Chapter 13
Features of Spin Transport in Magnetic Nanostructures with Nonmagnetic Metal Layers

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13.1 Introduction

Coupling between spin currents and localized magnetic moments in magnet (M)/normal (nonmagnetic) metal (N)-based multilayer magnetic nanostructures constitutes the basis of the mutual control between electric current and static or dynamic magnetic states. Herewith, magnetic layers include magnets with the exchange interactions both ferromagnetic (F) and antiferromagnetic (AF) types (e.g., ferrimagnetic compounds like YIG, Gd_3 Fe_5O_{12} and AFs Fe_3O_4, $NiFe_2O_4$, NiO [1, 2]), and normal metals are, usually, heavy metals with strong spin-orbit coupling (e.g., Pt, Ta, W). The mentioned interconnection in these magnetic nanostructures occurs via the interface scattering of the spin-polarized current and its *s-d* exchange interaction with static or dynamic magnetic states [3–6]. The impact of the spin current on the magnetic states is manifested through the spin-transfer torque, and the impact of the localized magnetic momentum on the spin current is manifested via the spin-dependent interface scattering accompanied by magnetoresistance effect. The spin polarization can be induced by effective bias fields of different origins including fields caused by an exchange interaction and the strong spin-orbit coupling. The entire spin-coherent region is limited in size by spin-flip relaxation processes.

In the case of static magnetic states, the mutual influence of the spin current and magnetic ordering can be manifested as the magnetoresistance effect of the dependence of the spin current on the magnetization orientation in the magnetic layer and, vice versa, the dependence of the latter on the spin current [5, 6]. Such effects can constitute the base for magnetic writing techniques in nonvolatile memory

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technologies such as MRAM [7] and racetrack memories [8]. They also include the giant magnetoresistance (GMR) effect in metallic magnetic multilayers, which was commercially utilized in high-end magnetic recording media [9]. Obtaining the mentioned multilayer magnetic nanostructures with properties of electric-controlled magnetic switching and the magnetic-controlled spin current involves the description of features of the spin transport in magnetic heterogeneous nanostructures allowing for the compatibility conditions at the interfaces [4, 10]. This is usually solved within the Landauer-Büttiker formalism [11] and, more rigorously, using the nonequilibrium Keldysh-Green functions [12, 13].

In the case of the dynamic magnetic states, their interconnection with the control spin current is affected by the magnetic precession-induced spin pumping and the spin accumulation in the normal-metal layer at the interface [3, 10]. The action of the spin currents on the magnetic dynamics via the spin-transfer torque and the reciprocal process of spin pumping result in the effect of controlled magnetic auto-oscillations [14]. The magnetic dynamic damping is related to the spin-pumping effect at the M|N interface that can be compensated by the spin-transfer torque from the spin current of the converted input current. This spin transfer is governed by the reflection and transmission matrices of the system, analogous to the scattering theory of transport and interlayer exchange coupling. Due to interfacial processes, M|N coupling becomes important in the limit of ultrathin ($\leq$ 10 nm) magnetic films and can lead to a sizable enhancement of the damping constant.

The abovementioned coupling effects at interfaces can occur in the magnetic nanostructures with both ferromagnetic (*F*) and antiferromagnetic (*AF*) exchange interactions, which are realized in ferro-, ferri-, and antiferromagnetic materials. Normal metal layers are medium for the spin currents, which can be converted from the control charge current by the spin-orbit interaction, especially the spin Hall and the spin-orbit Rashba [15, 16] effects.

The paper is organized as follows. In Sect. l, the spin-dependent transport in the F/N-based magnetic nanostructure is studied for the static magnetization. In the modified Stoner model with potential barrier dependent on the physical parameters including the magnetization directions, the chemical potentials of the layers, and the contact conductances, the parametrically dependent scattering of spin-polarized current is investigated. The mentioned parameters are determined by the spin-polarized kinetic equations in the framework of the Keldysh-Green function approach. It is considered both in single and composite F/N-based magnetic nanostructures. In Sect. 2, features of the interconnection between magnetization dynamics and the spin currents are studied in the F/N-based nanostructures. The process of the magnetization precession-induced pumping spin current in the nonmagnetic layers is considered as the result of the parametric time dependence of the interfacial scattering with the precession as the parameter. It is shown that the spin pumping slows down the precession corresponding to an enhanced Gilbert damping constant in the Landau-Lifshitz-Gilbert model. The spin current related to the spin pumping, which flows back into the ferromagnetic layers and driven by the accumulated spins in the normal-metal layers, is also discussed.

13.2 Spin Transport in the Case of Static Magnetic Field

13.2.1 Features of Spin-Dependent Electric Current in the F/N Bilayers

Characteristic features of the spin-dependent transport and the interfacial scattering in multilayer magnetic nanostructures with nonmagnetic metal (N) interlayers are manifested in the model bilayer nanostructure F/N depicted in Fig. 13.1.

These features are related to the conditions under which long-range spin effects are observable in normal metals. Spins injected into a normal-metal layer relax due to unavoidable spin-flip processes

Naturally, the dwell time on the layer must be shorter than the spin-flip relaxation time in order to observe nonlocality in the electron transport. For a simple F/N double heterostructure (F/N/F) with antiparallel magnetizations, the condition can be described as follows [17]. The spin current into the normal-metal layer is roughly proportional to the particle current, $e(ds/dt)_{tr} \sim I = V/R$, where s is the number of excess spins on the normal-metal layer, V is the voltage difference between the two reservoirs coupled to the normal-metal layer, and R is the F|N contact resistance. When the layer is smaller than the spin-diffusion length, the spin-relaxation rate is $e(ds/dt)_{rel} = -s/\tau_{sf}$, where τ_{sf} denotes the spin-relaxation time on the layer. (Otherwise, this simple approach breaks down since the spatial dependence of the spin distribution in the normal metal should be taken into account [18].) The number of spins on the normal-metal layer is equivalent to a nonequilibrium chemical potential difference $\Delta\mu = s\delta$ in terms of the energy level spacing δ(the inverse density of states) (more generally the relation between $\Delta\mu$ and s is determined by the spin susceptibility). The spin accumulation on the normal-metal layer significantly affects the transport properties when the nonequilibrium chemical potential difference is of the same order of magnitude or larger than the applied source-drain voltage, $\Delta\mu > eV$ or $\delta\tau_{sf}/\hbar > R/R_K$, where $R_K = e^2/h$ is the quantum resistance. Thus, spin accumulation is only relevant for sufficiently small normal-metal layers and/or sufficiently long spin-accumulation times and/or good contact conductances.

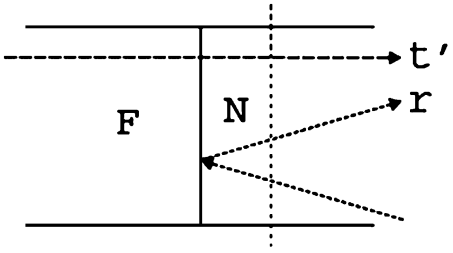

Fig. 13.1 A contact between a ferromagnetic (F) and a normal (N) metal layers. At the normal-metal side, the current is denoted as the dotted line. The transmission coefficient from the ferromagnet to the normal metal is t', and the reflection matrix from the normal metal to the normal metal is r

The spin-dependent current in the model structure F/N (Fig. 13.1) is expressed via the 2×2 distribution matrix $f(\varepsilon)$ in spin space at a given energy ε in the layer. The external reservoirs are assumed to be in local equilibrium so that the distribution matrix is diagonal in spin space and attains its local equilibrium value $f = 1f(\varepsilon, \mu_\alpha)$, where 1 is the unit matrix, $f(\varepsilon, \mu_\alpha)$ is the Fermi-Dirac distribution function, and $f(\varepsilon, \mu_\alpha)$ is the local chemical potential in reservoir α. The direction of the magnetization of the ferromagnetic layers is denoted by the unit vector $\boldsymbol{m}_\alpha$.

The 2×2 nonequilibrium distribution matrices in the layers in the stationary state are uniquely determined by current conservation

$$\sum_\alpha I_{\alpha\beta} = \left(\frac{\partial f_\beta}{\partial t}\right)_{rel}, \tag{13.1}$$

where $I_{\alpha\beta}$ denotes the 2×2 current in spin space from layer (or reservoir) α to layer (or reservoir) β and the term on the right-hand side describes spin relaxation in the normal layer. The right-hand side of Eq. (13.1) can be set to zero when the spin current in the layer is conserved, i.e., when an electron spends much less time on the layer than the spin-flip relaxation time τ_{sf}. If the size of the layer in the transport direction is smaller than the spin-flip diffusion length $l_{sf} = \sqrt{D\tau_{sf}}$, where D is the diffusion coefficient, then the spin relaxation in the layer can be introduced as $(\partial f^N/\partial t)_{rel} = (1\mathrm{Tr}(f^N)/2 - f^N)/\tau_{sf}$.

13.2.2 Passing the Electric Current Through the F|N Contact

The dependence of the spin current through the F|N interface on the distribution functions in the bilayer magnetic nanostructure F/N is described on the base of the method of the nonequilibrium Green function theory and Keldysh formalism. The Green functions are determined by the Kadanoff-Baym equations, representing the quantum kinetic equations for one-particle propagators. The electron subsystem of the bilayer F/N nanostructure subjected to an external field φ in the modified Stoner model is described by the Hamiltonian, which in the representation of the field operators of annihilation (ψ) and creation ($\psi^\dagger$) is given by

$$H(t) = \int dx \psi^\dagger(x) h(x,t) \psi(x) + \frac{1}{2} \times \int dx_1 dx_2 \psi^\dagger(x_1) \psi^\dagger(x_2) w(x_1, x_2) \psi(x_2) \psi(x_1), \tag{13.2}$$

where

$$h(r,t) = -\frac{1}{2}\nabla^2 + V^p(r) + V^s(r) + \varphi(r,t) \tag{13.2}$$

Here $x = (r, s)$ is the collective space-spin coordinate, the one-particle potential (13.2) contains spin-independent ($V^p(r)$) and spin-dependent ($V^s(r)$) parts of the barrier potentials in the interface F|N, and $w(r_1, r_2)$ is the operator of the two-particle interaction. The spin-dependent potential is $V^s(r) = (\sigma \cdot \boldsymbol{m})W^s(r)$, where σ is the Pauli matrix and $\boldsymbol{m}$ is the unit vector of the magnetization existing only in the ferromagnetic layer (F) and vanishing in the normal-metal layer (N).

The physical properties of the system are described by the one-particle nonequilibrium Green function, i.e., the expectation value of the time contour-ordered product of creation and annihilation field operators $\psi^\dagger(i)$ and $\psi(i)$ ($i = (1, 2, \ldots)$), respectively, with the collective coordinate, $i = (x_i, z_i)$, where $x_i = (r, s)$ is position-spin component and z_i is the time component. The latter takes real and imaginary values in intervals $[t_0, t]$ and $[t_0, t_0 - i\beta]$ (β is inverse temperature), respectively. The imaginary time interval corresponds to the equilibrium state. The conjugation of the mentioned time intervals forms the so-called Keldysh contour C consisting of forward and backward real-time branches and the thermal imaginary track. This nonequilibrium Green function can be presented as [12, 13]

$$G(1,2) = -i\frac{\mathrm{Tr}\left[T_C e^{-i\int_C H(z)\mathrm{d}z\psi(1)}\psi(1)\psi^\dagger(2)\right]}{\mathrm{Tr}\left[T_C e^{-i\int_C H_0(z)\mathrm{d}z\psi(1)}\right]} == -i\left\langle \psi_H(1)\psi_H^\dagger(2)\right\rangle. \quad (13.3)$$

where T_C is contour time-ordering operator on the Keldysh contour C with $H_0(z) = H(z)|_{z\in[t_0, t_0 - i\beta]}$ and the subscript H in the right-hand side denotes the Heisenberg representation. Due to (13.3), the Green function can be presented as

$$G(1,2) = \theta(z_1, z_2)G^>(1,2) + \theta(z_2, z_1)G^<(1,2), \quad (13.4)$$

where $\theta(z_1, z_2)$ is the contour step function equal to 1 or 0 versus z_1 or z_2 and is later on the contour C, and the greater ($G^>$) and lesser ($G^<$) Green functions are determined by the relations

$$G^>(1,2) = -i\left\langle \psi_H(1)\psi_H^\dagger(2)\right\rangle, \quad G^<(1,2) = i\left\langle \psi_H^\dagger(2)\psi_H(1)\right\rangle, \quad (13.5)$$

where an extra sign is introduced due to the interchange of the fermionic operators by the contour-ordering operator T_C. The Green function provides a direct access to observable physical quantities of the system. For example, the equal-time limit gives directly the particle spin density at the space-time point 1

$$\langle n(1)\rangle = \left\langle \psi_H^\dagger(1)\psi_H(1)\right\rangle = -G^<(1, 1^+) \quad (13.6)$$

(a superscript "+" means infinitesimal). The spin current density is determined as

$$I(1) = -\left\{\partial_t G^>(t, t') + \partial_{t'} G^<(t, t')\right\}_{t'=t} \quad (13.7)$$

The Green functions are described by the equation

$$\{i\partial_{z_1} - h(1)\}G(1,2) = \delta(1,2) + \int d3w(1,3)G(1,3;2,3^+) \tag{13.8}$$

following the Schrödinger equation for wave functions. Here, the two-particle Green function

$$G(1,3;2,3^+) = \left\langle \psi_H^\dagger(2)\psi_H(1)\psi_H(3)\psi_H^\dagger(3^+)\psi_H^\dagger(2)\right\rangle \tag{13.9}$$

expresses via the variation derivative with respect to the variation in the infinitesimal external potential ν of the one-particle Green function by the relation [12]

$$G(1,3;2,3^+) = i\langle n(3)\rangle G(1,2) + \frac{\delta G(1,2)}{\delta\nu(3)} \tag{13.10}$$

Consequently, Eq. (13.8) can be represented as self-contained variation derivative equation with the matrix representation

$$\left(L^0 + L^1\right)G = 1, \tag{13.11}$$

where the matrix L^0 is determined by matrix elements

$$L^0(1,2) = \left(i\partial_z - h^*(1) + \int d3w\left(1,3\right)\langle n(3)\rangle\right)\delta_{1,2} \tag{13.12}$$

not containing the variation derivative. The matrix L^1 is determined by matrix elements

$$L^1(1,2) = i\delta_{1,2}\int d3\, w(1,3)\frac{\delta}{\delta\nu(3)} \tag{13.13}$$

which are proportional to the variation derivative.

The expression for the functional derivative

$$\frac{\delta}{\delta\nu(3)}G = \frac{\delta}{\delta\nu(3)}G\left[G^{-1}\right]G \tag{13.14}$$

(the bracket separates the expression experiencing the variation differentiation) allows to represent the operation of the differential matrix L^1 on G via the self-energy matrix Σ not containing functional derivatives:

$$L^1G = \Sigma G, \quad \Sigma = L^1G\left[G^{-1}\right] \tag{13.15}$$

Finally, the matrix Eq. (13.11) reduces to the system

$$\left(L^0 + \Sigma\right)G = 1 \tag{13.16}$$

$$\Sigma = -L^1 G\left[L^0 + \Sigma\right] \tag{13.17}$$

The first-order approximation with respect to the interaction w determines the Hartree-Fock self-energy matrix $\Sigma^{\mathrm{HF}} = -L^1 G[L^0]$ with matrix elements

$$\Sigma^{\mathrm{HF}}(1,2) = -\delta(1,2)\int d3w(1,3)\langle n(3)\rangle + iw(1,2)G(1,2) \tag{13.18}$$

Here the first term describes the classical Hartree potential at 1 produced by the charge density throughout the space, and the second term is the space-nonlocal exchange potential originating from the Pauli exclusion principle and antisymmetry of the wave functions. Due to (13.16) the self-energy matrix in second-order approximation is determined by the equation

$$\Sigma = -L^1 G\left[L^0\right] + L^1 G\left[L^1 G\left[L^0\right]\right] \tag{13.19}$$

with matrix elements of the form

$$\begin{aligned}\Sigma^2 = \Sigma^{\mathrm{HF}}(1,2) + G(1,2)\int d3d4w(1,3)w(2,4)G(4,3)G(3,4^+)\\ -\int d3d4G(1,3)w(1,4)G(3,4)G(4,2)w(3,2)\end{aligned} \tag{13.20}$$

Here the second and third terms describe the correlation and scattering effects, respectively.

In the stationary situation, the Green function is determined via its energy Fourier transform G_E which without the subscript will be considered below. Entering in the given bilayer model *F/N*, the Cartesian coordinate system with the axis x along the interface plane normal and the axes y and z in the interface plane, the Green function can be represented by decomposition into quasi-one-dimensional modes as

$$G_{ss'}(1,1') = \sum_{nm,\,\alpha\beta} G^{\alpha\beta}_{nsms'}\chi^n_s(\rho,x)\chi^{m*}_{s'}\left(\rho',x'\right)e^{i\alpha k^n_s x - i\beta k^m_{s'}x'}, \tag{13.21}$$

where the indices $\alpha, \beta = (+,-)$; the signs "+" and "−" denote right-going (+) and left-going (−) modes, respectively; $\chi^n_s(\rho,x)$ is the transverse wave function; and k^n_s denotes the longitudinal wave vector for an electron in transverse mode n with spin s. Then, from the definition of the current through the Green function, at the spatial independence of the transverse wave function $\chi^n_s(\rho,x)$, it can be obtained the expression

$$I_{ss'}(x) = ie\sum_{n\alpha\beta}\left(\alpha v^n_s - \beta v^m_{s'}\right)G^{\alpha\beta}_{nsms'}(x,x)\int d\rho\chi^n_s(\rho,x)\chi^m_{s'}(\rho,x), \tag{13.22}$$

describing the spin current, where $v^n_s = \hbar k^n_s/m$ is the longitudinal velocity for an electron in transverse mode n with spin s. In a normal metal, the transverse states and the longitudinal momentum are spin-independent, and the spin current simplifies to

$$I_{ss'}(x) = 2ie\sum_{n\alpha} \alpha v_s^n G^{\alpha\alpha}_{nsms'}(x,x), \tag{13.23}$$

which is used in the calculation of the spin current on the normal side of the contact.

Using the representation

$$G^{\alpha\beta}_{nsms'}(x,x') = -i\left(\frac{g^{\alpha\beta}_{nsms'}(x,x')}{\sqrt{v_s^n v_{s'}^m}} + 1\delta_{ss'}\frac{\alpha\delta_{\alpha,\beta}\mathrm{sign}(x\text{-}x')}{v_s^n}\right), \tag{13.24}$$

where the latter term does not contribute to the current on the normal-metal side, whence it follows the expression

$$I_{ss'}(x) = 2e\sum_{n\alpha} \alpha v_s^n g^{\alpha\alpha}_{nsms'}(x,x) \tag{13.25}$$

for the spin current on the normal-metal side.

The complete description of the spin current through the F|N interface involves taking into account the connection between waves propagating to the right (left) on the right-hand side of the contact $\psi_R^+\left(\psi_R^-\right)$ and waves propagating to the right (left) on the left-hand side of the contact $\psi_L^+\left(\psi_L^-\right)$. This is described by the transfer matrix M obeying the relation

$$\begin{pmatrix} \psi_R^+ \\ \psi_R^- \end{pmatrix} = M \begin{pmatrix} \psi_L^+ \\ \psi_L^- \end{pmatrix}, \tag{13.26}$$

which in terms of the transmission (t) and reflection (r) coefficients takes the form

$$M = \begin{pmatrix} t - r'\left(t'\right)^{-1} r & r'(t')^{-1} \\ -(t')^{-1} r & (t')^{-1} \end{pmatrix}. \tag{13.27}$$

Here the transmission and reflection coefficients enter in definition of the scattering matrix

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}, \tag{13.28}$$

where $r_{nm}^{s\sigma}$ is the reflection matrix for incoming states from the left in mode m and spin σ to mode n with spin s and $t_{nm}^{s\sigma}$ is the transmission matrix for incoming states from the left transmitted to outgoing states to the right. In addition, r' is the reflection matrix for incoming states from the right reflected to the right, and t' is the transmission matrix for incoming states from the right transmitted to the left. The Green function to the left ($x = x_2$) of the interface is expressed via the Green function to the right ($x = x_1$) of the interface by the relation

$$g^{\sigma\sigma'}_{nsms'}(x = x_2, x') = \sum_{ls'',\sigma''} M^{\sigma\sigma''}_{nsls''} g^{\sigma''\sigma'}_{ls''ms'}(x = x_1, x'), \tag{13.29}$$

which after redefining, $g_{2(1)} = g(x = x_{2(1)}, x' = x_{2(1)})$, takes the matrix form $g_2 = Mg_1M^{\dagger}$.

In the approximation of isotropic quasi-classical Green functions in nanolayers 1 and 2, $(G_1)^{\alpha\beta}_{nsms'} = \delta_{n,m}\delta^{\alpha\beta}(G_1)^{\alpha\beta}_{ss'}$. In the representation of the retarded (G^R), advanced (G^A), and Keldysh (G^K) Green function, the total Green function has the form $G = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix}$. Here $G_R = -G_A = -1$, $G_{K,\ 1(2)} = h_{1(2)}1$, where the two-dimensional matrix h is related to the nonequilibrium distribution functions $h_{1(2)} = 2(f(\varepsilon)_{1(2)} - 1)$. Herewith

$$g^{-+}_{R,1} = 2r,\ g^{+-}_{R,2} = 2r' \tag{13.30}$$

and

$$g^{--}_{K,1} = t'h_2t'^{\dagger} + h_1r^{\dagger},\ g^{--}_{K,2} = th_1t^{\dagger} + r'h_2r'^{\dagger} \tag{13.31}$$

Inserting the expression (13.31) into (13.25) results in the expression

$$I = \frac{e}{h}\left\{\sum_{mn}\left[t'^{nm}f^F\left(t'^{mn}\right)^+ - \left(Mf^F - \tau^{nm}f^N(\tau^{mn})^+\right)\right]\right\}, \tag{13.32}$$

which describes the current through interface on its normal-metal side. Here $r^{nm}_{ss'}$ is the reflection coefficient for electrons from transverse mode m with spin s' incoming from the normal-metal side reflected to transverse mode n with spin s on the normal-metal side, and $t'^{nm}_{ss'}$ is the transmission coefficient for electrons from transverse mode m with spin s' incoming from the ferromagnet transmitted to transverse mode n with spin s on the normal-metal side.

13.2.3 Spin Parametric Dependence of Spin Current

The relation (13.32) between the current and the distribution functions has a simple form after transforming the spin-quantization axis. Disregarding spin-flip processes in the contacts, the reflection matrix for an incoming electron from the normal metal transforms is

$$r^{nm} = \sum_s u^s r^{nm}_s, \quad u^{\uparrow(\downarrow)} = (1 \pm \sigma \cdot \boldsymbol{m})/2, \tag{13.33}$$

where $r^{nm}_{\uparrow(\downarrow)}$ are the spin-dependent reflection coefficients in the basis where the spin-quantization axis is parallel to the magnetization $\boldsymbol{m}$ in the ferromagnet and σ is a vector of Pauli matrices. Similarly for the transmission matrix $t^{nm}_{\uparrow(\downarrow)}$

$$t'^{nm}\left(t'^{mn}\right)^{+} = \sum_{s} u^{s}\left|t'^{nm}_{s}\right|^{2} \tag{13.34}$$

From the unitarity of the scattering matrix, it follows that the general form of the relation (13.32) reads

$$eI = \sum_{s=\uparrow,\downarrow} G^{s}u^{s}\left(f^{F} - f^{N}\right)u^{s} - G^{\uparrow\downarrow}u^{\uparrow}f^{N}u^{\downarrow} - \left(G^{\uparrow\downarrow}\right)^{*}u^{\downarrow}f^{N}u^{\uparrow}, \tag{13.35}$$

where it is introduced the spin-dependent conductance parameters

$$G^{\uparrow(\downarrow)} = \frac{e^{2}}{h}\left[M - \sum_{nm}\left|r^{nm}_{\uparrow(\downarrow)}\right|^{2}\right] = \frac{e^{2}}{h}\sum_{nm}\left|t^{nm}_{\uparrow(\downarrow)}\right|^{2} \tag{13.36}$$

and the mixing conductance

$$G^{\uparrow\downarrow} = \frac{e^{2}}{h}\left[M - \sum_{nm} r^{nm}_{\uparrow}\left(r^{nm}_{\downarrow}\right)^{*}\right] \tag{13.37}$$

The precession of spins leads to an effective relaxation of spins non-collinear to the local magnetization in ferromagnets, and consequently the distribution function is limited to the form $f^{F} = 1f^{F}_{0} + \sigma \cdot \boldsymbol{m}f^{F}_{s}$. Such a restriction does not appear in the normal-metal layer and f^{N} can be any Hermitian 2×2 matrix.

The relation between the current through a contact and the distributions in the ferromagnetic layer and the normal-metal layer is determined by four parameters, the two real spin-dependent conductances ($G^{\uparrow}$, $G^{\downarrow}$) and the real and imaginary parts of the mixing conductance $G^{\uparrow\downarrow}$. These contact-specific parameters can be obtained by microscopic theory or from experiments. The spin conductances $G^{\uparrow}$ and $G^{\downarrow}$ describe spin transport for a long time [19]. The mixing conductance is relevant for transport between non-collinear ferromagnets. Note that although the mixing conductance is a complex number, the 2×2 current in spin space is Hermitian, and consequently the current and the spin current in any direction given by (13.35) are real numbers. Due to the definitions of the spin-dependent conductances (13.36) and the "mixing" conductance (13.38)

$$2\mathrm{Re}G^{\uparrow\downarrow} = G^{\uparrow} + G^{\downarrow} + \frac{e^{2}}{h}\sum_{nm}\left|r^{nm}_{\uparrow} - r^{nm}_{\downarrow}\right|^{2} \tag{13.38}$$

and consequently, the conductances should satisfy the relation 2 Re $G^{\uparrow\downarrow} \geq G^{\uparrow} + G^{\downarrow}$.

In terms of a scalar particle and a vector spin contribution $\boldsymbol{I} = (1\boldsymbol{I}_0 \pm \sigma \cdot \boldsymbol{I}_s)/2, \boldsymbol{f}^{N(F)} = 1\boldsymbol{f}_0^{N(F)} \pm \sigma \cdot \boldsymbol{s}_s(\boldsymbol{m})\boldsymbol{f}_s^{N(F)}$, and the particle current is described by the expression

$$I_0 = \left(G^{\uparrow} + G^{\downarrow}\right)\left(f_0^F + f_0^N\right) + \left(G^{\uparrow} - G^{\downarrow}\right)\left(f_s^F - \boldsymbol{m} \cdot \boldsymbol{s} f_s^N\right) \tag{13.39}$$

The familiar expressions for collinear transport are recovered when $\boldsymbol{m} \cdot \boldsymbol{s} = \pm 1$. The spin current is described by the expression

$$\begin{aligned} \boldsymbol{I}_s = \boldsymbol{m}\big[&\left(G^{\uparrow} - G^{\downarrow}\right)\left(\left(f_0^F - f_0^N\right) + f_s^F\right) + \left(2\mathrm{Re}G^{\uparrow\downarrow} - G^{\uparrow} - G^{\downarrow}\right)\boldsymbol{m} \cdot \boldsymbol{s} f_s^N\big] \\ &-2\boldsymbol{s}\mathrm{Re}G^{\uparrow\downarrow} f_s^N + (\boldsymbol{m} \times \boldsymbol{s})2\mathrm{Im}G^{\uparrow\downarrow} f_s^N. \end{aligned} \tag{13.40}$$

The first two terms point in the direction of the magnetization of the ferromagnet $\boldsymbol{m}$, the third term is in the direction of the nonequilibrium spin distribution $\boldsymbol{s}$, and the last term is perpendicular to both $\boldsymbol{s}$ and $\boldsymbol{m}$. The last contribution solely depends on the imaginary part of the mixing conductance. This term can be interpreted by considering how the direction of the spin on the normal metal layer $\boldsymbol{s}$ would change in timekeeping, with all other parameters constant. The cross product creates a precession of $\boldsymbol{s}$ around the magnetization direction $\boldsymbol{m}$ of the ferromagnet similar to a classical torque while keeping the magnitude of the spin-accumulation constant. In contrast, the first three terms represent diffusion-like processes, which decrease the magnitude of the spin accumulation. Due to the abovementioned relation $2\,\mathrm{Re}\,G^{\uparrow\downarrow} \geq G^{\uparrow} + G^{\downarrow}$, the nonequilibrium spin distribution f_s^N propagates easier into a configuration parallel to $\boldsymbol{s}$ than parallel to $\boldsymbol{m}$, since these processes are governed by positive diffusion-like constants $2\,\mathrm{Re}\,G^{\uparrow\downarrow}$ and $2\,\mathrm{Re}\,G^{\uparrow\downarrow} - G^{\uparrow} - G^{\downarrow}$, respectively.

13.2.4 Dependence of Spin Current on Contact Type

The four conductance parameters $G^{\uparrow}$, $G^{\downarrow}$, $\mathrm{Re}G^{\uparrow\downarrow}$, and $\mathrm{Im}G^{\uparrow\downarrow}$ depend on the microscopic details of the contact between ferromagnetic and normal-metal layers, which is characterized by elementary model contacts of a diffusive, a ballistic, and a tunnel types.

In the case a diffusive contact between a normal metal (N) and a ferromagnet (F) with conductances G_D^N and G_D^{Fs}, respectively, the spin-dependent resistance of the whole contact is the sum $\left(G_D^{Fs}\right)^{-1} + \left(G_D^N\right)^{-1}$, consequently the whole conductance $G_D^s = G_D^{Fs} G_D^N / \left(G_D^{Fs} + G_D^N\right)$, where $s = (\uparrow, \downarrow)$. These spin-dependent conductances fully describe collinear transport (in the absence of spin-flip scattering).

For non-collinear magnetizations, the mixing conductance, which is also needed, can be derived from the scattering matrix. The latter follows from the diffusion equation, describing the scattering properties of the contact by a spatially dependent distribution matrix. The current density (i) on the normal side of the contact ($x < 0$) is $i(x < 0) = \sigma^N \partial_x f$, where σ^N is the conductivity of the N layer. Consequently, the total

current in the N layer with the length L^N is $I(x<0) = G_D^N\left(L^N\partial_x\right)f$, where f is the spatially dependent distribution matrix on the normal side in the contact. In the normal-metal part, the boundary condition is $f(x = -L^N) = f^N$. In a ferromagnet, spin-up and spin-down states are incoherent, and hence spins non-collinear to the magnetization direction relax and only spins collinear with the magnetization will propagate sufficiently far away from the F|N interface. It is assumed that the ferromagnet is sufficiently strong and that the contact is longer than the ferromagnetic decoherence length $\xi = \sqrt{D/h_{ex}}$, where D is the diffusion constant and h_{ex} is the exchange splitting. The decoherence length is typically very short in ferromagnets, $\xi = 2$ nm in Ni wires.

The distribution function on the ferromagnetic side is represented by a two-component distribution function $f(x>0) = u^{\uparrow}f^{\uparrow} + u^{\downarrow}f^{\downarrow}$, where $u^{\uparrow}$ and $u^{\downarrow}$ are the mentioned spin-projection matrices. Here a spin accumulation collinear to the magnetization direction in the ferromagnet is taken into account. The boundary condition determined by the distribution function in the ferromagnetic part is thus

$$f^{\uparrow}\left(x = L^F\right) = f^{F\uparrow},\ \ f^{\downarrow}\left(x = L^F\right) = f^{F\downarrow}, \tag{13.41}$$

where L^F is the length of the F layer.

In assumption that the resistance of the diffusive region of the contacts is much larger than the contact resistance between the N and F layers, the total current in the ferromagnet is

$$I(x>0) = G_D^{F\uparrow}u^{\uparrow}\partial_x f^{\uparrow} + G_D^{F\downarrow}u^{\downarrow}\partial_x f^{\downarrow} \tag{13.42}$$

The distribution function is continuous across the F|N interface, $f(0^+) = f(0^-)$. Current conservation on the left ($x<0$) and on the right ($x>0$) of the normal metal-ferromagnet interface dictates the equation, $\partial_x I = 0$, which together with the boundary conditions $f(x = -L^N) = f^N$, $f(0^+) = f(0^-)$, and (13.41) uniquely determines the distribution functions and hence the conductance in the diffusive contact. Then the current on the normal side of the contact becomes

$$\begin{aligned} eI = {} & G_D^{\uparrow}u^{\uparrow}\left(f^F - f^N\right)u^{\downarrow} + G_D^{\downarrow}u^{\downarrow}\left(f^F - f^N\right)u^{\downarrow} \\ & + G_D^N\left[u^{\uparrow}\left(f^F - f^N\right)u^{\downarrow} + u^{\downarrow}\left(f^F - f^N\right)u^{\downarrow}\right] \end{aligned} \tag{13.43}$$

The current in a diffusive contact thus takes the generic form (13.35) with $G^{\uparrow} = G_D^{\uparrow}$, $G^{\downarrow} = G_D^{\downarrow}$, and $G^{\uparrow\downarrow} = G_D^N$. The mixing conductance is thus real and only depends on the normal conductance. The latter results can be understood as a consequence of the effective spin relaxation of spins non-collinear to the local magnetization direction. Those spins cannot propagate in the ferromagnet, and consequently the effective conductance can only depend on the conductance in the normal metal.

In the case of the ballistic contact, the reflection and transmission coefficients appearing in (13.36) and (13.37) are diagonal in the space of the transverse channels

since the transverse momentum is conserved. In a simplified model [20], the transmission channels are either closed $t = 0$ or open $t = 1$. The conductances (13.36) and (13.37) can then be found by simply counting the number of propagating modes. Then the spin-dependent conductance $G_B^{\uparrow(\downarrow)} = \left(e^2/h\right)N^{\uparrow(\downarrow)}$, where $N^{\uparrow}(N^{\downarrow})$ is the number of spin-up (spin-down) propagating channels. The mixing conductance is determined by $G_B^{\uparrow\downarrow} = \max\left(G_B^{\uparrow}, G_B^{\downarrow}\right)$ and is real. In a quantum mechanical calculation, the channels just above the potential step are only partially transmitting, and the channels below a potential step can have a finite transmission probability due to tunneling. Furthermore, the band structure of ferromagnetic metals is usually complicated and interband scattering exists even at ideal interfaces. The phase of the scattered wave will be relevant giving a nonvanishing imaginary part of the mixing conductance.

In the case of a tunneling contact, the transmission coefficients are exponentially small, and the reflection coefficients have a magnitude close to one. The spin-dependent conductance is

$$G_T^s = \frac{e^2}{h}\sum_{nm}\left|t_s^{nm}\right|^2. \tag{13.44}$$

For simple models of tunnel barriers, $r_s^{nm} = \delta^{nm}\exp i\phi^n = \delta r_s^{nm}$, where the phase shift ϕ^n is spin-independent. The expansion (38) in the small correction δr_s^{nm} leads to the expression $\mathrm{Re}G_T^{\uparrow\downarrow} = \left(G_T^{\uparrow} + G_T^{\downarrow}\right)/2$, where $G_T^{\uparrow}$ and $G_T^{\downarrow}$ are the spin-dependent tunneling conductances (13.44). Since the transmission coefficients in a tunnel contact are all exponentially small, the imaginary part of $G_T^{\uparrow\downarrow}$ is of the same order of magnitude as $G_T^{\uparrow}$ and $G_T^{\downarrow}$, but it is not universal and depends on the details of the contact.

13.3 Spin Transport in the Case of Dynamic Magnetic Field

13.3.1 Features of Coupling Spin Currents with Magnetic Dynamics

The interconnection between spin currents and the magnetic dynamics in F/N-based magnetic multilayer nanostructures underlies the current-controlled magnetic dynamics and utilization of the latter as new functionality in spintronic devices [21]. One is related to the *s-d* exchange interaction with localized spins and the spin-dependent scattering of spin-polarized electrons near the FlN interface. The impact of the spin current on localized spins occurs via a finite torque on the magnetic order parameter, and, vice versa, a moving magnetic order vector loses torque by emitting a spin current. The magnetic precession acts as a spin pump which transfers angular momentum from the magnetic into normal metal.

The technological potential of the mentioned magnetic nanostructures is related to utilizing transition metals (for instance, Co, Ni, Fe) that operate at ambient temperatures. Examples are current-induced tunable microwave generators (spin-torque oscillators) [22, 23] and nonvolatile magnetic electronic architectures that can be randomly read, written, or programmed by current pulses in a scalable manner [24]. The interaction between currents and magnetization can also cause undesirable effects such as enhanced magnetic noise in read heads made from magnetic multilayers [25].

In the framework of the Landau-Lifshitz-Gilbert model, the impact of the spin current on the magnetic dynamics, caused by the spin transfer, reduces to change of fundamental parameters such as the gyromagnetic ratio and Gilbert damping parameters. This spin transfer is governed by the reflection and transmission matrices of the system, analogous to the scattering theory of transport and interlayer exchange coupling. In the case when the normal-metal layers adjacent to the ferromagnetic layers are perfect spin sinks, the spin accumulation in the normal metal vanishes [26]. In the opposite case, the spin accumulation accompanies the spin diffusion, which gives essential contribution to the total spin current and its interconnection with magnetic dynamics.

Spin pumping by a precessing ferromagnet is, in some sense, the reverse process of current-induced magnetization dynamics. When the pumped spin angular momentum is not quickly dissipated to the normal-metal atomic lattice, a spin accumulation builds up and creates reaction torques due to transverse-spin backflow into ferromagnets. The interplay between magnetization dynamics and the nonequilibrium spin-polarized transport in heterostructures determining magnetic properties will be considered for the case of F/N-based nanostructures below.

13.3.2 Precession-Induced Spin Pumping Through F/N Interfaces

Characteristic properties of the precession-induced spin pumping are manifested in the model *N/F/N* magnetic junction schematic of which is displayed in Fig. 13.2. The ferromagnetic layer F is a spin-dependent scatterer that governs electron transport between (left (L) and right (R)) normal-metal reservoirs. The 2×2 operator $\boldsymbol{I}_l$ for the charge and spin current in lth lead ($l = L, R$) can be expressed in terms of operators $a_{\alpha m,\ l}(E)$ and $b_{\alpha m,\ l}(E)$ that annihilate a spin-α electron with energy E leaving (entering) the lth lead through the mth channel:

$$I_l^{\alpha\beta}(t) = \frac{e}{h} \sum_m \int dE dE' e^{i(E-E')t/\hbar} \left[a^\dagger_{\beta m,l}(E) a_{\alpha m,l}\left(E'\right) - b^\dagger_{\beta m,l}(E) b_{\alpha m,l}\left(E'\right) \right]. \tag{13.45}$$

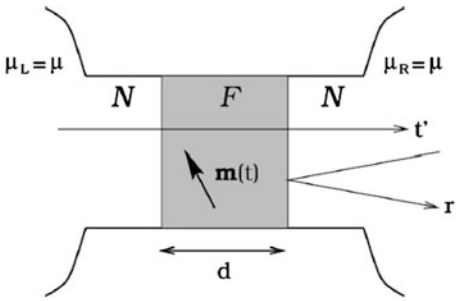

Fig. 13.2 Ferromagnetic layer (F) sandwiched between two normal-metal layers (N). The reflection and transmission amplitudes r and t' govern the spin current pumped into the right lead

When the scattering matrix $s^{\alpha\beta}_{mn,ll'}(t)$ of the ferromagnetic layer varies slowly on the time scales of electronic relaxation in the system, an adiabatic approximation may be used. The annihilation operators for particles entering the N layers are then related to the operators of the outgoing states by the instantaneous value of the scattering matrix $b_{\alpha m,l}(E) = s^{\alpha\beta}_{mn,ll'}(t)a_{\beta n,l'}(E)$. In terms of $a_{\alpha m,\ l}$ only, we can evaluate the expectation value $\left\langle I^{\alpha\beta}_l(t)\right\rangle$ of the current operator using $\left\langle a^{\dagger}_{\alpha m,l}(E)a_{\beta n,l'}\left(E'\right)\right\rangle = f_l(E)\delta_{\alpha,\beta}\delta_{m,n}\delta_{l,l'}\delta\left(E-E'\right)$, where $f_l(E)$ is the isotropic distribution function in the lth reservoir. When the scattering matrix depends on a single time-dependent parameter $X(t)$, then the Fourier transform of the current expectation value $I_l(\omega) = \int dte^{i\omega t}I_l(t)$ can be written as

$$I_l(\omega) = g_{X,l}(\omega)X(\omega) \tag{13.46}$$

In terms of a frequency ω- and X-dependent parameter

$$g_{X,l}(\omega) = -\frac{e\omega}{4\pi}\sum_{l'}\int dE\left(-\frac{\partial f_{l'}(E)}{\partial E}\right)\sum_{mn}\left(-\frac{\partial s_{mn,ll'}(E)}{\partial X}s^{\dagger}_{mn,ll'}(E) - \text{H.c.}\right). \tag{13.47}$$

Equation (13.46) is the first-order (in frequency) correction to the dc Landauer-Büttiker formula [11]. At equilibrium $f_R(E) = f_L(E)$, (13.46) is the lowest-order nonvanishing contribution to the current. Furthermore, at sufficiently low temperatures, we can approximate $\partial f_l(E)/\partial E$ by a δ-function centered at Fermi energy. The expectation value of the 2×2 particle number operator $Q_l(\omega)$(defined by $I_l(t) = dQ_l(t)/dt$) in time or by $I_l(\omega) = -i\omega Q_l(\omega)$ in frequency domain) for the lth reservoir is then given by

$$Q_l(\omega) = \left(\frac{e}{4\pi i}\sum_{mnl'}\frac{\partial s_{mn,ll'}(E)}{\partial X}s^{\dagger}_{mn,ll'}(E) + \text{H.c.}\right)X(\omega), \tag{13.48}$$

where the scattering matrices are evaluated at the Fermi energy. Because the prefactor on the right-hand side of (13.46) does not depend on frequency ω, the equation is also valid in time domain. The change in particle number $\delta Q_l(t)$ is proportional to the modulation $\delta X(t)$ of parameter X, and the 2×2 matrix current (directed into the normal-metal leads) reads

$$I_l(t) = e\frac{dn_l}{dX}\frac{dX(\mathrm{t})}{dt}, \tag{13.49}$$

where the "matrix emissivity" into lead l is

$$\frac{\partial n_l}{\partial X} = \frac{1}{4\pi i}\sum_{mnl'}\frac{\partial s_{mn,ll'}(E)}{\partial X}s^{\dagger}_{mn,ll'}(E) + \mathrm{H.c.} \tag{13.50}$$

If the spin-flip scattering in the ferromagnetic layer is disregarded, the scattering matrix $\boldsymbol{s}$ can be written in terms of the spin-up and spin-down scattering coefficients $\boldsymbol{s}^{\uparrow(\downarrow)}$ using the projection matrices $\boldsymbol{u}^{\uparrow} = (1 + \boldsymbol{\sigma}\cdot\boldsymbol{m})/2$ and $\boldsymbol{u}^{\downarrow} = (1 - \boldsymbol{\sigma}\cdot\boldsymbol{m})/2$:

$$s_{mn,ll'} = s^{\uparrow}_{mn,ll'}u^{\uparrow} + s^{\downarrow}_{mn,ll'}u^{\downarrow} \tag{13.51}$$

The spin current pumped by the magnetization precession is obtained by identifying $X(t) = \varphi(t)$, where φ is the azimuthal angle of the magnetization direction in the plane perpendicular to the precession axis. For simplicity, we assume that the magnetization rotates around the y-axis: $\boldsymbol{m} = (\sin\varphi, 0, \cos\varphi)$. Using (13.51), it is then easy to calculate the emissivity (13.50) for this process:

$$\frac{\partial n_l}{\partial \varphi} = -\frac{1}{4\pi}\left[A_r\sigma_y + A_i\left(\sigma_x\cos\varphi - \sigma_y\sin\varphi\right)\right], \tag{13.52}$$

where $A_r(A_i) =$ Re (Im)$[g^{\uparrow\downarrow} - t^{\uparrow\downarrow}]$. Expanding the 2×2 current into isotropic and traceless components,

$$I = \frac{1}{2}I_c - \frac{e}{\hbar}\sigma\cdot\boldsymbol{I}_s. \tag{13.53}$$

Here the charge current I_c and spin current $\boldsymbol{I}_s$ are identified. Due to Eqs. (13.49), (13.52), and (13.53), the charge current vanishes, $I_c = 0$, and the spin current

$$\boldsymbol{I}_s = (A_i\cos\varphi, A_r - A_i\sin\varphi)\frac{\hbar}{4\pi}\frac{d\varphi}{dt} \tag{13.54}$$

can be rewritten as

$$\boldsymbol{I}_s^{\mathrm{pump}} = \frac{\hbar}{4\pi}\left(A_r\boldsymbol{m}\times\frac{d\boldsymbol{m}}{dt} - A_i\frac{d\boldsymbol{m}}{dt}\right) \tag{13.55}$$

This current into a given N layer depends on the complex-valued parameter $A = A_r + iA_i$ (the "spin-pumping conductance") and the time-dependent order parameter of the ferromagnet $\boldsymbol{m}(t)$. In addition, $A = g^{\uparrow\downarrow} - t^{\uparrow\downarrow}$ depends on the scattering matrix of the ferromagnetic layer since

$$g^{\sigma\sigma'} = \sum_{mn} \left[\delta_{mn} - r^{\sigma}_{mn} \left(\mathrm{r}^{\sigma'}_{mn} \right)^* \right] \tag{13.56}$$

is the dimensionless dc conductance matrix [10, 26] and $t^{\uparrow\downarrow} = \sum_{mn} t'^{\uparrow}_{mn} \left(t'^{\downarrow}_{mn} \right)^*$. Here $r^{\uparrow}_{mn}$ ($r^{\downarrow}_{mn}$) is a reflection coefficient for spin-up ~spin-down electrons on the normal-metal side, and $t'^{\uparrow(\downarrow)}_{mn}$ is a transmission coefficient for spin-up/spin-down electrons across the ferromagnetic film from the opposite reservoir into the normal-metal layer, where m and n label the transverse modes at the Fermi energy in the normal-metal films. The magnetization can take arbitrary directions; in particular, $\boldsymbol{m}(t)$ may be far away from its equilibrium value. In such a case, the scattering matrix itself can depend on the orientation of the magnetization, and one has to use $A(\boldsymbol{m})$ in Eq. (13.55).

When the ferromagnetic film is thicker than its transverse spin coherence length $d > \pi/(k^{\uparrow} - k^{\downarrow})$, where $k_F^{\uparrow(\downarrow)}$ are the spin-dependent Fermi wave vectors, $t^{\uparrow\downarrow}$ vanishes [26], the spin pumping through a given FlN interface is governed entirely by the interfacial mixing conductance $A = g^{\uparrow\downarrow} = g_r^{\uparrow\downarrow} + ig_i^{\uparrow\downarrow}$, and we can consider only one of the two interfaces, as it is presented in Fig. 13.3.

The spin current (13.55) leads to a damping of the ferromagnetic precession, resulting in a faster alignment of the magnetization with the (effective) applied magnetic field $\boldsymbol{H}_{eff}$. The pumped spins are entirely absorbed by the attached ideal reservoirs. Thereto the enhancement rate of damping is accompanied by an energy flow out of the ferromagnet, until a steady state is established in the accompanied F/N system. For simplicity, assume a magnetization which at time t starts rotating

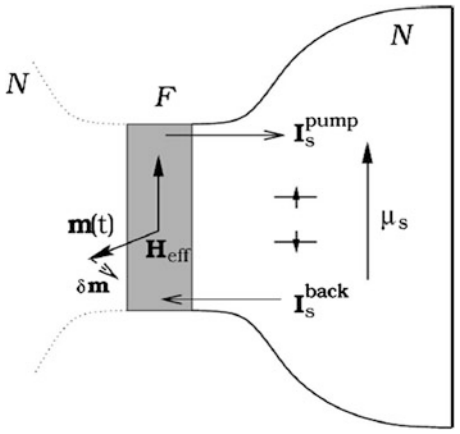

Fig. 13.3 Ferromagnetic nanostructure with the unit magnetization vector $\boldsymbol{m}(t)$ precessing around the external magnetic field $\boldsymbol{H}_{eff}$ and inducing the spin pumping $\boldsymbol{I}_s^{pump}$ through the interface F/N. The spin accumulation $\boldsymbol{\mu}_s$ results in the back spin current $\boldsymbol{I}_s^{back}$

around the vector of the magnetic field, $\boldsymbol{m}(t) \perp \boldsymbol{H}_{eff}$. In a short interval of time δt, it slowly (i.e., adiabatically) changes to $\boldsymbol{m}(t + \delta t) = \boldsymbol{m}(t) + \delta \boldsymbol{m}$. In the presence of a large but finite nonmagnetic reservoir without any spin-flip scattering attached to one side of the ferromagnet, this process can be expected to induce a nonvanishing spin accumulation $\boldsymbol{\mu}_s = \int d\varepsilon \mathrm{Tr}[\sigma \boldsymbol{f}(\varepsilon)]$, where σ is the Pauli matrix vector and $\boldsymbol{f}(\varepsilon)$ is the 2×2 matrix distribution function at a given energy ε. For a slow enough variation of $\boldsymbol{m}(t)$, this nonequilibrium spin imbalance must flow back into the ferromagnet, canceling any spin current generated by the magnetization rotation, since, due to the adiabatic assumption, the system is always in a steady state.

For the spins accumulated in the reservoir along the magnetic field, $\boldsymbol{\mu}_s \| \boldsymbol{H}_{eff}$ flow of N_s spins into the normal metal transfers the energy $\Delta E_N = N_s \mu_s/2$ and angular momentum $\Delta L_N = N_s \hbar/2$ (directed along $\boldsymbol{H}_{eff}$). By the conservation laws, $\Delta E_F = -\Delta E_N$ and $\Delta L_F = -\Delta L_N$, for the corresponding values in the ferromagnet. Using the magnetic energy, $\Delta E_F = -\gamma \Delta L_F H_{eff}$, where γ is the gyromagnetic ratio of the ferromagnet, it can be found that $N_s \mu_s/2 = \gamma N_s \hbar H_{eff}/2$. Then $\mu_s = \hbar \gamma H_{eff} = \hbar \omega$, where $\omega = \gamma H_{eff}$ is the Larmor frequency of precession in the effective field. The spin-up and spin-down chemical potentials in the normal metal split by $\mu_s = \hbar \omega$, the energy corresponding to the frequency of the perturbation.

The above mentioned the backflow of spin current $\boldsymbol{I}_s^{\mathrm{back}}$, which equals to the pumping current $\boldsymbol{I}_s^{\mathrm{pump}}$ described by the expression

$$\boldsymbol{I}_s^{\mathrm{back}} == \frac{1}{2\pi}\left(g_r^{\uparrow\downarrow}\boldsymbol{\mu}_s + g_i^{\uparrow\downarrow}\boldsymbol{m} \times \boldsymbol{\mu}_s\right) = \frac{\hbar}{4\pi}\left(g_r^{\uparrow\downarrow}\boldsymbol{m} \times \frac{d\boldsymbol{m}}{dt} - g_i^{\uparrow\downarrow}\frac{d\boldsymbol{m}}{dt}\right) \tag{13.57}$$

Here, it is used that $\mu_s = \hbar \omega$ and $\boldsymbol{\mu}_s \perp \boldsymbol{m}$, since by the conservation of angular momentum, the spin transfer is proportional to the change in the direction $\delta \boldsymbol{m} \perp \boldsymbol{m}$. Thus, for the case of a single and finite reservoir, Eq. (13.55) is recovered. It is easy to repeat the proof for an arbitrary initial alignment of $\boldsymbol{m}(t)$ with $\boldsymbol{H}_{eff}$.

The expressions for the adiabatic spin pumping are not the whole story, since spin-flip scattering is essential. In this case, the spin build-up occurs in the normal metal at dynamic equilibrium. Then, the contribution to $\boldsymbol{I}_s$ due to the spin-accumulation-driven current $\boldsymbol{I}_s^{\mathrm{back}}$ back into the ferromagnet:

$$\boldsymbol{I}_s = \boldsymbol{I}_s^{\mathrm{pump}} - \boldsymbol{I}_s^{\mathrm{back}}, \tag{13.58}$$

which vanishes in the absence of spin-flip scattering.

The spin current out of the ferromagnet carries angular momentum perpendicular to the magnetization direction. By conservation of angular momentum, the spins ejected by $\boldsymbol{I}_s$ correspond to a torque $\boldsymbol{T} = -\boldsymbol{I}_s$ on the ferromagnet. If possible, interfacial spin-flip processes are disregarded, and the torque t is entirely transferred to the coherent magnetization precession. The dynamics of the ferromagnet can then be described by a generalized Landau-Lifshitz-Gilbert (LLG) equation [5]

$$\frac{d\boldsymbol{m}}{dt} = -\gamma \boldsymbol{m} \times \boldsymbol{H}_{eff} + \alpha_0 \boldsymbol{m} \frac{d\boldsymbol{m}}{dt} + \frac{\gamma}{M_s V} \boldsymbol{I}_s, \tag{13.59}$$

where α_0 is the dimensionless bulk Gilbert damping constant, M_s is the saturation magnetization of the ferromagnet, and V is its volume. The intrinsic bulk constant α_0 is smaller than the total Gilbert damping $\alpha = \alpha_0 + \alpha'$. The additional damping α' caused by the spin pumping is observable in, for example, FMR spectra here.

13.3.3 Spin-Accumulation-Driven Backflow in the F/N Bilayer

The precession of the magnetization does not cause any charge current in the system. The spin accumulation or nonequilibrium chemical potential imbalance $\boldsymbol{\mu}_s$ in the normal metal is a vector, which depends on the distance from the interface x x, $0 < x < L$, where L is the thickness of the normal-metal film (see Fig. 13.4). When the ferromagnetic magnetization steadily rotates around the z axis, $\boldsymbol{m} \times \partial_t \boldsymbol{m}$ and the normal-metal spin accumulation $\boldsymbol{\mu}_s(x)$ are oriented along z. There is no spin imbalance in the ferromagnet, because $\boldsymbol{\mu}_s(x)$ is perpendicular to the magnetization direction $\boldsymbol{m}$. The time-dependent $\boldsymbol{m}_s$ is also perpendicular to $\boldsymbol{m}$ even in the case of a precessing ferromagnet with time-dependent instantaneous rotation axis, as long as the precession frequency ω is smaller than the spin-flip rate τ_{sf}^{-1} in the normal metal.

The spin accumulation diffuses into the normal metal as

$$i\omega \boldsymbol{\mu}_s = D\partial_x^2 \boldsymbol{\mu}_s - \tau_{sf}^{-1} \boldsymbol{\mu}_s, \tag{13.60}$$

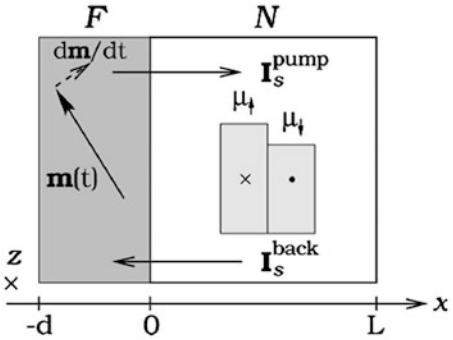

Fig. 13.4 Schematic view of the *F-N* bilayer. Precession of the magnetization direction $\boldsymbol{m}(t)$ of the ferromagnet F pumps spins into the adjacent normal-metal layer N by inducing a spin current $\boldsymbol{I}_s$ pump. This leads to a buildup of the normal-metal spin accumulation $\boldsymbol{\mu}_s$ which either relaxes by spin-flip scattering or flows back into the ferromagnet as $\boldsymbol{I}_s^{back}$

where D is the diffusion coefficient. The boundary conditions are determined by the continuity of the spin current from the ferromagnet into the normal metal at $x = 0$ and the vanishing of the spin current at the outer boundary $x = L$:

$$x = 0: \quad \partial_x \boldsymbol{\mu}_s = -2(\hbar NSD)^{-1} \boldsymbol{I}_s, \quad x = L: \quad \partial_x \boldsymbol{\mu}_s = 0 \tag{13.61}$$

where N is the (one-spin) density of states in the layer and S is the area of the interface. The solution to (13.60) with the boundary conditions (13.61) is

$$\boldsymbol{\mu}_s(x) = \frac{\cosh k(x-L)}{\sinh kL} \frac{2\boldsymbol{I}_s}{\hbar NSDk} \tag{13.62}$$

Using relation $D = v_F^2 \tau_{el}/3$ between the diffusion coefficient D, the Fermi velocity v_F, and the elastic scattering time τ_{el}, we find for the spin-diffusion length $\lambda_{sd} = v_F \sqrt{\tau_{el}\tau_{sf}/3}$. An effective energy level spacing of the states participating in the spin-flip scattering events in a thick layer can be defined by $\delta_{sd} = (NS\lambda_{sd})^{-1}$. The spin-accumulation-driven spin current $\boldsymbol{I}_s^{\text{back}}$ through the interface reads

$$\begin{aligned} \boldsymbol{I}_s^{\text{back}} = \frac{1}{8\pi} \Big[& 2g_r^{\uparrow\downarrow} \boldsymbol{\mu}_s(0) + 2g_i^{\uparrow\downarrow} \boldsymbol{m} \times \boldsymbol{\mu}_s(0) \\ & + \left(g^{\uparrow\uparrow} + g^{\downarrow\downarrow} - 2g_r^{\uparrow\downarrow}\right) \left(\boldsymbol{m} \cdot \boldsymbol{\mu}_s(0)\right) \Big]. \end{aligned} \tag{13.63}$$

Substitution of (13.62) into (13.63) gives total spin current

$$\boldsymbol{I}_s = \boldsymbol{I}_s^{\text{pump}} - \frac{\beta}{2} \left[2g_r^{\uparrow\downarrow} \boldsymbol{I}_s + 2g_i^{\uparrow\downarrow} \boldsymbol{m} \times \boldsymbol{I}_s + \left(g^{\uparrow\uparrow} + g^{\downarrow\downarrow} - 2g_r^{\uparrow\downarrow}\right) (\boldsymbol{m} \cdot \boldsymbol{I}_s) \boldsymbol{m} \right], \tag{13.64}$$

where the spin current returning into the ferromagnet is governed by the backflow factor $\beta = (\tau_{sf}\delta_{\text{sd}})/(\hbar \tanh (L/\lambda_{sd}))$. When the normal metals are shorter than the spin-diffusion length ($L \ll \lambda_{sd}$), $\beta \to \tau_{sf}\delta/h$, where $\delta = (NSL)^{-1}$. Basically, β is therefore the ratio between the energy level spacing of the normal-metal layer with thickness $L_{sf} = \min(L, \lambda_{sd})$ and the spin-flip rate.

By inverting Eq. (13.64), the total spin current $\boldsymbol{I}_s$ can be expressed in terms of the pumped spin current $\boldsymbol{I}_s^{\text{pump}}$ ([26])

$$\boldsymbol{I}_s = \left[1 + 2g_r^{\uparrow\downarrow} + \frac{\left(\beta g_i^{\uparrow\downarrow}\right)^2}{1 + \beta g_r^{\uparrow\downarrow}} \right]^{-1} \left(1 - \frac{\beta g_i^{\uparrow\downarrow}}{1 + \beta g_r^{\uparrow\downarrow}} \boldsymbol{m} \times \right) \boldsymbol{I}_s^{\text{pump}} \tag{13.65}$$

Then, substituting (13.55) into (13.65) results in total spin current $\boldsymbol{I}_s$, which is described by the equation of the form (13.55) but with a redefined spin-pumping conductance $A' = A'_r + A'_i$

$$\boldsymbol{I}_s = \frac{\hbar}{4\pi} \left(A'_r \boldsymbol{m} \times \frac{d\boldsymbol{m}}{dt} - A'_i \frac{d\boldsymbol{m}}{dt} \right) \tag{13.66}$$

Here A' is function of the mixing conductance $g^{\uparrow\downarrow}$ and the backflow factor β,$A' = A'(g^{\uparrow\downarrow}, \beta)$. For realistic $F|N$ interfaces $g_i^{\uparrow\downarrow} \ll g_r^{\uparrow\downarrow}$, so that $g^{\uparrow\downarrow} \approx g_r^{\uparrow\downarrow}$ and, consequently, A_i' vanishes. Substituting (13.66) into (13.59) renormalizes its Gilbert dumping constant, α_0, so that $\alpha_0 \to \alpha = \alpha_0 + \alpha'$, where

$$\alpha' = \frac{g_L g^{\uparrow\downarrow}}{4\pi\mu}\left[1 + g^{\uparrow\downarrow}\frac{\left(\tau_{sf}\delta_{sd}/h\beta g_i^{\uparrow\downarrow}\right)^2}{\tanh(L/\lambda_{sd})}\right]^{-1} \tag{13.67}$$

is the additional damping constant due to the interfacial F|N coupling. Here, g_L is the g factor and μ is the total layer magnetic moment in units of μ_B. When $L \to \infty$, (13.67) reduces $\alpha' = g_L g_{eff}^{\uparrow\downarrow}/4\pi$, where $1/g_{eff}^{\uparrow\downarrow} = 1/g^{\uparrow\downarrow} + R_{sd}$ and $R_{sd} = \tau_{sf}\delta_{sd}/h$ is the resistance (per spin, in units of h/e^2) of the normal-metal layer of thickness λ_{sd}. It follows that the effective spin pumping out of the ferromagnet is governed by $g_{eff}^{\uparrow\downarrow}$, i.e., the conductance of the $F|N$ interface in series with diffusive normal-metal film with thickness λ_{sd}.

The second factor on the right-hand side of (13.67) suppresses the additional Gilbert damping due to the spin angular momentum that diffuses back into the ferromagnet. Because spins accumulate in the normal metal perpendicular to the ferromagnetic magnetization, the spin-accumulation-driven transport across the F|N contact, as well as the spin pumping, is governed by a mixing conductance.

13.4 Conclusions

The spin transport in the F/N-based magnetic nanostructures in magneto-static and magneto-dynamic cases has been studied in the framework of the modified Stoner model. Using the modified quantum kinetic equation for the nonlinear Green functions and the spin-dependent scattering matrix, the spin currents through and near the F|N interface are described. In the magnetostatic case, the parametric dependence of the spin current on the relative orientation of the spin polarization and magnetization is shown. In the magneto-dynamic case of the magnetization precession, the precession-induced spin pumping into the normal-metal layer is described. The accompanying effect of the spin accumulation and the spin backflow exerted via the spin torque on the magnetization precession is considered.

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