Peirce on Diagrammatic Reasoning and Semeiotic

Javier Legris[1(✉)] and Cassiano Terra Rodrigues[2]

[1] Instituto Interdisciplinario de Economía Política de Buenos Aires, CONICET-Universidad de Buenos Aires, Av. Córdoba 2122, C1120 AAQ Buenos Aires, Argentina
jlegris@retina.ar

[2] Departamento de Filosofia, Pontifícia Universidade Católica de São Paulo, R. Monte Alegre, 984 - Perdizes, São Paulo, SP 05014-901, Brazil
ctrodrigues@pucsp.br

Abstract. Charles Sanders Peirce (1839–1914) is one of the "grounding fathers" of mathematical logic, having developed all of the key formal results of modern logic. He did it firstly (from 1860 on) in the algebraic tradition of mathematical logic stemming from Boole, combining it with the logic of relations, explicitly developed by Augustus De Morgan. From this, Peirce obtained a system that included quantifiers—a term he seems to have invented—and relative predicates. Developing his own system of relative terms, Peirce started from Boole's system, trying to apply it to De Morgan's logic of relations. Indeed, Peirce's aim is to include the logic of relations into the calculus of algebra using his own system of algebraic signs. On the one hand, Peirce's algebraic notation will be presented, specially: (a) relative terms as iconic representations of logical relations; (b) Peirce's quantifiers and the passage from a linear notation to a diagrammatic one. On the other hand, Peirce's graphical notation will be presented, specially: (a) his Alpha and Beta systems, which are fully compatible with what is nowadays called first-order logic, (b) and his unfinished Gamma system, designed for second-order logic and modal logic.

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1 Introduction: Peirce's Early Algebraic Logic

Peirce refuses Boole's identification of logical relations with equations. Holding that inclusion between classes is previous to identity, and *implication* is previous to both, Peirce aims to develop an abstracter calculus so that all logical relations can be defined solely upon the formal characters of one single fundamental relation. So, he replaces the identity sign "=" used by Boole by the specific sign "−<" (the 'craw foot') for the fundamental subsumptive operation of *illation*, which encompasses the logical relations of conditionality, inclusion and consequence [1: 360].

Next, Peirce deals with the composition of relations with classes, and not strictly relational composition, that is, composition of a relation with another, as in De

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Morgan's system. So, Peirce works with expressions like "lover of ____", or "giver to ____ of ____", rather than De Morgan's expressions with verbs like "____ loves ____" or "____ gives ____ to ____". Thus, the traditional interpretation of propositions as subject-predicate structures is maintained, but transformed, since not restricted to the predication of only one subject. A proposition is a blank-predicate-form, a kind of *icon*, from which the subject-terms are dropped of, with resulting gaps for the insertion of individual variables, the signs for which are *indexes*. Quantifiers can then be introduced, since to express Boole's algebra in relative terms, particularly hypothetical and particular propositions, existential quantification is needed, e.g., a relative term for *case of the existence of* _____; or for *what exists only if there is not* _____; or else *case of the non-existence of* ____; or still *what exists only if there is not*____ [1: 423].

Peirce's fully-fledged theory of multiple quantification in algebraic notation uses the Greek letters $\prod$—for logical *product*—and $\sum$—for logical *sum*—respectively to designate the *universal quantifier* and the *existential quantifier*. Then, there are juxtaposed subscript letters functioning as indexes, that is, specific deictic signs indicating specific items within the defined universe of discourse. As individual variables, indexes show which terms are bound together by a certain relation and in which specific order. For instance, if l denotes the relation of loving, then l_{ij} signifies "*i* loves *j*", with "*i"* and "*j"* as indexes for whatever individuals are in this love relation. So, a propositions like, e.g., "Everybody loves Chaplin", can be symbolized as $\prod_i l_{iC}$ (*C* as an index for the individual Chaplin); or else "Everyone loves someone" is rendered as $\prod_i \sum_j l_{ij}$ (*j* for *jemand*, German for *someone*). Peirce elsewhere remarks a proper notation necessarily includes icons and indexes, so $\prod$ and $\sum$ were chosen to make the notation as *iconic* as possible. For him, "every algebraical equation is an icon, in so far as it exhibits, by means of the algebraical signs (which are not themselves icons), the relations of the quantities concerned" [2: 13]. In Peirce's semeiotic, an icon is a sign that formally resembles its object. So, it better conveys the very movement of thought by "carrying the mind from one point to another", e.g., from the premises to the conclusion [2: 10].

Towards the end of the 19th century, Peirce developed a *diagrammatical* system he himself considered his *chef d'oeuvre* in logic: his *Existential Graphs* (EG). Developing it more or less at the same time as his conception of logic as semiotic, that is, the "*quasi*-necessary" and general doctrine of signs, Peirce considered it as *the logic of the future*, abandoning his early algebraic attempts for *philosophical* reasons.

As special kinds of icons, diagrams resemble their objects only in the aspects that attention needs to be drawn upon, that is, "only in respect to the relations of their parts that their likenesses consists" [2: 13]. So, the EG system is more capable than linear notations to lay bare the inferential movement of thought, showing how formal relations can be inter-derived from one another. Generalizing, all logical inferences can be semiotically interpreted as a sort of *diagrammatic experimentation upon signs*, which are essentially iconic. This point inserts Peirce in a long Western tradition of symbolic thought not restricted to linguistic analysis [3].

Peirce's graphical system includes: (a) his Alpha and Beta systems, which are fully compatible with what is nowadays called first-order logic, (b) and his unfinished Gamma system, designed for second-order logic and modal logic. The system has only three rules—scroll, cut, and line of identity—that permit experimentation and transformation of diagrams. The Existential Graphs system is truly a *topovisual* logical system [4], where only the connections and relations between parts are important, the rules of transformation of which make up the laws of the system. From this, Peirce developed inference rules that anticipated more recent and known systems of diagrammatization.

A less known subject is Peirce's distinction between mathematic and logic, which the graphs make more explicit. Mathematics is for Peirce the science that *draws* necessary conclusions from hypothetical diagrammatic structures, while logic is the science of *drawing* necessary conclusions. In other words, mathematics is the most abstract exercise of reasoning itself, based on a principle of parsimony, the most general of all theoretical activities. Logic, in its turn, is a normative science seeking to determine how we ought to reason, with concerns that can be said of a rhetorical nature [5]. Logic analyses reasoning, breaking it in its least constitutive steps to understand its logical movement. So, a formal system of signs has different uses for each science. Notwithstanding the difference, both logic and mathematics find diagrams most profitable, because all necessary reasoning is iconic, as said. As icons exhibiting the logical connexions among relations, diagrams allow for passing from simultaneity to sequentiality. Peirce's distinction between *theorematic* and *corollarial* deductions is understandable in this context: a theorematic deduction consists in adding elements to the diagram to see what would result of such modification. It is thus a creative abductive experimentation upon the diagram, "the heuretic part of mathematical procedure" [6: 49]. In corollarial deduction, the procedure starts from the observation of a diagram such as it is, without any modification, to affirm the conclusion. The conclusion, therefore, is necessarily obtained only without any further adjunction just by logical development of the diagram. Now, for Peirce, "reasoning essentially consists in the observation that where certain relations subsist certain others are found" [7: 164]. The distinction between the two forms of deductive reasoning shows that necessary reasoning is not limited to the strict drawing of consequences, but it is also a constructive activity of formal representations, by means of observing and modifying other such representations. Both mathematics and logic are experimental activities upon signs in general, and diagrams particularly. By studying and experimenting upon diagrams, we come to understand the very *semiotic* nature of mind itself. Thus, Peirce's arguments for iconicity also work for stressing creativity and discovery in mathematical and logical sciences.

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