Chapter 8
Prospective Mathematics Teachers' Perspectives on Technology

Mar Moreno and Salvador Llinares

Abstract This chapter examined prospective secondary mathematics teachers' perspectives on the role that technological resources play in supporting students' learning. In particular, we study prospective teachers' pedagogical reasoning in order to understand their decisions about the use of technology and their effects on students' mathematics learning. We analysed prospective secondary teachers' lesson plans on teaching mathematics through problem solving by integrating technology. Prospective secondary mathematics teachers' perspectives on the use of technology for supporting students' mathematical learning varied in two dimensions: (i) how technological resources are used, and (ii) what mathematical activity that prospective teachers should present to support students' learning. These dimensions are related to the idea of instrumental integration that is used to describe how teachers organize the conditions for instrumental genesis. We identified three ways of integrating technological resources.

Keywords Technology · Instrumentalisation · Mathematical activity
Teachers' perspective

8.1 Introduction

This research focuses on prospective secondary mathematics teachers' learning when using and integrating technology to support students' mathematics learning and reasoning (Goos, 2008; Niess, 2005; Tondeur, van Braak, Sang, Voogt, Fisser, & Ottenbreit-Leftwich, 2012; Wilson, Lee, & Hollebrands, 2011). Reviews on technological pedagogical content knowledge (Voogt, Fisser, Pareja Roblin,

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Tondeur, & Van Braak, 2012) show that pedagogical beliefs affect how teachers integrate technology. Within teacher education contexts, it is necessary to study prospective teachers' pedagogical reasoning in order to understand their decisions about the use of technology, and how prospective teachers' technological reasoning affects their decision making while using technology (Yigit, 2014). Technology, in this study, refers to the use of applets and dynamic geometry software to design dynamic representations of tasks. Using technology resources in teaching is related to the increasing emphasis on how prospective teachers can learn to engage students in meaningful mathematics tasks using technological tools (Stohl, 2005). Technology resources are tools that can help prospective teachers enact their perspectives on teaching and learning in lesson planning. This can be done if prospective teachers unpack mathematical contents into their constituent parts to define learning goals in their lessons (Morris, Hiebert, & Spitzert, 2009).

During lesson planning, prospective teachers' pedagogical reasoning can come up within the context of learning how to use technology resources to engage students in meaningful mathematical tasks. When prospective teachers are doing lesson planning, they should make decisions about how to use technological resources and have to determine the nature of the problem solving activity they aim to achieve. Lesson planning, as an activity in teacher education programs, involves a psychological process in which prospective teachers visualize the future, inventories means and ends, anticipate students' strategies, and constructs a framework to guide their future actions (Santos-Trigo & Camacho-Machín, 2009; Schoenfeld, 2011), and also a phenomenological approach in which they tell us what they plan to do. In the activity of lesson planning, prospective teachers should design instructional activities to address different mathematical contents by aligning instructional activities with learning goals, anticipating students' responses, thinking about assessment tasks to determine if students understand the learning concepts.

The use of technology for teaching through problem solving, underlines some aspects of the mathematical activity such as visualization, representations, formulation and conjectures, and generalization (Moreno-Armella & Santos-Trigo, 2016) that should be taken into account in lesson planning. These aspects are different from the mathematical activity generated on "paper and pencil" problem solving (Santos-Trigo, 2007; Santos-Trigo & Camacho-Machín, 2009). The transformation of mathematical problems that aims at creating learning opportunities for students to learn mathematics is a context in which the prospective teachers' approach toward technology appears. For this reason, lesson planning is an adequate context to study prospective teachers' pedagogical reasoning and how they learn to teach (Morris et al., 2009).

Therefore, the goal of this research is to identify prospective secondary mathematics teachers' perspectives on the role that technological resources play in supporting students' learning, when they planning a lesson that integrates technology through problem solving.

8.2 Theoretical Framework

Research on prospective teachers' learning attempts to explain how they acquire knowledge, beliefs, values and attitudes of their profession. Nowadays, the attempts to introduce technological resources in mathematics teaching raise new challenges for teachers and teaching (Goos et al., 2010). Using technology in teaching can imply using new kinds of mathematical tasks, modifying the nature of mathematical activities in classroom based on a set of pedagogical principles. From a teacher learning's perspective, the way in which prospective teachers learn to integrate technological resources in mathematics teaching could be mediated by their beliefs about the nature of mathematics, mathematics learning and mathematics teaching. So, learning to teach mathematics when digital tools are presented should make prospective teachers rethink the nature of the mathematical activity during problem solving and they should reflect on the role played by the teacher. In this study, we bring together two aspects of work that address how prospective teachers learn to integrate technological resources in mathematics teaching. The first one focuses on how prospective teachers' perspective can condition their learning to teach. The second focuses on the process of how prospective teachers organize the conditions for instrumental genesis of the technology (Chai, Koh, & Tsai, 2013).

From this perspective, learning to use technological resources in mathematics teaching may show different "prospective secondary teachers' perspectives" about teaching and learning. These perspectives could be considered as cognitive references through which prospective teachers learn to make decisions on teaching (Simon & Tzur, 1999). Simon and colleagues (Simon & Tzur, 1999; Tzur, Simon, Heinz, & Kinzel, 2001) conceptualise the expression "teachers' perspective" as a structure of pedagogical conceptions—knowledge and beliefs, which are responsible for organizing some aspects of their practice. Teachers' perspectives influence their learning and their cognitive references to make sense of learning contexts. In our study, we focused on the perspectives underlying prospective mathematics teachers' activity in lesson planning. For designing activities that integrate technological resources in their lesson planning, prospective teachers need to anticipate information about students' understanding. When prospective teachers anticipate students' answers, they might adjust learning opportunities. Regarding prospective teachers' activities in lesson planning to introduce technological resources, we consider that it is possible to identify aspects related to traditional, perception-based, and conception-based perspectives characterized in a different context (Tzur et al., 2001). Tzur et al. (2001) point out that from a traditional perspective teaching could be characterized by teachers' attempt to transmit particular mathematical ideas to students. While from a conception-based perspective, teachers attempt to orchestrate conditions that engage students in actively seeing and connecting those ideas, seeing mathematics as a web of conceptions that students abstract through reflection (Olive, Makar, Hoyos, Kor, Kosheleva, & Sträßer, 2010).

Secondly, prospective teachers' who are learning to use technology to support student's mathematical understanding and to develop problem solving skills could be placed in the intersection of research on how prospective teachers organize the conditions for instrumental genesis of the technology proposed to the students and the extent to which mathematics learning is fostered through instrumental genesis. In this study, instrumental genesis is understood to be the shaping of thinking by the tool in the construction of mental schemes and instrumentalisation as analogous to activities that involve the shaping of the tool by users (Goos et al., 2010; Healy & Lagrange, 2010).

The way in which an artefact becomes part of an instrument in the hands of a student is called instrumental genesis (Drijvers, Kieran, & Mariotti, 2010). In this case, the way in which prospective teachers design students' learning opportunities by integrating technological resources could support or not students' instrumental genesis. The role played by the prospective teachers' lesson plan in sharpening the instrumental genesis (in its double role of instrumentation as the way the applets—as an example of artefact—affect students' behaviour and thinking, and instrumentalisation concerns the way the students' thinking affects the use of applets) will define these students' learning opportunities. Since instrumental genesis consists in developing students' cognitive schemes and techniques, prospective teachers' perspective on the nature of mathematical knowledge and the role of technological resources in the teaching and learning of mathematics, reflected in lesson planning, will define opportunities to interrelate technical and conceptual elements during problem solving. Furthermore, when prospective teachers anticipate key moments in problem solving situations in which students interrelate technical and conceptual elements, they could define the institutional conditions to support the enhancement of instrumental genesis. The way in which prospective teachers consider the interrelation between technical and conceptual elements, in their lesson plans, the interaction between the techniques involved in using the applets—as an artefact—and the students' mathematical thinking becomes apparent. Additionally, when prospective teachers had to think about key moments in problem solving situations to orchestrate students' collective instrumentation, they had to anticipate ways of didactic configurations (additional tasks, type of questions, and so) considering the various stages of a mathematical situation. These aspects define the ways prospective teachers could orchestrate students' collective instrumentation (Bueno-Ravel & Gueudet, 2009).

8.3 Method

8.3.1 Participants and Context

The participants were 25 prospective secondary school mathematics teachers enrolled in a course on mathematics teaching in a postgraduate teacher education program. The prospective teachers were graduates in mathematics, engineering, and —computer sciences. They had different levels of knowledge about the use of technology as resources for teaching.

The postgraduate program granted them the qualifications required to teach mathematics in Secondary Education and included courses of mathematics education, mathematics, pedagogical studies—psychological and sociological studies—and eight weeks of teaching practices in secondary school classrooms. The mathematics education subjects represented 30% of the program's workload. Courses in mathematics education are designed to provide prospective teachers with the knowledge of teaching and learning mathematics.

The course on mathematics teaching and technology (a mathematics education course) lasted 50 h (four hours per week for 13 weeks). In this course, prospective mathematics teachers analysed curricular standards, tasks and lessons from mathematics textbooks, they also had the opportunity to explore applets for teaching mathematics, discussed class-teaching situations (teaching cases) in which technology was integrated and analysed the consequences on students' mathematical activity when technology was integrated into mathematics teaching which focused on problem solving. Geogebra was a technological resource introduced during some of these sessions. In these sessions, prospective teachers engaged in exploring different mathematics contents with applets to understand the opportunities and constraints that could be likely to create whilst using technology in mathematics teaching and learning. When they had to plan a lesson, using technology that focused on problem solving, they needed to understand how technology resources offered opportunities and constraints to students' learning. Prospective teachers read and discuss several research papers related to mathematics teaching and technology (Santos-Trigo & Camacho-Machín, 2009; Stein, Engle, Smith, & Hughes, 2008).

8.3.2 Instrument

As an assessment task at the end of the course, every prospective secondary mathematics teacher was asked to select a problem from a secondary mathematics textbook and modify it to plan a lesson focused on problem solving and integrating the use of technology. Prospective teachers had to modify the problem to create opportunities that favoured students' instrumental genesis to support aspects of the mathematical activity such as making and proving conjectures, using multiple representations, facilitating experimentation and particularization, generating connections and generalization. Prospective teachers are required to use some technological resources (applets or dynamic geometry) in their lesson plans to support students' mathematical activity. They had to anticipate students' answers. For this purpose, prospective teachers had to highlight the learning goals of the lesson, and solve the problem. Prospective teachers used the following template:

1. Anticipate ways in which students could solve the problem to examine if they were aligned with the achievement of the goals.

2. Identify features of mathematical activity (specialize? particularize, making conjectures and testing conjectures, ways of communicating, etc.) and possible evidence of students' learning.
3. Anticipate key moments in the resolution process to pose new challenges to students. Prospective teachers had to anticipate mathematical processes, which could be enacted during problem resolution, to identify the strengths and limitations involved in using the various representations and consequently plan how to encourage students to formulate and pursue questions in an attempt to establish mathematical relations.
4. Anticipate which students' answers could reflect different understanding and provide comments on the type of help to students, and indicate additional tasks and intentional and systematic organization of the various artefacts in guiding students' instrumental geneses, through instrumental orchestration.

8.3.3 Analysis

We analysed the lesson plans by attending to: (1) learning goals defined by the mathematical activity, which prospective teachers expected to develop, (2) how technology was used, and (3) how students' instrumental genesis was considered, including their arguments for using technology and the implications of its use.

The problem in the lesson plan was classified with regard to its cognitive demand, as high-level or low-level in relation to the mathematical activity that prospective teachers were expected to generate. Problems were classified with a high level cognitive demand when the questions required students to make connections between multiple representations engaged students in the conceptual ideas underlying the procedures, provided a context to go from specification—to generalization (Stein, Grover, & Henningsen, 1996). This type of problem could require that students experiment to make a conjecture and prove it. In this case, prospective teachers used the problem to support the students' reflections about relations between different mathematics concepts and representation registers. Problems used in this type of lesson plans and how they were described allowed students to set goals and engage in activities to solve them. We infer from these features a conception-based perspective in which mathematics is "thought as a web of conceptions that humans abstract through reflection" (Tzur et al., 2001). This approach underlines the interrelation between technical and conceptual elements as evidence of instrumental genesis defining the teacher's intention to support the interactions between the students and the artefact with a particular learning goal in mind. Prospective teachers who designed this type of lesson underscored the closely related co-emergence of the technical and conceptual aspects during the problem solving.

On the other hand, a problem in the lesson plan was classified with low-level cognitive demand when it only required students to reproduce previously learned facts, using a procedure to calculate, without providing any explanation. This approach defines the use of technological resources as a tool to only "display" the mathematical subject matter. This perspective does not take into account the students' understanding neither the potential of different technological tools, like dragging or visualizing relations between different types of registers (analytical-algebraic and geometrical).

Furthermore, the way in which prospective secondary teachers used technology affordances, like dragging objects, using sliders and quantify parameters, informed us about their ideas on how to promote students' mathematical activity and the role played by technological resources (that is to say, how the genesis of instrumentalisation is handled by prospective teachers in lesson planning). We focused on the arguments given by prospective teachers to justify the role of technological resources, during lesson planning, on problem solving in relation to students' learning (the relation between tools and mathematics learning). That is to say, how prospective teachers considered the use of digital resources with a mathematical intention (the instrumentation). We compared the descriptions of how prospective teachers proposed to use technological resources, during the lesson, to identify the reasons for using a given technological resource. This focus allowed us to infer the relation between the technological resources introduced during problem solving, the learning objective defined, and how they anticipated the students' answers.

Finally, prospective teachers' pedagogical reasoning in lesson planning were compared in an attempt to identify the differences and similarities of possible pattern groups in the data provided by prospective teachers.

8.4 Results

Based on these lesson plans, we identified three groups of prospective teachers taking into account two dimensions to characterize their perspectives. The first dimension is related to the way prospective secondary teachers considered the mathematical activity when students are engaged in problem solving using technology: ways of supporting mathematical relations, mathematical properties that could be emphasised using technology, using particular cases to make conjectures and so on. The second dimension is linked to how technology is used. That is to say, how the genesis of instrumentalisation is handled by prospective teachers in justifying the lesson plan. In other words, how prospective teachers orient students towards the use of an artefact (instrumentalisation) and towards the problem solving (the instrumentation). The way in which the use of technological resources was planned allows us to relate prospective teachers' reasoning based on the nature of the mathematical activity proposed to students. We present below three cases to show the different perspectives of prospective teachers.

G1—Technological resources to "display"

In the first group (n = 13 prospective teachers), the use of technological resources in the lesson plan of problem solving was anecdotic. Prospective teachers in this group used the technology resource only to present some aspects of the problem without being related to the nature of mathematical activity that could be generated. The prospective teachers used the technological resource only as a tool for illustrating the problem but not for reasoning with it. For example Jesus, one of the prospective teachers in this group, planned to use the technological resources to "illustrate" the topic that had previously been introduced in his explanations. Jesus' problem was suitable for a class of 14–15 year-old students. His goal was to "illustrate" how to calculate areas and perimeters of a 2D-shape (Fig. 8.1) (Calculate the area in which a tied horse could eat grass if there were two stakes that conditioned the horse's movements).

Jesus planned to use Geogebra to draw the geometrical figure that defined the horse' grazing field and to verify the calculations previously made by hand. For this prospective secondary teacher, the use of technology did not influence the nature of the student's mathematical activity and went on to solve the problem without technology. He justified his lesson plan by defining technology as an "illustrating and proving" tool.

Jesus stated:

> As we can see, the maximum area in which the horse could move along is delimited by these two sectors of circles, the pink one with radius equals seven and the blues one with radius equals two. Therefore, the total amount of grass that the horse is likely to eat is the sum of the interior circular sectors. Using algebra, the areas of each sector would be the pink sector area = (3/4) * π * (7^2) and the blue sector area = (1/4) * π * (2^2), and the total area would be the sum of the two sectors' area, approximately, 118.59 m^2. If we use dynamic software like Geogebra, it is easy to see and verify that the result is the same! (Added emphasis)

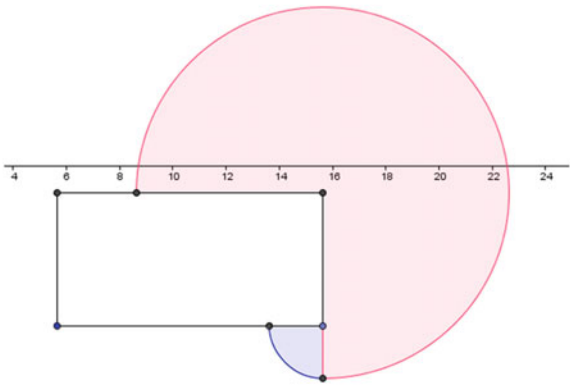


Fig. 8.1 Representation of Jesus' horse problem

When Jesus anticipated students' responses to exemplify difficulties in achieving the goals, he identified technical and procedural difficulties without indicating other high-level mathematical activities such as conjecturing, testing, particularizing and generalization:

> May be students have difficulties imagining the conditions of situations – how to go from one circumference to another, or how to consider the relation between the wall and the rope, … We could try to unlock these difficulties by posing questions like…:
>
> 1. Imagine that you are tied to a rope and you try to turn the corner, what happens?
> 2. What is the radius of the small circumference?
>
> For a possible generalization:
>
> 1. Would it always be the same if I put the horse on any other vertex?

To generate learning opportunities for students, prospective teachers in this group used the problems in their lesson plans without recognizing the potential of technological resources to modify students' mathematical activity. This feature makes transparent the potential of technological resources to visually represent geometrical invariants amidst simultaneous variations induced by, for example, dragging activities. So, for these prospective teachers, it was not possible to consider the utilities of Geogebra in interrelating the hypothetical mathematical conceptions that could have been developed (the question in the problems could be solved without the use of the technological resources). Consequently, it was not possible to talk of instrumental genesis. For example, Jesus focused on procedural aspects to calculate the areas and Geogebra was a tool used to validate the results previously obtained by a "paper and pencil solution":

> The purpose of this activity is to correctly represent and calculate the areas. The action followed by the teacher was to guide them to discover the steps that should be followed. This strategy is the most optimal. Thus, in situations that require such representations and calculations, students will know how to proceed. Even to use Geogebra to validate results.

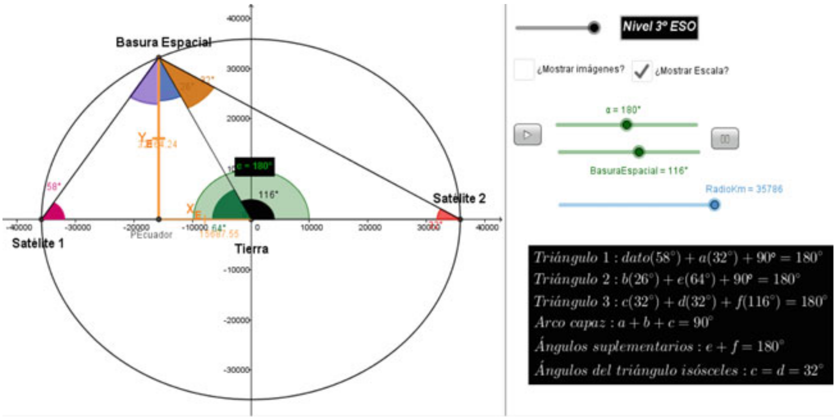
G2—Initiating the design of learning opportunities to support instrumental genesis

A second group of prospective teachers (n = 6 prospective teachers) planned to use technological resources to create learning opportunities for students to generate a mathematical activity that focused on the variability and relations between representation modes. These prospective teachers used sliders and dragging object as a means to discover mathematical relations. The problem used as a key element in the lesson plan generated a context in which the students' instrumental action would favour students' reflection about the relation between the action and the conceptual elements involved. These prospective teachers took advantage of potential offered by technological tools and provided the context for students to experiment and be able to relate solutions to different modes of representation or discover properties.

For example, David presented a modelling problem from an applet with Geogebra in which students (14–15 years old) had to connect the description of the situation with the use of an applet (Fig. 8.2) in a trigonometric lesson (isoptic, set of all the points from which a segment AB is seen under a given angle).

David initially proposed solving the problem without technology, and using technology to validate the calculations

> (With Geogebra, using the algebraic menu to get the values. Check the values obtained in the previous section with Geogebra.)



(With Geogebra, use the algebraic menu to get the values)

1. *Check that the values you got in the previous section correspond to what Geogebra shows?*
   How much are a (purple angle), b (blue angle), and c (brown angle)? And its sum?
2. *Change the values of α and find the value for which the problem is as simple as possible (it is not worth the Space Debris to be on top of one of the satellites)*
   a) *How much are angles a (purple angle), b (blue angle) and c (brown angle)? And its sum?*
   b) *How much is now Ye? And Xe?*
   c) *How much is the difference between α angle and the Space Debris angle?*
3. *Leaving α fixed, change the Space Debris angle*
   a) *Is there any other value for which you get the same result as in the previous case? What is the difference between these two angles?*
   b) *Is there any other value of the Space Debris angle for which you get the same Xe (ignoring the sign)? What is the relationship between these angles?*
   c) *And for Ye angle? What is the relationship between those angles?*

Fig. 8.2 Some questions in the David's problem to support the experimentation and the connection between representations, using sliders to make conjecture

He proposed to use the applet to generate a learning context to go beyond calculating. Using the Geogebra menu, he represented geometrical invariants from simultaneous variations induced by dragging. This prospective teacher introduced conditions in his lesson plan to generate opportunities to generate a "more or less stable sequence of interaction between the user and the artefact with a particular goal" (Drijvers, Kieran, & Mariotti, 2010, p. 109). In this case: modify values and notice the relation between new values. The goal of this sequence of questions in the lesson plan is: to identify invariants in the situation as a way of making mathematical conceptions emerge (in this example the mathematical notion of isoptic curve: for a given curve C, consider the locus of point P from where the tangents from P to C meet at a fixed given angle). The goal of the instrumented action scheme is to make the student notice the relation between the variability of parameters in the situation and the pattern that emerges from the mathematical conception in organizing this situation. The applet is designed to facilitate that student observe the connections between the graphical representation of the situation and the analytical expressions of the mathematical equations. Furthermore, the prospective teacher uses sliders to create a context to conjecture new relations between given values. This pedagogical use of sliders added a new aspect to the student's mathematical activity, conjecturing relationships between variables to modify the given values. The use of the applet create new learning situations for students enhancing mathematics activities as conjecturing relations between the given values that are not presented when the problem is enacted without technology.

However, when prospective teachers in this group anticipated students' responses, they only considered a procedural perspective of the students' mathematical activity. For example, when David anticipated students' answers he focused his attention only on identifying the equations, on the difficulties in solving systems of equations, and in handling the applet. This prospective teacher indicated the following as possible difficulties:

* Set the equation of the first triangle (data + a + 90° = 180°)

* Set the equation of the second triangle (b + 90° + e = 180°)

* Identify the congruence of angles in isosceles triangle (c = d)

* Set the equation of arc capable (a + b + c = 90°)

That is to say, while David could conceptualize a teaching situation through problem solving with the support of an applet, favouring certain mathematical processes as conjecturing, noticing the invariable in the situation, and setting connections between representations, he was only able to anticipate difficulties in identifying equations, in solving systems of equations and technical difficulties in handling the applet. When he indicated the student's difficulties, he focused on procedural elements but not on conceptual elements. In this case, the prospective teacher does not rely on the capacities of technological resources to generate learning opportunities in relation to the meaning of the capable arc and the properties of the angles inscribed on a circumference spanning the same arc.

This is apparent in the prospective teacher's behaviour: based on the different perspectives of his lesson plan, he anticipated students' mathematical thinking to be independent from the mathematical knowledge considered in the lesson. In this context, for example, Lourdes, a prospective teacher modified a problem of first and second grade equations to introduce Geogebra to facilitate the connections between different solutions. The problem is addressed to define a difference variable from the experimentation about particular cases

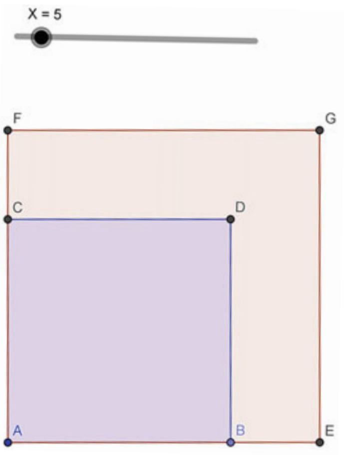
- Calculate the length of the side of a square, if by increasing its length by two centimetres, its area increases by 24 cm^2
- Construct the difference function of areas, represent it and obtain the solution
- Relate the dynamic model to the graphical representation of the function.

She guided the construction of the square and proposed to use sliders to approach the resolution of the problem (Fig. 8.3).

Lourdes use the technological resources to link different representation modes,

> We can take advantage of the potential of this program [Geogebra] to link the algebraic expression of the area difference function and the equation corresponding to the problem, as well as to establish connections between different resolutions.

This prospective teacher's approach to students' mathematical activities allows for the possibility of establishing connections between algebra and geometry.



Step 5: Create the difference variable
Create the variable dif= polygon2-plygon1, which gives us the difference between the areas of the enlarged square and the original square.
Step 6: We move the slider until we find the solution to the problem, which will be the value that the slider takes when, dif = 24. The solution is X = 5
Therefore, the side of the original square measures 5 cm and its area is 25 cm^2, while the side of the enlarged square measures 7 cm and its area is 49 cm^2

Fig. 8.3 Part of the Lourdes' square problem

In addition, it creates opportunities for students to guess the difference between areas by increasing the side length of the square. However, when she anticipated students' activity and the possible difficulties that students could face, her focus concentrated on the procedural aspects, not clearly explaining how to establish the relationship between the representation registers and the properties of the area function:

> Some students may correctly perform the resolution using the dynamic model, but do not reach the same solution from the graphical representation of the function of the difference of areas. Students may find the cut-off point of the graph of the function with the vertical line x = 24, instead of the point of intersection with the horizontal line y = 24. As the point of intersection is (24,100), the student would say that the side of the initial square measures 100 cm, which is the ordinate of that point. This indicates that the student has a good understanding of the geometric elements, but not the concept of function or the graphical representation of functions.
>
> Therefore, it interchanges the meaning of the coordinates of the points in the represented graph. In this situation, I would pose the following question:
>
> What do the points on the graph represent?
>
> I would ask him about the meaning of different particular points, so that he would arrive at the general idea. Then I would ask him for the meaning of the abscissa point 24, so that he would realize his error.
>
> Finally, I would ask: What point will give us the answer to the problem?

This prospective teacher focused exclusively on the meaning of ordered pairs did not take advantage of the potential of the relationship between the geometric screen, the graphical representation of the area function and the possibility of generating a table of values. This potential of the technological resource would have helped students to deduce the functional relation and the effect of the change of the value of the variables in the area of the square.

Prospective teachers in this group plan a lesson in which to integrate the potential of technological resources to favour the student's instrumental genesis. The problems used and their justifications of how to modify the cognitive demand of the problem are aimed at developing schemes and techniques. In particular, they were able to generate learning opportunities to identify invariant organization in a given situation. For that reason, they consider the conditions required for students to generate sequence of interactions using applets with a particular goal. With these characteristics of the lesson plans, prospective teachers support the co-emerge of technical and conceptual aspects; orchestrate conditions to engage students in seeing patterns and connecting ideas. However, this focus on the instrumental genesis in the lesson plan disappears when anticipating students' strategies and difficulties. When focusing on student's mathematical activity prospective teachers' perspective shifts to students' abilities to execute mathematical procedures. This difference between the perspective on the lesson plan and on student's activity reflects a dichotomy in how prospective teachers learn to integrate the technological resources when learning to teach.

G3—Integrating an epistemological stance about mathematical knowledge and students' mathematical activity

Finally, there is a third group of prospective teachers (n = 6 prospective teachers) that integrate their epistemological stance about mathematical knowledge in the lesson plan and how to anticipate students' mathematical activity. This approach showed an integrated perspective on the way of approaching mathematical activities with the support of technological resources and the cognitive stance on students' learning. For instance, Pablo a prospective teacher in this group chose a problem, in his lesson plan (for 14–15 years old students), which consisted of sub-problems from a particular case to general case:

> Calculate the length of the median in an equilateral triangle and the radius of inscribed and circumscribed circles to the triangle. Starting with an equilateral triangle (the length of the side is $10\sqrt{3}$ cm) and then with an isosceles triangle with the length of the different side is 12 cm.

Pablo's lesson plan is based on generating a mathematical situation, to support secondary school students in identifying the properties of mathematical objects and anticipate definitions. Pablo posed the problem, identified key moments of the resolution, which could be useful in getting over students' obstacles and difficulties, to generalize properties (Santos-Trigo & Camacho-Machín, 2009). For example, when Pablo anticipated the students' answers, he pointed out that some students would think that the property for the equilateral triangles works for all triangles. He identified relations between the mathematical contents and limitations of the properties that could be mobilised in resolving the problem (Fig. 8.4):

> In the hypothetical situation, a secondary school student would use the property, which is valid for equilateral triangles, with isosceles triangles, "**I will ask him to argue his answer and after then, he should make the construction**. For me, **it is a key moment for generating conflict and contradiction**. If **I supposed that the student shows me a good construction, which works, I would vary the length of one side and things began to fall down**."

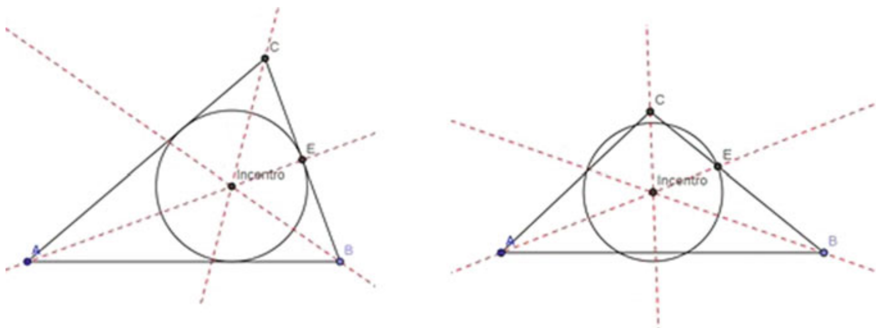


Fig. 8.4 Particular constructions that show limitations of the properties

Pablo anticipated possible challenges and difficulties for secondary students. He knew the difference between a geometrical construction and a simple representation or drawing. Dynamic software like Geogebra offers the possibility of taking advantage of its dynamism to enhance mathematical activity. For this reason, it is very important to consider the justification and argumentation that demonstrate what someone is exposing. Pablo indicated in his report that it is necessary that students realize that there are properties that only met some types of triangles. Pablo argued that point using the following figures constructed with Geogebra. Pablo displayed this fact modifying the lengths of the triangle sides.

For Pablo, using dynamic software could be suitable for learning mathematics and problem solving and let secondary students establish the differences between what it is a simple drawing and a "geometrical construction of mathematical objects". That fact underlines the role of technology in helping secondary school students to go more in depth into the knowledge of geometrical thinking (in this particular case, providing sense of the idea of geometric construction). It is exemplified by Pablo, in the particular case of an isosceles triangle whose construction coincides with the equilateral triangle when Pablo modifies the length of the side AC from $10\sqrt{3}$ to 11, it shows that the construction is not correct and students will have to look for other properties in constructing the circumscribed circle.

The characteristic of this lesson plan is that Pablo uses the technological resource to generate situations in which students can reflect on many particular cases to abstract the mathematical conception. The possibility of generating cognitive conflict when the generalization of a mathematical relation is not fulfilled is considered as a context that supports the student's reflection. In Pablo's lesson plan, Geogebra supports the generalization processes from sets of particular cases. Furthermore, when Pablo anticipates students' answers to the problem, he considers the potential of the dragging tool in Geogebra to represent geometrical invariants to induce visually the abstraction of the mathematical conceptions.

8.5 Discussion and Conclusion

This study examines how prospective secondary school mathematics teachers use technology resources, like applets and software of dynamic geometry as Geogebra and technological affordances as dragging objects, to quantify parameters and use sliders to support students' mathematics learning. The study uses Simon and Tzur's (1999) theoretical construct "teacher's perspective" to focus on how technology is used in a lesson plan and documented different ways in which prospective teachers use technology. These prospective teachers' perspectives are cognitive references through which they make decisions on teaching and it allows us to relate their epistemological stance about school mathematics (what type of mathematics activity could be supported) and what is the focus of students' mathematics learning.

We use two dimensions to define prospective teachers' perspectives. These dimensions take into account the nature of the student's mathematical activity that could be supported by the technological resources and how the prospective teachers plan to use the technological resources in problem solving. These dimensions are related to the idea of instrumental integration used to describe "how the teacher organize the conditions for instrumental genesis of the technology proposed to students and to what extent (s) he fosters mathematics learning through instrumental genesis" (Goos et al., 2010). We have characterized three ways in which technological resources, when integrated in prospective teachers' perspectives, show how they plan a lesson and anticipate students' mathematical activity.

Some prospective teachers pay more attention to the results than to the process of solution, and attach less importance to the students' mathematical activity such as conjecturing, proving, arguing, and connecting different representation modes. These prospective teachers turned technological resources into an end for itself and its use was anecdotic throughout the development of the mathematical activity and problem solution. On the other hand, some prospective teachers organised their lesson plan considering the mathematical activity generated by taking advantage of the potentials of the technological resource and by identifying key moments during problem solution. The prospective teachers' lesson plan is based on dragging objects, quantifying parameters and using sliders to support students' mathematical activity as conjecturing and testing, identifying properties and so on. For example, the identification of key moments in the problem solving process allowed prospective teachers to focus on the study of particular cases as an initial step in the search of properties, facilitating the connection between ways of representations and looking for problem alternative solution. However, other prospective teachers transformed problems in their lesson plans by using dynamic representations of problems to support mathematical relations, but when they anticipated students' mathematical activity they only took into account procedural mathematical aspects. These prospective teachers could be considered as those who have not yet established a bridge between the discipline's epistemological stance and the students' cognitive dimensions in learning to use the technological resources to support the mathematical learning in problem solving context (Santos-Trigo, Moreno-Armella, & Camacho-Machin, 2016).

These results suggest that learning to integrate technology in mathematics teaching aimed at promoting the development of the mathematical activity is a complex process. The different ways in which prospective teachers may consider technology as a pedagogical resource to support students' learning provide means of tracing learning trajectories of how prospective teachers learn about mathematics teaching (Stohl, 2005). Furthermore, we argue that the variability in which prospective teachers thought about technology and the role played by technology in problem solving could also be explained by the prospective teachers' beliefs about learning (Lin, 2008), and the nature of the mathematical task. This last issue emphasizes the need to carry out more research on the relations between knowledge, beliefs, and nature of the task in the lesson about how to use technology to support students' mathematics learning.

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