Chapter 7
Contemporary Framing of Technology in Mathematics Teaching

Rose Mary Zbiek

Abstract Many types of powerful digital technologies have been part of secondary school mathematics classrooms and mathematics teacher education in many places for decades. However, research on technology in teacher preparation continues to be sparse and challenging to synthesize. To organize and probe ideas, researchers and practitioners need better ways to frame their work. In this chapter, a blend of three conceptual tools is connected to existing literature to describe prospective secondary mathematics teachers' (PMSTs') professional growth in technology, content, and pedagogy in integrated and dynamic ways. The blending of Technological, Pedagogical, and Content Knowledge (TPACK); Mathematical Understanding for Secondary Teaching (MUST); and Play, Use, Recommend, Incorporate, and Assess (PURIA) perspectives underscores the complexity of learning to teach mathematics with technology.

Keywords Teacher knowledge · Technology use · Secondary mathematics Mathematics teaching

7.1 Introduction

In research and practice, technology is both an object of learning and a pedagogical tool in the education of prospective secondary mathematics teachers (PSMTs) as PSMTs learn of technology and learn with technology. Diversity of technology leads to a body of literature that is challenging to synthesize in the interest of framing future studies and of informing practice. This chapter is not a research synthesis or another way to frame what PSMTs need to know and do with technology. The goal of this chapter is to integrate existing frameworks to better understand research and practice around technology as both object and tools in the

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education of PSMTs. Underlying the integration is an assumption that PSMTs enter their preparation programs knowing something about each of technology, mathematics content, and pedagogy. Articulation and integration of the frameworks precedes a discussion of the implications of the amalgamated perspective.

7.2 Technology Opportunities and Options

In 2017, *technology* refers to an amazing array of products. Although it takes many forms, for the purposes of this chapter, "technology" refers to digital resources. Technology in mathematics education could be technology used for mathematical purposes, such as computer algebra systems, or for communication purposes, such as word processing packages and social media, or for other purposes, such as video analysis software that can be used by PSMTs to play and mark excerpts in the study of their classroom practice. Dick and Hollebrands (2011) refer to the first type of technology as "mathematical action technologies" (e.g., computer algebra system, dynamic geometry, graphics calculator, spreadsheet, online applets) and underscore, as do others, that not all of these products were developed initially for educational purposes. Bowers and Stephens (2011) refer to the second type of technology as "communication and visualization technology". PSMTs might enter teacher education programs with varying experience with one or more products in each of these general genres.

Technological tools exist in many different physical forms and can serve a variety of mathematical, pedagogical, or communicative purposes. Tools with very similar purposes can exist in different media. For example, graphing utilities can be found as phone applets, programs on laptop, and features of handheld calculators. Moreover, one tool might be used for different purposes. One example is the difference between a computer algebra system (CAS) used while solving a mathematics problem and the same CAS used to create a file in which a tutorial or assessment is embedded. As these nuances imply, one improvement in conducting, reporting, and synthesizing research and practice is to be attentive and clear about the technology being used and its mathematical, pedagogical, or communicative purpose.

7.3 Work of Educating Prospective Mathematics Teachers

As practitioners, mathematics educators prepare PSMTs to be the best possible teachers that they can be. As researchers, mathematics educators seek to understand not only how PSMTs develop their practice but also the nature of their knowledge, beliefs, identities, and other personal characteristics. PSMT preparation happens within the contexts of universities and schools and across mathematics content courses, pedagogy courses, and practical experiences in schools or in other educational venues. Accounts of technology use in secondary mathematics teacher education often fall in the important but sometimes challenging to defend overlap between a faculty member's research and the courses or programs in which the faculty member teaches.

In content courses, pedagogy courses, and practical experiences, discussions of technology can focus on PSMTs as learners with technology as the object of instruction or as individuals whose learning about other things is supported by technology. PSMT educators include a broad group of all who contribute to a teacher's development, such as, in a typical teacher education program, mathematics and statistics instructors, pedagogy course instructors, and field supervisors. The genres and specific pieces of technology PSMT educators choose to use and how they use those technologies can vary greatly. Pedagogy courses and practica also address the use of technology by the secondary school students whom the PSMTs instruct.

Although experiences are often spread across content courses, pedagogy courses, and practical venues, PSMTs must connect ideas across content, pedagogy, and practice to make sense of their preparation and to act on it in their own practice. For this reason, a body of technology-related literature, such as that referenced by Huang and Zbiek (2017), should be revisited in terms of what it reveals about how PSMTs develop understandings of content, pedagogy, and technology based upon what they know as they enter their teacher education programs—and how the PSMTs integrate new ideas and understandings.

Huang and Zbiek (2017) describe the process that led to the 18 articles they synthesize. Their multi-layered process for selection of the articles ensured that the cited studies were reported in detail in journal articles appearing in internationally circulated venues, were clear in their identification of technology, and were studies that focused specifically on prospective secondary mathematics teachers. The cited studies would ostensibly be useful in addressing fundamental questions that should be asked about the literature regarding technology and teacher preparation and how PSMTs develop understandings and improve their teaching practices. The task of posing such questions is problematic, however, in (at least) three ways. First, the very definition of *technology* is elusive. Second, mathematics teacher educators use a wide range of tools within the three contexts. Third, the secondary school student can be either an active user of the technology or a learner who benefits from the teacher's use of the tools.

A contrast of two of the 18 studies underscores the complexity of synthesizing the literature. For example, both Davis (2011) and Star and Strickland (2007) conducted their studies in the context of PSMTs developing their understanding of pedagogy. However, the two studies differed greatly in the technology used. Davis used computer algebra systems; Star and Strickland used video recordings of lessons. The two studies also differed in the aspects of pedagogy they targeted. Davis considered textbooks; Star and Strickland studied professional noticing. The mathematics content foci also differed. Davis' PSMTs considered the algebra and function strand across many lessons in a textbook; Star and Strickland's PSMTs viewed recorded lessons on angles, arc lengths, secants, and tangents. In Star and

Strickland's work, the secondary school students were not active users of the video. In Davis' work, secondary school students were expected to engage actively with the computer algebra systems.

As the comparison of Davis (2011) and Star and Strickland (2007) suggests, the literature on technology in secondary mathematics teacher education can be challenging to synthesize, but it can be synthesized. Huang and Zbiek (2017) chose to organize it around mathematics content courses, pedagogy courses, and practica. That parsing of the literature is useful in applying the results of the research to teacher education programs and course instruction. The same literature might be organized differently by attending less to the three typical venues of teacher education and more to PSMTs' learning, asking a question: *In general, of technology tools, mathematics content, and pedagogy, which is novel to PSMTs?* Asking this question synthesizes literature in a spirit of understanding and support of PSMTs continued development.

7.4 Probing the Literature About Technology and Teacher Preparation

A synthesis of literature used either to conceptualize new studies or to inform practice requires more than a collection of descriptive paragraphs of findings from individual studies. It requires having conceptual tools to frame and explain what existing studies offer. Because PSMTs are differently experienced with any one tool and its uses, and it is important to recognize what part of the experience is new to the PSMT, each of the following sections assumes that one of the three elements—technology, content, or pedagogy—is a novel piece for PSMTs. Each section then describes a conceptual tool that can be used to explore and describe in depth the subtleties and the nuances of the technology, the content, or the pedagogy. In a later section, the three conceptual tools are considered collectively.

7.4.1 When the Technology Is the Novelty

When digital technologies exploded in the late twentieth century, the tools were the novelty in classrooms for practicing teachers and also in teacher preparation. This is the setting, for example, during which technology became available for use in education. The Technological, Pedagogical, and Content Knowledge (TPCK or TPACK) framework (Mishra & Koehler, 2006) emerged as a useful conceptual tool to describe and support teacher development. Building from Shulman's (1986) seminal work around Pedagogy and Content Knowledge (PCK), Mishra and Koehler develop TPACK to capture the kinds of knowledge needed by a teacher to integrate technology into classroom practice. The framework includes

Technological Knowledge (TK), Technological Content Knowledge (TCK), and Technological, Pedagogical, and Content Knowledge (TPACK).

Bowers and Stephens (2011) call mathematics teacher educators to interpret TPACK as "an orientation that views technology as a critical tool for identifying mathematical relationships" (p. 290). The view of TPACK as orientation followed the prolonged attempts of these researchers to identify a set of skills and knowledge at the center of technology, content, and pedagogy that would be the skills and knowledge identified as TPACK. Bowers and Stephens began to ask a different question: "What factors *do* affect prospective teachers' development of a TPACK orientation?" (p. 290). The factors they identified included such things as a teaching style rich in "what if questions". Bowers and Stephens' identification of such factors led them to their conclusion that orientation rather than knowledge was critical. An improvement in discussions of technology as a novel agent in PSMT preparation is to recognize orientation as well as knowledge. Orientations, beliefs, and knowledge must be acknowledged and leveraged in teaching mathematics and in preparing PSMTs.

For situations in which technology is the novel element, mathematics teacher educators need to emphasize orientation as well as knowledge. Research that illuminates not only what orientations, knowledge, and beliefs PSMTs have but also how to address and leverage these things in teaching mathematics is critical. TPACK was explicitly explored in six of the 18 works cited by Huang and Zbiek (2017). The other 12 cited works were situated in content courses, pedagogy courses, and practical experiences but did not address exclusively, if at all, PSMTs' knowledge related to technology, pedagogy, and mathematics. For example, Davis (2015) considered how PSMTs read, evaluated, and adapted a textbook lesson that used computer algebra systems. In Davis' study, pedagogy was the novelty, while the technology and the mathematics content were more familiar.

7.4.2 When the Content Is the Novelty

Technology is not always that novel element for PSMTs, although it might be novel to the mathematics educators who work in PSMT preparation. Technology allows PSMTs to learn unfamiliar mathematics content and to learn new things about familiar mathematics content, making the mathematics content the novel element. Examples include such things as how PSMTs might understand and compose transformations of the plane differently after working with figures in dynamic geometry environments or with graphs in a Cartesian coordinate context. Another example might be how simulations and manipulations of samples in a dynamic statistics setting affects how PSMTs conceptualize and describe sampling distributions.

PSMTs learn not only new content but also have new opportunities to engage in mathematics. For example, PSMTs likely are familiar with exponential functions and their graphs. They might, however, not anticipate certain actions, such as

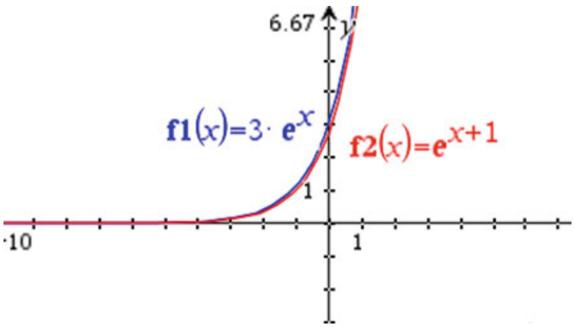

Fig. 7.1 Nearly the same graphs of $f(x) = 3e^x$ and $g(x) = e^{x+1}$

noticing that the graphs of $f(x) = 3e^x$ and $g(x) = e^{(x+1)}$ appear to be nearly identical, as in Fig. 7.1. This technology-based observation gives cause for a justification of whether these two graphs coincide. The observation also prompts the question of whether there are other pairs of functions of the form $\mathrm{f(x)} = ae^x$ and $\mathrm{g(x)} = e^{(x+b)}$ that have nearly the same graph. There is opportunity here to engage in the mathematical activity of noticing of the structure of mathematical systems. The structure of the symbolic representations is in contrast to the structure of transformations. Both graphs can be seen as transformations of the basic function, $p(x) = e^x$, with the non-trivial caveat that two transformations seem to map the basic function to the same graph—as shown in Fig. 7.1—but the transformations themselves are not equivalent. PSMTs engage in Mathematical Reasoning in terms of both Justifying/Proving and Reasoning When Conjecturing and Generalizing as PSMTs generalize their initial observations, test their claims, and symbolically verify their results.

As TPACK helps to sorts out different types of knowledge or orientations when technology is the new element, the conceptual tool does not allow nuances in mathematics content and action to be readily acknowledged. A framing different from TPACK—a tool that captures mathematical content, activities and the context of teaching—is needed. Mathematical Understanding for Secondary Teaching framework (MUST) (Heid & Wilson, 2015) is one such tool designed exclusively for mathematical knowledge for secondary mathematics education.

MUST is relevant to study and practice of secondary mathematics teacher education because it is based on the work of secondary school mathematics teachers. The emphasis on mathematics that is useful in teaching is a key point of the framework. Technology used for content development in teacher preparation serves well to extend and connect ideas that are common to school mathematics. The MUST framework captures the mathematics in Situations that were developed around incidents that happened in classroom settings and other venues of the daily work of teaching. The inspirational incident is what the MUST researchers called the Prompt. For example, consider the Prompt from the Division Involving Zero Situation, as shown in Fig. 7.2. The Situation is relevant to discussion of technology in secondary mathematics teacher preparation because not only might one

Fig. 7.2 Text for prompt from the division involving zero situation (adapted from Heid & Wilson, 2015, p. 95)

On the first day of class, preservice middle school teachers were asked to evaluate 2/0, 0/0, and 0/2 and to explain their answers.

There was some disagreement among their answers for 0/0 (potentially 0, 1, undefined, and impossible) and quite a bit of disagreement among their explanations:

- Because any number over 0 is undefined;
- Because you cannot divide by 0;
- Because 0 cannot be in the denominator;
- Because 0 divided by anything is 0; and
- Because a number divided by itself is 1.

use a variety of digital tools to explain the three indicated divisions in the Prompt but also one might encounter the situation within a classroom.

What might teachers use to address this Prompt? The answer to that question for a Prompt is found in the Foci. For example, two of the five foci for the Division Involving Zero Situation are the following:

- Mathematical Focus 2. One can find the value of whole number division expressions by finding either the number of objects in a group (a partitive view of division) or the number of groups (a quotitive view of division).
- Mathematical Focus 3. The mathematical meaning of $^{a}/_{b}$ (for real numbers a and b and sometimes, but not always, with $b \neq 0$) arises in several different mathematical settings, including slope of a line, direct proportion, Cartesian product, factor pairs, and area of rectangles. The meaning of $^{a}/_{b}$ for real numbers a and b should be consistent within any one mathematical setting.

The robustness of MUST relies on the quality of the Foci. The Foci for each Situation were created by a team of researchers from Penn State University and the University of Georgia and reviewed by another team of researchers from the two institutions. The Foci were also reviewed by groups of mathematics education researchers, of mathematicians, of mathematics teachers, and of mathematics supervisors at specified project meetings and public gatherings.

Qualitative analysis of hundreds of Foci produced in this way led to identification of three perspectives on mathematical understanding: Mathematical Proficiency, Mathematical Activity, and Mathematical Context of Teaching. The perspective of Mathematical Proficiency includes six aspects: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition, and historical and cultural knowledge. That is, Mathematical Proficiency was well captured by approximately six elements that echo the strands of mathematical proficiency in *Adding It Up* (National Research Council, 2001), with the addition of a strand of historical and cultural knowledge. The use of labeling from *Adding It Up* with slightly different descriptions was intentionally done both to allow for a connection between primary school and secondary school settings and to avoid offering a set of similar categories differently named.

The perspective of Mathematical Activity includes ideas offered under a variety of labels, such as mathematical practices (Common Core State Standards Initiative, 2010), process standards (National Council of Teachers of Mathematics, 2000), habits of mind (Cuoco, Goldenberg, & Mark, 1996), and specific areas such as mathematical modeling (Organisation for Economic Co-operation and Development, 2013). It extends to such things as symbolic insight (e.g., Bowers & Stephens, 2011). It includes Mathematical Noticing, Mathematical Reasoning, Mathematical Creating, and Integrating Strands of Mathematical Activity. Its first three areas can be further subdivided. For example, Mathematical Reasoning includes Justifying/Proving, Reasoning when Conjecturing and Generalizing, and Constraining and Extending.

Interestingly, in terms of technology in secondary mathematics teacher preparation, relatively little research is found in the first two MUST Perspectives. For example, Huang and Zbiek (2017) cite only two studies (Cory & Garofalo, 2011; Zengin & Tatar, 2015) that focus on Mathematical Proficiency. Both studies address conceptual understanding. A similar observation might be made about works related to Mathematical Activity. Notably, Huang and Zbiek cite only one study that addresses any form of Mathematical Activity. Zembat (2008) considers Mathematical Reasoning.

The small number of items related to Mathematical Proficiency and Mathematical Activity among the works cited by Huang and Zbiek (2017) might reflect the landscape of the 18 papers. Mathematical Context of Teaching, the third MUST perspective, is a view of the context for mathematical content in the MUST framework and might best be considered with Content issues. However, the details of the studies seem to fit better with pedagogy, for perhaps reasons that reveal both questions about research in the field and concerns about a dichotomy regarding technology use in secondary mathematics teacher preparation.

7.4.3 When the Pedagogy Is the Novelty

The main assumptions in Sects. 7.4.1 and 7.4.2 respectively, are that technology is the novel element in the teacher's practice and that mathematics is the novel element. These assumptions often work well in professional development with experienced mathematics teachers but they are not the totality of what is needed in work with PSMTs. PSMTs likely are digital natives who are familiar with school mathematics content and with a variety of mathematics, communication, or other technologies. For them, pedagogy is the new and intriguing piece.

As the third MUST perspective, Mathematical Context of Teaching considers mathematical work directly connected to the teaching of mathematics. The strands of this perspective are: Probe Mathematical Ideas, Access and Understand the Mathematical Thinking of Learners, Know and Use the Curriculum, Assess the Mathematical Knowledge of Learners, and Reflect on the Mathematics of Practice.

Works cited by Huang and Zbiek (2017) fell into only two of these categories. Two works addressed accessing the mathematical knowledge of learners (Akkoç, 2015; Santagata, Zannoni, & Stigler, 2007), and five of them explored accessing and understanding the mathematical thinking of learners (Hähkiöniemi & Leppäaho, 2011; Lee, 2005; Rhine, Harrington, & Olszewski, 2015; Star & Strickland, 2007; Wilson, Lee, & Hollebrands, 2011). The evidence suggests the need for mathematics teacher educators to help PSMTs to develop some aspects of mathematics understandings for secondary teaching (e.g., questioning, student thinking) and perhaps work with particular genres of technologies (e.g., video) of communication and collaboration technology. Using MUST to frame content issues within research and practice suggests there are aspects of mathematical understanding for teaching that need attention as they relate to pedagogy, especially in terms of technology.

It is important to keep in mind that in thinking about pedagogy the attention is not on the teachers (the PSMTs in this case) and their characteristics but on their teaching. An analysis of the current published studies about technology in teacher education from the sources used by Huang and Zbiek (2017) suggest that there is a growing body of literature about the use of digital video to capture, present, and study teaching.

In a synthesis of the research literature on technology, Zbiek and Hollebrands (2008) probe the literature about technology in the teaching and learning of mathematics to answer the question of how one learns to teach with technology—a question that essentially places pedagogy as the novel element in the teacher's practice. The curious answer to that question in a word is PURIA—a path to teaching with technology based in name and in spirit on Beaudin and Bowers' (1997) discussion of how teachers become proficient in using computer algebra systems. PURIA is an acronym representing modes (Play, Use, Recommend, Incorporate, Assess) through which Beaudin and Bower claimed teachers must pass in order to become highly proficient classroom technology users.

Although they located no studies that explicitly generated or tested the theory/framework at the time, Zbiek and Hollebrands (2008) argue that the literature to that date indicates that teachers do grow pedagogically (and technologically?) across these five realms. First, a person must Play with the technology and Use the technology for personal purposes. Although use of the word "Play" might give a different impression, the idea is that a person typically begins with open-ended opportunities to try what the technology can do.

Regarding mathematics technology, consider the introduction of computer algebra systems into a school. The person might attend a CAS workshop and be handed a CAS-capable calculator. The person's first instinct might be to Play with the device, figuring out how to turn it on, type some garble, enter an arbitrary function rule to see what a graph looks like. In these actions the person is not trying to do mathematical work, but rather attempting to see how the technology works and what it might do. Later that day, the person might then think about how the device might be used to solve a system of equations by graphing and then symbolically. Although he or she may fumble with keystrokes and menu options, the person is now working with a mathematical purpose and his or her intent it to Use the technology for personal mathematical purposes.

An example of the integration of a communication/collaboration tool might start with the mathematics teacher educator who Plays by drawing silly faces and saves and erases them when he or she first encounters an interactive white board. The next week, the mathematics teacher educator might Use it to draw and save a geometric diagram to show the class and have students mark as they work on proving a particular theorem. In this way, the mathematics teacher educator as the teacher of the lesson is making the interactive white board as a pedagogical tool for his or her own use but not yet making the interactive white board something to be learned by his or her PSMTs. In essence, the Play and Use phases seem indicative of opportunities for teachers (including teacher educators) to develop TK.

Following Play and Use, the person is prepared to Recommend use of the technology to others, Incorporate the technology into practice*, and* Assess students' use of the technology. These are the phases in which the person engages with others—and especially with students, as we might see by the continuations of the interactive whiteboard (IWB) and computer algebra system (CAS) examples in the next two paragraphs.

The individual who has begun to use CAS for his or her own purposes might next Recommend it to others. For example, the teacher might give it to a small group of students so they can learn how to produce solutions to the systems on their way to answering the question: "How many solutions can a system of two linear equations in two unknowns have?" The teacher would observe what happens and have some idea of how CAS was helpful—or not—for the students. A next move might be for the teacher to Incorporate the technology into a lesson on the number of solutions of a system so that all students might use it, and then the teacher might use it in other lessons as she or he Incorporates the technology into her or his practice. With reflection on these classroom experiences to Assess the technology's use and potential, he or she might conclude that the use is productive and then refine how he or she employs it in similar lessons.

The mathematics teacher educator who has started to use an interactive white board (IWB) as an instructor might next Recommend its use to a small group who is preparing a presentation for their class. The Incorporate move might then be the mathematics teacher educator offering a lesson in which all PSMTs or groups in the class have to use the IWB as part of their work, perhaps in lieu of a non-interactive presentation tool (e.g., PowerPoint or Prezi projected on a standard screen). The Assess piece might come with reflection upon whether and how the PSMTs used the IWB to enhance their presentations and to engage their classmates in the conversation.

Two points need to be made about PURIA before a discussion of how it might be integrated with MUST and TPACK. First, the framework as proposed by Beaudin and Bower (1997) has not been tested as a model of learning. Such empirical verification, though desirable, would be a long-term, demanding research effort that would likely involve more than one study. Zbiek and Hollebrands (2008) examined existing literature and noted how the compiled findings of the literature, at that time, supported PURIA as a framework for how teachers—including PSMTs—move from gaining initial knowledge of technology to developing classroom practice and pedagogy around the technology. Implicit in the following section is the observation that recent research in technology in teacher education also supports the PURIA framework.

7.5 Conceptual Tools to Inform Practice and Inspire Research

The blending of TPACK, MUST, and PURIA underscores the complexity of learning to teach mathematics with technology. The extent to which particular mathematics, specific technology, or pedagogy is the most novel and critical element depends on the individual PSMT.

To illustrate how the three conceptual tools are useful in understanding how PSMTs develop as teachers of mathematics with technology, consider a report by Bowers and Stephens (2011). The researchers argue that four of the resulting categories (TK, TCK, TPK, and TPACK) could be conceived as levels. Although one might be skeptical about whether these four TPACK orientation categories truly are levels in the classic sense of a level theory, connecting Bowers and Stephens' work to MUST and PURIA is enlightening, and perhaps fortifies their argument that the four categories are indeed levels.

Bowers and Stephens' bar graph in Fig. 7.3 illustrates the number of PSMTs that the researchers coded as being in each of four TPACK orientations. Although the numbers are small, the data invites the question of not only why are more of the PSMTs seeming yet to reach TPACK—the unspoken highest level—but also what might be the trajectory of their learning. The PSMTs arguably had some course experience with technology, so the claim here is not that this is a natural

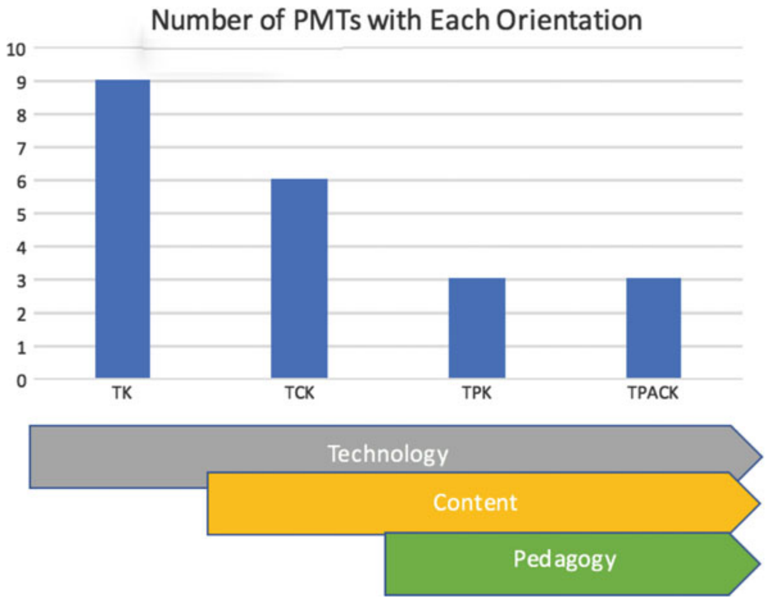

Fig. 7.3 Bowers and Stephens' (2011) coding results with banners representing focus on technology, content, and pedagogy

progression but rather it is a progression that agrees with multiple conceptual tools. A first look at the data suggests the relative prominence of technology, the moderate role of content, and the lesser role of pedagogy, in terms of what dominates the PSMTs' orientations. The horizontal arrows in Fig. 7.3 visually convey these relative roles.

Suppose the Technology, Content, and Pedagogy labels in Fig. 7.3 are replaced by "Play with Technology," "Use as Tool," and "Recommend/Implement," respectively, as shown in Fig. 7.4. This move, which might seem arbitrary at first, yields a revelation. Technology was present in all of the orientations—perhaps due to choices that Bowers and Stephens made. Replacing "Technology" with "Play with Technology" means the longest horizontal arrow could represent the development of technology orientations experienced through their teacher preparation program or by virtue of being digital natives. "Use as Tool," which replaces "Content," could be the use of tool to learn new mathematics or to understand familiar technology better. It also might be to learn about teaching and pedagogy through technology. Perhaps the orientation towards content follows from attention to PSMTs experience with technology as Use as Tool. The replacement of "Pedagogy" with "Recommend/Implement" might suggest that pedagogy-related orientations build on technology, which is part of all four orientations. In addition, the smaller number of PSMTs coded with TPK than coded with TCK might be a sign that Technology and Content orientations naturally and/or more productively precede Pedagogy orientations.

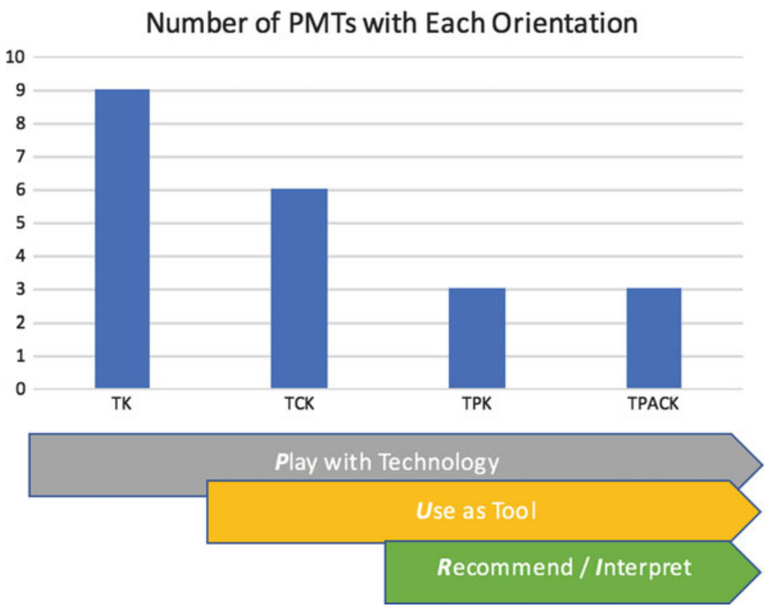

Fig. 7.4 Bowers and Stephens' (2011) coding results matched with PURIA modes

Researchers and practitioners interested in the development of PSMTs have provided some pools of related literature among a collection of studies that are largely descriptive of PSMTs and disparate in terms of the types of technology used. Examples of this tendency of existing literature are the studies cited by Huang and Zbiek (2017), as well as the earlier studies considered by Zbiek and Hollebrands (2008). Interesting work yet to be done, in addition to connecting the pieces of the current literature landscape, could undertake the overarching question of along what trajectory(-ies) do the technology, content, and pedagogy understandings and orientations of PSMTs develop and co-evolve. The claims stated in the previous paragraph are tentative. They raise a number of questions and suggest a number of subsequent studies aimed at exploring further how prospective teachers develop knowledge and orientations and integrate technology into their emerging classroom practices. Yet, the way in which components of the TPACK framework and those of the PURIA framework align provide support for both frameworks as feasible ways to look at PSMT development.

The reader might be wondering how MUST then fits into the framework picture with PURIA and TPACK. First, MUST as a way to refine how we look at Mathematical Proficiency and Mathematical Activity can be used to determine the extent to which technology is used for particular mathematical purposes. MUST's Mathematical Context of Teaching perspective touches on skills and actions often associated with pedagogy. For example, student thinking, school curriculum, and assessment—which correspond to Access and Understand the Mathematical Thinking of Learners, Know and Use the Curriculum, and Assess the Mathematical Knowledge of Learners—are common topics in many pedagogy courses. The implementation of these moves often fit within education but the corresponding mathematical understandings needed to execute these things are often left unaddressed in both content and pedagogy courses. They perhaps surface in practica as PSMTs must react to student thinking and assessment in authentic, open-ended ways. Use of MUST could help to focus both technology use and research questions about technology use in mathematics pedagogy courses and practical experiences.

Another point about MUST is that if technology is used for all of the elements of Mathematical Proficiency and Mathematical Activity, it seems that mathematics technology has the potential to be used for more than answer checking or the execution of basic procedures (a criticism often offered regarding the use of technology in mathematics instruction). Attention to the Mathematical Context of Teaching seems to allow for the blended use of mathematics tools and communication/collaboration tools. Focusing mathematics education research on both the type of mathematical work in which people engage and the nuances of pedagogy might be a productive way to simulate key aspects of the related work of teaching in pedagogy courses and of studying and improving practice in practica.

The final point regards connecting Figs. 7.3 and 7.4. The TPACK elements shown in Fig. 7.3 and the PURIA aspects shown in Fig. 7.4 are not necessarily mathematics-specific. While MUST aspects of mathematical understanding do not appear explicitly in Figs. 7.3 and 7.4, these figures are consistent with the idea of

PSMTs as learners coming to terms with particular technologies as tools for doing mathematics—and perhaps as tools for teaching mathematics—in the spirit of what Guin and Trouche (1999) name *instrumental genesis*. Instrumental genesis suggests that the PSMTs would have their ideas within each MUST perspective influenced by what technology offers and that they would begin to tamper with their tools to adjust the technology to their mathematical and pedagogical purposes. Integrating MUST, PURIA, and TPACK to the extent that PSMTs draw fluidly upon ideas within each of technology, content, and pedagogy and across these areas, using technology as a tool for both mathematics and pedagogy and to do so in reflective practice might be the ideal for which secondary mathematics teacher education research could reveal insights and secondary mathematics education practice could aspire to achieve and disseminate.

7.6 Implications

The extent to which different conceptual tools such as MUST, PURIA, and TPACK co-inform the work of teaching mathematics with technology in mutually complementary ways is an indicator of how robust they might be as tools to conceptualize and report research and inform and improve practice. Past and current literatures suggest that technology work with PSMTs happen around content courses, pedagogy courses, and practical experiences. Bowers and Stephens' (2011) data provide a rough snapshot of where a group of PSMTs are in their developing not simply knowledge but orientations towards technology use in the teaching of mathematics. Evidence from this chapter's compilation of empirical findings, literature, and theory—and perhaps the author's and reader's practice—indicate that informal experiences with technology (***P***lay) and then work with technology to do mathematics (***U***se) precede use of the technology with others in small ways (***R***ecommend) and then in major ways (***I***ncorporate). The observation suggests that PSMTs should encounter multiple forms of technology in all venues of their preparation, including mathematics courses, pedagogy courses, and practica and have time to work with these tools and provide time for PSMTs to develop the technology as their own tools and incorporate them into their daily work. Technology needs to be incorporated in teacher preparation in comprehensive ways for learning and for teaching. Importantly, mathematics teacher educators cannot assume that technology—or mathematics content, or pedagogy is the novel element for PSMTs—an assumption that was valid and necessary at the time that TPACK first emerged. The observations about the extent to which MUST's Mathematical Context of Teaching overlaps with pedagogy are indicators that more could be done in terms of how technology might serve to help PSMTs weave together their understandings of mathematics and technology.

The seemingly low number of studies located by Huang and Zbiek (2017), which amounted to less than two articles per year, raises several questions. First, where is the research on technology in secondary mathematics teacher preparation?

Is it not being submitted? Not being accepted? Or, is it shared more readily in venues other than journal articles? International norms and experiences with journal editing suggest each of these three questions raises a serious issue. Now is the time to contribute to the ongoing investigation of PSMTs' knowledge and skill in and across content, pedagogy, and technology and to the needed exploration of how PSMTs develop practice that incorporates technology as a full partner with content and pedagogy.

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