{J} wc_j = 1\right)$, $WM = \{wm^1, wm^2, \ldots, wm^T\}$ be the weighting set of M stages $\left(wm^t \in [0, 1], \sum_{t=1}^{T} wm^t = 1\right)$, $P = \{p(s^1), p(s^2), \ldots, p(s^N)\}$ be the probability set of N states $\left(p(s^n) \in [0, 1], \sum_{n=1}^{N} p(s^n) = 1\right)$. Let $X^{tn} = (x_{ij}^{tn})_{I \times J}$ be the DM matrix in the stage m^t in the state s^n, which is showed in Table 1. $x_{ij}^{tn} = \left(Ex_{ij}^{tn}, En_{ij}^{tn}, He_{ij}^{tn}\right)$ is the decision value of alternative a_i with respect to attribute c_j in stage m^t in state s^n.

Table 1. MSRDM evaluation information.

		c_1				…	c_J			
		s^1	s^2	…	s^N	…	s^1	s^2	…	s^N
m^1	a_1	x_{11}^{11}	x_{11}^{12}	…	x_{11}^{1N}	…	x_{1J}^{11}	x_{1J}^{12}	…	x_{1J}^{1N}
	a_2	x_{21}^{11}	x_{21}^{12}	…	x_{21}^{1N}	…	x_{2J}^{11}	x_{2J}^{12}	…	x_{2J}^{1N}
	…	…	…	…	…	…	…	…	…	…
	a_I	x_{I1}^{11}	x_{I1}^{12}	…	x_{I1}^{1N}	…	x_{IJ}^{11}	x_{IJ}^{12}	…	x_{IJ}^{1N}
…	…	…	…	…	…	…	…	…	…	…
m^T	a_1	x_{11}^{T1}	x_{11}^{T2}	…	x_{11}^{TN}	…	x_{1J}^{T1}	x_{1J}^{T2}	…	x_{1J}^{TN}
	a_2	x_{21}^{T1}	x_{21}^{T2}	…	x_{21}^{TN}	…	x_{2J}^{T1}	x_{2J}^{T2}	…	x_{2J}^{TN}
	…	…	…	…	…	…	…	…	…	…
	a_I	x_{I1}^{T1}	x_{I1}^{T2}	…	x_{I1}^{TN}	…	x_{IJ}^{T1}	x_{IJ}^{T2}	…	x_{IJ}^{TN}

3 A MSRDM Method for Normal Cloud Model with Three Reference Points

3.1 The Setting Method of Three Reference Points

In MSRDM problems, the performance of an alternative will change dynamically as time goes on. Therefore, current situation, development trend and decision goal fluctuate with the change of stage. Considering a single reference point is difficult to evaluate the current situation, dynamic and inspiring nature comprehensively. Thus, it is difficult to systematically describe the development trend and characteristics.

The idea of setting the three reference points is showed in Fig. 1. The developmental reference point (DRP) is set by the performances of the previous stage. Compared with the DRP, the progress from the previous stage to the present stage can be obtained. The state reference point (SRP) is set by the expected performance of multiple states in the present stage. Unlike with the SRP, the extent to which one alternative is better than the expected performance of multiple states can be obtained. The promoting reference point (PRP) is set by the potential and the performance of the previous stage, which can be seen as the goal. The PRP can be used to adjust the degree and direction of the effort. Compared with the PRP, the degree of effort in the present stage can be obtained. In order to fully compare the advantages and disadvantages of MSRDM information, this paper sets up three reference points, i.e., development, multistate and promotion.

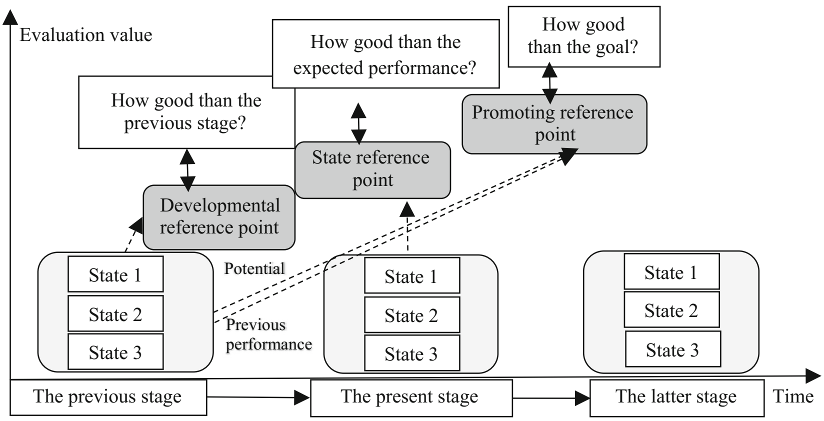

Fig. 1. The idea of setting the three reference points

State Reference Point 1. In multiple natural states, the expected value is the probability-weighted average of all possible values. It represents the central tendency of the values in multiple states. The expected value is what one expects to happen on average. If the value of an alternative is higher than the expected value, it will be a gain for the alternative to the expected value. The SRP is set by the average value of the expected values of alternatives in one stage.

Definition 6. The SRP of the attribute c_j at stage m^t is defined as

$$r_j^{ts} = \frac{1}{I}\sum\nolimits_{i=1}^{I}\sum\nolimits_{n=1}^{N} p(s^n)\cdot x_{ij}^{tn} = \left(Ex_j^{ts}, En_j^{ts}, He_j^{ts}\right) \tag{6}$$

Developmental Reference Point 2. From the viewpoint of development, the actual development level of an alternative at the present stage can be obtained by comparing the data with the data of the previous stage. The DRP is set by the performance of the previous stage. Thus, the progress of the present stage can be obtained by comparing it with the DRP. Compared with the previous stage, the greater the gain at the present stage, the better the development level of the present stage.

Definition 7. The DRP of the attribute c_j at stage m^t in natural state s^n is defined as

$$r_j^{tnd} = \frac{1}{I}\sum\nolimits_{i=1}^{I} x_{ij}^{t-1,n} = \left(Ex_j^{tnd}, En_j^{tnd}, He_j^{tnd}\right) \tag{7}$$

Promoting Reference Point 3. From the viewpoint of promotion, reasonable goals should be set up to motivate people's subjective initiative. The PRP is set by the potential and performance of the previous stage. It actually is an estimate of the present stage based on the resources and historical foundations. The degree of realization of the PRPs reflects the degree of effort and the potential. Compared with the PRP, the greater the gain of the alternative, the better.

Definition 8. The maximum growth potential of the alternative a_i with respect to the attribute c_j at stage m^{t-1} in natural state s^n is defined as

$$\tau_{ij}^{t-1,n} = \frac{max_j x_{ij}^{t-1,n}}{x_{ij}^{t-1,n}} \tag{8}$$

Definition 9. The average maximum growth potential of the attribute c_j at stage m^{t-1} in natural state s^n is defined as

$$\tau_j^{t-1,n} = \frac{1}{I}\sum\nolimits_{i=1}^{I} \left(\frac{\max_j x_{ij}^{t-1,n}}{x_{ij}^{t-1,n}}\right) \tag{9}$$

Definition 10. The PRP of the attribute c_j at stage m^t in natural state s^n is defined as

$$r_j^{tnp} = \tau_j^{t-1,n}\frac{1}{I}\sum\nolimits_{i=1}^{I} x_{ij}^{t-1,n} = \left(Ex_j^{tnp}, En_j^{tnp}, He_j^{tnp}\right) \tag{10}$$

3.2 The Value Function for Normal Cloud Model Under Three Reference Points

Comparing with three reference points can measure the performance of alternatives from different perspectives. The performance of one alternative at the current stage can be measured by comparing with the SRP. The performance of one alternative at the current stage can be measured by comparing with the SRP. The development performance of one alternative from the previous stage to the present stage can be measured by comparing with the DRP. Whether one alternative reaches the expected potential level can be measured by comparing with the GRP. When compared with the three reference points, the better the gain is, the better the alternative is. Then a value function is defined to obtain the gain or loss from a reference point. Take the DRP as an example.

Definition 11. The value function with the DRP is defined as

$$v_{ij}^{tnd} = \begin{cases} \left(1 - sim\left(x_{ij}^{tn}, r_j^{tnd}\right)\right)^{\alpha} & Ex_{ij}^{tn} \geq Ex_j^{tnd} \\ -\theta\left(1 - sim\left(x_{ij}^{tn}, r_j^{tnd}\right)\right)^{\beta} & Ex_{ij}^{tn} < Ex_j^{tnd} \end{cases} \tag{11}$$

where $sim\left(x_{ij}^{tn}, r_j^{tnd}\right)$ is the similarity between the attribute value and the DRP, which can be calculated by (2). When $Ex_{ij}^{tn} \geq Ex_j^{tnd}$, the value is a gain. The bigger the similarity between the attribute value and the DRP, the smaller the value of x_{ij}^{tn} as compared with the DRP r_j^{tnp}. When $Ex_{ij}^{tn} < Ex_j^{tnd}$, the value is a loss. The bigger the similarity between the attribute value and the DRP, the bigger the value of x_{ij}^{tn} compared with the DRP r_j^{tnp}.

The value functions for the SRP and the PRP are the same as the DRP.

3.3 The Aggregation Method for Three Kinds of Prospect Values

The prospect values with respect to each reference point can be calculated by Eq. (3). Then we can get $v_{ij}^{ts}, v_{ij}^{td}, v_{ij}^{tp}$. The comprehensive prospect values based on multiple reference points can be obtained by:

$$v_{ij}^{t} = \lambda_j^{ts} \cdot v_{ij}^{ts} + \lambda_j^{td} \cdot v_{ij}^{td} + \lambda_j^{tp} \cdot v_{ij}^{tp} \tag{12}$$

Where $\lambda_j^{ts}, \lambda_j^{td}, \lambda_j^{tp}\left(\lambda_j^{ts} + \lambda_j^{td} + \lambda_j^{tp} = 1, \lambda_j^{ts}, \lambda_j^{td}, \lambda_j^{tp} \in [0, 1]\right)$ represent the preference coefficients of different reference points at stage m^t. The preference coefficients can be given by decision makers in accordance with the actual situation. The coefficient of preference can also be determined according to the connections between the three kinds of prospect values.

The synthetic degree of grey incidence can describe the overall relationship of closeness between sequences [25]. So, we take the synthetic degree of grey incidence to obtain the preference coefficients of different reference points.

Let $X_j^{ts} = \left(v_{1j}^{ts}, v_{2j}^{ts}, \ldots, v_{Ij}^{ts}\right), X_j^{td} = \left(v_{1j}^{td}, v_{2j}^{td}, \ldots, v_{Ij}^{td}\right), X_j^{tp} = \left(v_{1j}^{tp}, v_{2j}^{tp}, \ldots, v_{Ij}^{tp}\right)$ be the sequences of the attribute c_j at stage m^t with respect to the three reference points.

The synthetic degree of grey incidence between each of the two-reference points can be calculated as $\rho_j^{tsd}, \rho_j^{tsp}, \rho_j^{tdp}$ [25]. Thus, the preference coefficients of different reference points can be obtained by:

$$\lambda_j^{ts} = \frac{1}{2} \cdot \frac{\rho_j^{tsd} + \rho_j^{tsp}}{\rho_j^{tsd} + \rho_j^{tsp} + \rho_j^{tdp}} \tag{13}$$

$$\lambda_j^{td} = \frac{1}{2} \cdot \frac{\rho_j^{tsd} + \rho_j^{tdp}}{\rho_j^{tsd} + \rho_j^{tsp} + \rho_j^{tdp}} \tag{14}$$

$$\lambda_j^{tp} = \frac{1}{2} \cdot \frac{\rho_j^{tsp} + \rho_j^{tdp}}{\rho_j^{tsd} + \rho_j^{tsp} + \rho_j^{tdp}} \tag{15}$$

The prospect values of alternatives at stage m^t can be obtained by

$$v_i^t = \sum\nolimits_{j=1}^{J} wc_j \cdot v_{ij}^t \tag{16}$$

The prospect values of alternatives at all stages can be obtained by

$$v_i = \sum\nolimits_{t=1}^{T} \sum\nolimits_{j=1}^{J} wm^t \cdot wc_j \cdot v_{ij}^t \tag{17}$$

3.4 The Two-Stage Optimization Model

In MSRDM problems, the weights of attributes and stages can be given by decision makers. In some situations, it is difficult to determine the exact weights for decision-makers. Inappropriate weight setting may lead to errors in DM results. In this paper, we add the decision maker's judgment of weights to the priori information set. In this way, the weight optimization model can be more objective to determine the attribute weight, and we also take into account the influence of the subjective weights by DMs.

According to the idea of minimax reference point optimization [26], we designed the two-stage optimization model to solve the weights. The idea of the two-stage optimization model is showed in Fig. 2. The abscissa represents the weights of attributes, and the ordinate represents the weights of stages.

In the first stage of the two-stage optimization model, the maximum value of each alternative at each stage is obtained. In the second stage of the two-stage optimization model, the biggest distance between the maximum value and the actual value is minimized step by step. For example, the biggest distance in Fig. 2 is ε_3 at first. After optimization by modeling, the maximum distance is changed to ε_2. Finally, the biggest distance between the maximum value and the actual value is minimized. All the values of alternatives are as close as possible to the maximum value.

For the convenience of calculation, the prospect values of alternatives are standardized, the annotation is unchanged. Then we have $v_{ij}^t \in [0, 1]$.

The first stage model $M1$ is used to calculate the maximum value of each alternative at each stage. M1 is defined as

$$max(v_i^t) = max \sum\nolimits_{j=1}^{J} wc_j \cdot v_{ij}^t \tag{18}$$

$$\begin{cases} \sum_{j=1}^{J} wc_j \cdot v_{ij}^t \le 1,\ i = 1, 2, \ldots, I \\ wc_j \in H_1,\ j = 1, 2, \ldots, J \end{cases} \tag{19}$$

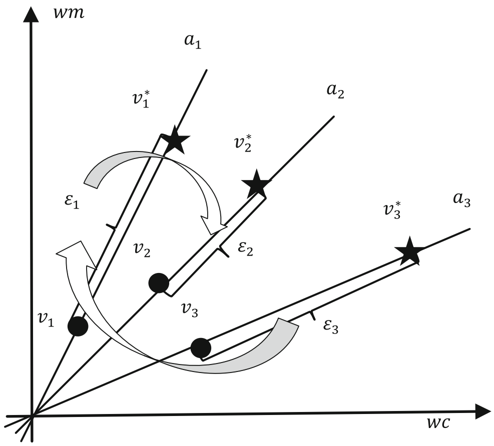

Fig. 2. The idea of the two-stage optimization method

The maximum value of each alternative at each stage can be calculated as v_i^{t*}. Each alternative can get its maximum value if model $M1$ is bounded. Because $v_{ij}^t \in [0, 1]$ and $wc_j \in [0, 1]$, the value of $\sum_{j=1}^{J} wc_j \cdot v_{ij}^t$ must be in the range [0, 1]. Attribute weights meet the prior set H_1, which can be expressed in 5 forms [27]. The prior set about attribute weights is usually decided by multiple decision makers.

The second stage model $M2$ aims to minimize the biggest distance between the maximum value and the actual value. M2 is defined as:

$$min(\varepsilon) \tag{20}$$

$$\begin{cases} \sum_{t=1}^{T} wm^t \cdot \left(v_i^{t*} - \sum_{j=1}^{J} wc_j \cdot v_{ij}^t \right) \leq \varepsilon, i = 1, 2, \ldots, I \\ \sum_{t=1}^{T} \sum_{j=1}^{J} wm^t \cdot wc_j \cdot v_{ij}^t \leq 1 \\ wc_j \in H_1, j = 1, 2, \ldots, J \\ wm^t \in H_2, t = 1, 2, \ldots, T \end{cases} \tag{21}$$

The minimum value of ε can be obtained from solving model $M2$. Constraints are the following conditions. The biggest distance between the maximum value and the actual value is less than or equal to ε. Each alternative is effective. Because $v_{ij}^t \in [0, 1], wc_j \in [0, 1]$ and $wm^t \in [0, 1]$, the value of $\sum_{t=1}^{T} \sum_{j=1}^{J} wm^t \cdot wc_j \cdot v_{ij}^t$ must in the range [0, 1]. The attribute weight and the stage weight satisfy the prior set H_1 and H_2 respectively. The prior sets about attribute and stage weights are usually decided by multiple decision makers.

Then the final weight can be calculated as wm^{t*} and wc_j^*, the final ranking value can be calculated as v_i. The bigger the value of v_i, the better the alternative a_i is.

$$v_i = \sum\nolimits_{t=1}^{T} \sum\nolimits_{j=1}^{J} wm^{t*} \cdot wc_j^* \cdot v_{ij}^t \tag{22}$$

3.5 The DM Procedure of the MSRDM Method

The DM procedure to solve the MSRDM problems with three reference points is demonstrated in the following steps.

Step 1. Set three reference points.

The three reference points are obtained from (6)–(10).

Step 2. Calculate the prospect values.

Use (2) to calculate the similarity between x_{ij}^{tn} and the reference points. Then the prospect values under each reference point can be obtained from (11).

Step 3. Calculate the preference coefficients and the comprehensive prospect values.

Calculate the synthetic degree of grey incidence between each of the two-reference points [25]. Then, use (13)–(15) to calculate the preference coefficients, and use Eq. (12) to calculate the comprehensive prospect values.

Step 4. Solve the attribute weights and the stage weights.

Build model $M1$ (18)–(19) to get the maximum values v_i^{t*} of each alternative at different stages. Build model $M2$ (20)–(21) to solve the attribute weights and the stage weights.

Step 5. Calculate the final ranking value.

The final ranking values can be calculated from (22). The best alternative is max (v_i).

4 Numeral Example Analysis

4.1 Numeral Example Background

A pharmaceutical company carried out a risk assessment of the quality of products. There are 15 products to be evaluated, which comprise the set of alternatives $A = \{a_1, a_2, \ldots, a_I\}(I = 15)$. Evaluation attributes comprise the set of attributes $C = \{c_1, c_2, \ldots, c_J\}(J = 4)$. Three natural states comprise the set of natural states $S = \{s^n | n = 1, 2, \ldots, N\}(N = 3)$. The evaluation information from $T = 4$ stages comprises the decision-making matrix $\left(x_{ij}^{tn}\right)_{I \times J}$. Attribute weights meet the prior set $H_1 = \left\{\sum\nolimits_{j=1}^{J} wc_j = 1; wc_j \geq 0.15; wc_1 + wc_3 \leq 0.45\right\}$, Stage weights meet the prior set $H_2 = \left\{\sum\nolimits_{t=1}^{T} wm^t = 1; 0 < wm^1 \leq 0.2; wm^2 > 0.15; wm^1 + wm^2 \leq 0.45; wm^3 \geq 0.2; wm^4 \geq 0.3\right\}$.

There are four attributes to describe the products to be evaluated. c_1 represents the management level of raw material. It can be evaluated by decision makers with the linguistic sets (very good, good, medium, poor, very poor). c_2 represents the qualified rate of product quality. It can be obtained according to the data of product inspection. c_3 represents technological level. It can be evaluated by decision makers with the linguistic sets (very good, good, medium, poor, very poor). c_4 represents economic benefit. It can be obtained from the annual profit ratio.

There are three natural states: s^1 = *the low* risks tate, $s^2 =$ *the medium* risk state, $s^3 =$ *the high* risk state. According to historical data, we have $P = \{p(s^1) = 0.65, p(s^2) = 0.25, p(s^3) = 0.1\}$. The DM matrixes at different stages are showed in the appendix as Tables 2, 3 and 4.

4.2 The Calculation Process

Step 1. Set three reference points.

(1) Use (6) to calculate the SRP in each stage.

Take the attribute c_1 as an example; the SRPs of attribute c_1 in four stages are showed in Fig. 3. The SRPs in stage m^1, m^2, m^3, m^4 are represented as $r_1^{1s} = (6.283, 3.431, 0.691), r_1^{2s} = (6.048, 3.32, 0.724), r_1^{1s} = (5.721, 3.214, 0.767), r_1^{1s} = (6.016, 3.342, 0.73)$.The symbols (".", "+", "Δ", "○") in Fig. 3 represent the SRP in stage m^1, m^2, m^3, m^4. Comparing the values of Ex in SRPs $(r_1^{1s}, r_1^{2s}, r_1^{3s}, r_1^{4s})$, we obtain that $Ex_1^{1s} > Ex_1^{2s} > Ex_1^{4s} > Ex_1^{3s}$. The SRP in stage m^1 is bigger than the other three reference points. The SRP in stage m^3 is smaller than the other three reference points.

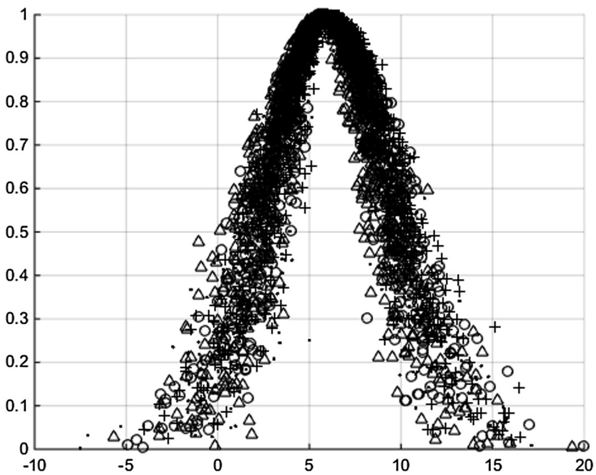

Fig. 3. The SRPs in different stages

(2) Use (7) to calculate the DRP in each state in each stage.

Take the attribute c_2 in stage m^1 as an example. The DRP in different states are represented as $r_2^{11d} = (0.923, 0.069, 0.018), r_2^{12d} = (0.923, 0.042, 0.21), r_2^{13d} = (0.899, 0.065, 0.019)$. We can find that $Ex_2^{11d} \cong Ex_2^{12d} > Ex_2^{13d}$. But the cloud drops of r_2^{12d} are more dispersed than the other two $(He_2^{12d} > He_2^{13d} > He_2^{11d})$.

(3) Calculate the PRP in each state in each stage by Eq. (10) (omitted).

Take the attribute c_3 in stage m^1 as an example. The PRP in different states are represented as $r_3^{11p} = (8.253, 4.843, 0.926), r_2^{12p} = (8.404, 5.1, 0.948), r_2^{13p} = (6.839, 4.163, 0.949)$. We can find that $Ex_3^{12p} > Ex_3^{11p} > Ex_3^{13p}$.

Step 2. Calculate the prospect values.

Calculate the similarities and the prospect values (omitted). Take the prospect values in stage m^1 as an example. The prospect values are showed in Fig. 4. "s, d, p" in Fig. 4 represent the state, development and PRP.

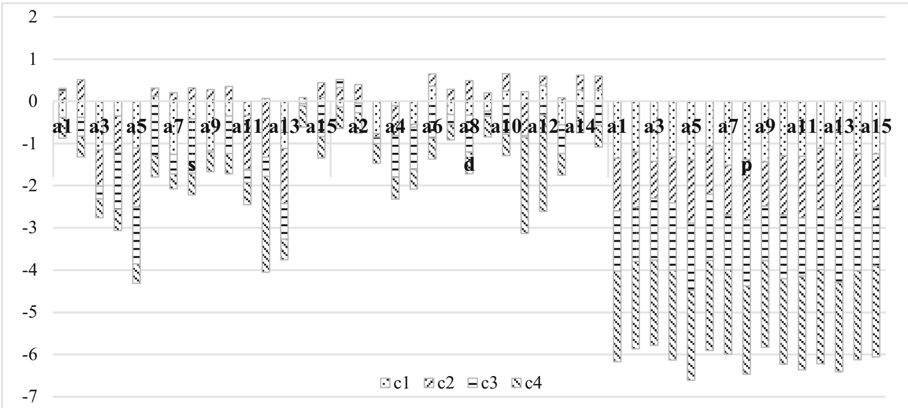

Fig. 4. The stacked column diagram of the prospect values in stage m^1

Compared with the SRPs, the prospect values of several alternatives with respect to attributes c_1 and c_2 are greater than 0, which means gains. This shows that the performances of these alternatives are higher than the expected performance of multiple states.

Compared with the DRPs, the prospect values of several alternatives with respect to attributes c_1, c_2 and c_3 are greater than 0, which means gains. This shows that the performances of these alternatives are higher than the levels of the previous stage. This means that these alternatives are working harder at the present stage than in the previous stage.

Compared with the PRPs, the prospect values are always lower than 0, which means losses. This shows that the performances of alternatives do not reach their potential. This means that these alternatives have not fully exploited their potential.

Step 3. Calculate the preference coefficients and the comprehensive prospect values.

Calculate the preference coefficients by using (13)–(15). The comprehensive prospect values based on multiple reference points can be obtained from (12) based on the preference coefficients (omitted).

Step 4. Calculate the attribute weights and the stage weights.

Solve model $M1$ and $M2$. The attribute weights and the stage weights can be obtained as $WC = \{0.15, 0.345, 0.15, 0.355\}$ and $WM = \{0.2, 0.25, 0.2, 0.35\}$. We can find that the weights of c_2 and c_4 are bigger than c_1 and c_3. The weights of stage m^4 is bigger than the other three stages.

Step 5. Calculate the final ranking value.

The final ranking value can be calculated from (22) and we get the ranking values of alternatives (as showed in Fig. 5).

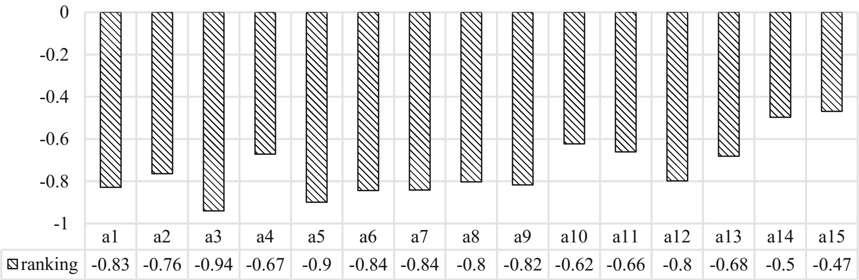

Fig. 5. The ranking values of alternatives

Figure 5 shows that the best alternative is $a15$; the performances of alternative $a15$ are good under different reference points. The prospect values of alternative $a15$ are gains from the SRP and the DRP. This shows that the performance of alternative $a15$ is better than the expected performance of all alternatives in one stage and it is better than in the previous stages. The contribution of alternative $a15$ is greater, thus it is the best one.

5 Conclusions

The paper aims to propose a new MSRDM method for normal cloud model considering three reference points. The progress, current performance and the degree of effort are the criteria for measuring the multistage development of things. In this paper, the SRP is proposed to measure the current performance of an alternative under multiple states. The DRP is proposed to measure the progress in the previous stage. The PRP is proposed to measure the degree of effort and potential. Thus, a value function is defined to obtain the gain or loss from a reference point for normal cloud model. The prospect values under three reference points are aggregated by the synthetic degree of grey incidence. Then a two-stage weight optimization model is built to obtain the attribute weights and the stage weights based on the idea of minimax reference point optimization. The numeral example analysis shows its feasibility and validity for solving the MSRDM problems.

There are many interesting issues related to the problem with multiple reference points. The MCDM/GDM problems with multiple reference points will be researched in future. And a cloud model is a useful tool to deal with large quantity of data. We will develop a more appropriate method to deal with the cloud model.

Acknowledgements. This work was supported by the National Natural Science Foundation of China [71171112, 71502073, 71601002]; Funding of Jiangsu Innovation Program for Graduate Education ("the Fundamental Research Funds for the Central Universities") [KYZZ16_0147].

Appendix

	p1	p2	p3	p1	p2	p3
	c1			c2		
a1	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55)	0.945,0.022,0.009	0.94,0.03,0.008	0.91,0.03,0.009
a2	8.07,2.75,0.19	5.7,1.93,0.47	5,1.82,0.55)	0.95,0.023,0.009	0.95,0.022,0.01	0.935,0.035,0.01
a3	5.7,1.93,0.47	5,1.82,0.55)	4.3,1.93,0.47	0.925,0.037,0.008	0.92,0.052,0.012	0.905,0.052,0.013
a4	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55)	0.91,0.03,0.01	0.91,0.06,0.01	0.88,0.065,0.011
a5	5.7,1.93,0.47	5.7,1.93,0.47	5,1.82,0.55)	0.925,0.037,0.009	0.92,0.022,0.011	0.89,0.03,0.012
a6	8.07,2.75,0.19	6.69,2.27,0.35	5.7,1.93,0.47	0.945,0.037,0.008	0.94,0.037,0.01	0.905,0.038,0.011
a7	5,1.82,0.55)	5.7,1.93,0.47	5.7,1.93,0.47	0.94,0.015,0.009	0.94,0.03,0.01	0.91,0.033,0.011
a8	6.69,2.27,0.35	5,1.82,0.55)	5,1.82,0.55)	0.97,0.021,0.009	0.96,0.022,0.01	0.92,0.031,0.01
a9	5.7,1.93,0.47	5.7,1.93,0.47	5.7,1.93,0.47	0.94,0.024,0.008	0.94,0.037,0.009	0.915,0.041,0.01
a10	6.69,2.27,0.35	5.7,1.93,0.47	5.7,1.93,0.47	0.955,0.175,0.009	0.95,0.022,0.009	0.911,0.033,0.011
a11	6.69,2.27,0.35	5.7,1.93,0.47	5.7,1.93,0.47	0.9150.033,0.010	0.91,0.025,0.01	0.89,0.031,0.011
a12	8.07,2.75,0.19	6.69,2.27,0.35	5.7,1.93,0.47	0.925,0.028,0.009	0.92,0.023,0.009	0.905,0.025,0.011
a13	5.7,1.93,0.47	5.7,1.93,0.47	5,1.82,0.55)	0.93,0.035,0.01	0.92,0.025,0.01	0.905,0.031,0.01
a14	6.69,2.27,0.35	6.69,2.27,0.35	5.7,1.93,0.47	0.94,0.033,0.009	0.93,0.023,0.01	0.911,0.025,0.011
a15	6.69,2.27,0.35	6.69,2.27,0.35	5.7,1.93,0.47	0.955,0.031,0.009	0.94,0.028,0.009	0.91,0.031,0.011
	c3			c4		
a1	6.69,2.27,0.35	5.7,1.93,0.47	6.69,2.27,0.35	87.5,0.425,0.849	72.5,3.822,0.833	57.5,3.822,0.283
a2	8.07,2.75,0.19	5.7,1.93,0.47	5.7,1.93,0.47	85.5,0.425,0.849	74.5,1.274,0.849	65.5,0.425,1.416
a3	5.7,1.93,0.47	6.69,2.27,0.35	6.69,2.27,0.35	85,0.849,0.708	73.5,1.274,0.849	68,2.548,0.708
a4	5,1.82,0.55)	4.3,1.93,0.47	5,1.82,0.55)	87.5,1.274,0.566	72,2.548,0.425	65.4.247,0.142
a5	5,1.82,0.55)	4.3,1.93,0.47	5,1.82,0.55)	87,1.699,0.425	72,3.397,0.142	65.5,4.671,1.167
a6	5.7,1.93,0.47	5,1.82,0.55)	4.3,1.93,0.47	84.5,2.123,0.283	72.5,2.123,0.566	68.5,2.973,0.566
a7	8.07,2.75,0.19	5.7,1.93,0.47	5.7,1.93,0.47	82.5,2.123,0.283	76,0.849,0.991	71,0.849,1.274
a8	5.7,1.93,0.47	5,1.82,0.55)	5,1.82,0.55)	85,1.699,0.425	71.5,1.274,0.849	68,1.699,0.991
a9	6.69,2.27,0.35	8.07,2.75,0.19	5.7,1.93,0.47	84.5,2.973,0.667	73,1.699,0.708	67.5,2.973,0.566
a10	5,1.82,0.55)	6.69,2.27,0.35	5,1.82,0.55)	87,1.699,0.425	74.5,1.274,0.849	67,3.397,0.425
a11	5.7,1.93,0.47	8.07,2.75,0.19	6.69,2.27,0.35	80,0.849,0.708	69,2.548,0.425	63.5,3.822,0.283
a12	6.69,2.27,0.35	5.7,1.93,0.47	5.7,1.93,0.47	75,0.849,0.425	70,2.548,0.849	61.5,3.822,0.425
a13	5.7,1.93,0.47	6.69,2.27,0.35	4.3,1.93,0.47	85.5,2.123,0.283	72,0.849,0.991	61,1.699,0.991
a14	6.69,2.27,0.35	6.69,2.27,0.35	4.3,1.93,0.47	84,1.699,0.425	72.5,2.123,0.566	67.5,2.123,0.849
a15	5.7,1.93,0.47	8.07,2.75,0.19	5.7,1.93,0.47	84,1.699,0.425	72,2.548,0.425	64.5,4.671,1.167

Table 2. The evaluation information in stage m^2

with	p1	p2	p3	p1	p2	p3
	c1			c2		
a1	5.7,1.93,0.47	5.7,1.93,0.47	5.7,1.93,0.47	0.915,0.037,0.01	0.91,0.03,0.011	0.89,0.033,0.011
a2	6.69,2.27,0.35	5,1.82,0.55)	5.7,1.93,0.47	0.91,0.045,0.01	0.93,0.045,0.009	0.91,0.045,0.011
a3	6.69,2.27,0.35	5,1.82,0.55)	5.7,1.93,0.47	0.94,0.045,0.009	0.93,0.022,0.009	0.915,0.032,0.106
a4	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55)	0.92,0.026,0.009	0.91,0.023,0.01	0.895,0.033,0.011
a5	6.69,2.27,0.35	5,1.82,0.55)	5,1.82,0.55)	0.905,0.038,0.009	0.9,0.022,0.01	0.88,0.025,0.011
a6	5.7,1.93,0.47	5.7,1.93,0.47	5,1.82,0.55)	0.91,0.035,0.01	0.9,0.037,0.012	0.85,0.045,0.014
a7	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55	0.95,0.035,0.009	0.925,0.03,0.01	0.875,0.033,0.011
a8	5.7,1.93,0.47	5,1.82,0.55)	4.3,1.93,0.47	0.950.035,0.009	0.925,0.022,0.01	0.89,0.026,0.011
a9	5.7,1.93,0.47	5,1.82,0.55)	4.3,1.93,0.47	0.935,0.025,0.008	0.92,0.037,0.009	0.885,0.038,0.011
a10	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55	0.95,0.015,0.009	0.93,0.022,0.01	0.89,0.035,0.011
a11	5.7,1.93,0.47	5.7,1.93,0.47	4.3,1.93,0.47	0.945,0.035,0.009	0.925,0.021,0.01	0.895,0.035,0.012
a12	6.69,2.27,0.35	6.69,2.27,0.35	5.7,1.93,0.47	0.93,0.025,0.009	0.925,0.023,0.01	0.887,0.035,0.011
a13	5.7,1.93,0.47	6.69,2.27,0.35	5,1.82,0.55	0.905,0.035,0.009	0.9,0.031,0.009	0.875,0.035,0.011
a14	8.07,2.75,0.19	5.7,1.93,0.47	5,1.82,0.55	0.955,0.021,0.008	0.94,0.031,0.009	0.915,0.035,0.01
a15	6.69,2.27,0.35	5,1.82,0.55	5.7,1.93,0.47	0.965,0.015,0.009	0.95,0.032,0.009	0.925,0.035,0.011
	c3			c4		
a1	5,1.82,0.55)	5.7,1.93,0.47	5,1.82,0.55)	83,0.849,1.416	72,1.699,0.566	66.5,3.822,0.566
a2	6.69,2.27,0.35	6.69,2.27,0.35	5.7,1.93,0.47	81,1.699,1.132	75,0.849,0.849	67,1.699,1.274
a3	5.7,1.93,0.47	6.69,2.27,0.35	5,1.82,0.55)	83,0.849,1.416	71,0.849,0.849	52.5,2.123,1.132
a4	6.69,2.27,0.35	8.07,2.75,0.19	5.7,1.93,0.47	83.5,2.123,0.991	72.5,1.274,0.708	67.5,2.123,1.132
a5	5.7,1.93,0.47	6.69,2.27,0.35	5,1.82,0.55)	86,1.699,1.132	74,0.849,0.849	65,3.397,0.708
a6	(5,1.82,0.55)	(5.7,1.93,0.47	(5.7,1.93,0.47	(77,0.849,1.416	(74,0.849,0.849	(62.5,2.973,0.849
a7	(5.7,1.93,0.47	(6.69,2.27,0.35	(4.3,1.93,0.47	(77,5.096,1.167	(76,2.548,0.283	(55.5,2.973,0.849
a8	(6.69,2.27,0.35	(6.69,2.27,0.35	(5,1.82,0.55)	(67.5,1.274,1.274	(59,1.699,0.566	(57,1.699,1.274
a9	(5.7,1.93,0.47	(6.69,2.27,0.35	(4.3,1.93,0.47	(82,0.849,1.416	(70,0.849,0.849	(60.5,2.973,0.849
a10	(6.69,2.27,0.35	(8.07,2.75,0.19	(5.7,1.93,0.47	(81.5,1.274,1.274	(70,3.397,,0.833	(59.5,5.521,1.167
a11	(5.7,1.93,0.47	(6.69,2.27,0.35	(4.3,1.93,0.47	(82,1.699,1.132	(71.5,2.123,0.425	(63.5,3.822,0.566
a12	(8.07,2.75,0.19	(8.07,2.75,0.19	(5,1.82,0.55)	(79.5,1.274,1.274	(73.5,2.123,0.425	(60.5,3.822,0.566
a13	(5.7,1.93,0.47	(6.69,2.27,0.35	(4.3,1.93,0.47	(80.5,2.973,0.708	(71,1.699,0.566	(61,1.699,1.274
a14	(8.07,2.75,0.19	(6.69,2.27,0.35	(5,1.82,0.55)	(81.5,3.822,0.425	(71,1.699,0.566	(63.5,4.671,0.283
a15	(6.69,2.27,0.35	(6.69,2.27,0.35	(5.7,1.93,0.47	(83.5,3.822,0.425	(70.5,1.274,0.708	(65,3.397,0.708

Table 3. The evaluation information in stage m^3

with	p1	p2	p3	p1	p2	p3
	c1			c2		
a1	5,1.82,0.55	5,1.82,0.55	4.3,1.93,0.47	0.975,0.015,0.009	0.96,0.027,0.01	0.935,0.031,0.011
a2	5.7,1.93,0.47	5,1.82,0.55	4.3,1.93,0.47	0.97,0.018,0.009	0.95,0.025,0.012	0.925,0.03,0.013
a3	5,1.82,0.55	5,1.82,0.55	4.3,1.93,0.47	0.91,0.055,0.008	0.91,0.037,0.01	0.89,0.04,0.012
a4	5.7,1.93,0.47	5.7,1.93,0.47	5,1.82,0.55)	0.97,0.014,0.01	0.96,0.035,0.011	0.93,0.04,0.011
a5	6.69,2.27,0.35	5,1.82,0.55	5.7,1.93,0.47	0.91,0.036,0.01	0.9,0.024,0.015	0.88,0.028,0.016
a6	5.7,1.93,0.47	5,1.82,0.55	5.7,1.93,0.47	0.915,0.023,0.008	0.91,0.035,0.012	0.875,0.04,0.013
a7	5.7,1.93,0.47	5.7,1.93,0.47	5,1.82,0.55)	0.925,0.016,0.008	0.92,0.045,0.01	0.089,0.046,0.011
a8	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55	0.935,0.023,0.009	0.92,0.026,0.009	0.089,0.026,0.01

(*continued*)

Table 3. (*continued*)

with	p1	p2	p3	p1	p2	p3
a9	5.7,1.93,0.47	6.69,2.27,0.35	4.3,1.93,0.47	0.905,0.025,0.009	0.9,0.036,0.01	0.087,0.041,0.011
a10	6.69,2.27,0.35	5,1.82,0.55	4.3,1.93,0.47	0.935,0.035,0.009	0.92,0.042,0.011	0.885,0.045,0.012
a11	5,1.82,0.55	5,1.82,0.55	5.7,1.93,0.47	0.955,0.025,0.009	0.94,0.025,0.011	0.92,0.03,0.012
a12	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55	0.925,0.031,0.009	0.93,0.032,0.014	0.915,0.035,0.015
a13	5.7,1.93,0.47	6.69,2.27,0.35	5.7,1.93,0.47	0.935,0.025,0.009	0.93,0.023,0.01	0.915,0.03,0.011
a14	6.69,2.27,0.35	6.69,2.27,0.35	5.7,1.93,0.47	0.943,0.011,0.007	0.94,0.023,0.01	0.92,0.025,0.011
a15	5.7,1.93,0.47	5.7,1.93,0.47	5,1.82,0.55	0.953,0.011,0.007	0.95,0.015,0.01	0.93,0.014,0.012
	c3			c4		
a1	5.7,1.93,0.47	6.69,2.27,0.35	5,1.82,0.55)	70.5,0.425,1.416	65,0.849,1.699	60.5,1.274,1.274
a2	6.69,2.27,0.35	6.69,2.27,0.35	5,1.82,0.55)	68.5,2.973,0.566	64.5,2.973,0.991	57.5,4.671,0.142
a3	5.7,1.93,0.47	6.69,2.27,0.35	5,1.82,0.55)	70,1.699,0.991	62,0.849,1.699	56,4.247,0.283
a4	5.7,1.93,0.47	5,1.82,0.55)	4.3,1.93,0.47	80,1.699,0.991	68.5,2.123,1.274	62.5,2.973,0.708
a5	5.7,1.93,0.47	5.7,1.93,0.47	4.3,1.93,0.47	77.5,2.123,0.849	66.5,2.973,0.991	59,5.096,1.333
a6	5,1.82,0.55)	5,1.82,0.55)	3.31,2.27,0.353	80.5,1.274,1.132	69.5,2.973,0.991	56.5,1.274,1.274
a7	5.7,1.93,0.47	5.7,1.93,0.47	5,1.82,0.55)	76,1.699,0.991	68.5,2.123,1.274	67.5,2.123,0.991
a8	6.69,2.27,0.35	6.69,2.27,0.35	5.7,1.93,0.47	76.5,2.973,0.566	70,4.247,0.566	62.5,2.123,0.991
a9	6.69,2.27,0.35	6.69,2.27,0.35	5.7,1.93,0.47	80,4.247,0.142	71.5,1.274,1.557	63,2.548,0.849
a10	8.07,2.75,0.19	6.69,2.27,0.35	5,1.82,0.55)	80,2.548,0.708	67.5,2.123,1.274	62,2.548,0.849
a11	5.7,1.93,0.47	6.69,2.27,0.35	4.3,1.93,0.47	77.5,2.123,0.849	71.5,1.274,1.557	67,1.699,1.132
a12	5.7,1.93,0.47	6.69,2.27,0.35	5,1.82,0.55)	78.5,2.973,0.566	73.5,1.274,1.557	66,2.548,0.849
a13	5,1.82,0.55)	5,1.82,0.55)	4.3,1.93,0.47	82.5,4.671,0.667	67.5,2.123,1.274	63,2.548,0.849
a14	5.7,1.93,0.47	5,1.82,0.55)	3.31,2.27,0.353	81.5,2.973,0.566	68,2.548,1.132	62.5,2.973,0.708
a15	6.69,2.27,0.35	6.69,2.27,0.35	4.3,1.93,0.47	82.5,2.123,0.849	70,5.945,1.333	62.5,3.822,0.425

Table 4. The evaluation information in stage m^4

with	p1	p2	p3	p1	p2	p3
	c1			c2		
a1	5.7,1.93,0.47	5,1.82,0.55	5,1.82,0.55	0.965,0.011,0.01	0.96,0.025,0.011	0.945,0.035,0.012
a2	5.7,1.93,0.47	5.7,1.93,0.47	5,1.82,0.55	0.965,0.012,0.01	0.955,0.026,0.013	0.915,0.03,0.021
a3	8.07,2.75,0.19	6.69,2.27,0.35	5.7,1.93,0.47	0.921,0.045,0.009	0.92,0.034,0.014	0.89,0.36,0.018
a4	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55	0.95,0.012,0.009	0.945,0.021,0.014	0.925,0.03,0.015
a5	5.7,1.93,0.47	5.7,1.93,0.47	5,1.82,0.55	0.92,0.033,0.011	0.91,0.022,0.014	0.895,0.032,0.019
a6	6.69,2.27,0.35	8.07,2.75,0.19	6.69,2.27,0.35	0.91,0.025,0.011	0.91,0.042,0.012	0.855,0.045,0.015
a7	5,1.82,0.55	5.7,1.93,0.47	5,1.82,0.55	0.93,0.011,0.01	0.92,0.035,0.011	0.895,0.038,0.016
a8	6.69,2.27,0.35	5.7,1.93,0.47	5.7,1.93,0.47	0.94,0.025,0.01	0.935,0.025,0.015	0.89,0.035,0.018
a9	5,1.82,0.55	5.7,1.93,0.47	5,1.82,0.55	0.92,0.026,0.01	0.92,0.024,0.012	0.85,0.035,0.015
a10	5.7,1.93,0.47	5,1.82,0.55	5,1.82,0.55	0.94,0.025,0.01	0.93,0.023,0.011	0.89,0.055,0.021
a11	5,1.82,0.55	6.69,2.27,0.35	5.7,1.93,0.47	0.96,0.02,0.01	0.935,0.012,0.012	0.875,0.034,0.015
a12	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55	0.93,0.025,0.012	0.925,0.015,0.014	0.885,0.025,0.018
a13	6.69,2.27,0.35	5.7,1.93,0.47	5,1.82,0.55	0.95,0.02,0.01	0.93,0.032,0.01	0.905,0.045,0.015
a14	5.7,1.93,0.47	6.69,2.27,0.35	5.7,1.93,0.47	0.95,0.015,0.012	0.925,0.012,0.01	0.89,0.025,0.018
a15	6.69,2.27,0.35	6.69,2.27,0.35	5.7,1.93,0.47	0.958,0.012,0.01	0.925,0.025,0.011	0.885,0.031,0.016
	c3			c4		
a1	5.7,1.93,0.47	6.69,2.27,0.35	4.3,1.93,0.47	71.5,2.973,1.132	64,4.247,0.283	60,4.247,0.142

(*continued*)

Table 4. (*continued*)

with	p1	p2	p3	p1	p2	p3
a2	5.7,1.93,0.47	6.69,2.27,0.35	5,1.82,0.55)	69.5,6.37,1.167	60,4.247,0.283	54,4.247,0.142
a3	6.69,2.27,0.35	6.69,2.27,0.35	5,1.82,0.55)	69,3.397,0.991	64,5.096,1.167	55.5,4.671,1.167
a4	8.07,2.75,0.19	8.07,2.75,0.19	5.7,1.93,0.47	77,1.699,1.557	70,1.699,1.132	62.5,2.123,0.849
a5	5.7,1.93,0.47	6.69,2.27,0.35	5,1.82,0.55)	79,2.548,1.274	69.5,1.274,1.274	62,2.548,0.708
a6	5,1.82,0.55)	6.69,2.27,0.35	5,1.82,0.55)	84.5,3.822,0.849	72.5,2.123,0.991	66.5,2.973,0.566
a7	5,1.82,0.55)	5,1.82,0.55)	4.3,1.93,0.47	76,0.849,1.84	73.5,2.973,0.708	67.5,2.123,0.849
a8	6.69,2.27,0.35	6.69,2.27,0.35	5,1.82,0.55)	72.5,2.123,1.416	69,4.247,0.283	61,2.548,0.708
a9	5,1.82,0.55)	5.7,1.93,0.47	5,1.82,0.55)	78,5.096,0.425	74,3.397,0.566	61.5,2.973,0.566
a10	5.7,1.93,0.47	5.7,1.93,0.47	4.3,1.93,0.47	78.5,2.973,1.132	70,1.699,1.132	60,4.247,0.142
a11	6.69,2.27,0.35	8.07,2.75,0.19	5.7,1.93,0.47	78,2.548,1.274	72.5,2.123,0.991	67,1.699,0.991
a12	6.69,2.27,0.35	6.69,2.27,0.35	5,1.82,0.55)	75.5,2.973,1.132	72.5,3.822,0.425	67.5,1.274,1.132
a13	8.07,2.75,0.19	8.07,2.75,0.19	5.7,1.93,0.47	81.5,2.973,1.132	72.5,2.123,0.991	61.5,2.973,0.566
a14	5.7,1.93,0.47	6.69,2.27,0.35	5,1.82,0.55)	82,1.699,1.557	74,3.397,0.566	67.5,2.123,0.849
a15	6.69,2.27,0.35	6.69,2.27,0.35	4.3,1.93,0.47	78.5,1.274,1.699	73,2.548,0.849	67.5,2.123,0.849

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