Operational Modal Parameters Identification Using the ARMAV Model

Heraldo N. Cambraia, Leonardo M. L. Contini and Paulo R. G. Kurka

Abstract Applied system identification is an important issue in science and engineering. Experimental modal analysis is used to describe the dynamical behavior of structures, in general, for a given set of input and output data. This article deals with multidimensional modal parameters identification valid for output-only data—operational modal analysis (OMA). This approach is interesting when the input is not known or difficult to be measured. A linear, time-invariant and finite dimensional mechanical system is considered, which is described mathematically by an autoregressive-moving-average-vector (ARMAV) model, excited by unknown operating forces assumed to be a white Gaussian process—a persistent excitation. The focus of the study is, both, theoretical and practical aspects, of the use of the ARMAV model in OMA. Specifically, it discusses the need of using an output-vector as reference for output-only parameters identification scheme. The model order is identified by inspection of the most significant singular values of a block Hankel matrix derived directly from the formulation of the model. The AR parameters matrices of the ARMAV model, contained in a companion matrix, are determined via least-squares technique. Natural frequencies, damping factors and modal shapes are identified by means of eigenvalues and eigenvectors of that companion matrix. Examples using computational simulated data are presented.

Keywords Operational modal analysis · ARMAV · Least squares approach

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1 Introduction

Mathematical modeling is an analytical approach used to describe the dynamic behavior of a natural phenomenon based on physical laws. System identification is an experimental approach, where parametric models are fitted from measured data [1]. Both approaches are important in system analysis, design and control problems.

Modal parameters can be used in analysis, modal updating via finite elements, damage detection and control. Modal parameters identification techniques, in the time domain, are classically based on the information contained in the impulse response functions (IRF) or in the input-output relationship [2]. In general terms, a modal identification test is conducted under certain laboratory conditions, where the structure is fixed to a test bench and hammers or actuators are used to produce controlled types of input forces, which are required to match a linear time-invariant mathematical model, covering a certain frequency range of interest. However, in many applications, the real operating conditions may differ significantly from those applied during the modal tests, where the input forces are not known, or just impossible to be measured. Parameters identification based on the knowledge of output-only responses, without using excitation information, is known as operational modal analysis (OMA) [3]. The subject is of actual scientific and industrial interest in mechanical and civil engineering opening a way for damage detection and structural health analysis [1, 4–6].

OMA is present in several practical engineering applications. Lardies and Ta [5] have used OMA to assess the structural health and damage detection of stay cables in cable stayed bridges.

Vu et al. [6] proposed a method for the automatic identification procedure to discriminate physical modes from spurious ones using a multivariate autoregressive (AR) model whose parameters are estimated via a least squares (LS) method. Zaghbani and Songmene [7] proposed a methodology based on OMA to compare the modal parameters of machine tools, demonstrating how OMA can be industrially exploited. Rainieri and Fabbrocino [8] present a literature review on automated operational modal-based damage detection for civil engineering structures. Ramos et al. [9] performed structural identification of monuments in Portugal by OMA to assess damage by means of vibration signature.

According to Peeters and Roeck [10], there are many methods used to perform the OMA parameters identification. Formally, for a completely unknown input, it can be assumed that the system is excited by a white Gaussian process known as a persistent excitation. A multivariate linear time-invariant autoregressive-moving-average-vector (ARMAV) model can be used to fit the data, adopting a least squares, maximum likelihood or prediction method as optimization criterion to calculate the model's parameters [1, 3]. Maximum likelihood optimization procedure leads to a highly non-linear minimization problem in order to calculate the parameters of the model. The solution of such a problem has a very high computational cost, especially for the multivariable parameters case.

The focus of the present paper is on, both, theoretical and practical aspects of the use of the ARMAV model in OMA. Despite OMA parameters identification is a well documented subject, some problems remains to be studied. Specifically, the need of using a output vector as reference for an output only parameters identification scheme. Practical aspect consists in the computational implementation of an ARMAV model identification algorithm.

In the present technique, the OMA parameters are identified from eigen decomposition of a companion matrix that contains the AR coefficients of the ARMAV model obtained via least squares optimization of a block Hankel matrix formed by correlation matrices between output measured data. The problem with the least squares approach is the adopting an initial over parametrization of the model order resulting in a number of spurious numerical modes that must be separated from the true modes of the system. The correct order of the ARMAV model is identified via inspection of the more significant singular values of the block Hankel matrix above mentioned, using singular value decomposition (SVD).

The performance of the presented technique is demonstrated using data generated by means of computational simulation. Impulse responses (which have a type of self-reference given by their impulsive force) and input-output data without using excitation information are considered.

The paper is organized as follows: Next section present the multivariate ARMAV model. The algorithm is then introduced. An application based on simulation using data from mechanical system is discussed. Finally, it is brings the main conclusions of the work.

2 The ARMAV Model

The autoregressive-moving-average-vector (ARMAV) model is largely used in multivariate system identification [3]. ARMAV model can represent a multivariate time series from a linear time-invariant dynamical system by means of a multivariate difference equation as,

$$\mathbf{y}(k+p) - \sum_{i=1}^{p} \boldsymbol{\alpha}_i \mathbf{y}(k+p-i) = \sum_{i=1}^{q} \boldsymbol{\beta}_i \mathbf{e}(k+q-i) \tag{1}$$

where $\mathbf{y}(k) = \{y_1(k)\ \ldots\ y_m(k)\}^T$ are the $m \times 1$ vectors representing the measurements of m outputs variables of the system at discrete time $k\Delta t$, with the superscript "T" denoting vector transposition. The vector $\mathbf{e}(k) = \{e_1(k)\ \ldots\ e_m(k)\}^T$ is a non-observable stochastic $m \times 1$ vector process of with zero mean and nonsingular $m \times m$ covariance matrix Σ, representing the extraneous noise contained in the measurements. The limits p and q represent, respectively, the orders of the autoregressive (AR) and moving-average (MA) matrix parameters. The generic scalar elements $\boldsymbol{\alpha}_i$'s and $\boldsymbol{\beta}_i$'s are, respectively, the p AR and the q MA parameter matrices of dimension $m \times m$.

Multivariable ARMAV model described by Eq. (1) can be converted to following first order difference equations as,

$$\mathbf{Y}(k+1) = \boldsymbol{\alpha}\,\mathbf{Y}(k) + \boldsymbol{\beta}\,\mathbf{E}(k) \tag{2}$$

where the $mp \times 1$ vectors are,

$$\mathbf{Y}(k) = \left\{ \mathbf{y}^T(k) \quad \mathbf{y}^T(k+1) \quad \cdots \quad \mathbf{y}^T(k+p-1) \right\}^T \tag{3}$$

$$\mathbf{E}(k) = \left\{ \mathbf{e}^T(k) \quad \mathbf{e}^T(k+1) \quad \cdots \quad \mathbf{e}^T(k+q-1) \right\}^T \tag{4}$$

and the AR parameters are contained in the following $mp \times mp$ companion matrix as,

$$\boldsymbol{\alpha} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \boldsymbol{\alpha}_p & \boldsymbol{\alpha}_{p-1} & \boldsymbol{\alpha}_{p-2} & \cdots & \boldsymbol{\alpha}_1 \end{bmatrix} \tag{5}$$

and the MA parameters are contained in the $mp \times mq$ matrix as,

$$\boldsymbol{\beta} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\beta}_q & \boldsymbol{\beta}_{q-1} & \boldsymbol{\beta}_{q-2} & \cdots & \boldsymbol{\beta}_1 \end{bmatrix} \tag{6}$$

Let's define the following vectors,

$$\mathbf{Y}^{fut}(k) = \left\{ \mathbf{y}^T(k) \quad \mathbf{y}^T(k+1) \quad \cdots \quad \mathbf{y}^T(k+p-1) \right\}^T \tag{7}$$

$$\mathbf{Y}^{pas}(k) = \left\{ \mathbf{y}^T(k) \quad \mathbf{y}^T(k-1) \quad \cdots \quad \mathbf{y}^T(k-s+1) \right\}^T \tag{8}$$

$$\mathbf{E}^{fut}(k) = \left\{ \mathbf{e}^T(k) \quad \mathbf{e}^T(k+1) \quad \cdots \quad \mathbf{e}^T(k+p-1) \right\}^T \tag{9}$$

where $\mathbf{Y}^{fut}$ and $\mathbf{Y}^{pas}$ are, respectively, $mp \times 1$ and $ms \times 1$ vectors and $\mathbf{E}^{fut}$ is $mp \times 1$, with the superscripts *fut* and *pas* denoting, respectively, future and past data.

Now, for a quantity of measured data of N_p points, post-multiplying Eq. (2) by $\mathbf{Y}^{pas\,T}(k-1)$ and taking the expectation values and assuming that the process $\mathbf{E}^{fut}$ and $\mathbf{Y}^{pas}$ are uncorrelated, i.e., $E\left[\mathbf{E}^{fut}(k)\mathbf{Y}^{pasT}(k-1)\right] = \mathbf{0}$, results in,

$$E\left[\mathbf{Y}^{fut}(k+1)\ \mathbf{Y}^{pasT}(k-1)\right] = \boldsymbol{\alpha}\ E\left[\mathbf{Y}^{fut}(k)\ \mathbf{Y}^{pasT}(k-1)\right] \tag{10}$$

where E means the expectation operation.

Let's define the following matrices,

$$\mathbf{H}^{(1)} = E\left[\mathbf{Y}^{fut}(k)\ \mathbf{Y}^{pasT}(k-1)\right] = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 & \cdots & \mathbf{R}_s \\ \mathbf{R}_2 & \mathbf{R}_3 & \cdots & \mathbf{R}_{s+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_p & \mathbf{R}_{p+1} & \cdots & \mathbf{R}_{p+s-1} \end{bmatrix} \tag{11}$$

and

$$\mathbf{H}^{(2)} = E\left[\mathbf{Y}^{fut}(k+1)\ \mathbf{Y}^{pas^T}(k-1)\right] = \begin{bmatrix} \mathbf{R}_2 & \mathbf{R}_3 & \cdots & \mathbf{R}_{s+1} \\ \mathbf{R}_3 & \mathbf{R}_4 & \cdots & \mathbf{R}_{s+2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{p+1} & \mathbf{R}_{p+2} & \cdots & \mathbf{R}_{p+s} \end{bmatrix} \tag{12}$$

as block Hankel matrices of dimension $mp \times mr$ formed by $m \times m$ covariance matrices defined as,

$$\mathbf{R}_i = E\left[\mathbf{y}(k)\ \mathbf{y}^T(k-i)\right] \tag{13}$$

where i means the correspondent lag of $\mathbf{R}_i$, for a quantity of lags used to build the matrices $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$ equal to $N_{lags} = p + s$.

Equation (10) can be rewritten as,

$$\mathbf{H}^{(2)} = \boldsymbol{\alpha}\ \mathbf{H}^{(1)} \tag{14}$$

Assuming matrix $\mathbf{H}^{(1)}$ to be nonsingular, it follows that the companion matrix $\boldsymbol{\alpha}$ can be calculated by solving the overdetermined system of linear equation as,

$$\boldsymbol{\alpha} = \mathbf{H}^{(2)}\mathbf{H}^{(1)T}\left(\mathbf{H}^{(1)}\mathbf{H}^{(1)T}\right)^{-1} = \mathbf{H}^{(2)}\mathbf{H}^{(1)+} \tag{15}$$

where $\mathbf{H}^{(1)+} = \mathbf{H}^{(1)T}\left(\mathbf{H}^{(1)}\mathbf{H}^{(1)T}\right)^{-1}$ denotes the Moore-Penrose pseudo inverse of $\mathbf{H}^{(1)}$.

3 Reference-Vectors for OMA Identification Scheme

Theoretically, in classical modal analysis, the impulsive or white Gaussian forces, used as excitation for, respectively, IRF's or input-output modal tests, have constant spectrum. These signals work as a type of reference in the modal parameters identification, for a certain frequency range of interest. However, in OMA parameters identification, where the input forces are not known, it is important to define some coordinates of reference to calculate the modal parameters of the system. The output vector of dimension $m \times 1$ is defined as,

$$\mathbf{y}(k) = \begin{Bmatrix} \mathbf{y}_r(k) \\ \mathbf{y}_{nr}(k) \end{Bmatrix} \tag{16}$$

where $\mathbf{y}_r(k)$ is the reference-output vector of dimension $r \times 1$. The vector $\mathbf{y}_{nr}(k)$ of dimension $(m-r) \times 1$ represents the part of non-referenced of the output vector $\mathbf{y}(k)$. The relation between $\mathbf{y}_r(k)$ and $\mathbf{y}_{nr}(k)$ is given by,

$$\mathbf{y}_r(k) = \mathbf{L}\,\mathbf{y}(k) \tag{17}$$

with $\mathbf{L} = [\,\mathbf{I}_r \quad \mathbf{0}\,]$ of dimension $r \times m$.

In OMA parameters identification, the non-referenced covariance matrices defined by Eq. (13) must be substituted by referenced-covariance matrices between the complete output vector $\mathbf{y}(k)$ and the reference-output-vector $\mathbf{y}_r(k)$ defined as,

$$\mathbf{R}_i^r = E\left[\mathbf{y}(k)\ \mathbf{y}_r^T(k-i)\right] = \mathbf{R}_i\,\mathbf{L}^T = E\left[\mathbf{y}(k)\ \mathbf{y}^T(k-i)\right]\mathbf{L}^T \tag{18}$$

For the example, in the case of only one reference as the jth variable $y_j(k)$, the referenced-covariance matrix $\mathbf{R}_i^r$ becomes,

$$\mathbf{R}_i^j = \mathbf{R}_i\mathbf{L}^T = \begin{bmatrix} \mathbf{R}_i(1,1) & \cdots & \mathbf{R}_i(1,j-1) & \mathbf{R}_i(1,j) & \cdots & \mathbf{R}_i(1,m) \\ \mathbf{R}_i(2,1) & \cdots & \mathbf{R}_i(2,j-1) & \mathbf{R}_i(2,j) & \cdots & \mathbf{R}_i(2,m) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_i(m,1) & \cdots & \mathbf{R}_i(m,j-1) & \mathbf{R}_i(m,j) & \cdots & \mathbf{R}_i(m,m) \end{bmatrix} \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 0 \end{Bmatrix} = \begin{Bmatrix} \mathbf{R}_i(1,j) \\ \mathbf{R}_i(2,j) \\ \vdots \\ \mathbf{R}_i(m,j) \end{Bmatrix} \tag{19}$$

The above equation shows how referenced covariance matrix $\mathbf{R}_i^r$ can be obtained from non-reference covariance matrix $\mathbf{R}_i$.

4 Modal Parameters Identification

The input-output relationship, based on Eq. (1), can be written as,

$$\mathbf{y}(k+p) - \sum_{i=1}^{p} \boldsymbol{\alpha}_i \mathbf{y}(k+p-i) = \mathbf{u}(k) \tag{20}$$

where $\mathbf{u}(k)$ denotes the $m \times 1$ input-vector related to the external forces applied to the system.

In order to obtain a scheme to estimate the modal parameters of the mechanical system, the z-transform is applied to both sides of Eq. (20) giving the following equations,

$$\left[z^p\mathbf{I} - z^{p-1}\boldsymbol{\alpha}_1 - z^{p-2}\boldsymbol{\alpha}_2 - \cdots - \boldsymbol{\alpha}_p\right]\mathbf{Y}(z) = \mathbf{U}(z) \tag{21}$$

$$\left[z^p\mathbf{I} - z^{p-1}\boldsymbol{\alpha}_1 - z^{p-2}\boldsymbol{\alpha}_2 - \cdots - z\,\boldsymbol{\alpha}_{p-1} - \boldsymbol{\alpha}_p\right]\mathbf{H}(z) = \mathbf{I} \tag{22}$$

where $\mathbf{Y}(z)$ and $\mathbf{U}(z)$ are, respectively, the z-transform of $\mathbf{y}(k)$ e $\mathbf{u}(k)$ and $\mathbf{H}(z)$ is the $m \times m$ transfer function between $\mathbf{Y}(z)$ and $\mathbf{U}(z)$ in the z-domain.

Equation (22) can be re-written in terms of a companion matrix $\boldsymbol{\alpha}$ as,

$$\left\{\begin{bmatrix} z\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & z\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & z\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & z\mathbf{I} \end{bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{I} \\ \boldsymbol{\alpha}_p & \boldsymbol{\alpha}_{p-1} & \cdots & \boldsymbol{\alpha}_2 & \boldsymbol{\alpha}_1 \end{bmatrix}\right\}\left\{\begin{matrix} \mathbf{I} \\ z\mathbf{I} \\ \vdots \\ z^{p-2}\mathbf{I} \\ z^{p-1}\mathbf{I} \end{matrix}\right\}\mathbf{H}(z) = \left\{\begin{matrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{I} \end{matrix}\right\} \tag{23}$$

The above equation can be written in a more compactly form as,

$$[z\mathbf{I} - \boldsymbol{\alpha}]\tilde{\mathbf{I}}_z\mathbf{H}(z) = \tilde{\mathbf{B}}(z) \tag{24}$$

where

$$\tilde{\mathbf{I}}_z = \left\{\begin{matrix} \mathbf{I} \\ z\mathbf{I} \\ \vdots \\ z^{p-2}\mathbf{I} \\ z^{p-1}\mathbf{I} \end{matrix}\right\} \quad \text{and} \quad \tilde{\mathbf{B}}(z) = \left\{\begin{matrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{I} \end{matrix}\right\} \tag{25}$$

are $pm \times m$ matrices.

The eigenvalue problem of companion matrix $\boldsymbol{\alpha}$ can be written, from Eq. (24), as,

$$\left[z_j\mathbf{I} - \boldsymbol{\alpha}\right]\tilde{\varphi}_j = \mathbf{0} \tag{26}$$

which leads to the calculation of a quantity of mp z-poles z_j's, where $mp - n$ of then are computational poles and may be separated from the identification process.

The minimal order of the model n can be identified by inspection of the more significants singular values of matrix $\mathbf{H}^{(1)}$.

In general, mechanical systems are modeled in continuous-time in nature using for example Newton's second law. The relation between the continuous-time poles λ_l's and the discrete-time poles z_l's is given by,

$$\lambda_j = \frac{\ln(z_j)}{\Delta t} \quad \text{with} \quad j = 1:n \tag{27}$$

where Δt is the sampling time interval.

The natural frequencies ω_j and modal damping ξ_j, for the case of underdamped vibratory systems, are estimated from λ_j, respectively, according to,

$$\omega_j = |\lambda_j| \quad \text{and} \quad \varsigma_j = \frac{\text{Re}(\lambda_j)}{\omega_j} \tag{28}$$

where symbol || denotes absolute value.

Finally, the mp eigenvectors ϕ_j of the companion matrix $\boldsymbol{\alpha}$, from Eq. (26), can be used to estimate the mode-shapes ϕ_j of the mechanical system using the following relation [2],

$$\phi_j = \mathbf{I}_z \phi_j \tag{29}$$

where ϕ_j is identified as,

$$\phi_j = (\mathbf{I}_z^T \mathbf{I}_z)^{-1} \mathbf{I}_z^T \phi_j \tag{30}$$

where ϕ_j is a $pm \times 1$ vector and ϕ_j is a $m \times 1$ mode-shape vector.

5 The ARMAV Algorithm

The ARMAV algorithm for OMA parameters identification consists in the following steps:

(1) Calculation of the matrices $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$, as Eqs. (11) and (12), for a total of a number of lags equal to $N_{lags} = p + s$, using referenced-covariance-matrices $\mathbf{R}_i^r$ obtained by Eq. (18) for a quantity of N_p measured data,

(2) Calculation of the companion matrix $\boldsymbol{\alpha}$ that contains the matrices parameters of the ARMAV model by Eq. (15),

(3) Calculation of a quantity of mp eigenvalues z_i's and the associated mp eigenvectors ϕ_i of the companion matrix $\boldsymbol{\alpha}$, with $i = 1, \ldots, mp$. The poles mp λ_i's of the mechanical system described in continuous time are calculated according to Eq. (27). For underdamped systems, the natural frequencies ω_j and damping factors ξ_j are calculated by Eq. (28),

(4) The mode-shape vectors and ϕ_j are obtained using Eq. (30),
(5) The minimum order of the system n can be obtained by means of inspection of the number of repeated poles identified by the former step or by the number of significant singular values of Hankel block matriz $\mathbf{H}^{(1)}$.

6 Examples of Application

In order to show the capabilities of the present OMA parameters identification technique using the ARMAV model, a SIMO numerical experiment is conducted. The collection of impulse responses and output-only data are obtained by numerical simulation from the five degrees of freedom mass-spring oscillator without damping, as shown in Fig. 1.

6.1 OMA Parameters Identification Using IRF's

Data of a SIMO, 1-input and 5-outputs, test are then numerically simulated with the unit impulse force acting in block 1. In the present test, it is adopted a number of 2000 data samples for each term $h_{ij}(k)$ for a quantity of 500 lags to build the covariance matrix $\mathbf{R}_i^1$ and the order of AR part of the model $p = 4$, resulting in a pair of matrices $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$ both of dimension 20×496. The time sampling interval Δt used is 0.025 s. Table 1 shows the exact and identified modal parameters.

The order of the system is identified to be equal to 10 by inspection of most significant singular values of matrix $\mathbf{H}^{(1)}$ is shown in Fig. (2). Based on this criterion, it is adopted $p = 2$ in the identification process resulting the five modes and modal identified parameters present in the Table 1.

Figure 3 shows the five identified mode shapes associated to five natural frequencies as compared to exact modes derived from numerical simulation.

Theoretically, it is important to note that the modal parameters identification using IRF's data, using the present method, does not require the use of reference-vectors in the identification scheme, as discussed in previous section. This type of data has their own references due to the impulsive forces as integrant part of calculation of IRF's.

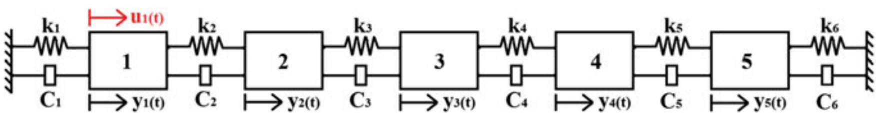

Fig. 1 Five degrees of freedom oscillator system

Table 1 Exact and identified modal parameters

Mode number	Exact natural frequency (Hz)	Identified natural frequency (Hz)	Error (%)
1	5.2105	5.2105	0
2	10.0658	10.0658	0
3	14.2353	14.2353	0
4	17.4346	17.4346	0
5	19.4457	19.4459	0.001

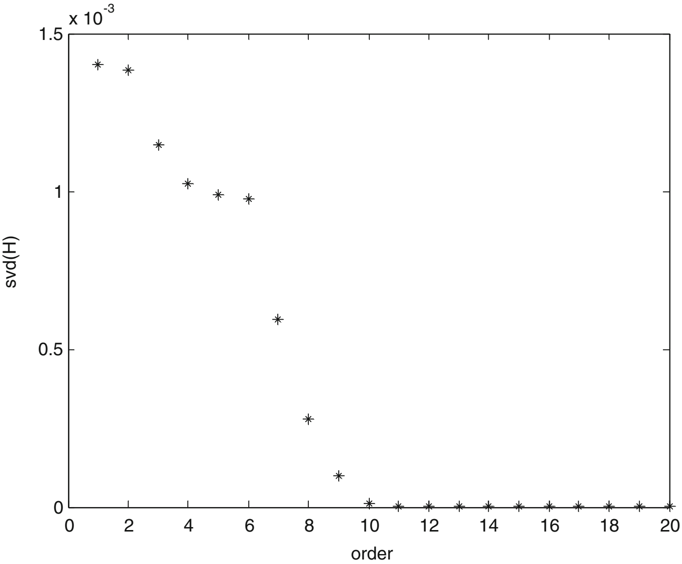

Fig. 2 Singular values of matrix $\mathbf{H^1}$

6.2 OMA Parameters Identification Using Output-Only Data

A SIMO 1-input and 5-outputs test is shown. Adopting as input $u_1(k)$, a mean zero white Gaussian noise signal with amplitude equal to 10 N, that acts on the blocks 1, the responses $\mathbf{y}(k)$'s are obtained by evaluating the following sum of convolution,

$$y_i(k) = \sum_{s=0}^{N_p-1} h_{i1}(s)\; u_1(k-s) \quad i = 1, \ldots, 5 \tag{31}$$

In the present test, it is adopted a number of 2000 data samples for each term $y_i(k)$ for a quantity of 500 lags to build the covariance matrix $\mathbf{R}_i^1$ and the order of

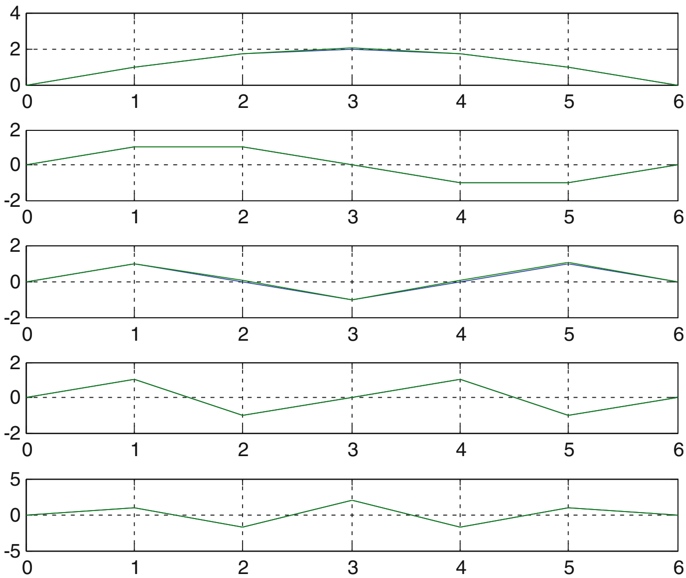

Fig. 3 Exact (green) and identified (blue) mode shapes

AR parameters of the model $p = 4$, resulting in a pair of matrices $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$ both of dimension 20×496. The time sampling interval Δt used is 0.025 s.

The order of the system is identified to be equal to 10 by inspection of most significant singular values of matrix $\mathbf{H}^{(1)}$ shown in Fig. 4. Based on this criterion,

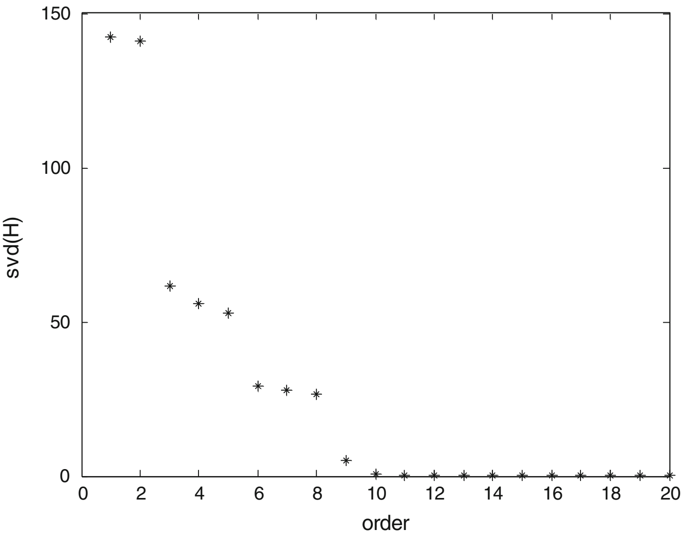

Fig. 4 Singular values of matrix $\mathbf{H}^{\mathbf{1}}$

Table 2 Exact and identified modal parameters

Mode number	Exact natural frequency (Hz)	Identified natural frequency (Hz)	Error (%)
1	5.2105	5.2049	0.1074
2	10.0658	10.0713	0.0546
3	14.2353	14.2413	0.0415
4	17.4346	17.4381	0.0207
5	19.4457	19.4479	0.0113

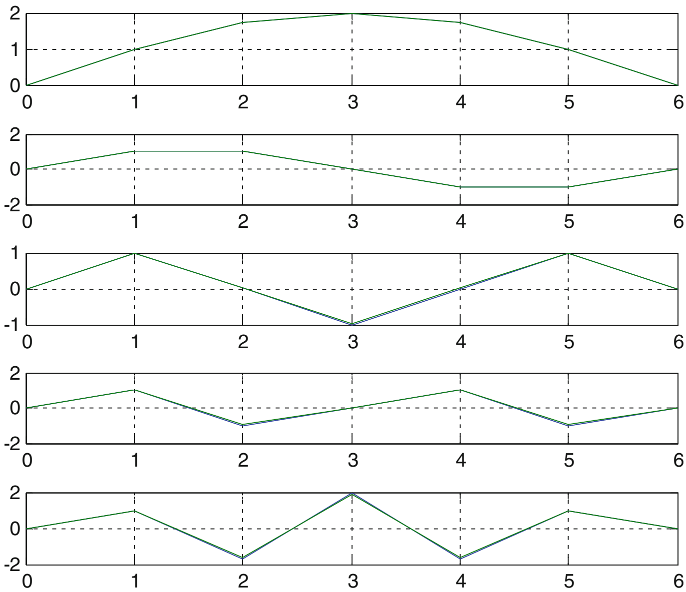

Fig. 5 Exact (green) and identified (blue) mode shapes

the order of AR parameters is changed to $p = 2$ resulting in a total of five identified modes. The exact and identified parameters present in the Table 2.

Figure 5 shows the five identified mode shapes associated to five first natural frequencies as compared to exact modes derived from numerical simulation.

7 Conclusion

OMA is a very attractive field in mechanical and civil engineering and several techniques has been proposed in the literature. The present OMA parameter identification, method based on ARMAV model, is very simple, has robust numerical properties and relatively low computational cost, using only linear algebra manipulations. The tests based on numerical simulated data show that the presented method can be regarded as a way to perform the modal identification—natural frequencies, damping factors and associated mode shapes. The need of an output-vector as reference in the output-only parameters identification is highlighted. The present paper encourages a future implementation of the present algorithm using a more precise (accurate) optimization technique for the parameters identification using, for example, the maximum likelihood technique.

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