A Multimoora Method Application with Einstein Interval Valued Fuzzy Numbers' Operator

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Abstract. Multi Criteria Decision Making (MCDM) is a process to decide the best possible ranking between alternatives. In fuzzy sense, fuzzy MCDM methods aim to get closer the best alternative. Interval valued intuitionistic fuzzy sets can be applied in MCDM and they form many fuzzy methods. One of them is MULTIMOORA which tries to deal with lessening incertitude options. MULTIMOORA method becomes important because of its usage compared to other MCDM methods. In this paper, the data in which Baležentis and Zeng (2013) studied is used and GIFTNOWGA operator which Baležentis and Zeng studied is replaced by Einstein operator to investigate how choosing different operators affect the final ranking of alternatives.

Keywords: MCDM · Decision making · Fuzzy · Multimoora Operator

1 Introduction

Decision making has been an interesting topic throughout centuries. This interesting topic mainly comes from the change of point of view by comparison in order to decide which alternative or situation fits in the best way [1].

MCDM is an important topic in the 21st century because of evaluating alternatives. Alternatives can have different proprieties to grab them and they might also have different features. In decision making process, it is essential that an operative process should be implemented in order to have a qualified decision making [2]. As another perspective, MCDM can be defined as an approach containing subjective rankings and evaluation in order to select the best [3].

Apart from crisp valued- MCDM methods, many studies about fuzzy MCDM methods have been developed to optimize all given criteria [4]. To give an example, Yang et al. have used MCDM to decide the best alternative of vendors by using integrated fuzzy MCDM techniques and they assumed that all criteria are independent via subjective preferences of experts [5]. As another example, Liang et al. used fuzzy linguistic assessment to evaluate personals by putting subjective weightings and ratings of decision makers into effect by implementing not only subjective but also objective

N. M. Durakbasa and M. G. Gencyilmaz (Eds.): ISPR 2018, *Proceedings of the International Symposium for Production Research 2018*, pp. 171–178, 2019.
https://doi.org/10.1007/978-3-319-92267-6_13

assessments [6]. In the same area, Dursun and Karsak have used another fuzzy MCDM approach of 2 tuple linguistic model to evaluate personnel by the aim of dealing with operational heterogeneity [7].

In the fuzzy MCDM approaches, the operator which Is used is also important. For example, Chiclana et al. have used the Ordered Weighted Geometric Operator (OWG) to obtain an incorporation of fuzzy problems [8]. Bordogna and Pasi evaluated linguistic aggregation operators to designate of the aggregation criteria [9].

MULTIMOORA method is an essential method by the way of MCDM. Baležentis et al. have implemented fuzzy MULTIMOORA method to assign indicators and situation of Lithuania in the European Union [10].

According to the studies explained above, it is aimed to use fuzzy MULTIMOORA method which is generally used in fuzzy MCDM and change the operator to observe final rankings of using Einstein operator compared to GIFTNOWGA operator in which Baležentis and Zeng used.

2 Background of the Multimoora Method

The MULTIMOORA (Multi-Objective Optimization by Ratio Analysis plus Full Multiplicative Form) method is an alternative for multi-criteria decision-making techniques based on weighting criteria, which was developed by Brauers and Zavadskas in 2010 [11]. MULTIMOORA has been applied in various areas where alternatives need be compared. MULTIMOORA method originated MOORA method and Full Multiplicative Form method for Multi-Criteria Decision Making [12]. Therefore, MULTIMOORA method consists of MOORA-Ratio System, MOORA-Reference Point System and Full Multiplicative Form method. In this context, the steps of MOORA-Ratio System, MOORA-Reference Point, Full Multiplication methods take place in the following section.

The MOORA method has three important assumptions. These assumptions include the use of metrics, which imply counting number, the availability of alternatives that take a discrete value, and the well-defined objectives [13]. After meeting assumptions, a decision matrix of alternatives and criteria are established. The decision matrix is in Eq. (1), with n-criteria and m-alternatives. The vector normalization method requires normalizing the decision matrix in the MOORA-Ratio method. Equation (2) normalizes the values of each alternative based on the criterion to obtain x_{ij^*} values.

$$\mathrm{X} = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} \tag{1}$$

$$x_{ij^*} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{j=m} x_{ij}^2}} \tag{2}$$

These indicators are added if the desirable value of indicator is maximum and subtracted if desirable value is minimum. The summarizing index of each alternative is given in Eq. 3 [13].

$$y_j^* = \sum\nolimits_{i=1}^{g} x_{ij^*} - \sum\nolimits_{i=g+1}^{n} x_{ij^*} \tag{3}$$

The performance values y_j^* are ordered from small to large to rank the alternatives. For preference problems, the alternative with the highest performance value should be preferred. The MOORA-Reference Point method uses the normalized decision matrix obtained by the Eq. (2) in the MOORA-Ratio method. The best values of each criterion are the reference values of the alternatives. In other words, if the criterion is benefit-oriented, it is the greatest value and if it is cost-oriented, it is the lowest value reference point. Tchebycheff min-max metric method is used to measure the difference between the reference points of the alternatives. The distance value d_{ij} is obtained by Eq. (4), where r_i is the reference point. Equation (4) and the reference series have the distances from the reference point of the alternatives by taking the absolute value differences of the normalization decision matrix values obtained from Eq. (2). Then the maximum distance values of the alternatives are determined. These values are ordered from small to large for alternative sorting. If the problem is a preference problem, the alternative with the smallest distance value should be preferred [13].

$$d_{ij} = min_j(max_i(|r_i - x_{ij^*}|)) \tag{4}$$

The Full Multiplication method is used in nonlinear, non-additive, preference or sorting problems where weighting and normalization cannot be used. The utility and cost performance values of the alternatives included in the decision matrix are obtained by multiplying the benefit side criterion values by Eq. (5) and the cost side criterion values by the Eq. (6). In these equations g = 1…, i are cost-oriented criteria, and k = i + 1,…, n are cost-oriented criteria. The utility performance value is compared to the cost performance value and the general performance value of the alternate is obtained as in Eq. (7). The alternatives are sorted from small to large according to their general performance values [11].

$$A_j = \prod\nolimits_{g=1}^{i} x_{gi} \tag{5}$$

$$B_j = \prod\nolimits_{k=i+1}^{n} x_{ki} \tag{6}$$

$$U_j = \frac{A_J}{B_J} \tag{7}$$

3 Methodology

Zavadskas et al. (2015) give two numerical examples of real-world civil engineering problems and rank the alternatives based on the suggested method. Then, they compare the results to the rankings yielded by some other methods of decision making with IVIF information. The comparison shows the conformity of the proposed IVIF-MULTIMOORA method with other approaches. The proposed algorithm is favourable because of the abilities of IVIFS to be used for imagination of uncertainty and the MULTIMOORA method to consider three different viewpoints in analysing engineering decision alternatives [11].

Wu et al. (2018) propose a strongly robust method to solve multi-experts multi-criteria decision making problems with linguistic evaluations. To enrich the computation and to improve the measures of probabilistic linguistic term set, they firstly define an expectation function of it. In addition, they advance three kinds of probabilistic linguistic distance measures reflecting on the difference of linguistic terms and probabilities at the same time to make up for the defects of the existing distance measures, and then propose the similarity and correlation measures. Integrating the subjective opinions with the correlation coefficients between criteria, they put forward a combined weight determining method. The robustness of the ranking method, MULTIMOORA, is enhanced by the improved Borda rule. Based on these research findings, a probabilistic linguistic MULTIMOORA method is proposed. Finally, the developed method is applied to an empirical example concerning the selection of shared karaoke television brands. The effectiveness of the proposed method is verified by some comparative analysis [16].

Aytekin (2016) identifies the importance weights of the patients, which are effective in preference of the hospital and the hospitals located in the city center of Eskişehir were listed with MULTIMOORA as the most criterion decision making technique in these factors. As a result, while it is determined that the most effective criteria for selecting hospitals is the availability of all kinds of services and specialists, it is seen that the competition levels of public hospitals were at a level that could compete with private hospitals [17].

There are lots of operators used in fuzzy decision making such as GITFNWGA (Generalized Interval-Valued Trapezoidal Fuzzy Numbers Weighted Geometric Aggregation), GITFNOWGA (The Generalized Interval-Valued Trapezoidal Fuzzy Numbers Ordered Weighted Geometric Aggregation) and GITFNHGA (The Generalized Interval-Valued Trapezoidal Fuzzy Numbers Hybrid Geometric Aggregation). Although there are lots of studies about fuzzy decision-making methods in the literature, there can be found limited studies about MULTIMOORA Method, to that end studies relating to Einstein Operator aggregation with MULTIMOORA. Balezentis and Zeng (2012) extend the MULTIMOORA Method with type 2 fuzzy sets with GITFNOWGA operator to select best candidate for a manager position in an R&D department [14].

This study aggregates type 2 fuzzy numbers with Einstein Operator to search whether the ranking of candidates will remain same or different and compares results with the study of Balezentis and Zeng (2012) [14]. There are four candidates named

A_1, A_2, A_3, A_4 and three decision makers labeled as DM_1, DM_2, DM_3. Decision makers assess the four candidates based on the five benefit criteria, which are proficiency in identifying research ideas (C_1), proficiency in administration (C_2), personality (C_3), experience (C_4), and self-confidence (C_5) [14]. Table 1 shows linguistic variables and respective generalized interval-valued trapezoidal fuzzy numbers [15].

Table 1. Linguistic term generalized interval-valued trapezoidal fuzzy number (Wei and Chen (2009))

Absolutely poor (AP)	[(0.0, 0.0, 0.0, 0.0; 0.8), (0.0, 0.0, 0.0, 0.0; 1.0)]
Very poor (VP)	[(0.00, 0.00, 0.02, 0.07; 0.8), (0.0, 0.0, 0.02, 0.07; 1.0)]
Poor (P)	[(0.04, 0.10, 0.18, 0.23; 0.8), (0.04, 0.10, 0.18, 0.23; 1.0)]
Medium poor (MP)	[(0.17, 0.22, 0.36, 0.42; 0.8), (0.17, 0.22, 0.36, 0.42; 1.0)]
Medium (F)	[(0.32, 0.41, 0.58, 0.65; 0.8), (0.32, 0.41, 0.58, 0.65; 1.0)]
Medium good (MG)	[(0.58, 0.63, 0.80, 0.86; 0.8), (0.58, 0.63, 0.80, 0.86; 1.0)]
Good (G)	[(0.72, 0.78, 0.92,0.97;0.8), (0.72, 0.78, 0.92, 0.97; 1.0)]
Very good (VH)	[(0.93, 0.98, 1.0, 1.0;0.8), (0.93, 0.98, 1.0, 1.0; 1.0)
Absolutely good (AG)	[(1.0, 1.0, 1.0, 1.0; 1.0), (1.0, 1.0, 1.0, 1.0; 1.0)]

Table 2 shows the evaluation of each decision makers for four candidates. Then, matrix A converts the linguistic terms into the generalized interval valued trapezoidal fuzzy numbers by Einstein Operator aggregation. Since all the criteria are benefit criteria, so we do not need to normalize them.

Table 2. Linguistics variables

		C_1	C_2	C_3	C_4	C_5
DM_1	A1	VG	VG	VG	VG	VG
	A2	G	VG	VG	VG	MG
	A3	VG	MG	G	G	G
	A4	G	F	F	G	MG
DM_2	A1	G	MG	G	G	VG
	A2	G	VG	VG	VG	MG
	A3	G	G	MG	VG	G
	A4	VG	F	MG	F	G
DM_3	A1	MG	F	G	VG	VG
	A2	MG	MF	G	MG	G
	A3	VG	VG	VG	VG	MG
	A4	MG	VG	MG	VG	F

[A]=	[(0,997	0,999	1	1	0,8);	(0,997	0,999	1	1	1)]
	[(0,986	0,993	1	1	0,8);	(0,986	0,993	0,9996	1	1)]
	[(1	1	1	1	0,8);	(1	1	1	1	1)]
	[(0,997	0,999	1	1	0,8);	(0,997	0,999	1	1	1)]
	[(0,99	0,998	1	1	0,8);	(0,99	0,998	1	1	1)]
	[(0,999	1	1	1	0,8);	(0,999	1	1	1	1)]
	[(0,997	0,999	1	1	0,8);	(0,997	0,999	1	1	1)]
	[(0,981	0,996	1	1	0,8);	(0,981	0,996	1	1	1)]
	[(0,998	1	1	1	0,8);	(0,999	1	1	1	1)]
	[(1	1	1	1	0,8);	(0,993	1	1	1	1)]
	[(0,997	0,999	1	1	0,8);	(0,999	0,999	1	1	1)]
	[(0,93	0,958	0,993	0,998	0,8);	(0,999	0,958	0,9935	0,9976	1)]
	[(1	1	1	1	0,8);	(0,976	1	1	1	1)]
	[(0,999	1	1	1	0,8);	(0,976	1	1	1	1)]
	[(1	1	1	1	0,8);	(0,976	1	1	1	1)]
	[(0,994	0,999	1	1	0,8);	(0,976	0,999	1	1	1)]
	[(1	1	1	1	0,8);	(1	1	1	1	1)]
	[(0,977	0,987	0,999	1	0,8);	(0,977	0,987	0,999	0,9998	1)]
	[(0,986	0,993	1	1	0,8);	(0,986	0,993	0,9996	1	1)]
	[(0,956	0,977	0,998	1	0,8);	(0,956	0,977	0,9975	0,9995	1)]

The four candidates are ranked according to Ratio System, cf. Eq. (4) which can be seen in Table 3.

Table 3. The ratio system

	RS_i	Distance	Rank
A_1	[(4.985,4.997,5,5,0.8), (4.962,4.997,5,5,1)]	4,533	4
A_2	[(4.961,4.98,4.999,5,0.8), (4.932,4.98,4.999,5,1)]	4,627	2
A_3	[(4.979,4.992,5,5,0.8), (4.957,4.992,5,5,1)]	4,586	3
A_4	[(4.858,4.929,4.991,4.997,0.8), (4.909,4.929,4.991,4.997,1)]	4,704	1

Table 4 shows the results of Reference Point System, four candidates are ranked based on their distance by Eq. (2) and Table 5 indicates the results of Full Multiplicative Form.

Table 4. Reference point

	$Max_j\{d(\beta_{ij}, \beta_j)\}$	Rank
A_1	0.103	1
A_2	0.108	3
A_3	0.105	2
A_4	0.119	4

Table 5. Full multiplicative form

	RS_i	Distance	Rank
A_1	[(0.838,0.920,1,1,0.8), (0.763,0.921,1,1,1)]	2.326995	4
A_2	[(0.811,0.895,1,1,0.8), (0.712,0.895,1,1,1)]	2.364969	2
A_3	[(0.832,0.913,1,1,0.8), (0.761,0.913,1,1,1)]	2.352266	3
A_4	[(0.681,0.784,1,1,0.8), (0.679,0.784,1,1,1)]	2.537938	1

Table 6 includes the results of Ratio System, Reference Point and Full Multiplicative Form; the MULTIMOORA ends with final ranking by dominance theory.

Table 6. Multimoora ranking

	Ratio system	Reference point	Full multiplicative form	MULTIMOORA (Final rank)
A_1	4	1	4	4
A_2	2	3	2	2
A_3	3	2	3	3
A_4	1	4	1	1

The results show that the final ranking of MULTIMOORA will remain differ with the use of Einstein operator. The results show that final ranking of Einstein operator is differ from the final ranking of GITFNOWGA operator conducted by Balezentis and Zeng (2012). It indicates managers should give importance to selection of the operator in multi criteria decision making methods. They should support the conclusion of the MULTIMOORA method with other experience and methods.

4 Conclusions

In today's world decision making problems have a considerable importance and this importance will continue to grow in the future. To that end, studies relating to decision making area will increase especially in fuzzy decision-making studies which consider uncertainty. Even though the literature consists of numerous studies about fuzzy decision-making methods, there can be found limited numbers of the MULTIMOORA method related studies. MULTIMOORA is rooted from MOORA method and Full Multiplicative Form method for Multi-Criteria Decision Making. Thus, MULTIMOORA method consists of MOORA-Ratio System, MOORA-Reference Point System and Full Multiplicative Form method. MULTIMOORA method is widely used to solve problems of preferencing and/or sequencing which facilitates jobs of decision makers.

In the context of this research, the data from the study of the MULTIMOORA Method with type 2 fuzzy sets and GITFNOWGA operator was conducted from Balezentis and Zeng (2012) is used to evaluate whether the final ranking of MULTIMOORA will remain be same/differ with the use of Einstein operator. The results show that final ranking of Einstein operator is differ from the final ranking of GITFNOWGA operator conducted by Balezentis and Zeng (2012). As result, the conclusions from this study illustrates that the selection of an aggregation operator is extremely crucial due to it may change the results of rankings.

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