Chapter 6
Diagnosis-Based Adaptations of Mathematics Lessons: Analysis of the Implementation by Prospective Teachers During Practical Phases

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Abstract What conclusions do prospective teachers draw from their students' current learning level for their own lesson planning? Why do they draw these conclusions? Within an empirical-qualitative study, 15 prospective teachers planned a mathematics lesson during their practical phase. They were asked to diagnose the learning level of their students and then modify the planned lesson, if they thought it to be necessary. First, a system of categories was developed, describing their interpretations of the diagnosis, the modifications of the planned lessons and their justifications. Afterwards, eight different, recurrent types of decisions were identified. With these results it was possible to generate a process model, which helps to understand how the prospective teachers came to their decisions.

Keywords Adaptive teaching · Diagnosis · Empirical-qualitative study
Mathematics lessons · Prospective teachers

6.1 Introduction

Planning lessons is an essential component of a teacher's professional duties (Baumert & Kunter, 2006). Due to the fact that there usually is heterogeneity within a class (Baumert et al., 2001) diagnosis as well as individual improvement, which can be implemented in the form of adaptive teaching, have gained in importance over the past few years. Furthermore, the importance of diagnostic competences on the part of the teaching staff has long been empirically proven (e.g. Karing, Pfost, & Artelt, 2011). Effective as well as lasting teaching and learning processes may be initiated by tying in with individual learning levels (Hußmann & Selter, 2013). Politics and society demand that prospective teachers should already be capable of

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diagnosing learning levels and using particular improvement measures at the end of their education (Sekretariat der Ständigen Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland, 2004). In Germany, the principles of these two educational aspects—diagnosis and adaptive teaching—which are taught at universities, must already be executed by the prospective teachers during the two practical phases of their education. During each practical phase, they gather experiences as a teacher for about five to seven weeks at a school.

The examination of this topic—the implementation of diagnosis and adaptive teaching by the prospective teachers during their practical phases—can be profitable because of a variety of reasons: The research of teachers' competences in the scope of diagnosis and the planning of adaptive lessons comprises a wide range of necessary skills. These include amongst others: planning lessons in general, evaluating the learning difficulties of a topic, appraising the students' prior knowledge and considering this prior knowledge while planning lessons. All of these skills must be combined when planning adaptive lessons (Heinrich, 2017). Thus, it is important to examine the adaptive planning competences of prospective teachers and to expand our comprehension of these. Furthermore, diagnosis and adaptive teaching are primarily theoretically developed concepts (Moser Opitz, 2010; Schwarzer & Steinhagen, 1975), whose implementation in actual classes has not been investigated yet. Finally, not only the planning of lessons, but also the diagnosis of the students' learning level were defined as a significant duty of the teacher education by the education ministers of the German federal states (Sekretariat der Ständigen Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland, 2004).

The main concern of this study is the following question: How do prospective teachers implement diagnosis and adaptive teaching in their practical phases? This scientific contribution addresses especially the following research questions: Which conclusions do prospective teachers draw from their students' current learning level for their own lessons? How do they justify these conclusions? Which argument types, each consisting of an interpretation of the diagnosis, a consequence for the lesson and its justification, can be identified? How can the process from a diagnosis to the adaptation of a lesson, which focuses on uncovered prior knowledge, be theoretically and empirically modelled?

6.2 Theoretical Framework

School education, which aims to support the learning processes of each individual student, requires the adjustment of lessons and the level of difficulty of questions and exercises to the students' individual learning conditions (Helmke, 2014). This teaching approach is based on the assumption that a person learns an ability, such as multiplication, better with a teaching method that is suitable for him or her than

with another method, which is per se just as good (Cronbach, 1975). The individual learning processes of the students are considerably too diverse and multilayered, so that the use of one certain teaching method cannot achieve a learning success with all members of a heterogeneous group (Beck et al., 2008). But such an adaptive education needs a precise diagnosis of these conditions, so that the improvement measures are suitable for each individual student (Hesse & Latzko, 2011). If the individual's learning conditions have not been unearthed, it is not possible to adjust lessons to that individual's needs. Therefore, teachers must be amongst others competent in diagnosing students' learning conditions.

The notion of diagnostic competence "(that, in English, might have some medical connotations) is used for conceptualizing a teacher's competence to analyse (sic!) and understand student thinking and learning processes without immediately grading them" (Prediger, 2010, p. 76). In general, there are different reasons to conduct a diagnosis. Usually they are used at the end of a certain subject to evaluate the students' learning gains. However, diagnoses can also be conducted to unearth the students' current learning level to optimize lessons. In this case the diagnosis is used either at the beginning of or during the covering of a specific subject. This second type of diagnosis is the one that is addressed in this study. Many authors and organizations attach great significance to the skill of diagnosing students' learning levels, for example the National Council of Teachers of Mathematics in its Standards and Principles: "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000).

Adaptive planning and adaptive teaching is another significant competence of teachers in this matter (Beck et al., 2008). This is the competence, which enables one to tie in with the individual learning conditions of the students. Overall, teachers have a lot of possibilities to react to the differences in their students' learning conditions (König, Buchholtz, & Dohmen, 2015). Ignoring these differences, which is the passive reaction form, could lead to an increase of these discrepancies. The substitutive reaction form describes organizational courses of action, which are supposed to lead to a homogenization of a learning group, such as the repetition of a grade or external differentiation. Here, the students are adjusted to the lessons. An adjustment of the lessons to the students seems to be more preferable. According to König et al. (2015) this is executed in the active reaction form. The lessons are adjusted to the students' needs and learning differences and this is what adaptive teaching is about.

Corno and Snow (1986) describe that adaptive teaching can be implemented on two different levels. They distinguish between micro- and macro-adaptations. The short-term adjustments teachers make during their lessons are called micro-adaptations. Usually these emerge from observations and subjective judgements (Schrader, 2013)—which are among implicit forms of diagnosis—because teachers have to analyze the learning conditions, the learning success as well as the learning difficulties of their students throughout the implementation of a lesson

(Schrader & Helmke, 2001). This is different when making far-reaching, long-term decisions, so called macro-adaptations, for an entire lesson or teaching unit. Here, teachers have enough time to conduct an explicit diagnosis of their students' learning conditions (ibid.).

So far has been discussed that in order to adjust a lesson to the individual students' needs teachers need to be competent in diagnosis their students' learning levels as well as in adaptive planning and teaching. In addition, teachers must be able to clarify the mathematical content. This is necessary to get an overview of what the students should learn during the lesson or unit and to already get an idea of the possible learning difficulties. Teachers also need to know how to identify the necessary subject-related learning conditions of the lesson to be planned, because these are the aspects that have to be looked at during the diagnosis. Figure 6.1 shows a theoretical modeling of the modification process of a lesson to the students' learning conditions. Of course the second step—planning a lesson—need not be executed before the diagnosis. Especially experienced teachers might be able to skip it. However, it is presumed that it is easier for prospective teachers to diagnose the needed learning conditions for a particular lesson that has already been planned than for a vaguely envisaged learning process.

The principles for the steps discussed above are all taught at the University of Oldenburg, where this study took place—this includes principles for teaching mathematics in general, but not for adaptive teaching specifically. But does this mean that prospective teachers automatically succeed when they try to implement these theoretically developed principles during their practical phases? According to Patry (2014) this is very unlikely. He states that scientific theories are usually broad and therefore rarely concrete. This means, there exists a gap between theoretical principles and practical implementation. This gap must be closed by the prospective teachers. Furthermore, they also need to pursue several goals at once, which are addressed in different theories, and they must revert to multiple of their own beliefs. In addition, acting adequately is very specific to each individual situation. In other words: Prospective teachers must react appropriately to a specific given situation, while they pursue a variety of goals. Thus, multiple action-guiding beliefs are activated in their minds, which they must cope, while they can only resort to very few theoretically developed principles that must be adapted to the specific situation. This means that a direct translation into an action is not possible without additional effort.

To sum up, prospective teachers are supposed to be able to diagnose the learning level of their students and react to it accordingly by the end of their education. To do so they need to possess a variety of competences, such as diagnostic or adaptive teaching competences. The problem is that it is not sufficient, if they only know the theoretical principles of these aspects. These theoretical considerations lead to the main concern of this study: How do prospective teachers implement diagnosis and adaptive teaching in their practical phases?

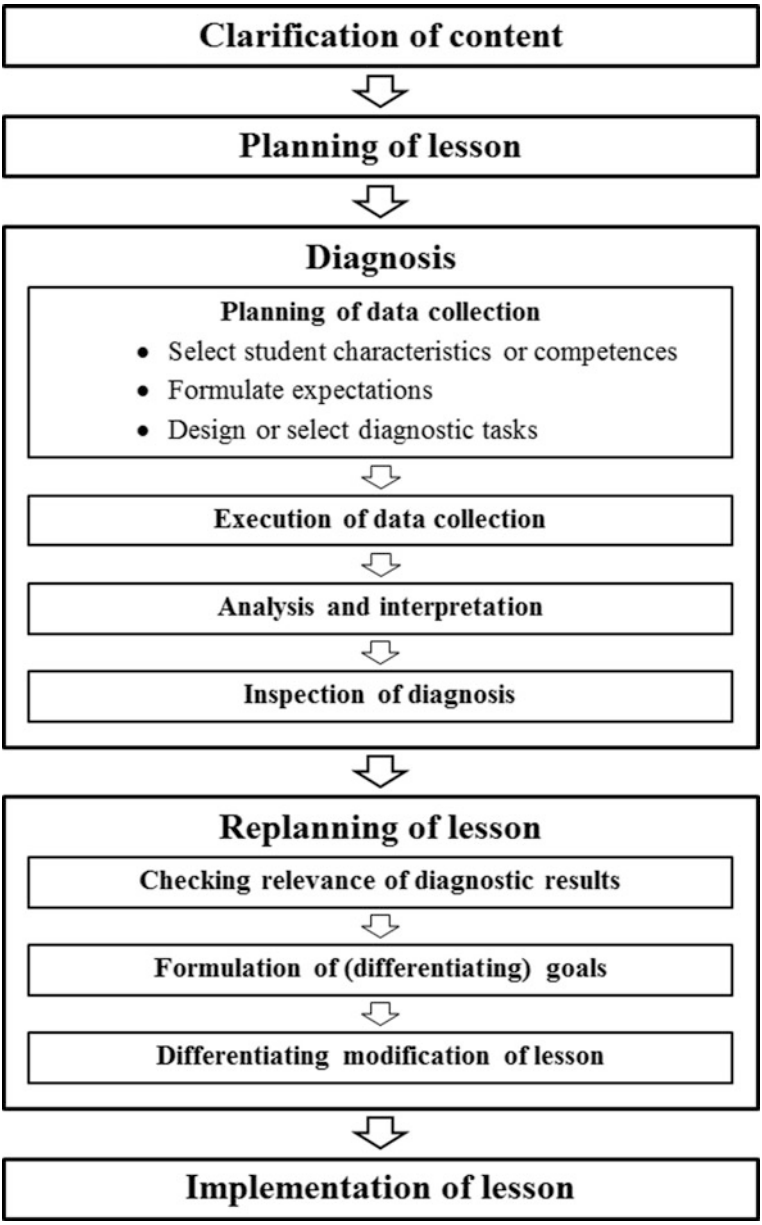

Fig. 6.1 Theoretical modeling of the modification process of a lesson to the students' learning conditions

6.3 Research Design

The presented research questions already indicate that the pursued issue is the *comprehension* of the individual actions and argumentations of prospective teachers when adjusting a lesson to their students' learning conditions. Therefore, an empirical-qualitative research approach was used (Mayring, 2014). The research of the implementation of adaptive teaching in practical phases by prospective mathematics teachers has to date not occurred (e.g. König et al., 2015). Hence, it was appropriate to use an explorative research design, which allows to develop new hypotheses in a relatively unexplored area, or to establish theoretical or conceptual requirements, so it is possible to formulate initial hypotheses (Bortz & Döhring, 2006). Below will be described what the participants of this study had to do and then the sample will be characterized.

The assignment addressed itself to prospective teachers, who were at that time about to start a practical phase at a secondary school, which lasted five to seven weeks. During this time they had one week to get to know the school, to observe some teachers in their classes and to decide in which classes they wanted to teach. In the second week they started to teach about one or two lessons per day. The prospective teachers worked on the following assignment in a class and a grade of their choice during the second or third week of their practical phase.

First the prospective teachers chose a specific mathematics lesson for this assignment. Then they began to plan it. During this step they were supposed to already think about the necessary subject-related learning conditions that the students needed to have in order to reach the goals of the lesson. Afterwards the prospective teachers designed a few diagnostic math problems to determine the learning level of their students in school. A few days before the implementation of the planned lesson they gave these diagnostic math problems to their students, which worked on them during a prior math lesson. After the prospective teachers collected the students' answers, they analyzed the students work and interpreted the results of the diagnosis. Then they were asked to modify their planned lesson with regard to the diagnostic results, if they thought this to be necessary. The last step was to implement the (possibly modified) lesson. During these steps the prospective teachers were not supervised and they were not allowed to accept any help from the experienced math teachers.

In addition, an open, partially standardized, guided interview was conducted after the implementation of the lesson, in which the prospective teachers' thoughts and decisions were put into focus. Here they talked about their interpretations of the students' learning level as well as their reasons for the chosen modifications. These interviews were videotaped and then transliterated by the author. Overall the following data was collected: the first teaching plan, the developed diagnostic tool including the students' work, the modified teaching plan and the interview transcripts. The research design and the research focus are shown in Fig. 6.2.

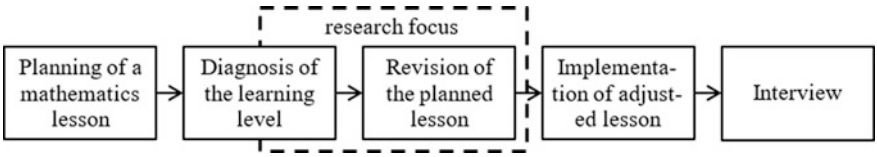

Fig. 6.2 Graphical representation of the empirical research design and the research focus

In February 2013, 15 prospective mathematics teachers participated in this study. Seven of them were female and eight were male. All prospective teachers studied mathematics to become a secondary school mathematics teacher. At the time twelve participants were in their seventh semester, three in their ninth (standard period of study in Germany: ten semesters). So, all of the prospective teachers had attended a lecture, which broached the issue of fundamental mathematics education. This included the justification and legitimation of mathematics as a part of the general education, reflections of the specifics of mathematical work, psychological principles of individual learning and social learning processes as well as consequences for the improvement of mathematical learning in the context of mathematics instruction. In addition, ten of them had gone to a lecture, which aimed to deepen their didactical understanding of either stochastics and analysis or geometry and algebra. Moreover, five prospective teachers already had participated in a seminar with an emphasis on diagnosis. Here they were taught how to develop diagnostic math problems and how to analyze students' answers. The fact that only five prospective teachers anticipated in this seminar led to the decision that all 15 prospective teachers had to attend a further seminar before entering the practical phase. In the course of the seminar they were shown criteria for "good" diagnostic math problems, they practiced developing such problems and how to analyze students' responses. However, the issue of adaptive teaching was not broached.

Still, it was—of course—possible that the prospective teachers identified the wrong learning conditions of the planned lesson or that they developed inadequate diagnostic math problems or that they analyzed the students' answers incorrectly or that they interpreted the results of the diagnosis wrong. This would be very unfortunate, but it does not affect the results of this study, because the focus is put on the decision process from the interpretation of the diagnostic results to the modifications of the planned lessons (see Fig. 6.2). At the end of the second step of the theoretical modeling of the modification process (see Fig. 6.1) the prospective teachers thought they had unearthed their students' learning conditions and this study wanted to understand what conclusions they drew from these. So at this point it does not matter, whether they identified the learning conditions correctly—of course for the implementation and the learning process of the students it makes a huge difference.

6.4 Analysis Method

Below the used methods of analysis are described. Here, the focus lies both on the approach of the collected data and the typification of arguments, because the used procedures were strongly adapted. However, the formation of the system of categories will not be depicted in detail, since Mayring's (2014) approach was implemented one-to-one.

6.4.1 Approach to the Collected Data

With a view to the collected data rose the question, how it could be compressed. This question came up because of two reasons: On the one hand, the data of the 15 prospective teachers was quite extensive. On the other hand, the participants of the study expressed many comments, which partly repeated themselves or were formulated in a different way—for example in the modified lesson plan and later during the interview. Here, it was the duty of the researcher to extract the statement that developed itself from these comments (Klein, 1980). The data was coded by the author of this contribution as well as another person with a mathematics education background.

For this purpose several procedures by Mayring (2014), which he proposed in the course of his *Qualitative Content Analysis*, were utilized. First of all a selection criterion was defined, which was determined by the theoretically derived subject of the creation of categories meaning the research questions of this study. This selection criterion allowed ignoring unimportant from the topic deviating text passages. The research questions suggested that only those statements were of interest, which addressed the results of the diagnosis, the prospective teachers' interpretations of the diagnosis results, the consequences for the planned lessons and the justifications for these consequences. All further comments were disregarded.

Hereupon the data set was worked through line by line (Mayring, 2014). As soon as a text passage complied with the selection criterion, which means that it could be assigned to one of the four described aspects above (result, interpretation, consequence, justification), it was color-coded and finally written out. Afterwards these comments were paraphrased. This included the elimination of all text components that lacked of content as well as the translation of all the remaining text components to a homogenous language level. For example, George's comment "However, it is also noticeable that a few students have problems to calculate the area of rectangles" was translated into the paraphrase "A few students have problems to calculate the area of rectangles". George expressed this in his second lesson plan. As he talked about the same diagnostic math problem during the interview, he mentioned "A few of them had difficulties with the calculations". This comment was also translated into the above paraphrase. Subsequently the originated paraphrases were reduced by combining those, which broached the same or at least a similar matter (Mayring, 2014). So from George's two comments, in which one was given in writing and the other one verbally,

derived with this procedure the above paraphrase. According to Klein (1980) this paraphrase represents a statement, which George expressed using different comments —as shown above. All of the text passages that were extracted from the data set during the first cycle were treated in this way.

During the last step of the reduction of the collected data the statements of the prospective teachers were grouped according to the four aspects of the selection criterion, before another cycle of the data set was used to search for connections among these arguments. This enabled the graphic representation of which consequences deduced with which justification from which interpretation of which diagnosis result. Thus at this point so called argument trees (Klein, 1980) were utilized, which visualized not only the individual statements of the prospective teachers, but also how these statements formed an argument with one another (see Fig. 6.3). These argument trees made it possible to see all of the prospective teachers' diagnosis results, interpretations, consequences and justifications on a single page.

Afterwards an inductive system of categories was developed from the data set on hand. For this process further techniques of the *Qualitative Content Analysis* by Mayring (2014) were resorted to. Here especially his technique *Summarization and Inductive Category Formation* was used to form categories in the following three dimensions: the interpretation of a diagnosis result, the consequence for the planned lesson and the justification for this consequence. A full display of the taken steps during the development of the inductive category systems would go beyond the scope of this contribution, especially because the steps were implemented just like Mayring (2014) suggests them. However, Fig. 6.4. will give an overview of the analysis steps.

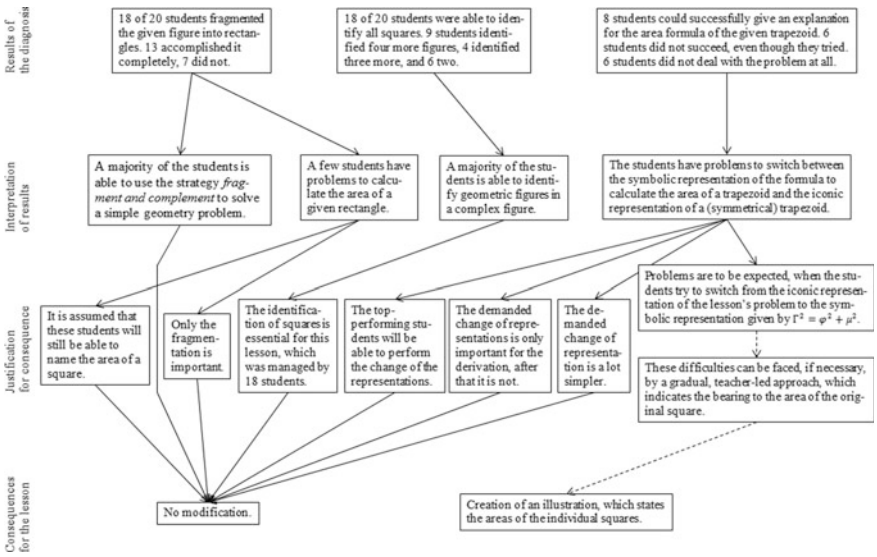

Fig. 6.3 Argument trees of George

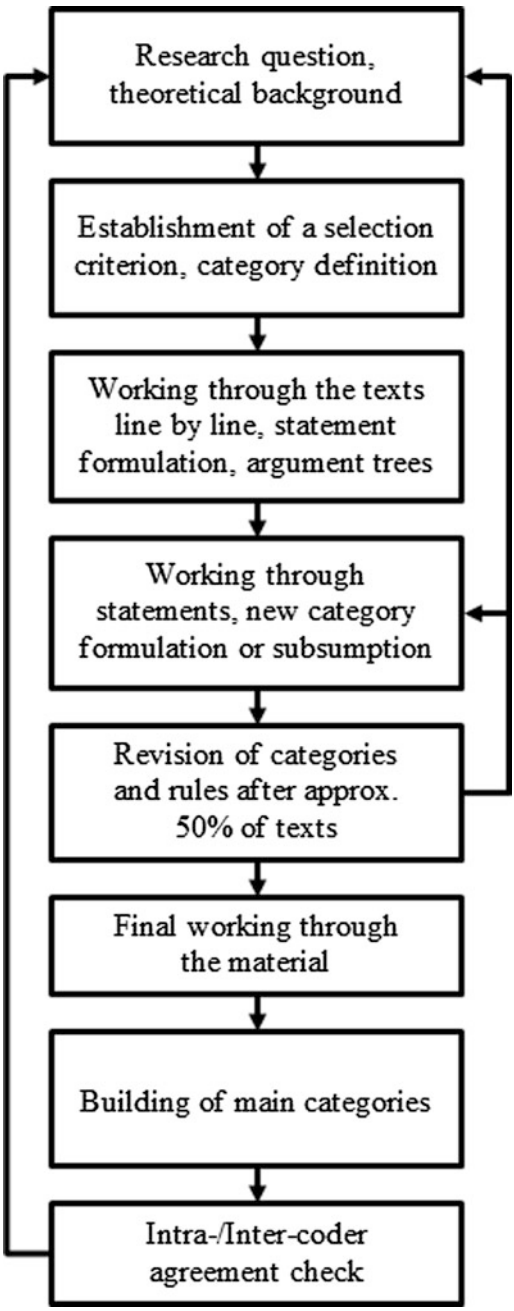

Fig. 6.4 Steps of inductive category development; based on Mayring (2014)

6.4.2 Typification

Below, how argument types were generated with the aid of the developed system of categories will be described. Overall this analysis procedure was oriented towards typification methods described by Kelle and Kluge (2010), but considerations of Mayring's (2014) *type-building content analysis* were partly taken into account as well. Kelle and Kluge (2010) formulate that a typology is the result of a grouping process. Thereby it is crucial that at the end internal homogeneity on the level of the type as well as external heterogeneity on the level of the typology is given. This means on the one hand that the individual components of a type should be quite similar, while on the other hand the types themselves should be very different from each other. Kelle and Kluge (2010) distinguish between one-dimensional typologies, which can be developed with regard to one single attribute, and multidimensional typologies, which are generated from a combination of attributes. In the second case the essential categories are combined and thereby an attribute space is created. This process can be illustrated clearly with the help of cross tables.

First of all, according to Mayring (2014) it is necessary to define the typification dimension plus the related specifications in order to work through the data set. In the study on hand, argument types were composed, which consist of a prospective teacher's interpretation of a diagnosis result, a consequence for the planned lesson and its justification. Consequently, the goal was not to assign a complete prospective teacher to one type of argument, but his or her arguments. So it was possible that the arguments of one prospective teacher would be allocated to multiple types. Thus, the typification dimension and its specifications were defined with the just mentioned aspects. A new cycle of the original data was not required, since the constructed argument trees already gave an excellent overview of all the existing arguments.

A typification process can be divided into four phases (Kelle & Kluge, 2010), which were all executed in the course of this study. First it was necessary to create relevant comparative dimensions, so that it was possible for categories to originate. These categories were needed both to identify similarities and differences between the arguments as well as to describe the developed types. This phase is similar to the above mentioned first step of Mayring (2014) and was already completed with the formation of the inductive category system.

During the second phase, the prospective teachers' arguments were grouped and empirical regularities analyzed. For this purpose the prospective teachers' statements were classified using the comparative dimension and the already developed categories. Via the use of multidimensional cross tables and the utilization of attribute spaces it was possible to determine all potentially appearing combinations of the categories as well as the actual frequency distribution of these combinations. The contrasting of arguments was also a part of this phase. This meant that arguments, which consisted of a certain combination of categories, were compared with one another to verify the above mentioned internal homogeneity of the originated types. In addition, it is essential to compare the types with each other to check if the

external heterogeneity is fulfilled, because the diversity of the data set should be represented in the developed argument types.

The analysis of the content-related context was the focus of the third phase. The goal of the typification is not just to describe the appeared frequencies, but also to understand and explain the prospective teachers' arguments. Again, both the individual arguments within a type and the types themselves were compared and contrasted. The result of this analysis was that (a) arguments were moved to different types, because they were more similar to the arguments there, (b) peculiar arguments were for the time being ignored and later examined separately and (c) multiple types were combined, because they were similar to one another. This led to a reduction of the attribute space and hence to a decrease of the number of appearing combinations of attributes.

In the concluding fourth phase, the developed types of arguments were characterized on the basis their combinations of attributes as well as the identified and reconstructed content-related contexts. Kelle and Kluge (2010) note that many researchers would forget that this phase is an independent analysis step and yet the characterization is essential for the copious description of the individual types and for the further classification of other arguments. But one should also have in mind when describing similarities that the elements of a type are not identical. They are only similar. Both Mayring (2014) as well as Kelle and Kluge (2010) recommend to choose an illustrative prototype, which resembles the respective type especially.

6.5 Results of the Empirical-Qualitative Study

Below, the results of this study will be illustrated. First, the categories of two of the three dimensions—consequences and justifications—are explicated. Second, the eight identified argument types are depicted. Third, the empirical modeling of the modification process is described.

6.5.1 Consequences for the Lesson Planning

First of all, the consequences that prospective teachers deduced from their diagnosis for their planned lessons are presented. The analysis of the data set indicated the following five categories in the dimension *consequences for the planned lesson*:

1. no modifications
2. modifications of the subject-related content
3. modifications of a teaching step
4. adding support for or simplifying of a math problem
5. adding a learning objective

A total of 51 statements were classified in this dimension. Most of the time the prospective teachers came to the conclusion that the diagnosis results indicated that no modification was necessary. The second category includes modifications such as removing subject-related content from the current topic, illustrating the link between two mathematical concepts, establishing the relationship to everyday life or adding a revision, in which subject-related content from past topics is supposed to be reactivated in the students' minds. The consequences that were assigned to the third category refer to adding exercise sheets, changing the group classification or the educational reserve, or adding or changing the teaching step of securing the results. Modifications like adding solution cards, aid cards, written or oral hints, or diagrams, which are to support the solving process of math problems, fall into the fourth category. In addition, simplifying math problems is also a modification in terms of the fourth category. Only one statement was classified into the fifth category, but it is highly probable that the other prospective teachers pursued additional learning objectives with their modifications as well—however, they did not express this explicitly.

Overall, the data showed a broad scope of lesson modifications by the prospective teachers. It is possible to understand their planning decisions, if these modifications are linked to the correspondent justifications and the interpretations of the diagnosis results. Moreover, this could lead to further considerations regarding the difficulties, with which the prospective teachers are confronted, and which planning decisions are preferable or rather critical. For this purpose it is necessary to examine the justifications for the consequences first.

6.5.2 *Justifications for the Consequences*

The analysis of the data led to three different categories in the dimension *justification for the consequences*, whereby the third category also has seven subcategories (see Table 6.1):

1. no or little deficits resp. good planning
2. diagnosis results are irrelevant for the lesson
3. diagnosis results are relevant for the lesson

All in all, it was possible to assign 80 statements to the ten categories and subcategories. Statements, which addressed that (a) an aspect of the lesson was already well planned, (b) a problematic diagnostic task did not reveal any information about the students' learning level or (c) the students had the necessary competences available, were assigned to the first category. The prospective teachers' justifications, which were classified into the second category, broached the irrelevance of uncovered competences or deficits. These are, for example, only needed for one of many possible solution approaches or for the derivation of a theorem, but not its application. Furthermore, the prospective teachers argued that

Table 6.1 Subcategories of category 3 and their anchor examples

Subcategories of category 3	Anchor examples
3.1. Too much or too little was planned	"We will not get that far during the lesson."
3.2. Content has already been taught	"This topic has already been covered thoroughly. That is nothing new."
3.3. Links should be illustrated or established	"The difference between addition and multiplication of fractions should be clarified."
3.4. Math problem is too difficult	"The problems must be simplified. Finding the correct strategy should be easier and less open."
3.5. Prior knowledge should be reactivated	"Previous knowledge should be reactivated respectively recalled."
3.6. Partner or group work as a solution	"The students will solve the problems in groups, so they can supplement their knowledge."
3.7. Joint start resp. joint accomplishment of goals	"I want everyone to be on the same level."

either the knowledge gaps were uncovered accidently or the tested aspect was supposedly easier implemented in the planned lesson. If the prospective teachers stated that the availability or the absence of a diagnosed competence was problematic for the planned lesson, because it was needed, for instance, for the used worksheet, the assigned tasks or the application of a theorem, these statements were grouped into the third category. It also includes statements, which attributed the examined deficits gaps certain relevance, because the students would probably not have been able to understand the content of the lesson or to solve the given math problems due to their knowledge gaps.

Usually further statements, which justified the consequences more precisely, followed the conclusion of the diagnosis results' relevance. These statements were summarized into the seven subcategories of the third category. In the course of the first subcategories the prospective teachers concluded that they would be able to either cover more content than they had anticipated or less. The second subcategory comprised statements, which referred to the fact that dealing with a certain topic to the given time was not appropriate according to the core curriculum or that the topic was already broached extensively. An example for this subcategory is the justification that the planned lesson focused on the link between two mathematical concepts, so the prospective teacher had to act on the assumption that the students had comprehended these two concepts. The statements of the third subcategory addressed the necessity of clarifying or establishing a link or a transition between two mathematical expressions, representations or concepts. Other statements of this subcategory emphasize the need to broach the prior knowledge or the everyday experience of the students more intensively.

When the prospective teachers argued that the solution process of a math problem was at that time too difficult for the students and therefore the task had to be changed, so that, for instance, the strategy development is easier and less open, their statements were assigned to the fourth subcategory. The fifth subcategory

contains justifications, which adverted to the necessity of reactivating prior knowledge in the students' minds. The statements of the sixth subcategory are again more multifaceted. Here, the prospective teachers brought forward the argument that the top-performing students would intercept the knowledge gaps by helping the under-performing ones due to the already implemented partner or group work. Further examples are the prospective teachers' assumptions that the students could solve the given tasks together, that the prior knowledge gaps could be closed or that the student could supplement their knowledge during the group work. All statements, which brought up the wish that all students should be at the same level or that they should have the correct solutions in their notebooks, were summarized into the seventh subcategory. The statements of this subcategory could also refer to the need that all students, and not only the top-performing ones, should accomplish certain goals of the lesson.

Consequently, similar to the consequences of the planned lessons, there was a grand variety of justifications for these consequences. As already mentioned the examination of the connection of all three dimensions—interpretation, consequence and justification—is important in order to comprehend the prospective teachers' planning decisions. Therefore these connections will first be illustrated in summary and then be more closely analyzed below.

6.5.3 Types of Arguments

A total of 104 arguments, each consisting of an interpretation of a diagnosis result, a consequence for the planned lesson as well as its justification, could be classified into eight different types (also see Fig. 6.5):

1. no modification due to problems with the diagnosis
2. no modification due to existing prior knowledge
3. no modification due to the irrelevance of the knowledge gap
4. no modification due to already planned group work
5. modification of the subject-related content to clarify a link
6. modification of the subject-related content to reactive prior knowledge
7. modification of a teaching step to establish similarities
8. simplification of math problems or adding support

In the course of the first argument type, the prospective teachers established that the negatively regarded results of the diagnosis had to be explained by problems with the diagnosis itself. Due to the fact that the prospective teachers were not able to determine their students' learning level, they did not modify the planned lesson at this specific point. The second type of argument occurred most often. A diagnostic task uncovered the availability of necessary, subject-related competences, so the prospective teachers did not make any modifications. The arguments of the third

		Diagnosis				
	Justifications	Problems with the diagnosis	Student knew or were able to do something.	Students were not able to do something.	Students were not aware of a link.	Students did not know something.
Consequences: None	No deficits (determinable)	5 from 4	28 from 15			
Consequences: None	Results irrelevant; already good planning		12 from 8			
Consequences: None	Results relevant, but group work is implemented		16 from 9			
Consequences: Content	Results relevant, so clarification of links			11 from 5		
Consequences: Content	Results relevant, so reactivation of prior knowledge			16 from 6		
Consequences: T. step	Results relevant, so common start/goal			5 from 2		
Consequences: Support	Results relevant, tasks are too difficult			11 from 6		

Fig. 6.5 Overview of the identified types of argument (T. step = teaching step; entries in the cells indicate the number of arguments that were contributed by the number of prospective teachers, for instance, "12 from 8" means that twelve different arguments from eight different prospective teachers were assigned to the correspondent argument type)

type did also not lead to any modifications although the diagnosis detected knowledge gaps; however, these were appraised to be irrelevant due to several reasons. In the fourth type, the prospective teachers concluded that a small or a large part of the class did not possess the necessary, subject-related competences. Still, they did not modify the planned lesson, because the given tasks were to be solved in partner or group work, so the prospective teachers assumed that the knowledge gaps would be—in whatever form—intercepted by this teaching method.

The diagnosis of the fifth argument type referred to the problem that the students either did not know the link between two mathematical concepts or they confused them with one another. With the justification that the link between these concepts should be established or clarified, the prospective teachers modified the

subject-related content of their lesson. The sixth argument type is characterized by a negatively interpreted diagnosis result as well as the conviction that the missing prior knowledge was already covered and therefore must be reactivated in the students' minds. Again, these considerations led to modifications of the subject-related content—usually broaching the prior knowledge in the course of a revision. Uncovered knowledge gaps were also the starting point of the seventh type of argument. Here, the prospective teachers took the decision to modify a teaching step of the planned lesson, such as additionally securing the results during the lesson, to ensure that all students accomplish a certain goal together. Both simplifying tasks and adding support for their solution process, for instance, by giving short oral or written hints, were the consequences of the eighth argument type to the absence of prior knowledge. The prospective teachers argued that the used math problems were too difficult for the students with regard to their learning level.

To this point the argument types were illustrated. Next, a part of the deeper analysis will be presented; whereby the focus is placed on the argument types 5 through 8. First, it can be observed that 13 of the 15 prospective teachers contributed at least one argument that was assigned to one of these four types. Hence, almost every prospective teacher actually decided to modify his or her lesson in some way. This is to be welcomed, since the reaction to an uncovered knowledge gap per se is something positive in general. It is satisfying that the prospective teachers recognize the need for action after they appraised the diagnosed deficits to be relevant. So in the case of the argument types 5 through 8 it is possible to speak of the implementation of adaptive teaching—at least to some extent.

If the arguments of the eighth type of argument, which address adding support for the solution process of exercises, are examined, one might think that the argument types 6 and 8 are quite similar, but this is not the case. The consequences of the sixth type wanted to intensively broach again the issue of basic concepts, such as relative frequencies, whereas the conclusion of argument type 8 aimed to remind the students of minor aspects. An example for the latter is the reminder of the scale factor, when the students were supposed to determine the equation of a parabola. The essential difference between the argument types 5 and 6 is that the prospective teachers whose arguments were assigned to type 5 did not only uncover and reactivate missing knowledge, but attributed it to the lack of knowledge of missing links or the confusion of concepts and reacted to this discovery.

Consequently, it can be positively mentioned that some of the prospective teachers actually distinguish between different kinds of mistakes and also react differently to those. If they diagnose the missing comprehension of a mathematical concept, they provide the needed knowledge instructively. However, if they come to the conclusion that their students confuse two concepts or are not aware of a link between them, they tackled these problems accordingly as well. Depending on the diagnosis result the prospective teachers recognize the need to build bridges (type 5), to close gaps (type 6), to establish a common ground (type 7) or to simplify or help with the given tasks (type 8).

Of course, there is always the danger to over- or underestimate the diagnosis results, especially the uncovered knowledge gaps, when modifying a planned lesson on the basis of diagnosis. This is due to the fact that the diagnosis is analyzed and interpreted by novices. Depending on how confident a prospective teacher feels relating to the own lesson planning as well as the content and the pedagogical content knowledge, the reaction to the results could be either insufficient or excessive. Both cases can be problematic. On the one hand, if the prospective teachers underestimate their students' knowledge gaps they might encounter the same problem like Bryan. His students were—according to his own statement—totally overstrained with the derivation of the formula for calculating angles between vectors in the three-dimensional space, because he underestimated the impact of their deficits. On the other hand, if the prospective teachers overestimate their student's knowledge gaps, they might have to make the same experiences as Paul. He had planned an extensive revision of the concepts of relative and absolute frequencies and had designed an exercise sheet as well. Later during the implementation of the lesson he realized that his students did not have deepened difficulties with the comprehension of the concepts themselves. They had only forgotten the word or the term for these concepts. In Bryan's case the students did not learn much during the lesson, while in Paul's case the lesson became less effective, because a lot of time was spent on an unnecessary revision.

6.5.4 Empirical Modeling of the Modification Process

A goal of the study on hand was also to empirically model the process, which leads from a diagnosis to adaptations of a lesson to the students' current learning level. Below, the used evaluation method are briefly illustrated, because due to the strong dependency of the exact approach on the developed system of categories as well as the identified types of arguments this illustration could not be given earlier. Afterwards, the results of these analyses are presented.

In the first step of the analysis the justifications of each argument type were examined more closely. During the process was checked whether these justifications were assigned to the same category or if they partly originated from different ones. In the first case the justification was converted into a polar question, which allowed the conclusion to the underlying justification. The second case was checked if the procedure of the first case was possible or if multiple polar questions had to be generated, so that the entire scope of the justification was still reflected. An example for the first case is the first argument type: *no modifications due to problems with the diagnosis*. The examination of the prospective teachers' justifications showed clearly that they asked themselves, whether their observed negative diagnosis results had to be attributed to problems with the diagnostic tasks or the implementation of the diagnosis. So, at this point the following polar question was developed: Were there any problems with the diagnosis? The third type of argument, *no modification due to the irrelevance of the knowledge gap*, is an example

for the second case. Most of the given justifications addressed the irrelevance of a knowledge gap by expressing that they did not belong to the prior knowledge that was needed for the lesson. These justifications led to the polar question: Are the results relevant for the planned lesson? However, some prospective teachers argued that the planned lesson already intercepts the uncovered deficits, which meant that they were irrelevant due to another reason. Therefore a further polar question was necessary: Does the lesson already react to the results?

During the second step of the analysis the developed polar questions were brought into an appropriate and conclusive order. For instance, it would not have been reasonable that the prospective teachers first asked themselves if the implemented group work already reacts to a knowledge gap and then consider whether this knowledge gap has to be attributed to problems with the diagnosis.

In the third step an empirical model was generated, which illustrates the decision process of the prospective teachers from the interpretation of the diagnosis results to the modification of their planned lesson. It shows the polar questions that the prospective teachers asked themselves and also to what conclusion the affirmation or the negation of certain questions led. So, the result of the empirical modeling is a process model (see Fig. 6.6), which reflects the 104 arguments of the participants of this study. It consists of ten different ways, because the argument types 3 and 4 allowed respectively two action-guiding motives.

First the prospective teachers seemed to ask themselves if the results of a diagnostic task were to be regarded positive. Here, there is reason to presume that they compared the results of the diagnosis to their expected learning difficulties or the necessary, subject-related prior knowledge. If this question was affirmed by the prospective teachers they had to clarify if there were still some students, who did not have the needed competences—or at least parts of them—available. If this was not the case, the planned lesson was not modified due to the reason that no deficits have been discovered (cf. type 2). However, if there were students with some knowledge gaps, even though the diagnosis results were appraised to be positive,

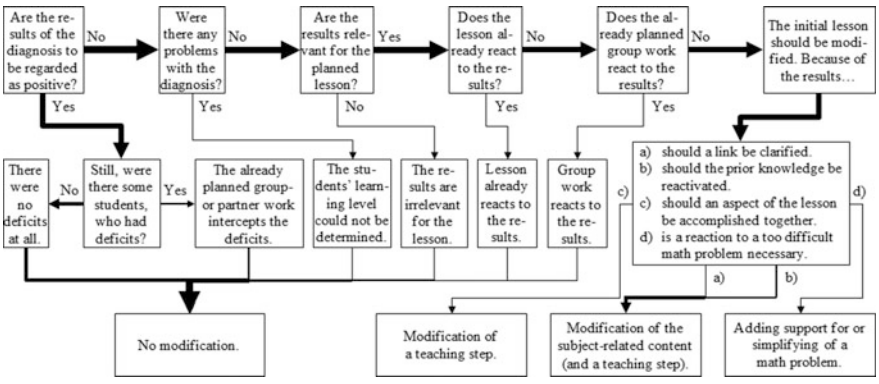

Fig. 6.6 Empirical modeling of the modification process; arrow thickness represents frequency

the planned lesson was nevertheless under no circumstances adapted. This decision was justified by stating that the already planned partner or group work would intercept these deficits (cf. type 4). All in all this implies that a positively appraised diagnosis result never led to a modification of the lesson, even if there were students, who did not possess the competences that were necessary to reach the lesson's goals.

In the case that the first question was answered in the negative—so the diagnosis results were not to be regarded as positive—the prospective teachers asked themselves, if the results were caused by problems with the implementation of the diagnosis or the used diagnostic tasks. If this was true, they concluded that the students' learning level could not be determined and therefore they were not able to adapt the lesson to their students' needs (cf. type 1). Could the negative diagnosis results not be attributed to problems with the diagnosis the prospective teachers contemplated, if the results were actually relevant for their planned lesson. If this was not the case the lesson was not modified either (cf. type 3). Were the uncovered knowledge gaps generally relevant for the lesson the prospective teachers considered, whether the lesson already intercepted these. An affirmation of this question did also not lead to an adaptation of the lesson (cf. type 3, again). The last question, whose affirmation resulted in the fact that no modifications were made, asked, if the already planned partner or group work could intercept the deficits of the students (cf. type 4).

If this was not true, the prospective teachers came to the conclusion that the initial lesson plan should be adapted on the basis of the diagnosis results. Depending on their motive, they (a) modified the subject-related content, (b) added support or simplified tasks or (c) adapted a teaching step. The wish to clarify or establish a link between two mathematical subjects (cf. type 5) or to reactivate prior knowledge (cf. type 6) always led to modifications of the subject-related content. If the prospective teachers wanted to accomplish goals or to start from the same initial point with the whole class, they adapted teaching steps (cf. type 7). Finally the awareness that the contemplated tasks were too difficult for the students resulted in adding support or simplifying tasks (cf. type 8).

Of course the model above neither claims that all students teachers have asked themselves all of these questions nor that they have done so in the suggested order. The goal was to develop a model that describes the appeared phenomena of this empirical-qualitative study. It makes it possible to recognize, for instance, how many polar questions have to be answered in a certain way so that the prospective teachers actually decided to modify their lessons on the basis of the diagnosis. Only when a diagnosis result was to be regarded positive, could not be attributed to problems with the diagnosis and was relevant for the planned lesson and when neither the planned lesson nor the planned partner or group work reacted to the results, the prospective teachers adapted their lessons to their students' needs. This phenomenon cannot be explained by the collected data, but many possible explanations are conceivable of which four will be addressed here.

1. The prospective teachers ponder thoroughly, whether an adaptation of the planned lesson would actually improve it. If they come to the conclusion that this is not the case, they look for an explanation or a justification with which they neglect the diagnosis results.
2. The prospective teachers concentrate on the (from their perspective) most essential prior knowledge gap. Other, less important gaps are, for instance, sourced out into the group work or described as irrelevant.
3. The prospective teachers do not know how they can or should react to the results of the diagnosis and therefore look for a justification, which explains why they do not react.
4. The quality of the prospective teachers' diagnoses is on such a low level that, on the one hand, there actually are problems with them and, on the other hand, they accidently uncover knowledge gaps, which are indeed irrelevant.

6.6 Conclusion and Outlook

Thus, the central results of this study were presented. The conclusion of this chapter begins with the explication of possible implications for the teacher education. Then an outlook will be given, which focuses on continuative research questions.

In summary, it was possible to show that the prospective teachers implement many of the single steps of the diagnosis and modification process to some extent well. However, it became also clear that the execution of the individual steps as well as the entirety of the process was partly problematic. The explanation approaches for the lack of reaction to some diagnosis results and the non-existent development of differentiating modifications, which were developed in the course of the empirical modeling of the modification process (see Fig. 6.6), give a first impression of the possible underlying difficulties. The question at this point is, whether the prospective teachers only lack practical experiences and exercises or indeed theoretical elements of knowledge as well. Either way, it therefore follows that first the individual steps of the modification process should be placed into focus. This means that both diagnosis and the differentiating, adaptive planning of lessons should be discussed and practiced separately, before the combination of these is tackled.

It is assumed that the theoretical level as well as the practical one is essential to learn the adaptive planning of lessons. On the one hand, various possibilities for differentiation and planning should be introduced in the theoretical part of the teacher education, whereupon their advantages and disadvantages should be discussed. On the other hand, diverse action alternatives should be talked about with the aid of concrete situations during the practical phases. The results of this study provide indications for this purpose. For example, the fourth type of argument, which does not modify the lesson due to already planned group work, leads to the opportunity to broach the issue of group work with the prospective teachers. When

is this teaching method suitable? Which advantages does it possess? Which disadvantages or dangers does it implicate? What is the best way to implement it? A goal of group work is, for instance, to combine subject-related and social learning (Barzel, Büchter, & Leuders, 2011). In order for this to be achieved, it is necessary to lay the needed foundations. In particular the point of this teaching method is that different solution approaches are pursued by the individual members of a group and later discussed by the whole group. However, Barzel et al. (2011) do not explicitly mention that the purpose of group work is to have the top-performing students fill in the knowledge gaps of the weaker ones. In similar ways it should be discussed with the prospective teachers, in which cases it is appropriate to repeat already covered knowledge, to simplify math problems, to add support for the solution process of math problems or to clarify links between two mathematical concepts.

The following focuses on the outlook of this study. First of all it should be mentioned, that the collected data itself allows many continuative research questions. The initial lesson plan could be analyzed with regard to the used teaching methods, the selected math problems or other, similar aspects. Furthermore, it is possible to investigate, whether the participants of the study identified the necessary, subject-related learning conditions of their lesson correctly. The developed diagnostic tasks could also be a subject of an examination. Here could be checked, if the prospective teachers tested all of their identified learning conditions or to what extend the used math problems meet the criteria for diagnostic tasks (e.g. Dannenhauer, Debray, Kliemann, & Thien, 2008). In addition, the students' solutions of the diagnostic tasks are available. With these it would be possible to survey, whether the prospective teachers analyzed their data correctly.

The examined sample of 15 prospective teachers of the Carl von Ossietzky University of Oldenburg is in view of the preconditions, to which they resorted during their practical phase, relatively homogeneous because the department of mathematics education puts emphasis on diagnosis and improvement. Though the data indicated a few critical aspects during the implementation of the given task by the prospective teachers, it also showed a variety of positive issues, such as their competence-oriented perspective on the diagnosis results. But the question is which results would be received, if the research design was used to study prospective teachers from another university, which focuses on other issues within the teacher education. It is also conceivable that one would get very different results, if the prospective teachers' task is given to experienced teachers.

Moreover, all the aspects that have been revealed by the qualitative-explorative study at hand, whose surface has only just been scratched, should be researched more deeply. For instance, it was possible to unearth the prospective teachers' justifications for their selected consequences. But it was not discovered, for what reasons they dismissed alternative planning possibilities. Furthermore, the second type of argument, for example, which does not modify the planned lesson due to existing prior knowledge, raises the question, if the role of this available knowledge changes in the consciousness of the prospective teachers. Does its importance increase, because they know that they can rely on it or does it decrease, because they concentrate on the knowledge gaps instead?

To conclude, it should be noted that the design is not only suitable for research purposes, but also for the education of prospective teachers. As already mentioned above, at first it is important to address and practice all the single steps that are necessary to planning a lesson, which considers the students' learning conditions, individually. Afterwards prospective teachers could be asked to implement the task of the research design in their practical phase in order to try the entirety of the process. Combined with a close supervision, which proposes ideas for improvement from a mathematics education point of view, this could initiate effective learning processes.

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