ICME-13 Monographs

Marilyn E. Strutchens
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Despina Potari
Leticia Losano *Editors*

Educating Prospective Secondary Mathematics Teachers

Knowledge, Identity, and Pedagogical Practices

ICME13
Hamburg 2016

Springer

ICME-13 Monographs

Series editor

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ISSN 2520-8322 ISSN 2520-8330 (electronic)
ICME-13 Monographs
ISBN 978-3-319-91058-1 ISBN 978-3-319-91059-8 (eBook)
https://doi.org/10.1007/978-3-319-91059-8

Library of Congress Control Number: 2018940414

Printed on acid-free paper

This Springer imprint is published by the registered company Springer International Publishing AG
part of Springer Nature
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

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Chapter 1
Introduction

Marilyn E. Strutchens, Rongjin Huang, Despina Potari and Leticia Losano

Keywords Field experiences · Prospective Teachers Knowledge Teachnologies · Tools and resources · Prospective Teachers Professional Identities

During topic study group 48 on Pre-service Mathematics Education of Secondary Teachers regular sessions, significant new trends and developments in research and practice on the mathematics education of prospective secondary teachers were discussed. An overview of the current state-of-the-art and recent research reports from an international perspective were provided. In keeping with the call for papers, presentations focused on similarities and differences related to the development of mathematics content and pedagogical content knowledge of teachers; models and routes of teacher education, curricula of mathematics teacher education; the development of professional identities as prospective mathematics teachers and a variety of factors that influence these different aspects; field experiences and their impact on prospective secondary mathematics teachers' development of the craft of teaching; the impact of the increasing availability of various technological devices and resources on preparing prospective secondary mathematics teachers; and others.

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_1

We received fifty-four 4-page submissions to TSG 48. From the submissions, 20 papers were selected for presentations during the regular meetings. Each of these 20 papers was scheduled in one of the following TSG sessions based on the topic of the paper: (1) Field Experiences (two sessions); (2) Prospective Teachers Knowledge (two sessions); (3) Technologies, Tools and Resources (one session); and (4) Prospective Teachers Professional Identities (one session). Also, we invited four speakers to submit an extended article, one for each of the major themes: Blake E. Peterson and Keith R. Leatham (field experiences), João Pedro da Ponte (teachers' knowledge), Rose Zbiek (tools, technologies, and resources), and Márcia Cristina de Costa Trindade Cyrino (teachers' identities). Thus, 24 papers were scheduled for the TSG 48 regular meetings.

In this monograph, we provide a subset of the papers presented during the conference. Each of the sixteen papers were extended by the authors and reviewed by the editors of this monograph and the authors of other papers. We have divided the monograph into four sections according to the four major themes from the TSG 48 mentioned above. Moreover, this monograph serves as an excellent companion to the Topical Survey: The Mathematics Education of Prospective Secondary Teachers Around the World (Strutchens et al., 2017), which synthesizes and discusses significant new trends and developments in research and practices related to various aspects of preparing prospective secondary mathematics teachers from 2005 to 2015.

As mentioned previously, this monograph is divided into four sections which are representative of the four themes from the topic study group 48 on Pre-service Mathematics Education of Secondary Teachers. Part I focuses on field experiences. While university coursework can provide knowledge about content and about teaching strategies, it is during clinical experiences that prospective teachers develop the craft of teaching—for instance, the ability to design lessons that involve important mathematical ideas, design or select tasks that will help students to access those ideas, and implement instructional strategies to successfully execute the lesson. Moreover, the Association of Mathematics Teacher Educators (2017) states the following:

> An effective mathematics teacher preparation program includes clinical experiences that are guided on the basis of a shared vision of high-quality mathematics instruction and have sufficient support structures and personnel to provide coherent, developmentally appropriate opportunities for candidates to teach and to learn from their own teaching and the teaching of others. (p. 26)

In part I, Peterson and Leatham describe how restructuring their program's student teaching experience from a traditional one student teacher per mentor teacher model to a paired placement model with two student teachers paired with one mentor teacher and how the new model impacted their student teachers' growth. Martin and Strutchens illustrate the power of the Mathematics Teacher Education Partnership's networked improvement community approach, by sharing the work of the clinical experience research action cluster which has employed improvement science methods to developed resources that support improved models for both student teaching and early field experiences, as well as professional development for mentor teachers. Kilic examines affording pre-service mathematics teachers with the opportunity to work with a pair of students for a semester and reflect on their own practices and students' performances. Mohr-Schroeder, Jackson, Cavalcanti, and Delaney examine how a robotics course in an educator preparation program that required a field experience in an informal learning environment impacted its participants. Heinrich analyze how prospective teachers implemented diagnosis and adaptive planning competences during field experiences.

Part II focuses on technologies, tools and resources. Although the use of various technologies in promoting mathematics learning in classrooms has been recommended for decades (e.g., International Society for Technology in Education, 2000; NCTM, 2014), how to prepare prospective teachers to teach mathematics using technologies effectively is still a challenging task (Huang & Zbiek, 2017). In this part, three chapters examine various aspects about how to prepare PSMT's use of technology. Building on extensive literature reviews, Zbiek propose and describe a blend of three conceptual tools to frame integrated and dynamic ways of preparing secondary mathematics teachers. These three conceptual tools frame what knowledge and skills in each of technology, content, and pedagogy PSMTs should have, and how they should acquire them through multiple venues. Moreno and Llinares explore prospective secondary mathematics teachers' perspectives on the role that technological resources play in supporting students' learning. Akcay and Boston focus on pre-service teachers' ability to integrate technology into instructional activities in ways that support students' mathematical thinking and reasoning, using the Instructional Quality Assessment to assess the cognitive demand of: (a) instructional tasks, (b) description of how tasks would be implemented or were implemented during the lesson, and (c) level of response expected from or produced by students.

In part III, the main focus is on teacher knowledge. Teacher knowledge has been considered as an important resource for mathematics teaching. The special features of this knowledge have been widely discussed and studied during the last two decades, and different theoretical frameworks have been developed. Many of which are related to the work of Shulman (1986), as for example the notion of mathematics knowledge for teaching of Ball, Thames, and Phelps (2008). Many studies have been undertaken in the area of prospective secondary mathematics teacher knowledge focusing on the measurement of this knowledge and on the process of its development (see Potari & Ponte, 2017). In the papers discussed in this part the emphasis is placed on how this knowledge can be developed in the context of teacher education.

For example, Potari and Psycharis examine the structure and quality of prospective mathematics teachers (PMTs)' argumentation when identifying and interpreting critical incidents from their initial field experiences. The study offers an analytical framework on the basis of argumentation structures and classification of warrants and backing to trace PSMTs noticing and the resources including knowledge on which this is framed. Lin, Yang, and Chang provide one example of PSMTs' survey study in one complete learning cycle, and summarize several criteria of evaluating how PSMTs conduct a study to understand students' mathematical thinking in a holistic perspective. Manouchehri, Yao, and Saglam investigate prospective secondary school teachers' knowledge of mathematical modeling and its development through especially designed instructional units in teacher education. Arnal-Bailera, Cid, Muñoz-Escolano, and Oller-Marcén explore prospective secondary school mathematics teachers' marking practices of students' work and implement teacher education activities aiming to develop these practices. The study of Ölmez focuses on how prospective middle school mathematics teachers reason with quantitative definitions for multiplication and fractions about proportional relationships.

Part IV focuses on teacher professional identities. Teachers' professional identities, as well as other related concepts such as self-perceptions, attitudes and beliefs, are a relevant area of study within mathematics teacher education research. According to Sachs (2005), "teacher professional identity stands at the core of the teaching profession. It provides a framework for teachers to construct their own ideas of 'how to be', 'how to act,' and 'how to understand' their work and their place in society" (p. 15). In this way, the professional identity of a prospective teacher is a complex notion since it addresses the complex and mutual relationships between the prospective teacher, the institutions involved in her pre-service education (universities, schools) and the society where she lives (Losano & Cyrino, 2017). Since professional identities "are not only an answer to the question 'Who am I at this moment?', but also an answer to the question 'Who do I want to become?'" (Beijaard et al., 2004, p. 122) analyzing how PSMTs develop their identities in the spaces and moments offered and promoted by pre-service education is an important issue. This is the direction taken by the articles included in this part.

Cyrino examines how the analysis of a multimedia case featuring one mathematics teacher's practice supported prospective mathematics teachers in the construction of their professional identity. Hine investigates prospective secondary teachers' self-perceptions concerning their readiness to assume a full-time mathematics teaching position at schools. Durandt and Jacobs analyzes the convictions and mindsets of prospective mathematics teachers towards modelling based on their initial engagement with a modelling task.

The sixteen studies presented in this monograph provide the field with models for studying and preparing secondary mathematics teachers. Many of the practices presented are innovative and have the potential to move the field forward. Much like the studies in Strutchens et al. (2017), most of the studies presented are qualitative in nature, providing the readers with thick descriptions of the context and conditions under which prospective secondary mathematics teachers are being prepared.

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Part I
Field Experience

Chapter 2
The Structure of Student Teaching Can Change the Focus to Students' Mathematical Thinking

Blake E. Peterson and Keith R. Leatham

Abstract This paper describes our efforts to change the focus of our student teaching experience by altering the structure of that experience. We provide evidence that the restructuring accomplished its purposes and, in so doing, addressed a number of problems with the traditional structure it replaced. In particular, we achieved less focus on issues of classroom management and student behavior, more focus on students' mathematics, and substantial opportunity to grapple with the elicitation, interpretation and use of student mathematical thinking during class discussion. Although there is still room for improvement, our model provides an existence proof that the focus of the student teaching experience can indeed be altered and improved.

Keywords Paired student teaching · Student mathematics · Field experience
Learning to teach

Field experiences are a common element of teacher preparation throughout the world (White & Forgasz, 2016). These experiences range from classroom observations early in a teacher preparation program to student teaching—a full-immersion experience where the prospective teacher teaches a full load of classes daily for several weeks. For many decades, student teaching has been the culminating experience for United States mathematics teacher education programs. Traditionally, this capstone experience places one student teacher (ST) in the classroom of one cooperating teacher (CT) for 10–15 weeks. In this setting, the ST observes the teaching of the CT for a few days and then begins to take on the responsibility of teaching some of the CT's classes. Within a few weeks the ST typically takes over most or all of the CT's load and responsibility. To oversee the student teaching experience, the university assigns a professor or graduate student,

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_2

referred to as a university supervisor (US), to observe the student teacher 3–4 times during the 15-week experience. Although the university oversees student teaching, the infrequent visits of the US leads to an experience that is dictated primarily by each individual CT, who may receive minimal compensation for his or her time and is primarily serving on a voluntary basis.

Although research has consistently documented that the student teaching experience is viewed as extremely valuable by both preservice teachers (Anderson & Stillman, 2013; Guyton & McIntyre, 1990) and teacher educators (Metcalf, 1991; Valencia, Martin, Place, & Grossman, 2009), the actual educative value of student teaching depends a great deal on the quality of that experience (Ronfeldt & Reininger, 2012), which varies considerably (Anderson & Stillman, 2013). Thus, the research community has long documented the problematic nature of the traditional student teaching experience (Brouwer & Korthagen, 2005; Feiman-Nemser & Buchmann, 1985; Greenberg, Pomerance, & Walsh, 2011). Although much of the literature just cited is focused on student teaching in general, we found it to be consistent with the problems we had seen in our experience working with student teachers who were being prepared specifically to teach mathematics in grades 7–12 (ages 12–18). In our paper, *Purposefully Designing Student Teaching to Focus on Students' Mathematical Thinking* (Leatham & Peterson, 2010a), we outlined some of those problems as discussed in the literature, argued that many aspects of these problems were a product of the existing structure of student teaching, and then proposed a new structure designed to address those problems. In this paper we summarize that argument, briefly describe the revised student teaching structure, and then focus on reporting what we have learned over the past decade by analyzing data from initial implementation of the structure. We discuss where the evidence suggests we should go next and implications our work has for others who desire to improve their student teaching experience.

2.1 The Need to Restructure Student Teaching

In Leatham and Peterson (2010a) we argued that the traditional structure of student teaching contributed to five problems with the quality of the student teaching experience:

1. Lackluster outcomes—the traditional student teaching structure lacks clearly delineated learning goals, which tends to lead to an "experience for experience sake" mindset by those involved and thus huge variation in the nature and quality of learning.
2. Survival over technique—the traditional student teaching structure tends to turn over to the student teacher the entire job of the cooperating teacher, and to do so relatively quickly, which puts STs in a "sink or swim" situation where they necessarily focus on surviving to live another day moreso than planning, enacting and reflecting on quality instructional activities. This focus on survival tends to emphasize classroom management.

3. Focus on self—the pressures and structure of traditional student teaching, with its focus on taking on the "job" of a teacher, turn STs inward moreso than outward, which often causes them to be more concerned about their own actions than those of their students.
4. Isolation—the structure of traditional student teaching mirrors the structure of traditional classrooms. Such classrooms are often more isolating than collaborative, and STs come to view teaching as work that is done primarily alone. Even collaboration with the cooperating teaching can often be more reactive than proactive depending on the role the CT chooses to play.
5. A class with no teacher—the traditional structure of student teaching places a majority of the learning experiences of the STs in the hands of CTs, who tend to see themselves more as experienced colleagues than as teacher educators. This lack of guidance leads to haphazard, unfocused learning and, again, "experience for experience sake."

Overall, the traditional structure of student teaching tends to focus STs more on *managing students* than on *facilitating student learning* (Anderson & Stillman, 2013). That is, the structure itself seems to inhibit STs from focusing on what is arguably the most important aspect of their role—helping students come to understand content.

Prior to restructuring our student teaching experience, we felt like our structure was very much a traditional structure with the accompanying problems just delineated. Before overhauling the structure, however, we decided to gather more evidence. Thus, in order to gain a better understanding of the structure and purpose of our own student teaching experience, we examined how CTs viewed this purpose (Leatham & Peterson, 2010b). We surveyed 45 of our previous CTs and found that most of them saw the primary purposes as giving STs an opportunity both to experience real classrooms with real students and to work with real teachers in learning how to manage these classrooms, thus confirming that our structure had created an environment where our CTs viewed student teaching as primarily experience for experience sake (Feiman-Nemser & Buchmann, 1985). With respect to survival, many CTs indicated that learning about classroom management was a primary purpose of student teaching. Furthermore, when asked "specific to mathematics teaching, what do you feel is the most significant contribution you make to the success of a student teacher?" half of the CT respondents made no mention of mathematics (Leatham & Peterson, 2010b). This finding seemed to indicate that they saw the purpose of student teaching to be primarily about teaching in general as opposed to teaching mathematics in particular. This view contrasted dramatically from our view, where student mathematical thinking was an integral part of the learning-to-teach experience. Thus, these CTs did not see our student teaching program as being about learning to craft and carry out mathematics lessons that effectively anticipated, elicited, and built on students' mathematical thinking—our desired purpose. Finally, our student teaching experience was determined almost completely by individual CTs, whose goals varied substantially. In addition, many CTs did not see themselves as teacher educators but saw their role as just providing a space for the student teaching experience to occur. Hence, we had a class with no teacher (Leatham & Peterson, 2010b) which contributed, in part, to the STs focusing on self and on survival over technique (Leatham & Peterson, 2010b).

2.2 Restructuring Student Teaching

In order to try to address the problems that we had identified in our existing student teaching experience, we began by discussing and then explicitly articulating our primary purpose for the experience: to learn how to anticipate, elicit and use students' mathematical thinking. Based on our own experiences with variations on the structure of student teaching (Peterson, 2005; Wilson, Anderson, Leatham, Lovin, & Sanchez, 1999) we discussed how we could alter the current structure of the experience in ways that would both address the problems and support our desired purpose. In this section, we describe the resulting revised structure; in the section that follows we present a theoretical argument for these choices.

In the new model, two STs are placed together with one CT for the 14-week experience. Two or three student teaching pairs from neighboring schools form a cluster, and all STs in a cluster are supervised by the same university supervisor (US). Figure 2.1 gives a basic outline of how the revised student teaching experience was structured (see Leatham & Peterson, 2010a for a full description of and rationale for the new structure). During each of weeks 3–5, each pair of STs jointly plans and then individually teaches a single lesson. The teaching of these first lessons are observed by the other members of the cluster, the US and CT. The observers take note of the students' mathematical thinking that emerges during the lesson and make that thinking and how it related to the goals of the lesson the focus of the post-lesson reflection meeting. We refer to this sequence of activities as the Teach/Observe/Reflect process.

In addition to preparing, teaching, reflecting on and observing lessons during those first five weeks, STs do several other learning-to-teach activities: observe their CT and other experienced teachers in the school and write reflection papers about these observations; complete a daily journal wherein they record goals for their day's observations, keep a record of their day's activities, and make note of students' mathematical thinking they observed and found to be of particular interest; and conduct student interviews, wherein they probe students with regard to their mathematical understanding on problems from a recent homework, quiz or test,

<table>
<tr><th>Week</th><th colspan="2">Learning-to-teach Activities</th></tr>
<tr><td>1</td><td>Become familiar with classroom & students</td><td rowspan="3">Focused Observations
Daily Journal
Student Interviews</td></tr>
<tr><td>2</td><td>Observe experienced teachers and write reflections</td></tr>
<tr><td>3-5</td><td>Teach/Observe/Reflect</td></tr>
<tr><td>6-13</td><td colspan="2">Teach “full” (half) load</td></tr>
<tr><td>14</td><td colspan="2">Teach/Observe/Reflect</td></tr>
</table>

Fig. 2.1 Outline of the new student teaching structure

then write a reflection paper about these interviews. The work produced from these learning-to-teach activities is submitted to the US to be reviewed and can be used as fodder for subsequent discussions with the ST.

After the first five weeks, the learning-to-teach activities described in the previous paragraph are suspended and the CT's full load of teaching is split between the two STs in the classroom. Each ST takes on the full responsibility of each of their assigned classes for the next 8 weeks of student teaching. During the final week of student teaching, the STs begin to turn teaching responsibility back to the CT and engage one last time in the Teach/Observe/Reflect process. The planning of lessons throughout the experience is done as a collaboration between the pair of student teachers, with feedback from the CT. The US observes the teaching of each ST every 1–2 weeks for the remaining 9 weeks of the student teaching experience, providing formative feedback following each observation.

2.3 How the New Structure Addresses the Problems

Our biggest disappointment with student teaching had been that there seemed to be too much focus on learning about classroom management and how to deal with student behavior problems and too little focus on learning about crafting quality mathematics lessons and how to deal with student mathematical thinking. As described above, we suspected (see Leatham & Peterson, 2010a) that there were structural elements of student teaching that contributed to this focus imbalance. So, we altered the structure in order to provide the space and support for a change of focus. Furthermore, through changing this focus, we hoped to change the outcome. Because of the results of the survey study (Leatham & Peterson, 2010b), we were deliberate in making our desired purpose explicit in the new structure. We wanted student mathematical thinking to become an integral part of the experience, for STs, CTs and USs to see mathematics, its teaching and its learning as problematic and something worthy of discussing rather than something that was taken for granted and in the background. We created a detailed syllabus that described the overall structure and purpose of the student teaching experience and took care to familiarize the STs, CTs and USs with their roles within that structure. In so doing we attempted to make more explicit and transparent the teaching role of both the CTs and the USs. We paired and clustered STs in order to decrease isolation and a focus on survival and increase space for conversation and reflection about mathematics teaching, particularly the practices of eliciting, interpreting and using student mathematical thinking. Bullough et al. (2003) argued based on their own implementation of student teaching pair, "If to learn to teach is to learn to manage … then partnership teaching has an obvious disadvantage. However, if student teaching's primary purpose is to … expand one's knowledge of methods and of children … then partnership teaching has an advantage" (p. 71). The learning-to-teach activities explicitly focused STs on their students' mathematical thinking. This focus was intended to facilitate our overall purpose as well as to decenter the STs away from a focus on self and toward a focus on their students, thus encouraging them to adopt their students' perspective (Arcavi & Isoda, 2007).

Having provided this focus and the space for conversation and reflection, we asked the following questions: By altering the structure of student teaching can we alter its focus away from classroom management and onto student mathematics? When given a structure that privileged mathematics, students' mathematics and the pedagogy of crafting and orchestrating good mathematics lessons, did such issues fill the provided space? If so, what does that focus look like and to what extent does it give STs the opportunity to reflect on issues of engaging students in meaningful mathematical activity?

2.4 Data Collection and Analysis

We collected data during the first two fall semesters of the implementation of the new structure. The participants during Fall 2006 consisted of a cluster of 3 pairs of STs, their 3 CTs and the US (the first author of this paper). The participants during Fall 2007 consisted of 2 clusters of 2 pairs of STs each, their 4 CTs (one of whom had been a CT the previous year as well) and two USs (one of whom is the second author of this paper). Data included the STs' daily journals, interviews with the STs and their CTs, audio-recordings of unstructured ST-CT conversations, and video-recordings of reflection meetings at the end of each Teach/Observe/Reflect process. Here we describe the nature of each of these types of data and the analysis of each data type used for this paper. We note that because this analysis is of data collected from the first two years of implementation and analyzed from various perspectives over the course of a decade, the results we report are not the product of years of fine tuning this new structure, and thus more likely the product of the structure itself.

Daily Journals: One of the learning-to-teach activities in which the STs participated was the keeping of a daily journal almost every day for 4–5 weeks at the beginning of the student teaching experience. Each day they were asked to list their learning goals and then plan what they would do each period, from observing their CT to observing another teacher in the school to planning a lesson. At the end of the day, they would then briefly report their thoughts and impressions from each of those activities. The most critical part of the daily journals was responding to the following prompt:

> Describe observed mathematical thinking where a student was either frustrated or appeared to have misconceptions. If you were to work with this student what questions would you ask and when would you ask them? If you were to use this student's thinking as part of a class discussion, how would you use it?

We analyzed STs' responses to this prompt by focusing on the nature of the student mathematics they identified and the ways the STs talked about how they might use that thinking.

Interviews: During the course of the student teaching experience, the STs and the CTs were all interviewed three times: once at the beginning of the semester, once after the completion of the first 5 weeks and once at the end of the semester. The focus of these interviews was to determine the participants' perceptions of the student teaching experience. The questions ranged from a general inventory of what aspects of the experience helped them grow as a teacher to the positives and negatives of specific aspects of student teaching (e.g., observing peers, keeping a journal, conducting student interviews). For the purposes of this paper we focused on inferring the STs' and CTs' views of the pairing aspect of the structure. We did so by qualitatively analyzing statements they made about the affordances and constraints of the pairing.

Reflection Meetings and Unstructured Conversations: The revised structure created the space for two different types of reflective conversations to occur. One type was the structured post-lesson reflection meeting that occurred each time a cluster of STs observed one of their peers teach. The other type was the unstructured, periodic conversation that happened whenever the pair of STs and the CT visited for more than 5 min. In order to get a sense of the extent to which these conversations did as was desired—privileged mathematics, students' mathematics and the pedagogy of crafting and orchestrating good mathematics lessons over issues of classroom management—we analyzed the nature of both types of reflective conversations. We used a statement (which was typically a sentence) as the unit of analysis. Using context as needed, each identified statement was coded for whether it related to (a) pedagogy (P), referring to the circumstances of the classroom or to specific pedagogical moves; (b) students (S), referring to student thinking or actions, either as an entire class or as individuals; (c) mathematics (M), referring specifically to mathematical content, topics or ideas; and (d) behavior (B), referring to issues of classroom management, student discipline and administrivia. As expected, many statements received multiple codes. For example, the ST statement "I really liked what Katherine said, 'Well, if I take my string and I measure my circumference first, then I can figure out my radius'" received an S because of the reference to what the student Katherine had said, and an M because of the reference to the relationship between the circumference and radius of a circle. There are no pedagogical or behavioral references in the statement, so it would be coded as SM. An example of a statement coded as PSM occurred when the STs were discussing a situation where the written mathematical sentence was $15 = 7 + 8$, but when a student had come to the board she had written the sentence as $7 + 8 = 15$. In the reflection meeting a ST asked, "Is there a reason you didn't focus on that? I mean did you want to make a statement about that at all, because I think that's an interesting idea that a student would bring up: you can just switch the things on the equals sign and it means the same thing. But then also that she has to rewrite it for her to make sense. Does that make sense?" This statement is coded with an S and M because they are talking about the mathematics that a student wrote on the board. This statement was also coded with a P because of the focus on the pedagogical decision around the student mathematics.

Having analyzed the reflection meeting data, we wondered whether these discussions really did differ from those that had occurred before we implemented our structure. Although we did not have reflection meeting data from before the implementation of the structure, we did have recorded unstructured conversations between CTs and STs from about a decade earlier (see Peterson & Williams, 2008). We applied the P, S, M, and B coding to the unstructured conversations from the earlier study and from our new structure and compared.

To better understand the nature of these conversations, both in the reflection meetings and in the unstructured conversations, we had two graduate students, who themselves, as undergraduate students, had student taught under the new structure, delve a bit deeper into the original data in two different ways. In a first approach, we took a closer look at the PSM-coded statements (all statements coded with P, S and M regardless of whether the B code was also applied) in the unstructured conversation data (The details of this approach can be found in Franc, 2013). We chose representative samples of PSM-coded statements from the two sets of unstructured conversation data and analyzed the statements for the extent to which they seemed to be aligned more with traditional or ambitious perspectives of students, pedagogy, and mathematics. Ambitious instruction occurs when students are engaged in problem-solving activities and teachers respond to student thinking generated in this setting (Kazemi, Franke, & Lampert, 2009). Ambitious teaching also aligns with the strands of mathematical proficiency outlined by Kilpatrick and his colleagues (National Research Council, 2001). Traditional instruction, on the other hand, promotes teacher-centered classrooms that use lectures and examples to teach mathematics. Such views place the teacher as the mathematical authority in the classroom and students as the recipients of mathematical knowledge disseminated by the teacher.

To gain a sense of how statements were coded as either ambitious or traditional we offer a few examples here relative to pedagogy. The statement “I felt like I got my message across better in 5th period because—did you see I did two examples for them?” was coded as traditional with respect to pedagogy because it was focused on the teacher presenting clear examples. The statement “Have them discuss their thoughts [on why rules of exponents work] with their partner before you have a class discussion so they’ve had someone validate their ideas,” on the other hand, was coded as ambitious with respect to pedagogy because it was focused on the teacher encouraging students to use their own reasoning to develop understanding. The statement “Don’t move on until their questions are answered because they’re going to have a lot of questions on [classifying functions]” was coded as neutral relative to pedagogy because there is no clear indication of how the student questions were to be answered. Similar reasoning was used to code statements as ambitious, traditional, or neutral relative to students and mathematics.

Statements that were coded as ambitious relative to pedagogy were often also coded as ambitious relative to both students and mathematics. The same was true for both the traditional and neutral codes. The coding scheme, however, was designed to allow the same statement to be coded differently relative to pedagogy, students, and mathematics. To further understand this coding variation, we offer an example of the coding of one such statement relative to pedagogy, students, and mathematics. As just mentioned, the statement "Don't move on until their questions are answered because they're going to have a lot of questions on [classifying functions]" was given a neutral code for its message about pedagogy because it is implying that teachers need to respond to students in some way, which is reasoning common to both ambitious and traditional teaching (although the nature of the response might be very different). It was also given a neutral code for mathematics because of the message that learning mathematics is not always easy—a message unique to neither ambitious nor traditional teaching. This same statement, however, was given an ambitious code for its message about students because it sends the message that student thinking should influence what happens in the classroom.

In a second approach to looking more closely at the reflection meetings we moved beyond an analysis of sentences (Leatham & Peterson, 2013) to a unit of analysis of conversation pieces (referred to as "chunks") formed by major topic changes (The details of this approach can be found in Divis, 2016). The length of the resulting 209 chunks ranged from a few sentences to several pages of transcript. Our desire here was to capture the extent to which these conversations focused on mathematics, students' mathematics and the pedagogy of crafting and orchestrating good mathematics lessons. Although these data were gathered in 2006 and 2007, we felt that *Principles to Action* (PtoA) (NCTM, 2014) was consistent with the vision we had when the new student teaching structure was implemented and provided detailed descriptions of important principles of effective mathematics instruction, including a list of teacher or student actions associated with each principle. Therefore, we coded these chunks according to the actions associated with each of the eight principles of effective teaching and learning (NCTM, 2014) (referred to hereafter as the PtoA Principles).

2.5 Results

We organize our results around three major outcomes of the new student teaching structure: (a) less focus on students' behavior problems; (b) more focus on students' mathematics; and (c) a focus on the practice of eliciting, interpreting and using student mathematical thinking. As we discuss these outcomes, we also highlight how they address the five problems of traditional student teaching mentioned earlier.

2.6 Less Focus on Behavior

Table 2.1 shows the results of our coding of the reflection meetings and of the unstructured conversations from the new structure and from the decade before it was implemented using the P, S, M, and B codes. As can be seen in the table, the

Table 2.1 Percentages of statements receiving pedagogy, students, mathematics or behavior codes (each statement could receive more than one of the P, S, M, or B codes.)

	Conversations		Reflection meetings
Code	1998	2006–2007	2006–2007
Pedagogy	87	82	71
Students	48	46	56
Mathematics	28	63	59
Behavior	18	5	4

percentage of statements receiving the behavior code dropped dramatically in the new structure from about 1 in 5 comments to only about 1 in 20 comments. Although conversations about student behavior still took place, they were clearly not a major focus.

During the first lessons of the semester (the ones followed by a reflection meeting), there were at least five adults in the classroom observing the lesson being taught, which we feel contributed to the decrease in student behavior issues and corresponding decrease in conversations about behavior in the reflection meetings. The researchers' presence is another likely influence on the conversation topics in these reflection meetings. It is interesting to note, however, that the decreased focus on behavior continued in the subsequent unstructured conversations where only the CT and STs were present. This result was a bit surprising given there was no instruction given regarding the expected focus of these conversations. We hypothesize a couple of possible contributing factors to the continued small emphasis on student behavior. One feature of the student teaching structure that may have contributed to this phenomenon of fewer conversations about student behavior is that the reflection meeting conversations set the pattern for the nature of conversations throughout the student teaching experience. A second feature of the structure that may have contributed to the minimal student behavior conversations was that, again, there may have been an actual decrease in student misbehaviors because of the presence of other adults (in this case a student teaching partner and the CT) in the classroom throughout the student teaching experience. Taken together, this decreased conversational emphasis on student behavior is an indication that the STs were less worried about survival—one of our goals with the restructure of the student teaching experience (Leatham & Peterson, 2010a).

With ST pairing being a possible contributor to decreased focus on issues of student discipline, we now examine the STs' perception of being paired. Our analysis of the interview data revealed that participants' perceptions of being paired were overwhelmingly positive. In fact, when ranking the value of the various learning-to-teach activities in which they engaged, being paired was by far the highest ranked overall. Having someone to talk to was the primary justification for liking the paired situation. This justification offered by the STs is evidence that the new structure met one of our goals of reducing the isolation of teaching (Leatham & Peterson, 2010a). The STs identified several different aspects of the dialog with their peers that were advantageous, namely preparing lessons together, observing someone else's teaching style, and just having someone to talk to who was having a similar experience. Some STs indicated that their lessons were better because they were prepared in collaboration. We suspect that these better-prepared lessons may have also contributed to a decrease in problems with classroom management.

The participants identified a few negative aspects of being paired, but these aspects seemed unavoidable when seeking the benefits of being paired. For example, several STs mentioned the challenge of planning a lesson with someone else. Another aspect related to learning to run the classroom. Specifically, the STs mentioned that because they had less time in front of the class they had fewer opportunities to deal with classroom management issues on their own. (Although some saw this aspect as negative, we saw their comments as further evidence that the pairing did indeed contribute to a decrease in focus on classroom management.) We see all of these perceived negatives as a small price to pay for the positives that were identified, particularly because the positives could not occur otherwise.

2.7 More Focus on Students' Mathematics

As shown in Table 2.1, the percentage of statements receiving the mathematics code more than doubled under the new structure, and from less than a third of the statements to more than half. These statements, however, were not typically about mathematics only (see Fig. 2.2). Although purely mathematical conversations (M codes) did increase under the new structure, the big gains were in conversations combining mathematics with students and/or pedagogy (PM, SM, and PSM codes). PSM-coded statements tripled in frequency. These gains in mathematics codes came primarily from a decrease in P and PS codes. In the unstructured conversations prior to the change in structure over 65% of the statements received P or PS with no reference to mathematics. After the change in structure, only about 35% of the statements received the P or PS codes. Since statements with a P or PS code are about what the teacher does generally or what the teacher does relative to the students without considering the student mathematical thinking, we believe that this decrease in the number of statements receiving P or PS codes is an indication of progress in the STs moving away from a focus on themselves—another of our goals in the restructuring of student teaching (Leatham & Peterson, 2010a). Furthermore, because the behavior codes were overwhelmingly applied to statements that were coded as P or PS, these data provide further evidence of a decreased focus on survival and an increased focus on the technique of using student mathematical thinking.

Analysis of the 2006–2007 journal data lends further insights into just *how* the STs in the new structure focused on students' mathematics. In an analysis of the nature of the mathematics they noticed, each response was coded as either having a conceptual (the reasoning behind the students' struggles) or a procedural (the details of the procedure with which they were struggling) focus. The ratio of overall responses coded as conceptual compared to procedural was about 24:5.

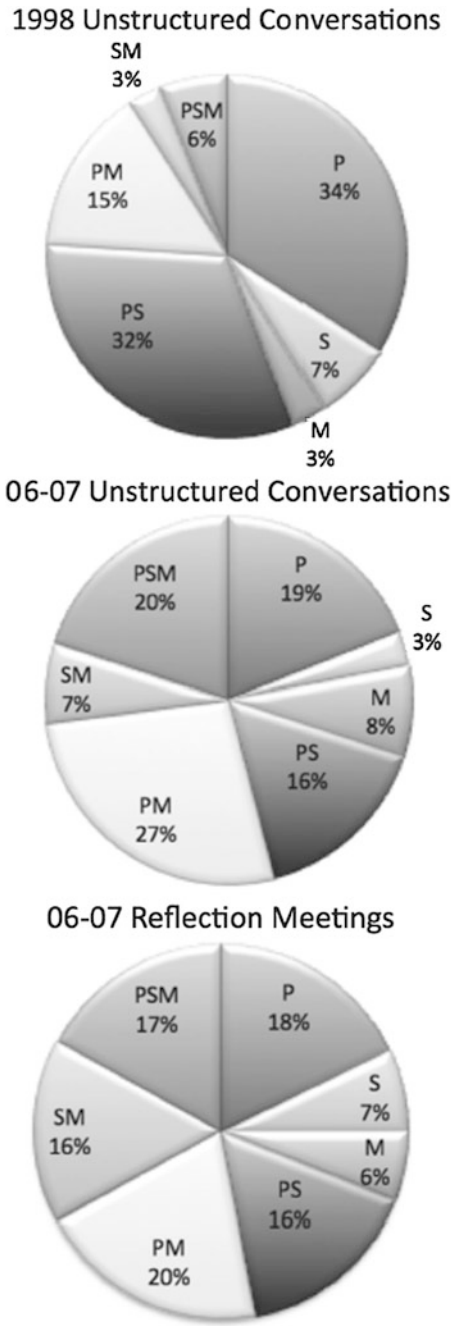


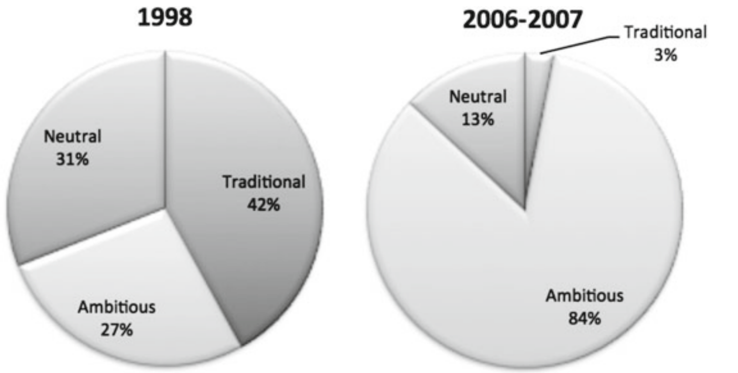
Fig. 2.2 Percentages of P, S and M code combinations across the various types of conversations

The emphasis of the STs on the conceptual understanding of the students is further evidence of how the student teaching structure provided space for the STs to focus on students' mathematics, and to do so in productive ways.

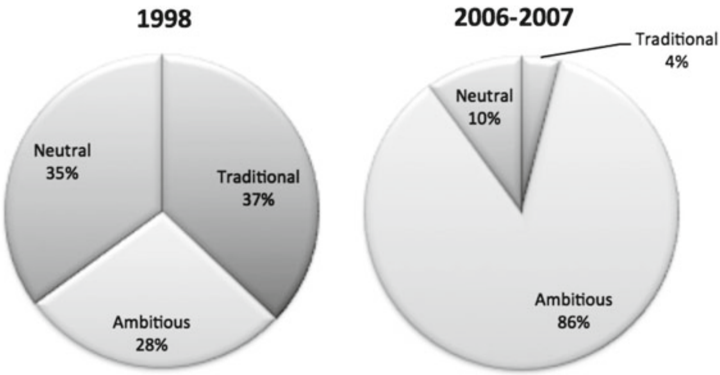
2.8 A Focus on Eliciting, Interpreting and Using Student Mathematics

Analysis of the journal data also provides insights into how the STs thought about using student mathematics. The way in which the STs talked about using the student thinking fell overwhelmingly in two categories—engaging students in a discussion about the students' thinking or explaining the students' thinking for the class. The specific ways in which they talked about engaging the class in the discussion involved asking students to make sense of the student thinking, characteristic of ambitious teaching. The teacher explanations included reteaching definitions, clarifying misconceptions, and creating new examples, characteristic of traditional teaching, as in these teacher explanations, it was the teacher rather than the students who was doing the cognitive work. The STs talked about discussing the student thinking about three times as often as they talked about explaining the thinking. This tendency toward engagement over explanation is evidence of the focus of the student teaching structure on eliciting and using students' mathematical thinking.

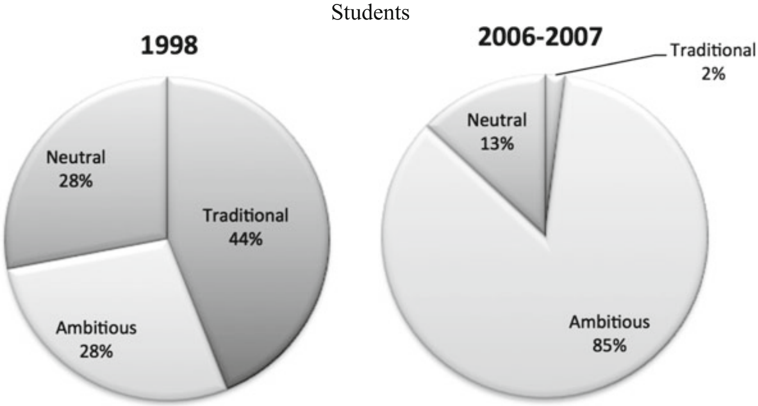
Our deeper analysis of the unstructured conversations provides further evidence that the structure was indeed focused on issues of engaging students in meaningful mathematical activity. We took a closer look at the statements in the unstructured conversation data that received P, S, and M codes (again, whether or not they received the B code) because, although these PSM-coded statements occurred more than three times as often in the newer structure, we wondered whether the nature of these statements were basically the same, just more common, or whether they were actually qualitatively different. As mentioned before, we analyzed the statements for the extent to which they seemed to be aligned more with traditional or ambitious perspectives of students, pedagogy, and mathematics. We found the results to be quite striking (see Fig. 2.3). Under the revised structure the unstructured conversations in which our STs engaged were overwhelmingly ambitious in nature. In addition to being less traditional in nature, they were also far less neutral as well. Our students were receiving much clearer messages about the nature of mathematics and of mathematics learning and teaching, and those messages were much more in line with the messages we wanted them to have the opportunity to hear. We also note that, although pairing STs in the new structure requires half as many CTs as before, many of the new-structure CTs were still rather traditional in their approaches to teaching. This fact provides further evidence of the influence of the structure itself on the nature of the student teaching experience. We believe that this increased emphasis on eliciting and using students' mathematical thinking as well



Pedagogy



Students



Mathematics

Fig. 2.3 The traditional, ambitious or neutral nature of PSM-coded statements from unstructured conversations before and after the restructuring of student teaching as the focus on ambitious teaching found in the unstructured conversations is evidence that the problem of lackluster outcomes has been partially overcome by the new structure.

Our deeper analysis of the reflection meetings also provides evidence that the structure was indeed focused on issues of engaging students in meaningful mathematical activity. Looking across the last two reflection meetings for all of the cluster groups, we examined chunks of conversation (as defined earlier) to determine if they were or were not focused on one of the principles in PtoA. In deciding whether a given principle was commonly discussed we looked not only at the frequency of chunks that received a certain sub-code but looked also at the length of chunks (as measured by a word count) and whether all three cluster groups had chunks receiving this code. In doing so, we found that seven of the eight PtoA principles were a significant focus of these conversations. The seven PtoA principles are (a) Establish Mathematics Goals to Focus Learning, (b) Implement Tasks That Promote Reasoning and Problem Solving, (c) Use and Connect Mathematical Representations, (d) Facilitate Meaningful Discourse, (e) Pose Purposeful Questions, (f) Support Productive Struggle in Learning Mathematics, (g) Elicit and Use Evidence of Student Thinking. The one principle that was only minimally discussed in the reflection meeting conversations was Build Procedural Fluency from Conceptual Understanding. The principle that was overwhelmingly the most commonly discussed was Elicit and Use Evidence of Student Thinking. These results provide evidence that making students' mathematical thinking the focus of conversation naturally led to a focus on mathematics teaching consistent with the PtoA Principles. The underlying "productive beliefs" (NCTM, 2014, p 11) of PtoA are about conceptual understanding, reasoning and sense making. Thus, putting STs in a position to reason about and make sense of student mathematical thinking also positioned them to be thinking about those students' conceptual understanding, reasoning and sense making—foundational ideas behind the PtoA Principles.

In addition to the chunks that were coded according to the NCTM principles, there were several internal codes that emerged from the data. The three most common internal codes were Mathematical Pedagogy, Student Teacher Mathematics and Managing Student Behavior. Although classroom management issues were discussed, they made up only a small portion of all of the conversations. Looking across the coding of all of the chunks it seemed clear that (a) the STs spent a significant amount of time talking about using student thinking; (b) they talked about mathematics from both pedagogical and personal perspectives; and (c) their discussions of managing student behavior were minimal.

2.9 Lessons Learned

We have been very pleased with how the restructuring of our student teaching program has played out over the past 10 years. Our own experience and the results of our data analysis have led us to conclude that the changes were for the better. Of the five common problems in mathematics student teaching outlined at the beginning of the paper, we believe we have made significant progress on four of them. As we have mentioned previously, our data have provided evidence that we have overcome lackluster outcomes, increased emphasis on technique over survival, decreased isolation and decreased a focus on self. Although we believe the structure itself begins to address the problem of a class with no teacher, we do not have evidence that more of the CTs view themselves as a teacher of their ST. This result is a limitation of the data we chose to gather.

With all of the positive results, we still see room for improvement. There are two areas, in particular, where we see the potential for productive change. First, although the structure creates much collaboration between paired STs, it does not create adequate CT-ST collaboration. In fact, although CTs saw many positive elements of the STs being paired, one CT noticed that "there's not as much communication between the cooperating teacher and the two student teachers" as there has been in the past, which was seen as a negative aspect of the paired STs. Because of this feedback, we would like to build into the structure the need for the STs to work more with the CT as they create and revise their lessons, particularly during the early weeks of student teaching. Second, the structure allowed us to elicit from the STs important ideas related to productive use of student mathematical thinking, but it did not necessarily create adequate space for us to leverage those ideas sufficiently to improve ST learning. For example, although the daily journals elicited important ideas related to students' mathematical thinking and how it might be used in classroom mathematics discourse, the structure does not capitalize on this observational focus. The journals were read weekly by the US, but there were no organized discussions of the STs' observations. Similarly, although the structure of the reflection meetings allowed important ideas to surface, the structure actually discouraged the CT and US from capitalizing on promising teaching opportunities at the moment they occur during these reflection meetings. For example, in order to encourage the STs to engage in a discussion among themselves about their observations, the CT and US were asked to withhold their comments until after the STs had asked their questions and made their comments. However, this structure prevented the CT or US from capitalizing on in-the-moment opportunities to engage the STs in considering in more detail some of the points they were discussing. (For documentation of a similar issue in the area of language arts, see Valencia, Martin, Place & Grossman, 2009.) Finding ways to maintain the current improvements while creating space for more deliberate use of STs' emerging ideas by the CT and US would likely make the restructured student teaching experience even more rich and rewarding for all involved.

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Chapter 3
Improving Secondary Mathematics Teacher Preparation Via a Networked Improvement Community: Focus On Clinical Experiences

W. Gary Martin and Marilyn E. Strutchens

Abstract The Mathematics Teacher Education Partnership is a consortium of over 90 U.S. universities and colleges, along with partner school districts, focused on improving the initial preparation of secondary mathematics teachers. The Partnership uses a Networked Improvement Community design that incorporates improvement cycles to develop adaptable interventions across contexts, as well to scale interventions across the Partnership to support comprehensive program improvement. Rather than addressing a single dimension of a secondary mathematics program, the Partnership is undertaking parallel lines of research in multiple areas. To illustrate the power of the approach, this chapter will more deeply explore one of those lines of research related to clinical experiences: A “research action cluster” (RAC) consisting of representatives of 24 university-led teams is working to improve the clinical experiences of secondary mathematics teacher candidates. This RAC has employed improvement science methods to developed resources that support improved models for both student teaching and early field experiences, as well as professional development for mentor teachers.

Keywords Mathematics education · Teacher preparation · Clinical experiences
Secondary mathematics · Improvement science

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_3

3.1 Introduction

3.1.1 Rationale

The U.S. continues to struggle to ensure that its students have the mathematical preparation needed for future success. For example, in the 2015 National Assessment of Education Progress, which periodically "measures students' knowledge and skills in mathematics and students' ability to apply their knowledge in problem-solving situations" (The Nation's Report Card, 2017), only 25% of twelfth-grade students demonstrated a level of proficiency needed for future success. Moreover, there has been little improvement in scores over the past decade (The Nation's Report Card, 2017). A similar result can be seen in results from the Programme for International Student Assessment in 2015, in which only 20% of U.S. 15-year old students exceeded the third proficiency level of six, and the U.S. average score fell in the bottom half of industrialized nations (National Center for Educational Statistics [NCES], 2017).

One explanation for the inadequate preparation of U.S. students in mathematics may be found in the significant shortage of well-prepared secondary mathematics teachers in the country. More than 1 in 6 secondary schools report "serious difficulties" in filling vacant mathematics teaching positions (Ingersoll & Perda, 2010). According to the NCES (Keigher, 2010), 1 in 12 secondary mathematics teachers leave the profession every year. The attrition rate is particularly high for beginning mathematics teachers; almost 1 in 7 leave teaching after their first year (Ingersoll, Merrill, & May, 2012). Moreover, quality of mathematics instruction continues to be a concern, as seen in two national surveys of practicing secondary mathematics teachers (Banilower et al., 2013; Markow, Macia, & Lee, 2012): only half reported using instructional practices and goals aligned with the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Thus, the preparation of secondary mathematics teachers in the U.S. requires addressing the interlocking issues of the quantity and quality of those entering the profession. The systemic nature of these issues is illustrated in Fig. 3.1, which depicts a downward cycle in mathematics teacher preparation in the U.S., adapted from Wilson (2011). The cycle begins at the top with the inadequate preparation of U.S. students in mathematics; note that K–12 denotes students in precollege education from kindergarten (K) through grade 12, the final grade in U.S. precollege education. Moving to the right, this implies that the pool of students who are adequately prepared to enter mathematics teaching as a career is quite small; moreover, well-prepared students have many options and so may not choose to enter teaching. Continuing to the lower right of the cycle, mathematics teacher preparation programs often do not provide candidates with the mathematics knowledge needed for teaching (cf. Ball, Thames, & Phelps, 2008). At the bottom of the cycle, candidates may not have clinical experiences that support their development of effective teaching practices (Horn & Campbell, 2015).

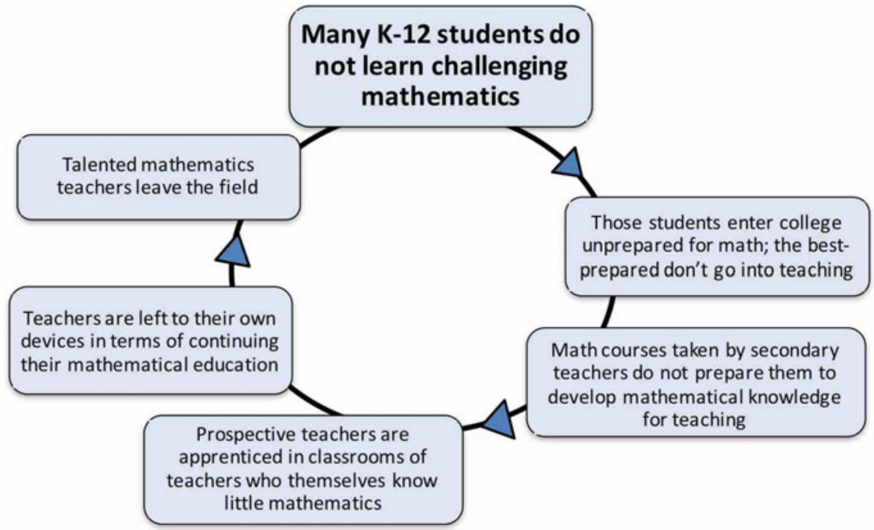

Fig. 3.1 The downward cycle in mathematics teacher preparation (adapted from Wilson, 2011)

Those candidates who enter the teaching profession, lower left of the cycle, often have little support for their continuing growth (Horn & Campbell, 2015), with the result that many talented teachers leave the profession. And we return to the top of the cycle, where many students continue to receive an inadequate preparation in mathematics.

3.1.2 Formation of the Partnership

To address the challenge presented in this downward cycle—the undersupply of new secondary mathematics teachers who are well prepared to help their students attain the goals of the CCSSM and other rigorous state mathematics standards—the Association of Public and Land-grant Universities (APLU) formed the Mathematics Teacher Education Partnership (MTE-Partnership), a national consortium of over 90 universities and over 100 school systems, as a project within its Science and Mathematics Teaching Imperative (SMTI), which focuses more generally on improving mathematics and science teaching. APLU is an organization of major state universities within the U.S., particularly focused on addressing issues related to higher education and its leadership.

The initial concept for the Partnership was formed at the 2011 SMTI Annual Conference, which focused on how higher education might respond to the just-released CCSSM, including necessary changes in teacher preparation. A group of attendees submitted a white paper to the SMTI executive committee proposing the formation of the project, and a planning team was formed to organize the Partnership. Funding from the National Science Foundation (#1147987) supported the development and launch of the network in Spring 2012, and subsequent grants from the Leona M. and Harry B. Helmsley Charitable Trust have supported its continuing development.

The goal of the Partnership is to "transform secondary mathematics teacher preparation" (MTE-Partnership, 2014, p. 1). University programs participate in the Partnership as a part of teams that include K–12 school districts and other partners involved in secondary mathematics teacher preparation, with a requirement that teams engage mathematics teacher educators, mathematicians, and K–12 personnel in their activities. The inclusion of multiple stakeholders in the efforts reflects the focus of the partnership on "develop[ing] and promot[ing] a common vision and goals for how to best prepare teacher candidates who can promote student success in mathematics" within a program, as well as engaging in mutual learning and sharing responsibility across the Partnership (MTE-Partnership, 2014, p. 2). There are currently 39 partnership teams across 31 states in the U.S. (see Fig. 3.2).

3.1.3 Research Design

About a year after its formation, the MTE-Partnership adopted the Networked Improvement Community (NIC) model developed and used by the Carnegie

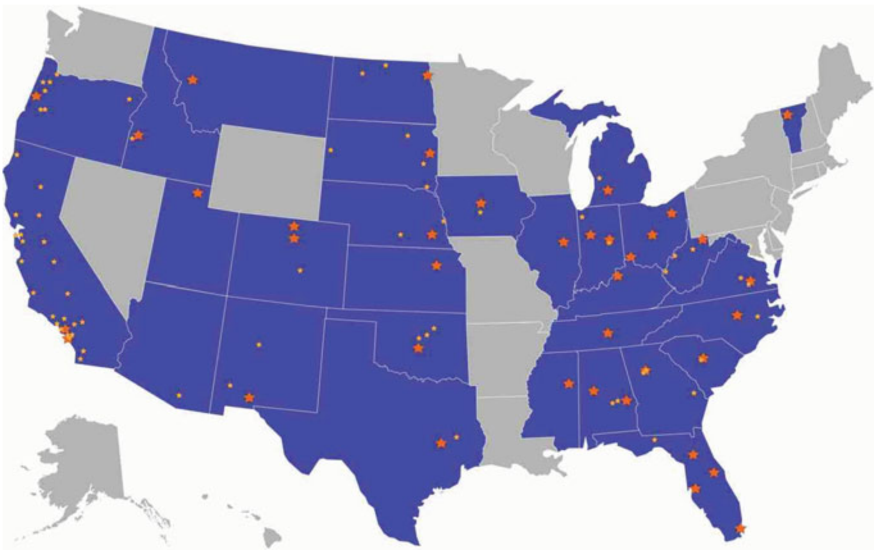

Fig. 3.2 Participation in the MTE-Partnership. Large stars represent lead institutions for a team, and small stars represent other participating universities and colleges

Foundation for the Advancement of Teaching in response to several design challenges identified by the planning team, including (a) the need to maintain the engagement of the teams in the work of the Partnership and (b) the need to maintain a focus on disciplined inquiry consistent with the mission of universities (Martin & Gobstein, 2015). This design supports active collaboration by the partnership teams to address significant issues in secondary mathematics teacher preparation using improvement science to ensure fidelity to academic standards of inquiry. While no explicit theoretical stance was adopted in the work, as its focus is more on building solutions to problems than on building theory, the emphasis on collaborative building of knowledge is consistent with social constructivism (Ernest, 1991).

NICs are distinguished by four essential characteristics (Bryk, Gomez, Brunow, & LeMahieu, 2015); each characteristic is described in the following, along with how the Partnership addressed that characteristic.

- **Focused on a specified common aim**: The Partnership is focused on the twin aims of producing mathematics teacher candidates who meet a "gold standard" of preparedness to address the Common Core and of increasing the quantity of well-prepared candidates by Partnership programs by 40% by 2020, as depicted in the left-most column of Fig. 3.3. Note that the improvement target was set through a collaborative process of collecting data from the individual teams and programs. Further information on the measures used to assess candidate quality is given in a later section of this chapter.

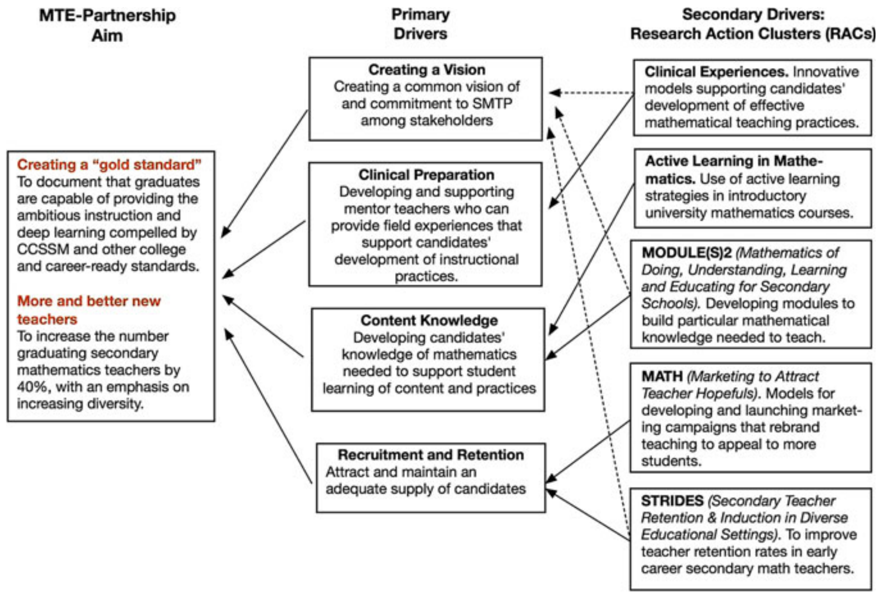

Fig. 3.3 The MTE-Partnership driver diagram (Martin & Gobstein, 2016)

- **Guided by a deep understanding of the problem and the system that produces it**: Over a period of nearly a year, the membership teams worked together to develop a shared vision for the Partnership, which is reflected in its *Guiding Principles for Secondary Mathematics Teacher Preparation* (MTE-Partnership, 2014). This document then formed the based for identifying challenges in secondary mathematics teacher preparation. A multi-step process described by Martin and Strutchens (2014) led to the identification of four significant problem areas of primary importance to the Partnership. In the second column of Fig. 3.3, these problems are restated in the positive as primary drivers, the Partnership's main areas of influence necessary to promote movement towards achieving the aim (Bryk et al., 2015). Note that these primary drivers are well-aligned with the Standards for Program Characteristics and Qualities in the *Standards for the Preparation of Teachers of Mathematics* released by the Association of Mathematics Teacher Educators (AMTE) (2017).
- **Disciplined by the rigor of improvement science**: The use of evidence to guide the development of interventions ensures that the changes being proposed are actually improvements. Moreover, use of an iterative cycle of prototyping, testing, and refining interventions, as seen in Fig. 3.4, has the potential to lead to timely solutions to important problems (Bryk et al., 2015). "Research action clusters" (RACs) have been organized to carry out the development of interventions. The current RACs are summarized in the third column of Fig. 3.3. More detail is provided in the following section.
- **Networked to accelerate the development, testing, and refinement of interventions and their effective integration into varied educational contexts**: Rather than trying to "control" variation, as typical in traditional

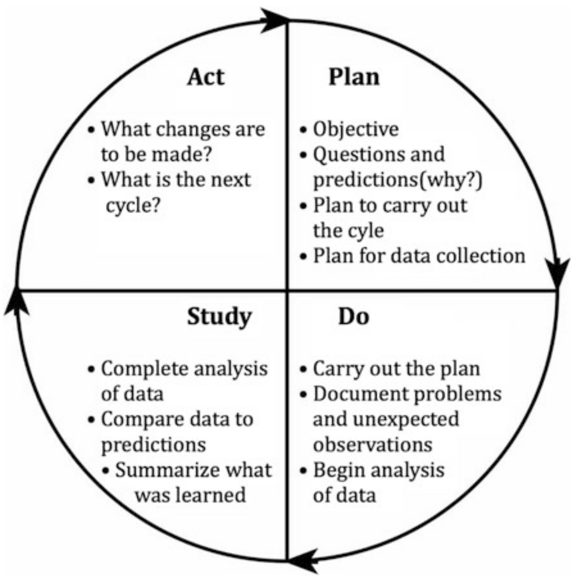

Fig. 3.4 The Plan-Do-Study-Act (PDSA) cycle (adapted from Langley et al., 2009)

educational research, the Partnership's design embraces variation to study how interventions need to be adapted to respond to the differing conditions under which they are used. As they are tested and refined, interventions can gradually spread across the network, supporting scale up (Bryk et al., 2015). Thus, rather than developing a "treatment" that is tested against a control group, the initial development and testing of an intervention begins in a small number of settings. As its efficacy is demonstrated, it is tested in an increasing number of settings, noting adaptations that are needed due to differences in the context. Eventually, the interventions designed should be useful by teams across the Partnership. Further note that the structure of the network allows a "divide and conquer" approach in which subsets of teams can address different problem areas, providing teams access to a wider range of interventions as the work of the RACs progresses.

3.2 Areas of Inquiry

3.2.1 Formation of Research Action Clusters

Working groups, each including teams from across the MTE-Partnership, were formed to further analyze the four primary drivers described in Fig. 3.3. In addition to conducting reviews of existing literature related to the driver diagram, a survey of Partnership teams provided more detail about particular challenges they faced in each area. This analysis resulted in a series of white papers that have guided the continuing work of the Partnership. Each working group proposed potential areas of action or "change ideas" for achieving their respective primary drivers. Across the working groups, an initial set of 13 proposed change ideas were put forward. Based on further analyses of priority and interest by the teams, this set was pared down to five. A "research action cluster" (RAC) was established by the Partnership to begin work on each of these change ideas. Partnership teams were invited to join these RACs in fall 2013; each team generally joined one or two RACs.

Note that one RAC was later disbanded due to its inability to form a clear plan of action, and an additional RAC was formed summer 2015 to address an emergent area of concern, induction of candidates into the profession. An additional working group is currently working to build the foundations for a new RAC that considers how programs can integrate findings from the existing RACs to support overall program transformation, with a focus on institutional change. Thus, the network is evolving based on the needs of its partner institutions. Each RAC incorporates the NIC design, using improvement cycles to develop interventions addressing an identified aim.

Figure 3.3 represents the present structure of the Partnership, including the current set of five RACs, how they are related to primary drivers identified for the Partnership, and the overall aim for the Partnership. Note that none of the change ideas related to Creating a Vision were initially addressed by a RAC; however, most of the other RACs indirectly address this primary driver, and the new RAC addressing program transformation may more directly address it. A brief summary of each of the RACs follows:

- The Marketing to Attract Teacher Hopefuls (MATH) RAC is developing marketing strategies to attract students to consider secondary mathematics teaching as a career.
- The Actively Learning Mathematics (ALM) RAC is focusing on improving the content preparation of candidates in introductory university mathematics classes, precalculus through calculus 2, using "active learning" strategies (Freeman et al., 2014) and incorporating the use of learning assistants (Webb, Stade, & Grover, 2014).
- The Mathematics of Doing, Understanding, Learning and Educating for Secondary Schools [MODULE(S^2)] RAC is producing modules or courses specifically aimed at developing mathematical knowledge for teaching (cf. Ball et al., 2008) in alignment with the recommendations of the *Mathematics Education of Teachers II* report (Conference Board of Mathematical Sciences, 2012). Initial development work has begun in the areas of transformational geometry, modeling, and statistics.
- The Clinical Experiences RAC is focusing on improving clinical experiences, including experimenting with new models for both student teaching (cf. Leatham and Peterson, 2010b) and early field experiences, as well as professional development for mentor teachers.
- The Secondary Teacher Retention and Induction in Diverse Educational Settings (STRIDES) RAC is considering ways to increase the number of years that early career secondary mathematics teachers completing Partnership programs remain in the field.

3.2.2 Collective Impact of the Research Action Clusters

In support of the MTE-Partnership aim and drivers, each RAC has developed its own aim and driver diagram for its area of concern. In essence, each RAC forms a NIC within the broader NIC, and in some cases subgroups within the RACs have further focused in on particular issues, thus creating a nested structure of improvement work. Collectively, these RACs address the downward cycle discussed at the start of this paper; Fig. 3.5 depicts the contribution of each RAC.

While the RACs are progressing at different rates, interventions found effective by the RACs in addressing significant problems in secondary mathematics teacher preparation are beginning to emerge and can be adopted by additional Partnership teams not involved in their development. For example, based on its research, the MATH RAC has produced the *Secondary Mathematics Teacher Recruitment Campaign Implementation Guide* (MTE-Partnership, 2015) which is designed to

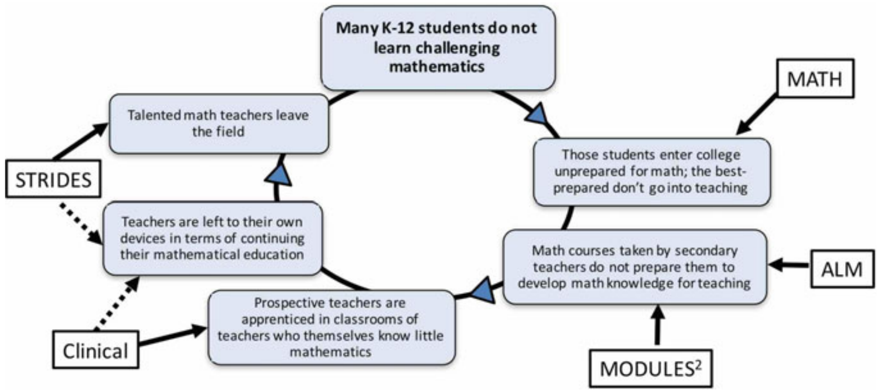

Fig. 3.5 Addressing the downward cycle in mathematics teacher preparation

"help faculty members and others within mathematics or STEM teacher education programs maximize their impact on teacher candidate recruitment" (Overview Module, p. 2). The RAC is also collecting specific examples of how the guide can be adapted in various contexts. The Actively Learning Mathematics RAC has developed professional development materials for instructors, a series of activities, and other supports for promoting active learning in introductory college mathematics. The MODULE(S^2) RAC has produced modules that instructors can use to increase the knowledge of geometry and statistics needed by secondary teachers; these materials are being tested by faculty members across the Partnership. The Clinical Experiences RAC has developed professional development and other materials to support the implementation of innovative approaches to early field experiences and to full-time internship experiences; more detail is provided later in the chapter.

3.2.3 Measures

The activities of the MTE-Partnership are designed to support progress towards meeting the aim established in the NIC design, and a suite of measures has been devised to track progress towards the overall MTE-Partnership aims. A measures working group, which includes members from each of the RACs, was established to guide this effort. To address the first aim of the Partnership to increase the supply of new secondary mathematics candidates, the group collects data on the production of teacher candidates by membership teams on an annual basis. Baseline data suggested that the Partnership produces about 15% of the supply of secondary mathematics teachers in the U.S. Teams also provided targets for increasing their candidate production, which led the MTE-Partnership to establish a target of increasing candidate production by 40% from 2014 to 2020, which would be about 20% of the national supply, assuming steady demand for teachers.

The measures working group is also identifying or developing measures that can be used to track progress towards the second Partnership aim of improving the quality of candidates graduated. Given that programs have existing measures in place, often required by certifying agencies, establishing common measures across the Partnership has been particularly challenging. A common observation protocol, the Mathematics Classroom Observation Protocol for Practices (MCOP2) (Gleason, Livers, & Zelkowski, 2015) was selected for use across Partnership programs. While programs may not be able to replace the protocols they currently use, they are being asked to use the MCOP2 with a sample of teacher candidates at the conclusion of their culminating student teaching experience as a common data point across programs. The MCOP2 is additionally used by several RACs to track their progress towards their specific RAC aims.

The measures working group has also developed a survey for teacher candidates completing Partnership programs to self-assess their preparedness as they begin their careers as secondary mathematics teachers based on the *Guiding Principles* (MTE-Partnership, 2014) and the Mathematics Teaching Practices (NCTM, 2014). In addition, the measures group oversees an annual program survey in which team leaders self-assess the effectiveness of their program in preparing candidates in alignment with the *Guiding Principles* (MTE-Partnership, 2014).

While each measure in isolation provides a limited picture of the quality of the candidates being produced by Partnership programs, triangulating the data across the measures may provide more complete evidence of programs' success in ensuring the quality of the teachers they produce. Additional measures are being considered to garner input about candidate quality from additional sources, such as candidates' eventual employers, and to address additional dimensions of candidate quality, such as mathematical knowledge for teaching. Such measures will add both depth and breadth in understanding the quality of candidates prepared by Partnership programs.

Finally, measures are central to the work of each of the RACs. Each RAC develops, adopts, or adapts measures that can be used to track progress as improvement cycles are implemented and guide decisions about changes that need to be made in the next improvement cycle. Moreover, as testing of the improvements scales up to additional sites, the evidence that is gathered across the range of contexts helps to document specific adaptations that may be needed to address various contextual factors. This ensures that the interventions can be scaled with integrity across the Partnership.

3.3 Research on Clinical Experiences

We now turn our attention to the research action cluster focused on clinical experiences. This is meant to serve as an example of how the MTE-Partnership design has supported the work in one particular research focus, as well as to provide information about the progress made in this research area.

3.3.1 Contextualizing Clinical Experiences

Clinical experiences of secondary teacher candidates, along with content knowledge and the quality of the prospective teachers, have been dubbed as the aspects of teacher preparation that are likely to have the strongest effects on outcomes for students (National Research Council [NRC], 2010). In addition, the *Report of the Blue Ribbon Panel on Clinical Preparation and Partnerships for Improved Student Learning* commissioned by the National Council for Accreditation of Teacher Education [NCATE] (2010) in the U.S. suggests a "*clinically based preparation* for prospective teachers, which fully integrates content, pedagogy, and professional coursework around a core of clinical experiences" (p. 8). Moreover, NCATE (2010) suggests that prospective teachers experience a clinical experience continuum in which a developmental sequence of teaching experiences during the teacher education program is delineated with experiences moving from the simplest, such as learning names, recording grades, and counting the number of students who will eat lunch prepared by the cafeteria or who brought their lunch from home, to the most complex, such as differentiating instruction, developing assessments, and designing and implementing unit plans. These experiences begin in a pre-teaching experience (mainly observational), next a practicum (perhaps teaching a lesson or working with small groups of students) connected to a methods course, and then finally an internship/student teaching experience (gradually taking on teaching responsibilities until the candidate is teaching a full load of classes and then gradually gives the classes back to the cooperating teacher).

In addition, teachers feel that clinical experiences are beneficial to their professional development:

> Study after study shows that experienced and newly certified teachers alike see clinical experiences (including student teaching) as a powerful—sometimes the single most powerful—component of teacher preparation. Whether that power enhances the quality of teacher preparation, however, may depend on the specific characteristics of the field experience. (Wilson, Floden, & Ferrrini-Mundy, 2001, p. 17)

During clinical experiences, prospective secondary mathematics teachers (PSMTs) develop *the craft of teaching*—the ability to design lessons that involve important mathematical ideas, design tasks that will help students to access those ideas, and to successfully carry out the lesson. This may include effectively launching the lesson, facilitating student engagement with the task, orchestrating meaningful mathematical discussions, and helping to make explicit the mathematical understanding students are constructing (Leatham & Peterson, 2010a, p. 115).

Even though it is desirable for prospective teachers to develop the craft of teaching as described, teacher preparation programs in the U.S. and many other countries find it difficult to place PSMTs with cooperating teachers who are prepared to foster their growth due to many cooperating teachers' lack of proficiency with this approach to teaching, which is in alignment with the National Council of Teachers of Mathematics [NCTM] (1989, 1991, 1995, 2000, 2014) standards documents and other calls (Boykin, 2014; Horn & Campbell, 2015) for inquiry-based and problem- and student-centered instruction. The cooperating teachers' lack of proficiency in using an inquiry approach to teaching may be attributed to their beliefs systems or lack of professional development related to the approach, or a combination of these factors and others.

Furthermore, a bidirectional relationship needs to exist between teacher preparation programs and school partners in which clinical experiences take place. This relationship should reflect a common vision and shared commitment to inquiry-based practices and other issues related to mathematics teaching and learning. Borko, Peressini, Romagnano, Knuth, and Willis (2004) asserted that compatibility of methods courses and student teaching experiences in which PSMTs participate on several key dimensions is essential for the settings to reinforce each other's messages, and thus work in conjunction, rather than in opposition, to prepare reform-minded teachers.

The Clinical Experiences RAC (CERAC) consists of 24 university led teams, each consisting of at least one mathematics teacher educator, a mathematician, and a school partner. Within the different partner-teams the relationship among the team members may vary. For example, for one team the mathematician is able to observe teacher candidates and participate in debriefings; the mentor teacher works well with the interns and the university supervisor, both in mentoring the teacher candidates and in providing information about the implementation of the paired-placement student teaching model in her classroom; and the university supervisor is a program faculty member who is heavily involved in the MTE-Partnership. In this case, the cooperating/mentor teacher does not receive a stipend for her role. The RAC is currently developing and testing models for clinical experiences following the NIC model in alignment with the MTE-Partnership's guiding principles (2014). This work includes fostering partnerships between institutions of higher education, schools and districts, and other stakeholders, in order to prepare teacher candidates who promote student success in mathematics, as described in the CCSSM and other college- and career-ready standards. Higher education faculty and partnering school districts and schools work together to actively recruit, develop, and support inservice master secondary mathematics teachers who can serve as mentors across the teacher development continuum from preservice to beginning teachers. Moreover, this RAC helps to ensure that teacher candidates have the knowledge, skills, and dispositions needed to implement educational practices (NCTM, 2014) found to be effective in supporting all secondary students' success in mathematics.

We are addressing a two-fold problem: (1) There is an inadequate supply of quality mentor teachers to oversee field experiences, particularly those who are well versed in implementing the CCSSM, including embedding the standards for mathematical practice into their teaching. (2) For most universities and their school partners a bidirectional relationship does not exist between the teacher preparation programs and school partners in which clinical experiences take place. Bidirectional relationships between universities and their school partners need to be built and should reflect a common vision and shared commitment to the vision of CCSSM and other issues in mathematics teaching and learning.

3.3.2 Structure of the Clinical Experiences RAC

CERAC is divided into three Sub-RACs, each focused on a particular model for clinical experiences: Methods, Paired Placement, and Co-planning and Co-teaching (CPCT). The Methods Sub-RAC focuses on aligning what is taught to teacher candidates during the coursework and the practicum work in K–12 schools with mentor teachers. Mentor teachers provide teacher candidates with opportunities to experience the authentic work of expert teachers. Furthermore, supervising teacher candidates can encourage the professional growth of mentor teachers (Feiman-Nemser, 1998; Rhodes & Wilson, 2009). Helping to name mentor teacher actions and talk with language used in the theoretical underpinnings more familiar to teacher educators and teacher candidates can better leverage the expertise of the mentor teachers as well as further develop their understanding of the theoretical and mathematical support behind their work. The paired placement model is a student teaching approach in which two prospective teachers are paired with a single cooperating teacher. The cooperating teacher provides purposeful coaching and mentoring, and the two pre-service teachers offer each other feedback, mentoring, and support (Mau, 2013). CPCT is a pedagogical approach that promotes the collaboration and communication between teacher candidates and mentor teachers who share a common space in the planning, implementation, and assessment of instruction (Bacharch, Heck, & Dahlberg, 2010).

In addition to the partnership's aim and driver diagram, each RAC has its own aim and driver diagram. The aim of the Clinical Experience RAC is as follows:

During student teaching, teacher candidates (TCs) will use each of the eight mathematics teaching practices (NCTM, 2014) at least once a week during full time teaching. Below is a list of the mathematics teaching practices (NCTM, 2014, p. 10):

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

The primary drivers for the Clinical experience RAC are:

(1) **Transparent and coherent system of mentor selection and support** (**cooperating teachers and university supervisors**), which is done within partnerships between school districts and universities focusing on professional development and program specific guidelines;
(2) **Interdependency of methods course and early field experiences**, which focus on embedding the standards for mathematical practice in instruction that utilizes the eight mathematics teaching practices to ensure that each and every student has access to meaningful mathematics learning;
(3) **Student teaching as clinical training**, which ensures that requirements for student teaching and feedback during student teaching emphasize the responsibility of teacher candidates to advance mathematics learning among secondary students through collaboration with more expert mentors in use of mathematics teaching practices;
(4) **Shared vision about teacher development**, which is designed to ensure that there is mutual agreement between district(s) and universities about what quality teaching of secondary mathematics looks like and how to further skills of all teachers (including teacher candidates) and see mentor teaching as part of career ladder;
(5) **Focus on access and equity**, which includes both quality of experiences and opportunities to learn for the students and the teacher candidates. The preparation of each new teacher of secondary mathematics represents an opportunity to disrupt long-standing teaching practices that contribute to inequities in learning outcomes.

Each Sub-RAC is implementing PDSA cycles based on their goals and objectives. There are overlapping areas that focus the RAC as a whole, such as NCTM's mathematics teaching practices, professional development for mentors around the CCSSM, mentoring mathematics teacher candidates, and outcome measures. There are also specific goals to be attained within each of the Sub-RACs, and each Sub-RAC is addressing specific research questions. The three Sub-RACs are using a set of common measures, including:

(1) the MCOP2 (Gleason, Livers, & Zelkowski, 2015), also used as a core measure by the Partnership;
(2) a survey of program completers designed by the MTE-Partnership to show how well prepared the teacher candidates feel based on the experiences that they had in their programs; and
(3) the Mathematics Teaching Practices Survey designed by the RAC to determine the level at which prospective secondary teachers are engaged with NCTM's (2014) Mathematics Teaching Practices.

Each sub-RAC is developing modules and tools that will enable other programs to implement the different approaches to field experiences that they are designing, including: Syllabus and Orientation Session for the Paired Placement Model, Mathematics Teaching Practice Survey, CPCT Workshops, CPCT Survey, and Standards for Mathematical Practice Module for Methods Courses with Pre- and Post-course Survey.

3.3.3 Paired Placement Sub-RAC

We now take a closer look at one of the Sub-RACs in order to better understand the work of the MTE-Partnership. The Paired Placement Sub-RAC is comprised of members representing five institutions and their school partners. The Sub-RAC focuses on the paired placement model for student teaching in which two prospective teachers are paired with a single cooperating teacher. The cooperating teacher provides purposeful coaching and mentoring, and the two pre-service teachers offer each other feedback, mentoring, and support (Leatham & Peterson, 2010b; Mau, 2013). As a Sub-RAC, we read articles (Goodnough, Osmond, Dibbon, Glassman, & Stevens, 2009; Leatham & Peterson, 2010a, b; Mau, 2013) to learn about the model. The research questions that guided the study are:

(1) What are the successes and challenges of implementation of the paired-placement model for clinical experiences at each different university?
(2) How do the successes and challenges of the paired-placement model compare across the various institutions involved in the study?
(3) What are attributes across the institutions that contributed to the successes of the paired-placement model?
(4) What are attributes across the institutions that contributed to the challenges of the paired-placement model?

One team implemented the model fall 2013 and reported to the other teams about its findings. Two additional teams used this information along with information from the literature to prepare mentor teachers and candidates for the experience Spring 2014. Teams also worked with their participants to adjust the model within their context utilizing PDSA cycles and monitored the process throughout the semester. Teams met via a conference call to discuss the results of the implementations and what they would do differently. During Fall 2014, teams built on these experiences to create professional development modules, syllabi, and measures. These materials were implemented during Spring 2015, utilizing suggested improvements from previous iterations. Teams implemented additional paired placements the following year: one during fall 2015, and six during spring semester 2016.

Through PDSA cycles and data collected from participants, we are learning much about the model. We have found that it allows teacher candidates to really focus on student learning and the craft of teaching. Teacher candidates and mentor teachers who have experienced this model believe that it benefits all of their growth in teaching as well as the students' growth in learning mathematics. They also stated that the model has helped them to become more collaborative. Our goal is to continue to refine the workshops and syllabi so that they can be adapted to different contexts.

3.4 Conclusions and Next Steps

3.4.1 Progress

The MTE-Partnership has made significant strides in defining a common vision for secondary mathematics teacher preparation, identifying major problems impeding progress towards the vision, developing interventions to address those problems, and identifying measures to track progress. The Partnership's design has undergirded this process. NICs combine the disciplined inquiry of improvement science with the power of networking to accelerate improvement (Bryk et al., 2015). Use of improvement cycles by the RACs has helped to ensure interventions are not just changes, but improvements, and the network provides opportunities to test them across multiple contexts to see how they may need to be adapted to be most effective.

In the case of the Clinical Experiences RAC, all of the members have found the NIC to be helpful in improving their field experiences for secondary mathematics teachers and have seen growth in the secondary mathematics teacher candidates based on the changes that have been implemented. The challenge will be to see how well the tools work in other settings with people who were not engaged in the development process.

The full power of the MTE-Partnership NIC, however, can be seen in the breadth of the network it has established. First, given the number of institutions involved, the network provides the capacity to simultaneously address multiple problems of practice through its set of five RACs. Second, while each team generally only has the capacity to directly participate in the research of one or two RACs, the network provides the opportunity for teams to learn from the efforts of the other RACs in which they are not participating. Thus, the network provides a rich collection of resources to which Partnership teams can contribute and from which they can draw in improving their programs. No single institution could hope to address such a broad scope of improvement efforts.

3.4.2 Challenges and Next Steps

There are, however, significant challenges in harnessing the network to achieve the MTE-Partnership's goal of transforming secondary mathematics. First, there have been continuing challenges in maintaining the Partnership. As Martin and Gobstein

(2015) note, “There has sometimes been competition between building participant identification with the overall MTE-Partnership network and the individual RACs in which they participate” (p. 488). Maintaining effective leadership structures within and across the RACs require continuing attention, along with ensuring effective communications strategies.

Second, a growing concern for the MTE-Partnership is how to effectively manage the knowledge that is being generated by the RACs so that it is accessible by non-RAC teams. This requires maintaining an accessible repository of current materials as well as access to relevant training and support. In addition, teams using the interventions collect relevant data so that their experiences with the interventions can be incorporated into the knowledge that is being generated. Some RACs are beginning to experiment with how to best manage that process, but a more general approach across the Partnership is needed.

Third, teams may not have the needed resources and supports to simultaneously implement the findings across the multiple dimensions of improvement. Their initial focus was likely on one or two RACs in whose development they participated, and they may not have the personnel, time, or resources needed to incorporate findings that are emerging from the other RACs. This has led to the establishment of a new Partnership focus on developing approaches to support teams in establishing “strategic pathways for improvement” to manage the overall process of improvement. Teams will need to prioritize the improvements they can address based on their needs and available resources. This will also involve increasing awareness of and support for secondary mathematics teacher preparation, such as building “buy in” of institutional leaders, recruiting additional faculty members to participate in the effort, and shoring up relationship with school districts to better collaborate with field experiences. We are working to launch a new RAC to build approaches for addressing this challenge.

Finally, equity and social justice are highlighted within the *Guiding Principles* (MTE-Partnership, 2014) as well within the aims of each of the RACs. However, a survey of participants in the Partnership revealed that there is some concern about whether these issues are receiving consistent focus and attention. Thus, the planning team has formed a working group to explore how we can better ensure that equity and social justice issues are effectively interwoven into the fabric of the MTE-Partnership research efforts.

These challenges point to the need for the Partnership to continually change and evolve to meet changing circumstances and needs. Even the foundational documents need to be revisited. For example, the release of AMTE’s (2017) *Standards for the Preparation of Teachers of Mathematics* raises the question of whether the *Guiding Principles* should be revisited to ensure that they adequately capture the best wisdom of the field. New priorities, such as the focus on program transformation and issues of equity and social justice, suggest that the Partnership’s aim and driver diagram may need to be revisited to ensure they effectively capture the Partnership’s most current thinking. The set of RACs has evolved over the past years, and it is likely that additional changes will occur as some RACs conclude their development of particular interventions, and as new needs are identified.

Moreover, new ways of interacting may be needed as the focus of teams moves beyond working with a RAC to improve some aspect of the program to overall program transformation.

3.4.3 Concluding Remarks

In conclusion, the NIC design has been very useful in framing the efforts of the MTE-Partnership to address significant problems related to the inadequate number of secondary mathematics teacher candidates who are prepared to support their students' success in mathematics. The Clinical Experiences RAC members have found working in a NIC to be beneficial in many ways, including identifying and solving problems of practice, collaborating on research projects and publications, and improving the relationships between school and district partners. We realized that even though our contexts may differ in subtle ways, we have enough issues and challenges in common to utilize PDSA cycles and common measures that could lead to transforming our programs. Indeed, we feel that the NIC model offers great potential in mobilizing networks of different types to address common problems in mathematics education and beyond.

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Chapter 4
Pre-service Teachers' Reflections on Their Teaching

Hulya Kilic

Abstract The aim of this study was to afford pre-service mathematics teachers with the opportunity to work with a pair of students for a semester and reflect on their own practices and students' performances. Seven pre-service teachers were matched with seven pairs of sixth graders, and they worked with them for 12 weeks. The data was collected through pre and post interviews with pre-service teachers, videos of their interactions with students, and their written reflections on these interactions. The analysis of videos and written reflections revealed that pre-service teachers benefitted from this setting such that they got better in communicating with students, estimating students' performances and providing appropriate scaffolding for students' understanding. During the post interviews pre-service teachers also noted that working closely with students helped them to learn about students' mathematical thinking and understanding as well as improve their teaching skills.

Keywords Pre-service · Reflection · Pedagogical content knowledge
Scaffolding · Middle school

4.1 Introduction

Teachers' professional knowledge and skills play an important role in students' learning (Darling-Hammond, 2010). For effective instruction, teachers need to facilitate and encourage students' understanding by paying attention to their cognitive needs and using appropriate teaching strategies (Sowder, 2007). The studies revealed that pre-service and novice teachers' knowledge and skills, specifically their pedagogical content knowledge (PCK) is immature (Morris, Hiebert, & Spitzer, 2009) however, having them to work with students, examine students' work or analyze lesson videos and reflect on them contributes to the development of

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_4

their PCK (Llinares & Valls, 2010; Stockero, 2008). Therefore, teacher training programs should enable pre-service teachers (PSTs) to have more teaching experiences and reflect on their practices to support their professional growth (Chamoso, Caceres, & Azcarate, 2012; Weiland, Hudson, & Amador, 2014).

Many scholars agreed that reflecting on one's teaching is important for effective teaching such that teachers have opportunity to observe how their teaching practices influence students' understanding and reshape their instruction accordingly (Goldman & Grimbeek, 2015; Lee, 2005; Wagner, 2006). Furthermore, the depth of such reflection is related to teachers' knowledge and experiences (Lee, 2005) such that PSTs' reflections on their own or others' practices are mostly the description of what happened during the lesson without making comments about or drawing conclusions from those instances (Chamoso et al., 2012; Goldman & Grimbeek, 2015). However, PSTs could be trained to become reflective through special methods courses or field experiences. The studies showed that such interventions contributed to PSTs' reflection as well as their PCK, thus, they began to pay more attention to students' thinking and consider how to facilitate students' learning and understanding (Santagata, Zannoni, & Stigler, 2007; Weiland et al., 2014).

In this study, we analyzed PSTs' interactions with a pair of sixth graders in terms of their noticing of students' mathematical thinking, scaffolding students' understanding and reflecting on their own practices and students' performances. We aimed to investigate how working closely with students would contribute to the development of PSTs' PCK, specifically their noticing skills, scaffolding, and reflective practices as well as how PSTs would evaluate the effects of this experience on their professional development. In this paper, I discussed the nature of PSTs' oral and written reflections on their interactions with students to exemplify possible contributions of this study to their PCK and also presented PSTs' views about such an experience to discuss perceived effects of this study to their PCK.

4.2 Theoretical Framework

Reflection and reflective practices of teachers and their roles in teachers' professional development have been discussed and studied by many scholars for decades after Dewey (1933) and Schön (1983) first mentioned about these terms (Sowder, 2007). Although scholars might have different views about the definition of reflection (Mewborn, 1999), it can be identified as a cyclic process of *thinking about* and *thinking of* own practices based on multiple perspectives (Ward & McCotter, 2004). In other words, reflection not only entails thinking about what happened but also planning and acting for the next step, so called reflective practices (Bergman, 2015; Mewborn, 1999; Zimmerman, 2002).

Rodgers (2002) proposed four phases of reflection as experience, description, analysis and experiment. She explained the first phase as *learning to see* where teachers are expected to be aware of significant instances in the learning environment and respond to them appropriately. She identified the second phase as *learning to describe and differentiate* where teachers are on the action just after recognition of that significant instance. She described the third phase as thinking about the instance critically from multiple perspectives and creating a theory about that instance and the last phase as taking an intelligent action based on previous phases (see for further details, Rodgers, 2002). That is, reflective teachers are expected to both think about their instructional actions, such as planning the lesson, developing materials and tasks, interacting with students and also how those actions influence their students' learning (Bergman, 2015). For instance, students' failure in understanding a particular concept might be an impetus (experience) for a teacher to think about (description) his teaching strategies. Then, he may think of and search for effective ways of teaching that concept (analysis) and then try out a new strategy in his class (experiment). After implementation, he again evaluates the effectiveness of that new strategy on students' understanding (cyclic process).

Although reflection seems to be a natural practice for teachers, developing a habit of reflection is not an easy task for many teachers (Mewborn, 1999). It entails cautious attention to what happens in classrooms and sustained efforts to promote students' learning and understanding and improve professional skills as teachers (Chamoso et al., 2012; Wagner, 2006). In order for effective teaching to occur, teachers need to be aware of how their instruction supports students' understanding and to take necessary precautions to improve students' learning (Darling-Hammond, 2010; Jansen & Spitzer, 2009), both in-service teachers and pre-service teachers should be given support to be reflective.

Indeed, teachers' awareness of students' understanding and reflective practices inform teachers' professional knowledge and skills, specifically their PCK because PCK involves knowing what teaching strategies and examples are more appropriate for students, which strategies or tools are more effective to teach a particular subject matter and how to address students' difficulties and misconceptions (Hill, Ball, & Schilling, 2008). When a teacher recognizes a significant issue about students' understanding, his reflective practices in terms of identifying the gap in students' thinking and using alternative ways to scaffold students' understanding mostly emerge from his *knowledge of content and students* and *knowledge of content and teaching* (Hill et al., 2008). Moreover, teachers' reflective practices and their noticing skills are interrelated because noticing entails paying attention to students' thinking, interpreting it, and making decision about how to respond to it (van Es & Sherin, 2008) where whole reflection process is about "seeing learning, differentiating its parts, giving it meaning, and responding intelligently" (Rodgers, 2002, p. 235). That is, once teachers begin to notice students' understanding, they begin to reflect on their earlier practices as well as think of their next actions (van Es & Sherin, 2008). Therefore, teachers' reflections on their teaching both help to evaluate teachers' PCK, including their noticing skills and also to improve their PCK (Chamoso et al., 2012; Goldman & Grimbeek, 2015). In this study, we accepted that PSTs' practices and reflections not only informed their reflective skills but also their noticing skills and PCK. However, in this paper, I will mainly focus on PSTs' reflections on students' understanding and their own scaffolding practices; and I also discuss how their reflections informed their PCK (Hill et al., 2008).

Because development of a habit requires more time (Sowder, 2007), PSTs could be given opportunities to be reflective during teacher education programs through several practices. The field experiences, methods courses or some other specialized courses have some potential to provide opportunities for PSTs to be reflective. In those courses, PSTs might be asked to take comprehensive field notes or watch their own or others' videos of teaching and then make comments on the teacher's and students' practices as well as students' understanding by providing justifications for their comments. The studies revealed that PSTs' practices contributed to their PCK to some extent in terms of their reflective thinking and noticing skills (Mewborn, 1999; Jansen & Spitzer, 2009; van Es & Sherin, 2008). Therefore, we offered an elective course for PSTs where they would work with a pair of students for a semester, videotape their interactions with students, analyze their own videos and write reflection paper for each interaction. For the reflection paper, we gave them a list of items to address, as done in other studies (e.g., Bergman, 2015; Chamoso et al., 2012), therefore, we did not use a holistic approach to analyze reflection papers but analyzed them item by item. However, in this paper, I will only present the findings of the items which specifically asked for PSTs' reflections on their own practices and students' understanding.

4.3 Methodology

4.3.1 Research Setting and Participants

This study was designed as a 14-week course in Spring 2014, aiming to provide opportunities for PSTs to improve their PCK as a part of a university-school collaboration program in a large university in Istanbul, Turkey. Because teaching experiences contribute more to development of PSTs' PCK (e.g., Hiebert, Morris, Berk, & Jansen, 2007) we set up an environment for PSTs where they could work with students. Seven female PSTs took the course, and each PST was matched with a pair of sixth graders from the collaboration school. They worked with the students once a week for 1-h period on the given tasks for 12 weeks. Four of the PSTs (PST A, PST B, PST C and PST G) were seniors and the others (PST D, PST E and PST F) were junior undergraduate students. They all took general pedagogy and mathematics courses prior to this study.

In the first week, we informed the PSTs about the study, their duties and responsibilities. We also interviewed them about their formal/informal teaching experiences, their expectations from the study, etc. For the rest of the study, the group (the research team and the PSTs) met before the implementation of the tasks and discussed how students might perform on the tasks and how to scaffold their understanding and eliminate their misconceptions. After the implementations, the group met again for oral reflection such that we asked PSTs to share whether they experienced any significant instances about students' performances, what inferences they made about students' understanding and how they addressed those significant instances. For 10-week period, PSTs worked on the tasks prepared by the research team, and they videotaped their interactions with students. They let students work individually first, then as a pair, and then they asked them what they did in order to scaffold their understanding and learning. After each implementation, PSTs wrote a reflection paper after watching their own videos and looking at students' work. The purpose of oral reflection sessions was to gather data about PSTs' noticing skills and reflective practices such that we wanted to compare whether the depth of oral reflections and the written reflections differed after watching their own videos or not. In their written reflections, we asked them to evaluate their own scaffolding practices as well as to write about the process of task implementation, students' performances on tasks, and their overall thoughts about the implementation by providing justifications for their comments. Briefly, in this research setting, we attempted to provide an opportunity for PSTs to experience all phases of reflection (Rodgers, 2002) and gathered data about their reflective skills through oral and written reflections.

As different from field experiences or methods courses, in this research setting, the PSTs had more opportunity to reflect on their own teaching because they just worked with a pair of students for 12 weeks. Thus, they were able to learn about their students' characteristics and capabilities after a couple of weeks, and also they had enough time to improve their reflection skills (Sowder, 2007). Moreover, PSTs videotaped all implementations and wrote their reflection reports after watching their videos rather than just reflecting based on their memories. Thus, they had opportunity to analyze themselves and students as an outsider, re-watch videos when necessary, and realize some instances that they did not pay attention to during implementations (Stockero, 2008).

We set up this study as an after-school program for the 6th grade students of the partner school. Therefore, we followed the same order of the topics in the mathematics curriculum such that the content of the tasks for the weeks 1–4 was numbers and geometry, for weeks 5–7 it was algebra and for weeks 8–10 it was fractions. Ten weeks later, we asked PSTs to prepare tasks for their own groups and implement them. They also videotaped the implementations and wrote reflections. In the last week, we asked them to make a presentation to summarize their experiences throughout the semester. Finally, we interviewed them regarding their thoughts about the study.

4.3.2 Data Analysis

We transcribed the videos of task implementations and pre and post reflection discussions. The PSTs' videos were transcribed and coded in terms of their ability to catch significant instances related to students' thinking and their scaffolding practices. Although some scholars developed coding schemes for the depth of the reflection (e.g., Chamoso et al., 2012; Lee, 2005; Ward & McCotter, 2004), they were not directly applicable to our context. Therefore, we developed our own coding scheme for PSTs' oral and written reflections about the implementations.

We focused on the depth of their reflection on the students' performance and their practices. The coding scheme for the reflections was as follows: (0) Do not make clear comments about students' understanding and her scaffolding practices (1) Makes valid comments students' understanding and her scaffolding but do not justify her reasoning (descriptive) (2) Makes valid comments about students' understanding and her scaffolding and justifies her reasoning (exploratory). For instance, we coded a statement like "One of my students did well but the other one did not" as Level 0 because the PST did not tell what the student did or did not do well. We coded a comment like "They had difficulty in dividing a whole number by a fraction … I told them to use fraction tiles …" as Level 1 because the PST did not tell more about the reasoning behind students' difficulty or her scaffolding. And finally, we coded the following comment "I realized that one of my students wrote that 2/3 = 9/10. I thought that she looked at the difference between numerator and denominator of given fraction and made her decision accordingly … Indeed, she explained her reasoning in a way that I already guessed so. Then, I …" as Level 2 because PST provide justification for her comments about students' understanding and her scaffolding. We came up with 90% or more agreement for each PST's reflection. We mostly had difficulty to differentiate Level 1 and Level 2 such that whether PSTs provided enough justifications for their comments or not. Later, we discussed the discrepancies and achieved a consensus in coding.

We also transcribed pre and post interviews to compare whether their thoughts about teaching and learning mathematics as well as their own knowledge and capabilities about teaching had changed over time or not. We used interview data to discuss PSTs' perceived gains from this study.

4.4 Findings

4.4.1 The Nature of Oral and Written Reflections

The results of the coding for oral and written reflections for 10 weeks are given in Table 4.1. The PSTs' written reflections on students' understanding were explanatory such that they mostly provided justifications for their comments by giving samples from students' work or short vignette between the students and themselves (L(2); 61%). However, their comments about their own scaffolding was mostly descriptive such that they did not write about the reasoning behind their scaffolding (L(1); 61%).

For instance, in one of the algebra tasks during the 7th week PST A's male student had difficulty in writing an equation for a given problem statement. The task was as follows: *Ali began a new diet program accompanied with an exercise program. Therefore, he is jogging in the park for three or four days in a week. This*

Table 4.1 Levels of pre-service teachers' oral and written reflections in weekly basis

	Oral reflection						Written reflection			
	Assumptions about students' performance (pre-discussion)			Reflections on implementations (post-discussion)			Students' performance		Own practices	
	L(0)	L(1)	L(2)	L(0)	L(1)	L(2)	L(1)	L(2)	L(1)	L(2)
Weeks 1–4	18	9		3	21	3	10	16	11	15
Weeks 5–7	5	7	9	7	5	9	8	12	14	6
Weeks 8–10	5	8	8	1	11	9	8	12	15	5
Total	28 (40%)	24 (35%)	17 (25%)	11 (16%)	37 (54%)	21 (30%)	26 (39%)	40 (61%)	40 (61%)	26 (39%)

Note PST G was absent during one of the oral reflections. She also submitted only six written reflections

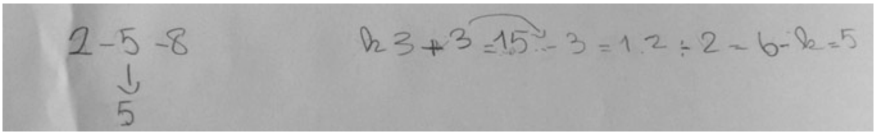

Fig. 4.1 PST A's student's answer for algebra task

week he went jogging for three days. Each day he ran 3 km more than the previous day. If he ran 15 km totally in this week, how many kilometers did he run in the second day? The student estimated how much he ran each day as 2, 5, and 8 and then wrote the equation as $k \cdot 3 + 3 = 15$ and solved it as $k \cdot 3 = 12 \;\; 12 : 2 = 6 \;\; 6 - k = 5$ as shown in Fig. 4.1.

In her written reflection, PST A provided the following short vignette about the discussion of this solution.

PST A: What did you think about this problem?
Student: I guessed the answer.
PST A: OK. But if you are given larger numbers [in the problem], it might be hard to estimate the answer. So, tell me what did you do here? [pointed to his equation]
Student: Because each day he ran 3 km more, I wrote +3.
PST A: Why did you write 3 here? [pointed to 3 in $k \cdot 3$]
Student: Because he ran 3 km more; like 3, 6, 9.
PST A: Do you need to write 3k for that?
Student: 3 km *plus* 3 days, *times*?? [He puzzled] I guess, it is wrong.
PST A: OK, let's read the problem and think about it again

Because the student estimated the answer immediately it was evident that he understood the problem statement. However, she asked about the equation he wrote and the solution of the equation. During the discussion, he realized his mistakes and set up the correct equation through PST A's prompting questions. PST A wrote the followings about the student's performance in her report:

> As inferred from the dialogue between us, he realized his mistakes when I asked about what his equation represented for. Later on, although he set up x for the first day, x + 3 for the second day and x + 3 + 3 for the third day, he initially could not figure out what to do with these expressions. When I read the problem statement again, he recognized that the sum of these expressions will be 15. However, he was not sure about how to add these terms. Then, he experienced difficulties in solving the equation. Although he was able to re-explain the solution to me at the end, I am not sure that he will not repeat the similar mistakes again… His failure in this task was not mainly because of his lack of knowledge but his misunderstandings of algebraic expressions and equations. He did not pay attention for what unknown value he is assigning a letter for and then set up an equation accordingly. He felt that he needs to set up an equation somehow by using the given information in the problem. Hence, he wrote k · 3 for "3 days", +3 for "3 km more" and 15 for "to put something the other side of the equation." He did the similar things in the previous problems. When we finished the discussion of the tasks I made him work on simple examples but I think he needs to be re-taught about variables, algebraic expressions, setting up and solving equations from the beginning.

The analysis of student's previous performances on algebra tasks supported PST A's comments about her students' mathematical understanding. Furthermore, during the pre-discussion, she said that the student had difficulty in writing and evaluating algebraic expressions and might fail to solve the given tasks. In terms of evaluation of her own scaffolding practices, she wrote the followings:

> I knew that he had some problems in algebra because he had some misunderstandings about algebraic expressions and equations. I suppressed my feelings about re-teaching of algebra, and I tried to guide him through questioning. I tried to help him to figure out what was unknown and how to set up an equation by reading the problem statement again. He answered all my questions and solved the tasks eventually, but I felt that he needs to do some exercises about rewriting verbal expressions as algebraic expressions and vice versa to understand it better.

PST A noted the student did not know much about writing and solving equations, and she tried to help him through questioning but she did not write much about the questions she asked him and why she asked such questions. During the post-discussion, PST A did not tell much about the student's performance but noted that he performed poorly as she expected. Similarly, most of the time, PSTs described what happened during the implementations in the post-discussions (54%) without giving the details about the students' performances or their scaffolding. Moreover, in some cases they did not tell much how their students performed on the tasks or what they did during the interactions but confirmed their peers' comments about the implementations such as "My students also solved the problem in that way" or "I agree with her", etc.

As shown in Table 4.1, as PSTs got to know the students they got better in making assumptions about their performances. In the first four weeks, they were not able to make clear comments about students' possible performances on the tasks (L(0) = 18) but in later weeks, they began to provide justifications for their assumptions (L(2) = 9 and L(2) = 8, respectively). Because in this setting we arranged three sets of tasks about the same content for weeks 5–7 and weeks 8–10, after the initial tasks of that particular content they were able to estimate students' performance better. As described above, PST A was able to estimate her student's performance on the problem solving tasks based on her earlier observations of that student on previous algebra tasks. That is, those sequential tasks contributed to development of PSTs' *knowledge of content and students* because they began to recognize what is difficult or confusing for students to understand in algebra or in fractions (Hill et al., 2008).

Furthermore, based on students' earlier performances, the PSTs tried to plan how to act during next implementations. For instance, in one of the fraction tasks of the 10th week, students were asked to fairly share 3 sandwiches between 6 children and 2 sandwiches between 3 children, etc. PST C said that her students could divide 3 sandwiches into halves for 6 children but they would write that each child would eat 1/6 of a sandwich rather than 1/2 of it because there would be a total of 6 pieces of sandwiches. She said that when it would be the case she would ask them what 1/6 represented as a fraction to make them realize their mistakes. Indeed, her students answered that problem in the way that PST C estimated, and she addressed that mistake in a way that she planned for as follows:

PST C: What does 1/6 represent for?
Student: One out of six.
PST C: How many sandwiches do you have?
Student: 3.
PST C: Is this [pointed out one of the rectangular shapes] a whole sandwich or is it a piece of it?
Student: Whole.
PST C: Then, you have 3 whole sandwiches. Do you get 3 [whole], if each child takes 1/6 [of whole]?
Student: No. [silence] This piece [pointed out one of the pieces in the rectangular shape] is 1/2 of the whole sandwich.
PST C: Right!

However, in some cases the PSTs had to handle unexpected situations and make new decisions immediately. For instance, one of PST G's student used "x" randomly while solving algebra tasks during the 7th week. For the task that was given as an example above, he wrote the followings: 1st day: 6x + 3x = 9 km = 9x, 2nd day: 9x + 3x = 12 km = 12x, 3rd day: 12x + 3x = 15 km = 15x. The student attempted to solve the other tasks in this fashion, too. During the oral reflection session, PST G said that she did not expect such solution from the student however she asked him what "x" and "3x" stood for in those equations. The student realized his mistake but he could not correct it. PST G noted that she talked about algebraic expressions and solving equations by giving simple examples. Then she asked him to solve the given tasks by setting up appropriate equations. She tried to guide him by reading the problem statement in a step by step manner and asking him what to do in each step. Although PSTs mostly preferred to ask students to tell what they did and then scaffold their understanding through prompting questions, as in PST

G's case, when they realized that there were major gaps in students' understanding of the given subject matter they preferred telling and showing type of scaffolding. That is, students' mathematical capabilities and earlier performances encouraged PSTs to think about and try out appropriate and effective ways of addressing students' mathematical understanding which is an indicator for PSTs' *knowledge of content and teaching* (Hill et al., 2008) as well as their reflective skills (Rodgers, 2002; Ward & McCotter, 2004). However, because of some restrictions of the research setting such that PSTs had to implement the tasks prepared by the research team and they were asked to avoid teaching the content but attempt to support students' understanding through questioning and manipulatives, they did not have much flexibility in deciding how to act in terms of what examples, representations or tools to use to teach particular content.

4.4.2 Pre-service Teachers' Perceived Gains

Because reflection is a cyclic process involving thinking about one's own actions and planning for next steps (Rodgers, 2002; Ward & McCotter, 2004), we recognized that not only PSTs' views about teaching and learning mathematics and their reflections mutually influence each other (Cavanagh & Garvey, 2012; Mewborn, 1999) but also their inferences about their own teaching shape their reflective practices (Bergman, 2015). Therefore, we wanted to analyze PSTs' perceived gains from this intervention study to validate contributions of this study to their PCK, specifically to changes in their actions during each phases (experience, description, analysis and experiment) of reflection (Rodgers, 2002).

During the pre-interview, I asked for PSTs' previous tutoring and practicum experiences, their views about the reasoning behind students' difficulties in mathematics, their knowledge about use of manipulatives in teaching mathematics and their expectations from this study. All of them had some tutoring experiences with students from different grade levels. Except PST D and PST E, they were going to schools for the field experiences in the semester that this study took place. They all agreed that students had some prejudices about mathematics such that it was hard and abstract. PST A and PST E also noted that the students gave up working on mathematics when they struggled with understanding the concept or the procedures. Students neither put much effort to understand mathematical concepts nor made practice of mathematical procedures. All PSTs emphasized that concrete materials should be used to teach some abstract mathematical concepts in middle schools. PST C said that she frequently used concrete materials while tutoring and her students understood the subject matter or problem better and did not forget it. That is, prior to study, all PSTs agreed that students struggled with mathematics but using concrete materials might help them to understand it.

The PSTs' major expectation from this intervention was to learn how to communicate with students. They wanted to learn about effective questioning and scaffolding. PST A and PST G also noted that they hoped to learn about students' difficulties and misconceptions and how to address them effectively. PST C, PST E and PST F told that they were not patient persons in general, therefore they wanted to learn to be more patient to the students.

During the post-interview, I asked what they learned from this intervention, how it was similar or different from their earlier teaching experiences, and whether their expectations from this study was satisfied or not. The PSTs noted that one-to-one interactions with students contributed to their professional knowledge more than field experiences. During the field experiences they mostly observe teachers such that they teach once or twice in a semester. Furthermore, PSTs compared their gains from this intervention with their tutoring experiences. They said that watching the videos of their interactions with students and writing a reflection on these forced them to think about and reshape their teaching practices as well as the way of their communicating with students. For instance, PST C said the followings:

> Initially, I was thinking that they could finish the given tasks immediately and I would just ask them to explain what they did and then rephrase their explanations. However, it was not the case. When they could not do the tasks, I began to be concerned about how to help them. When I watched the videos, I realized that I could not give appropriate examples and I mostly attempted to tell how to solve the task. As I got to know my students, I began to make better estimations about what might be difficult or confusing for them and I got ready for what to do when it happened…I also learned how to ask questions to students. They did not want to talk when I ask "why" and "how" types of questions directly. Therefore, I learned how to rephrase my questions without using "why" or "how" but still get the answers that I looked for.

As similar to PST C's comments, other PSTs also noted that their expectations from this intervention were mostly satisfied such that they began to feel more confident about how to communicate with students, their repertoire of students' difficulties or misconceptions in mathematics improved, and they learned how to scaffold students' mathematical understanding. In addition, PST C and PST E noted that they recognized how manipulatives facilitate students' understanding and increase their motivation. For instance PST E said the followings:

> I initially did not know much about how to use manipulatives but I witnessed that students understood how to compare fractions or add them up by using fraction strips. I also observed that one of my students was more active when use of manipulatives [were] involved in the task. I believe that concrete materials should be used in mathematics lessons because they motivate students to learn mathematics as well as help them to understand it.

Based on post-interview data, we inferred that having PSTs to work with a pair of students for a semester and then to reflect on the process contributed to their reflective skills because they had to pay attention to students' practices, think about how to address students' difficulties, assess the effectiveness of their scaffolding and plan for the next implementation. That is, they passed through all phases of reflection (Rodgers, 2002). Furthermore, PSTs' self-evaluation could be counted as an indicator of improvement in their PCK, specifically in their *knowledge of content and students* because they learned more about what was easy or difficult for students to understand in mathematics (Hill et al., 2008).

PSTs also noted that preparing tasks for their own students was a beneficial practice and even more, they said that the number of such practices should be increased. Although they mostly had positive feelings about the intervention, they complained about managing the time commitment. They were taking six or seven other courses in that semester, and they were not able to write the reflections as detailed as they wished to do so.

4.5 Discussion and Conclusion

In this study, we accepted that reflection, noticing skills and scaffolding practices are involved in teachers' PCK. Any actions aiming to improve or assess teachers' reflective skills, noticing or scaffolding would eventually contribute to or inform our understanding of teachers' PCK. Therefore, we designed a research setting that enabled us to learn about PSTs' PCK through their reflections, noticing and scaffolding practices. In this paper, I attempted to present findings about PSTs' reflections as they emerged from their one-to-one interactions with students and analysis of their own videos of those interactions.

The findings of this study revealed that PSTs' professional knowledge and skills could be improved through special courses where PSTs are given opportunity to make practice of teaching and reflect on their practices (Chamoso et al., 2012; Jansen & Spitzer, 2009; Llinares & Valls, 2010). The PSTs were asked to write reflection papers after watching their own videos and provide some justifications for their comments about students' performances and their own practices. The analysis of PSTs' written reflection revealed that use of videos of their own teaching helped them to analyze students' mathematical thinking and their own practices (Jansen & Spitzer, 2009; Stockero, 2008; Sun & van Es, 2015). They were able to detect the gaps in students' mathematical understanding correctly such that they provided evidences from their interactions with students and students' written work, as in PST A's reflection on the algebra tasks. PSTs' interpretations of students' thinking could be counted as a sign for their reflection skills (Goldman & Grimbeek, 2015; Rodgers, 2002; Ward & McCotter, 2004) as well as their PCK, because understanding and anticipating students' mathematics are involved in PCK (Hill et al., 2008).

Furthermore, the PSTs attempted to explain their actions and scaffolding practices in their written reflections which can be counted as a sign for their reflective skills since reflection entails critical analysis of own actions (Rodgers, 2002; Ward & McCotter, 2004) Although they did not give much details about the reasoning behind their actions, we could infer from the videos that they tried to facilitate students' understanding of mathematical concepts and procedures through questioning (Goldman & Grimbeek, 2015). PSTs' attempts also provided evidence of their PCK in terms of choosing appropriate examples, representations or tools to help students' understanding (Hill et al., 2008). Moreover, during the post-interview PSTs noted that this intervention contributed to their professional knowledge more than their field experiences because they had opportunity to observe their own teaching and how their comments and behaviors influence students' understanding. On the other hand, PSTs did not say much about their interactions during the oral reflections with respect to the written reflections. It might be because of external factors such as limited time given for oral reflections or internal factors such as lack of experiences that enable them to evaluate students' mathematics in-action.

The PSTs' perceived gains from this intervention in terms of learning about students' difficulties and misconceptions were compatible with what we found out in their oral and written reflections. As PSTs learned about their students' mathematical capabilities, they were able to make better estimations about their performances (Weiland et al., 2014). Furthermore, their comments about students' thinking were mostly valid as we elicited from students' written work and interaction videos. Therefore, in terms of enlarging the repertoire of students' possible difficulties and misconceptions in mathematics (Hill et al., 2008), we could conclude that this intervention contributed to PSTs' PCK, specifically to their *knowledge of content and students*.

During the post-interviews the PSTs noted that they began to feel better in communicating with students because they learned about the characteristics and the needs' of their students. It was evident both from implementation videos and PSTs' assumptions about students' performances on the tasks as given in the Table 4.1 and as discussed in PST C's assumptions about fraction task. Furthermore, they noted that they learned how to use manipulatives and how those manipulatives help students' learning and understanding. Hence, in terms of learning about instructional tools and how to use those tools, this intervention addressed their PCK, specifically their *knowledge of content and teaching* (Hill et al., 2008).

On the other hand, there were some limitations of this study such that the PSTs were taking several other courses therefore, in some weeks their written reflections were not as detailed or satisfactory as it was expected. For instance, they did not give the reasoning behind their scaffolding practices but preferred to describe what they did during the implementations. Therefore, we could not ensure that such intervention could yield a linear increasing pattern in terms of the depth of reflection or not. A further study might be conducted with a group of PSTs who do not have much work load to eliminate the possible "time barrier" for writing reflection papers. Furthermore, PSTs might be asked to prepare and implement their own tasks to provide more opportunity for *analysis* and *experiment* phases of reflection (Rodgers, 2002) as well as to improve their *knowledge of content and teaching* (Hill et al., 2008).

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Chapter 5
Gaining Valuable Field Experience Through the Use of Informal Learning Environments

Margaret J. Mohr-Schroeder, Christa D. Jackson, Maureen Cavalcanti and Ashley Delaney

Abstract Informal field experiences can be a powerful mechanism for exposing preservice and inservice teachers to unique opportunities to experience content in ways different than how they were trained and/or different than how their current environment supports it. Additionally, they provide a low-stakes environment to practice and hone important teaching skills and knowledge. In this qualitative study, we examine how a robotics course in an educator preparation program that required a field experience in an informal learning environment impacted its participants. Three themes emerged from their reflections and interviews: (a) development of a better understanding of STEM; (b) increased knowledge and enlightenment of instructional practices, especially the importance of asking good questions; and (c) students' increased interest and excitement in learning STEM content. Through participation in the robotics course and the required informal field experience, teachers learned more about classroom instruction, students, classroom management, and what sustained engagement looked like through this low-stakes authentic experience. Additionally, they saw firsthand the importance and necessity of creating a positive classroom community (e.g., growth mindset).

Keywords Informal learning · Field experiences · Preservice teachers STEM · STEM literacy

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_5

5.1 Introduction

Informal learning experiences provide opportunities for preservice and in-service teachers to apply what they learned in coursework in an authentic environment outside of the classroom (Jackson, Mohr-Schroeder, & Little, 2014). With this experience, teachers have an opportunity to reflect on their experiences and subsequently apply the lessons in ways that will transform education (Swick, 2001). Research suggests pedagogy rooted in informal learning environments, such as experiential pedagogy, grounds the learning in experience (Dewey, 1938; Root, 1997) for preservice and in-service teachers. The See Blue STEM Camp (Mohr-Schroeder et al., 2014) is one such informal learning environment where teachers can engage in authentic tasks that will increase their STEM literacy and help them become more effective STEM teachers. For preservice STEM teachers, participation in the informal learning experience exists within a broader context of a robotics course. Per the recommendations of widely cited research over the past 40 years, the robotics course offers a model that marries field experiences with methods courses (e.g., Darling-Hammond, 1994; Feiman-Nemser & Buchmann, 1983; Grossman, Hammerness, & McDonald, 2009) and provides varied learning environments (Wade & Yarbrough, 1996) for preservice teachers to bridge knowledge of teaching across content, theory, and field experiences. Additionally, the robotics course allows preservice STEM teachers to practice the application of this newfound knowledge of teaching with students in a field experience housed within an informal learning environment. The purpose of this paper is to share the knowledge gained by the preservice teachers' participation in a robotics course, offered by the College of Engineering, that included a field experience within an informal learning environment in a teacher preparation program.

5.2 Review of Literature

5.2.1 *Informal Learning Environments*

Informal learning environments include multiple settings including museums, libraries, camps, family adventures, after school programs, and daily activities such as cooking and gardening (National Research Council, 2009). Central to defining informal learning is its contrast to the formal learning that typically occurs in schools. "[Informal learning] is not typically classroom-based or highly structured, and control of the learning rests primarily in the hands of the learner" (Marsick & Watkins, 1990 p. 12). In other words, it is deliberate and purposeful, but not directed. Resting the locus of control within the learner helps minimize institutional agendas and systemic inequities to engage disenfranchised youth (Rahm & Ash, 2008) and adults (Marsick & Watkins, 2001). Informal learning is one of three essential pieces (the K–12 system and higher education, the other two) for successful integrating and implementing STEM education into U.S. schools (U.S. Department of Education, 2007) and teacher education (National Science Board, 2007). Learning environments beyond the walls of traditional education better situate learning into authentic contexts which helps facilitate the processes of application and generalization (Choi & Hannafin, 1995). Learning in such environments is shown to increase community involvement and activism as well as confidence and long-term success in adult learners (McGivney, 1999). The goals and skills (i.e. self-regulation) students engage in through informal learning experiences should complement their formal learning experiences (Boekaerts & Minnaert, 1999). Students of all ages benefit most when formal and informal environments complement each other providing rich and varied opportunities to learn (Eshach, 2007).

5.2.2 *Informal Learning Environments as Field Experiences for Preservice Teachers*

Informal field experiences for preservice teachers include field experiences that occur outside traditional clinical field experiences (Chambers & Lavery, 2012; Darling-Hammond, 2006; Tuchman & Isaacs, 2011). Traditionally, preservice teachers work with a cooperating teacher in a classroom during the school day observing and teaching a group of students. A focus on such environments for preservice teacher education still appears to dominate research. Although there is some research on informal learning environments such as museums and aquarium visits (e.g., Leinhardt, Crowley, & Knutson, 2003; Ramey-Gassert & Walberg, 1994), and tutoring (e.g., Worthy & Patterson, 2001), there is a dearth of literature on informal field experiences for preservice educators in STEM, with the exception of Jackson et al. (2014), Jackson et al. (2018), and Pop, Dixon, & Grove (2010). Research studies on informal learning environments have expanded to include ways preservice and inservice teachers can engage in such environments (e.g., Popovic & Lederman, 2015).

Informal learning environments allow the application of theory to practice (Bates, 2009; Jackson et al., 2014; Meredith, 2010; Olson, Cox-Petersen, & McComas, 2001) and provide an avenue for preservice teachers to be reflective (Meredith, 2010). Participation in informal learning environments as part of a teacher education program has largely existed within an academic school year (e.g., Olson et al., 2001). The effect of informal learning environments on preservice teachers include knowledge of content and teaching; awareness of strategies for providing inclusive, multicultural education; and preservice teachers' conceptualization of their future classroom. In addition, the design of informal learning environments has also been considered, including the importance of providing authentic contexts (Baldwin, Buchanan, & Rudisill, 2007; Boyle-Baise & Sleeter, 1998; Deering & Stanutz, 1995) and how the structure of informal learning environments as a field experience influence preservice teachers' beliefs and future practice. However, studies of informal learning environments as a field experience during the summer have not been documented.

Informal learning environments offer a setting for preservice teachers to gain different strategies, such as engaging the community (e.g., LaMasters, 2001) and one-on-one instructional planning (e.g., Worthy & Patterson, 2001), when compared to traditional field experiences. The goal of most field experiences is to provide preservice teachers an authentic environment of their future profession (Rethlefsen & Park, 2011). However, preservice teachers can view traditional field experiences as inauthentic, especially because they are always a visitor in the classroom (Alsup, Conard-Salvo, & Peters, 2008). Traditional field experiences are often limited to the encounters in the classroom during the regular school day. While this allows preservice teachers to engage with valuable situations that arise in the classroom setting (Brannon & Fiene, 2013), it creates uneven experiences due to the variations in quality of the cooperating teachers (LaMaster, 2001). These variations in quality are important because of the field experience's influence on preservice teachers' future teaching practices. New teachers are most likely to use practices they liked as students or implement practices of their cooperating teachers (Alsup et al., 2008). Therefore, research related to informal learning environments as a field experience, namely an *informal field experience*, for preservice teachers provides a low-stakes opportunity to examine how it can contribute to preparing future educators.

There are potential outcomes from participating in informal field experiences. For example, such experiences may strengthen content and pedagogical knowledge amongst participants. Klanderman, Webster Moore, Maxwell, and Robbert (2013) investigated the impact of college students' (i.e., elementary education majors, secondary mathematics education majors, and mathematics majors) participation in a single day Math Triathlon event for grades 7–8. They found the college students showed stronger knowledge of content and mathematics standards and better understanding of mathematics instruction after participating in the event. Other studies examining preservice teachers outside of the classroom setting revealed similar increases in content knowledge. Popovic and Lederman (2015) found that inservice secondary mathematics teachers identified overt examples of mathematics in exhibits (e.g., numbers, shapes, figures) at a museum. However, over time, the teachers moved to viewing the exhibits in terms of abstract mathematical concepts, and they made connections between the mathematics and science.

Preservice teachers' participation in an informal field experience may also inform their future teacher practice (Jackson et al., 2018; Johnson & Chandler, 2009). Johnson and Chandler (2009) found there were several advantages of participating in an informal learning environment as a field experience. The teachers in their study reported the informal field experience was a change from traditional classroom routines, and it provided more hands-on learning and an opportunity to vary instruction for diverse learners. Moreover, tutoring is a potential avenue for preservice teachers to engage in differentiation and learn how to meet the needs of individual students. Jackson et al. (2018) examined the way after school tutoring helped preservice mathematics teachers acquire knowledge and confidence to help students who struggle in mathematics. The implications of this work suggest ways field experiences in informal settings can provide preservice teachers with a more comprehensive understanding of teaching as a profession and the advantages of supporting learning in authentic contexts.

Informal learning environments and informal field experiences provide an opportunity for preservice teachers to engage in solving real world problems in authentic settings (e.g., Jackson et al., 2014; Popovic & Lederman, 2015). Everhart and McKethan (2004) suggest that utilization of field experiences promote more opportunities for preservice teachers to plan for teaching and to work with students. This increased opportunity is better promoted by integrating field experiences with preservice teacher training and experiences, especially those found within an informal learning environment. Olson et al. (2001) highlighted the importance of such integration, as a "desire to prepare teachers who not only understand research about the use of informal environments, but who actually implement recommendations by effectively using informal settings" (p. 169).

Finally, connections not only between STEM disciplines, but also between content and practice can arise through experiential learning in the field. While beneficial to students, nontraditional field experiences are additionally beneficial for preservice teachers developing skills beyond the traditional teaching methods found in a classroom setting such as lecture (Djonko-Moore & Joseph, 2016). STEM fields are not always represented to a great degree in lower and middle grade level instruction and content. To address this deficit, there is a need for preservice teachers to gain practical knowledge of how to integrate field experiences into their instruction for their own efficacy and to strengthen student learning in these areas of STEM (Kisiel, 2013). The existing literature does not extend the impact of informal learning environments and informal field experiences on preservice teachers more broadly to an integrated perspective on STEM rather than a siloed approach (Bybee, 2010; Hurd, 1998). Treating STEM as transdisciplinary subject of its own is important for understanding ways informal learning environments can support STEM literacy for preservice teachers.

5.3 STEM Literacy as a Theoretical Framework

An integrated approach to STEM education is needed to prepare STEM teachers to teach students in the 21st century. Traditionally, teacher preparation programs use field experiences as an important component of teacher education programs (He, Means, & Lin, 2006). However, teacher education coursework and classroom field experiences often do not align in their practices (Alsup et al., 2008), especially when focusing on integrating subjects. Effective teacher education programs require more extensive field experiences so that preservice teachers have opportunities to concurrently apply concepts they are learning about in coursework with teachers who model similar instructional strategies (Darling-Hammond, 2006).

By synthesizing perspectives on literacy within and across STEM disciplines, Cavalcanti (2017, p. 65–66) defined STEM literacy as the

> conceptual understandings and procedural skills and abilities for individuals to address STEM-related personal, social, and global issues" (Bybee, 2010, p. 31); the ability to engage in STEM specific discourse; a positive disposition toward STEM (e.g., Wilkins, 2000, 2010, 2015), including a willingness to engage and persist in STEM-related areas 66 (e.g., Wilkins, 2000, 2010, 2015); an understanding of the utility of applying STEM concepts to solve real world problems; and, an appreciation of how the processes and practices of STEM areas change as technologies and demands of modern society change.

It involves and is nestled in the transdisciplinary integration of STEM disciplines and the tools and knowledge necessary to apply STEM concepts to solve complex problems (Balka, 2011). An understanding of STEM literacy as a unique tool set to create and use knowledge of and across disciplines arises from applying the concept of literacy to disciplines individually and holistically (Mohr-Schroeder, Cavalcanti, & Blyman, 2015). For example, prospective and in-service teachers who have opportunities to experience and apply an integrative pedagogy develop a broader meaning of STEM and hone their STEM literacy skills than those who have a degree in a single STEM discipline. As a consequence, they can plan for varied STEM learning experiences which reflect diverse backgrounds of the students they serve. While research exists on how using an integrated approach to teach STEM subjects can increase student motivation and achievement, limited research exists on ways to support teacher development that integrates STEM disciplines (Honey, Pearson, & Schweingruber, 2014). Kisiel (2013) notes the importance of and shift toward integrating field experiences in providing unique instruction that integrates the content of a STEM discipline into a nontraditional learning environment.

5.4 Methodology

This project utilized qualitative methods to answer the following research question: How does a course within an education program that utilizes informal learning experiences as a field experience impact participants?

5.4.1 Population

The 38 participants in the study were undergraduate (n = 3) and graduate students (n = 20) seeking certification in mathematics or science education (grades 8–12), graduate students (n = 11) in a STEM Education doctoral program, and college-credit seeking high school students (n = 4) from a local STEAM (science, technology, engineering, arts, and mathematics) high school program. All participants were enrolled in a 4-week long hybrid (some face-to-face sessions in addition to asynchronous online modules) summer introduction to robotics course, offered by the College of Engineering and co-taught by an engineering professor and STEM education professor, at a large public university in the southeast region of the United States. Throughout the course, participants (a) gained familiarity with the interdisciplinary field of robotics and its growing impact on society; (b) developed the ability to direct robots using computer languages for communication; (c) gained familiarity with widely-used computer programming constructs including variables, assignment, looping, and conditional statements; (d) gained aptitude in understanding, designing, and evaluating patterns of logic and reasoning expressed as algorithms; (e) learned to practice argumentation and reflection on topics related to disciplinary content, including and especially ethics; and (f) became more comfortable and effective working in a team setting, particularly in analyzing and communicating logical and computational ideas with others.

Additionally, participants explored robots, engineering concepts, engineering design, and K–12 robotic curricular materials as they learned about basic robotics communication and programming. The EV3 Mindstorm Lego Robots were used for the course because most of the participants in the course had ties to K12 students, and the state in which this course took place hosted and regularly supported First Lego League competitions in which a majority of school districts participate in each year. EV3 Mindstorm Robots are made and sold by Lego Education. They are an easy-to-learn robot, using the Lego building concept and attaching a computer brick that can be programmed through their drag and drop software. Students begin with drag and drop programming and then, as they master building and programming processes, can move into more advanced algorithms. After building the EV3 robots, students were required to program their robots to meet various challenges. The early challenges (such as drawing a square) required students to use "blocks" to program their robot to move forward, backward, and turn. "Loops" and "switches" were used for more challenging tasks to incorporate the use of sensors (e.g., program robot to sense when it was 11 in. from the wall, and then turn and return to the original position).

5.4.2 See Blue STEM Camp

In order to apply what they learned in the first 3 weeks of the course, the course contained a field experience in an informal learning environment, the See Blue STEM Camp, during the final week. The See Blue STEM Camp (Mohr-Schroeder et al., 2014) is a week-long (5 days) summer day camp for rising middle level students (incoming grades 5–8). The middle grades students are recruited from area middle schools with a focus on female and students of color making up at least 50% of the camp population. The camp population between 2014 and 2016, when this study occurred, ranged between 144 and 240 students. The students who participated in camp did not necessarily like science, technology, engineering or mathematics, but rather, needed a positive, out of school experience with STEM and/or were provided the chance to participate in a summer camp.

The camp focuses on authentic hands-on sessions where students are given opportunities to engage in a variety of STEM fields. Additionally, during the camp the students participate in a daily session of Lego Robotics. Opposite Lego Robotics, students attend sessions focused on STEM content in authentic learning environments. For example, students go to the biology lab to learn about drysopholia (fruit flies) from a biology professor and his graduate students. All the topics and content in See Blue STEM Camp focus on the eight Standards of Mathematical Practice (Council of Chief State School Officers [CCSSO], 2010) from the *Common Core State Standards for Mathematics* and the eight science and engineering practices (NRC, 2011) from the *Next Generation Science Standards* (see Table 5.1).

The language of the standards for mathematical practice and the science and engineering practices reveal an extensive overlap to support students solving complex problems and participating in authentic learning experiences, thereby increasing their STEM literacy. Additionally, bridging these standards sets a foundation for integrating more interdisciplinary practice and teaching through informal field experiences within participating teachers' future practice.

Robotics course participants' roles in the STEM Camp were as "teacher leaders". As teacher leaders, they were assigned to one of four student groups; each group had 36 students. They traveled with the group all day, gaining exposure to STEM content during the STEM content session and then applying their robotics course knowledge the other half of the day during the robotics session. In their role as teacher leaders, participants were expected to practice their classroom management skills and help guide middle grades students through the STEM content investigations and robotics challenges using scaffolded questioning. The emphasis was on teacher leader as a facilitator, not as a presenter. Participants were also expected to help with managerial tasks such as keeping track of students, collecting daily

Table 5.1 Mathematical practices (CCSSO, 2010) and science and engineering practices (NRC, 2011)

Mathematical practices	Science and engineering practices
1. Make sense of problems and persevere in solving them	1. Asking questions and defining problems
2. Reason abstractly and quantitatively	2. Developing and using models
3. Construct viable arguments and critique the reasoning of others	3. Planning and carrying out investigations
4. Model with mathematics	4. Analyzing and interpreting data
5. Use appropriate tools strategically	5. Using mathematics and computational thinking
6. Attend to precision	6. Constructing explanations and designing solutions
7. Look for and make use of structure	7. Engaging in arguments from evidence
8. Look for and express regularity and repeated reasoning	8. Obtaining, evaluating, and communicating information

surveys, assisting with lunch time, conflict resolution, etc. The 5-day field experience, which counted towards teacher education program requirements, totaled 40 h. At the end of each day, participants completed a daily reflection.

5.4.3 Data Collection and Analysis

Data were collected over three summers (2014–2016) throughout participants' (henceforth, referred to as teachers given their role in the camp) participation in the See Blue STEM Camp via their robotics course. The teachers were asked to complete daily reflections at the end of each day for the first four days of STEM Camp. The reflection prompts focused on (a) what they learned at the camp, (b) what they liked about what they learned, (c) what they did not like about what they learned, and (d) what they would like to learn more about. At the end of the camp, the teachers reflected and synthesized their growth and learning in a two-page written final reflection. In addition, the teachers participated in a semi-structured interview about their experiences working and participating in the See Blue STEM Camp.

The data (written daily reflections, final reflection, and interviews) were analyzed using open coding to discover themes that directly emerged from the data (Strauss & Corbin, 1998). We first read through the daily reflections, final reflections, and interviews and wrote analytic memos (Maxwell, 2005) on the teachers' responses and grouped them together using a constant comparative method (Charmaz, 2006; Glaser, 1965; Glaser & Strauss, 1967). We then discussed how we grouped the responses and created themes based on the data. All disagreements were discussed until a consensus was reached. Once a consensus was obtained, we reviewed the themes and supporting data and identified three prominent themes that emerged from the data. All discrepancies were resolved during the final development of the overall themes.

5.5 Results and Discussion

> Friends and family wondered what on earth a Lego Robotics class would do to help me become a teacher, but every day in class and all throughout STEM Camp I learned so much about teaching strategies and how to be an effective instructor. (Teacher Reflection 2016)

As the teachers participated in the informal field experience of the See Blue STEM Camp via the robotics course, three prominent themes emerged from their reflections and interviews: (a) the teachers developed a better understanding of STEM; (b) teachers' instructional practices were enlightened; and (c) students' interest and excitement increased, which all positively influenced the teachers' development as future STEM teachers.

5.5.1 Understanding STEM

Prior to participating in the robotics course, most of the teachers did not have a clear understanding of STEM and what it looked like when students actively engaged in STEM activities. All of the teachers articulated that STEM was an acronym for science, technology, engineering, and mathematics. However, it was not until after they participated in the See Blue STEM Camp via the robotics course did they come to realize the true meaning of STEM. For example, one teacher stated, "They're really are all interconnected and kinda [sic] go together" (Teacher Interview 2015). Another teacher further elaborated that STEM is "interdisciplinary education" involving the four disciplines of science, technology, mathematics, and engineering, and you do not teach each discipline in isolation. Many of the teachers enrolled in the robotics course expressed they were extremely comfortable in their mathematics abilities (as current and future mathematics teachers), but they were not confident in their science and engineering abilities.

> All I am really good at (or really familiar with, I should say) is the mathematics part of STEM. The traditional mathematics classroom is what I was familiar with because that is all I had really ever seen. So needless to say I was a little nervous about stepping into a class and camp that dealt more with science and technology than mathematics. But now I've been through the class and camp, I see that they are all intertwined and I love it! (Teacher Final Reflection 2014)

Another teacher stated he was also confident in his mathematics skills as a future mathematics teacher, but he had not taken any science classes since high school and felt he had limited abilities in engineering and technology. But, he now felt "more connected to all of the disciplines."

Since many of the teachers were primarily only confident in their mathematics abilities (all but five were mathematics concentration) they deepened their content knowledge in various STEM disciplines as they participated in the informal field experience—See Blue STEM Camp. In one session that focused on energy, the teachers were surprised to discover that cement acts like a glue to hold concrete together. The teachers had the misconception that cement dries and that is why it hardens. They were shocked to discover this in fact was not true. Instead, the cement undergoes a chemical reaction, hence why cement needs to sit untouched while it cures.

The teachers' understanding of STEM was broadened not only from the STEM content sessions, but also from the middle level students participating in the camp, particularly during the robotics sessions. One teacher stated, "I didn't truly grasp the programming side until camp actually started. I would say the kids in the camp helped me more with understanding complex programming on the EV3s than anything else did" (Teacher Final Reflection 2014). One teacher felt uncomfortable knowing the students were able to pick up on the technology faster than he could. He confessed, "It made me feel a little inept because of how long it took me to program the robots to do a square compared to how quickly the students could do it" (Teacher Reflection 2014). But, a majority of the teachers were not intimidated by the students' knowledge. In fact, teachers were simply amazed at what the students could do and how quickly they picked up the programming language. Teacher knowledge deepened through their hands-on experience with the students. In fact, one teacher selected a pair of students and followed them through the process of programming to modeling their robots programming functions. He learned that "each robots' programming was slightly different in the number of increments increasing and decreasing based on the programming function they were attempting for their robots" (Teacher Reflection 2014). Another teacher stated she learned a lot helping students code their robots.

> I learned you can manually rotate a motor to see how many degrees it is turning. Also, the order in which the motor ports are selected in the code makes a difference. I'm not sure if this is also true for the EV3, but the NXT will not turn correctly if the ports are selected in the order opposite to the previous block in the code. (Teacher Reflection 2014)

The teachers recognized that not only were they teaching the students, the students were teaching them. "This is something that I think is very important for all future teachers to realize. Students will teach you just as much as you teach them" (Teacher Final Reflection 2014).

The many varied STEM experiences helped teachers better understand STEM. Subsequently, they became excited about the different ways they could take what they learned into their classrooms. The teachers were involved in activities ranging from extracting DNA, interacting with human organs, sending a magnetic ball through PVC and a copper pipe, and geocaching and mapping using Google Earth. They exclaimed how they would like to use all of the activities from STEM Camp in their classrooms. A teacher voiced some hesitation, but realized the importance of it.

> As a mathematician, we enjoy knowing a specific algorithm to solve a given problem. As a STEM educator and student, we must embrace several methods and different attempts to reach a certain result. I am nervous about working across disciplines because I am not an out of the box thinker. Recognizing this now is beneficial to my growth. (Teacher Final Reflection 2014)

In the preceding excerpt, the teacher distinguished between being a mathematics and being a STEM educator. The perspective the teacher shared reflects what many of the teachers realized; namely that STEM was more about the integration of the four subjects. Even more, the activities they participated in via the robotics course and the See Blue STEM Camp broadened their view of STEM. A teacher stated, "I have learned some great ways to introduce integrated STEM activities to students for engineering, robotics and many other content shifts" (Teacher Reflection 2015). A teacher summed it up by saying, "I love seeing STEM in action, not just theory" (Teacher Reflection 2015).

5.5.2 Instructional Practices

The majority of the teachers discussed the importance of including hands-on activities to keep students engaged in the lesson. While they had learned and been exposed to hands-on activities during their teacher preparation program, they saw limited formal classroom enactment of such tasks. From their experience with the camp, they recognized students learned more through hands-on tasks and that the tasks sustained student engagement with the topic. A teacher stated, “It seems like everyone had a lot of fun with the interactive stations. It is such a simple way to get students engaged, which is something I hope to bring to my classroom in the future” (Teacher Reflection 2014).

The potential for supporting implementation of hands-on activities with the use of questioning was effectively modeled during the sessions. Consequently, teachers were able to observe questioning as an effective instructional practice. Though the teachers had learned in their teacher preparation program the power of asking effective questions to push and challenge students’ thinking, they had the opportunity to witness first-hand how a question sparked discussion, engaged students, and deepened students’ understanding. For example, during one session of camp focused on modeling the systems of the human body, the presenter asked the students “what it felt like to be out of breath” and what was happening to the body during this time. The preservice teacher commented in his final reflection that this particular question generated a powerful discussion. The preservice teacher remarked, “Seeing this idea in practice was encouraging and made me realize even more why that is a good method of instruction, especially in teaching science” (Teacher Reflection 2016). However, some teachers were not accustomed to having students think for themselves and solve a task. They were used to being the sole source of knowledge; they found themselves having to refrain from giving students answers to STEM tasks. A teacher realized that he needed to work on his questioning strategies and let the students figure out the task for themselves so they could learn it.

> Sometimes it is best to have them walk you through the process and give some positive encouragement for what they did right. Usually having the students vocalize what they did gives them a way to hear when they actually missed a step. You can guide them with good questions if they still can’t figure it out. There is a difference in giving the answer directly and asking probing questions to check for understanding. (Teacher Reflection 2014)

In addition to questioning, the teachers saw the importance of differentiating instruction to meet the needs of all of the students. A teacher articulated, “The thing for me to remember was that there are many different personalities and ability levels, each with their own learning style and needs, and a teacher has to recognize them and treat them accordingly” (Teacher Reflection 2016). One way the teachers voiced this could be accomplished is to provide students multiple methods and/or strategies to solve a task. A teacher commented, “There are multiple ways to learn a topic, and therefore as a future teacher, it shows me how important it is to explain something to my kids in different ways to best meet their learning [needs]”

(Teacher Reflection 2014). Moreover, a teacher argued that some students may need additional scaffolding to help them solve a task, while others may be able to coach themselves through the task without any support. As one of the teachers was working with some students who were struggling on a robotics task, she informed them that students "who solve problems easily don't learn anything, while the ones who struggle to solve even one problem learn so much more and get so much more out of it" (Teacher Reflection 2016).

The teachers also gained first-hand experience on what it meant for a teacher to be flexible. They learned the importance of adapting their instruction in the moment. During STEM Camp one of the robotics instructors had to adapt his instruction due to materials not being assembled. The teachers were glad they had the opportunity to see how to handle situations when the lesson did not go as planned. "I gained insight in how to adapt to things not going right. [The instructor] was really good at adapting the plan and it is something that I would like to develop as a teacher" (Teacher Reflection 2014). One teacher realized he has to be more flexible in how he thinks about his own teaching. He remarked, "I have had a very rigid view of mathematics and unfortunately that has influenced the way I teach. Math does not always have to be black and white, right or wrong, although there are occasions for that. I need to allow for flexibility" (Teacher Final Reflection 2014).

Teachers recounted how participation in the informal field experience allowed them to observe the implementation of effective instructional practices and the importance of differentiation and flexibility in achieving instructional goals.

5.5.3 *Students' Excitement*

The teachers had an opportunity to witness students' excitement when learning STEM content. They expressed that seeing students' enthusiasm in these disciplines was rare. "I liked seeing students excited about learning! We do not see students interested in education and learning everyday, especially math" (Teacher Reflection 2014). Many of the teachers did not expect students to be so engaged in learning and enthusiastic about learning STEM concepts, especially since many of the students did not enroll in the camp because they enjoyed the STEM disciplines. One teacher commented, "I didn't expect them to be this excited, to be honest" (Teacher Interview 2015).

With the excitement, the teachers noticed how persistent the students were and refused to give up even when they were unsuccessful completing various tasks. The teachers were impressed by how the students "threw themselves with reckless abandon at some of the problems they faced" (Teacher Reflection 2016). In fact, one teacher articulated, "I was impressed with the persistence of many of the groups. Even when some kids got frustrated, they refused to give up. It was awesome to see!" (Teacher Reflection 2014). Another teacher commented,

> My blue group started the Green City Challenge today and everyone (including myself) seemed overwhelmed with all of the different tasks they could try with their robot at first. But, after a while the students were getting the hang of it and learning to take the programming step by step. They were getting so into it and it was great to see them cheering when they accomplished something new. (Teacher Reflection 2015)

The teachers were amazed at how the students took ownership of their learning. They stated the students would ask for help, but then would say, “never mind, I’ve got it.” The students realized they did not need the assistance of the teachers to complete the task. They recognized they could figure it out on their own. Therefore, after the students’ successful completion of each task, the teachers noticed they would jump up and down, smile, cheer, and take a “walk of victory.”

Through this informal field experience the teachers recognized that not only were they teaching the students, the students were also teaching them. One teacher remarked, “While I was teaching, I was also learning, and while the kids were learning, they were also teaching me. This is something that I think is very important for all future teachers to realize. Students will teach you just as much as you teach them” (Teacher Final Reflection 2014). Another teacher further expressed, “The amazing thing is I learned even more from the students during STEM camp. They were pointing things out to me that I didn’t even know existed, so I feel confident in my knowledge both of the robots and the programming as well” (Teacher Final Reflection 2014). To sum it up, a teacher concluded, “The neat thing is that I have learned so much more about programming from watching, and talking to the kids; obuchenie (teacher and students learning from each other)” (Teacher Reflection 2015).

5.5.4 Applying What Was Learned in Practice

> I could point out dozens of little incidents and moments all throughout the camp, but they can best be summarized as this: teaching is a wonderful burden. It has its triumphs and setbacks, endless frustrations and moments of sheer joy. The days are long but never quite long enough. The students are both the cause and the relief of your headaches, and you will curse them while you thank the Creator for each last one of them. (Teacher Reflection 2016)

Looking specifically at the preservice and inservice teachers included in this study (n = 23), the teachers stated that while they loved what they experienced in terms of instruction and integrated content during the robotics course and STEM camp, they find it difficult to integrate into their daily mathematics instruction.

> … yeah we made gak, and I really want to do that in class one day or maybe if we could have him come in or have someone come into do it just so they could see some hands on stuff, but it always ends up falling through because like we map tested for the past two days, I have to give a final next week, I have to give a test next week in my other classes; so it’s really hard to plan …. (Teacher Interview 2016)

Although the teachers found it challenging to do some of the hands-on activities they experienced during STEM Camp, they were able to implement several instructional strategies they learned about and reflected on.

> I'm able to bring in a lot more examples from outside. So before, especially with the older staff here, it's just very straightforward, but then now [sic] I'm able to kind of talk about some of the stuff where programming comes in; … sometimes I'll try and bring in questions like "where do you think this could come from" or how this could be useful where you would see it and so some of the students who have done the stem camp can go "oh yeah, we've done this before"; uh, especially when we start talking about rotations with graphing and everything like that; … so it's kinda those extension questions to bring on and then it kinda peaks the interest of the students that haven't done it like "oh, what are you talking about"; and then we have a robotics course uh through electives that a lot of the students take; I don't know if it's just a robotics course, but they get out the robots and do things with them; so it kinda makes those nice connections between their elective course and their core classes. (Teacher Interview 2016)

Unlike the teachers who taught mathematics, the STEAM teacher regularly integrated the content learned from the course and STEM Camp.

> Yeah, um since I've transferred to STEAM, I've strived to take a focus of project based learning and the whole idea of STEM camp is to engage students in different projects and different problems um instead of just worksheets of math and science and technology, just problems after problems, more of like real world problems where they have to assemble, disassemble and assemble a robot for instance. Or uh build a dam that can support so much water. So those kinds of problems are problems that we actually face um as people. We're changing the emphasis more towards just doing the content to applying the content in a real world setting. (Teacher Interview 2016)

School structure impacts feasibility of implementation of integrated content (Chiu, Price, & Ovrahim, 2015; Honey et al., 2014), which is evident in these two school settings. It is important to note that the largest impact of the camp was on all the teachers' instructional practices. The teachers voiced they were planning and implementing engaging lessons and asking purposeful questions. The teachers noticed and were surprised by *how* they asked questions to their students, which they attributed to being a participant in the camp.

> … I do a lot more scaffolding with my instruction, as well as get the students up a moving around to get them awake and their brain working at full capacity. (Teacher Interview 2016)

While the data collection methods do not offer a big picture regarding classroom impact, it does offer some direction towards how we can better support participants' integration of what they learned from the informal field experience.

5.6 Limitations

Although this course counted towards requirements for degree completion at the university, it was not a required course taken by all preservice and inservice teachers in our programs. Teachers who enrolled in the course self-selected into the course and had some sort of desire to learn more about robotics and STEM. Further, the data from this study included self-report data.

5.7 Conclusions and Implications

5.7.1 Conclusions

The See Blue STEM Camp was an informal learning environment in which teachers meaningfully engaged in as a field experience requirement in the robotics course they took in the summer. The teachers expressed that through participating in the informal field experience they learned more about classroom instruction, students, classroom management, and what engagement looked like through this authentic experience. They also saw firsthand the importance and necessity of creating a positive classroom community.

> I think my biggest take away [sic] from this course is the mindset that the students were allowed to have while completing assignments and practicing with their robots. (Teacher Reflection 2016)

> I was amazed by the engagement and motivation of the middle-school students whenever they were faced with challenges. The students definitely learned more than robotics in this camp; they also learned life lessons such as patience, determination, perseverance, and teamwork. (Teacher Reflection 2016)

Teachers expressed the students worked well together, especially surprising to most of the teachers since a majority of the students did not previously know one another. One teacher commented, "Pretty much everyone was getting along with their partners … more people [are] becoming friends" (Teacher Reflection 2015).

Through the informal field experience, teachers were exposed to and engaged in a transdisciplinary STEM experience that provided them with a low-stakes environment to hone their teaching skills and knowledge. The traditional field experience in siloed (e.g., mathematics, biology) classrooms continue to prohibit teachers from developing their own STEM literacy as teachers and learners. The informal field experience via the robotics course the teachers engaged in were aimed at providing an embedded pedagogy to increase STEM literacy and learning in context intended to influence the delivery of STEM learning in their classrooms. Informal field experiences, and informal learning environments in general, can be an important tool for providing preservice and inservice teachers with unique opportunities to experience content in ways different than how they were trained and/or different than how their school currently supports it. As one teacher proclaimed, "I've had very limited experiences with STEM in general, so everything I've been learning has been new" (Teacher Interview 2015). As educators, we need to engage teachers in experiences that foster their STEM literacy, which will ultimately support STEM teaching and learning. When the learning experiences integrate STEM-related content, gains are possible to support STEM teaching and learning.

> The course was great. I enjoyed my time in it and have taken away many things from it, not only in programming and robotics, but in other important areas like classroom practice. I would certainly encourage other … students to enroll in this course because the benefits are multifaceted and far reaching. Truly, it's a unique experience …. (Teacher Reflection 2016)

> The course increased my confidence about working in STEM environments and stimulating scientific thinking in students. That confidence was built during STEM camp with the skills I developed from the classroom/online portion of the course. It was all about asking the right questions of the students, to stretch their thought process. (Teacher Reflection 2016)

> After EGR 599 and STEM Camp, I have gained confidence in my ability to work across the STEM disciplines. The sessions showed me how diverse the STEM field can be and how each particular discipline contributes to learning and exploring the others. The professors did an excellent job of explaining their fields and how they fit into the interconnected idea of STEM. I feel that I have a better sense of the broad scope of applications of STEM. I also feel better prepared to help students understand the relevance and importance of developing a deeper understanding in each of the STEM components in order to become a well-rounded scientist or engineer or doctor. (Teacher Reflection 2016).

Overall, teachers consistently expressed surprise that they learned as much from the students as they learned from the session leaders. They were impressed by the students' knowledge, persistence, and ability to learn quickly. Teachers also expressed surprise in the relationships and camaraderie the campers formed. These observations are counter to the social stereotypes of STEM because the camper's demographics are more representative of the current U.S. population instead of the white, male, Einstein-like image that most envision for a scientist or STEM-person (Chambers, 1983; Picker & Berry, 2000). Teachers recognized the value of differentiation based on readiness as well as their environment. Facilitating student connections to their lives instead of forcing the teachers' ideas of right and wrong is a powerful shift in teaching practices toward an equity lens. This study did not examine teachers' conceptions of traditionally underrepresented populations in STEM. However, literature from other fields (Baldwin et al., 2007; Boyle-Baise & Sleeter, 1998; Deering & Stanutz, 1995) discusses the power of informal learning environments on teaching for social justice and equity. Research into how teachers encourage and promote STEM to underrepresented groups is needed and an essential area of further examination.

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Chapter 6
Diagnosis-Based Adaptations of Mathematics Lessons: Analysis of the Implementation by Prospective Teachers During Practical Phases

Matthias Heinrich

Abstract What conclusions do prospective teachers draw from their students' current learning level for their own lesson planning? Why do they draw these conclusions? Within an empirical-qualitative study, 15 prospective teachers planned a mathematics lesson during their practical phase. They were asked to diagnose the learning level of their students and then modify the planned lesson, if they thought it to be necessary. First, a system of categories was developed, describing their interpretations of the diagnosis, the modifications of the planned lessons and their justifications. Afterwards, eight different, recurrent types of decisions were identified. With these results it was possible to generate a process model, which helps to understand how the prospective teachers came to their decisions.

Keywords Adaptive teaching · Diagnosis · Empirical-qualitative study
Mathematics lessons · Prospective teachers

6.1 Introduction

Planning lessons is an essential component of a teacher's professional duties (Baumert & Kunter, 2006). Due to the fact that there usually is heterogeneity within a class (Baumert et al., 2001) diagnosis as well as individual improvement, which can be implemented in the form of adaptive teaching, have gained in importance over the past few years. Furthermore, the importance of diagnostic competences on the part of the teaching staff has long been empirically proven (e.g. Karing, Pfost, & Artelt, 2011). Effective as well as lasting teaching and learning processes may be initiated by tying in with individual learning levels (Hußmann & Selter, 2013). Politics and society demand that prospective teachers should already be capable of

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_6

diagnosing learning levels and using particular improvement measures at the end of their education (Sekretariat der Ständigen Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland, 2004). In Germany, the principles of these two educational aspects—diagnosis and adaptive teaching—which are taught at universities, must already be executed by the prospective teachers during the two practical phases of their education. During each practical phase, they gather experiences as a teacher for about five to seven weeks at a school.

The examination of this topic—the implementation of diagnosis and adaptive teaching by the prospective teachers during their practical phases—can be profitable because of a variety of reasons: The research of teachers' competences in the scope of diagnosis and the planning of adaptive lessons comprises a wide range of necessary skills. These include amongst others: planning lessons in general, evaluating the learning difficulties of a topic, appraising the students' prior knowledge and considering this prior knowledge while planning lessons. All of these skills must be combined when planning adaptive lessons (Heinrich, 2017). Thus, it is important to examine the adaptive planning competences of prospective teachers and to expand our comprehension of these. Furthermore, diagnosis and adaptive teaching are primarily theoretically developed concepts (Moser Opitz, 2010; Schwarzer & Steinhagen, 1975), whose implementation in actual classes has not been investigated yet. Finally, not only the planning of lessons, but also the diagnosis of the students' learning level were defined as a significant duty of the teacher education by the education ministers of the German federal states (Sekretariat der Ständigen Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland, 2004).

The main concern of this study is the following question: How do prospective teachers implement diagnosis and adaptive teaching in their practical phases? This scientific contribution addresses especially the following research questions: Which conclusions do prospective teachers draw from their students' current learning level for their own lessons? How do they justify these conclusions? Which argument types, each consisting of an interpretation of the diagnosis, a consequence for the lesson and its justification, can be identified? How can the process from a diagnosis to the adaptation of a lesson, which focuses on uncovered prior knowledge, be theoretically and empirically modelled?

6.2 Theoretical Framework

School education, which aims to support the learning processes of each individual student, requires the adjustment of lessons and the level of difficulty of questions and exercises to the students' individual learning conditions (Helmke, 2014). This teaching approach is based on the assumption that a person learns an ability, such as multiplication, better with a teaching method that is suitable for him or her than with another method, which is per se just as good (Cronbach, 1975). The individual learning processes of the students are considerably too diverse and multilayered, so that the use of one certain teaching method cannot achieve a learning success with all members of a heterogeneous group (Beck et al., 2008). But such an adaptive education needs a precise diagnosis of these conditions, so that the improvement measures are suitable for each individual student (Hesse & Latzko, 2011). If the individual's learning conditions have not been unearthed, it is not possible to adjust lessons to that individual's needs. Therefore, teachers must be amongst others competent in diagnosing students' learning conditions.

The notion of diagnostic competence "(that, in English, might have some medical connotations) is used for conceptualizing a teacher's competence to analyse (sic!) and understand student thinking and learning processes without immediately grading them" (Prediger, 2010, p. 76). In general, there are different reasons to conduct a diagnosis. Usually they are used at the end of a certain subject to evaluate the students' learning gains. However, diagnoses can also be conducted to unearth the students' current learning level to optimize lessons. In this case the diagnosis is used either at the beginning of or during the covering of a specific subject. This second type of diagnosis is the one that is addressed in this study. Many authors and organizations attach great significance to the skill of diagnosing students' learning levels, for example the National Council of Teachers of Mathematics in its Standards and Principles: "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000).

Adaptive planning and adaptive teaching is another significant competence of teachers in this matter (Beck et al., 2008). This is the competence, which enables one to tie in with the individual learning conditions of the students. Overall, teachers have a lot of possibilities to react to the differences in their students' learning conditions (König, Buchholtz, & Dohmen, 2015). Ignoring these differences, which is the passive reaction form, could lead to an increase of these discrepancies. The substitutive reaction form describes organizational courses of action, which are supposed to lead to a homogenization of a learning group, such as the repetition of a grade or external differentiation. Here, the students are adjusted to the lessons. An adjustment of the lessons to the students seems to be more preferable. According to König et al. (2015) this is executed in the active reaction form. The lessons are adjusted to the students' needs and learning differences and this is what adaptive teaching is about.

Corno and Snow (1986) describe that adaptive teaching can be implemented on two different levels. They distinguish between micro- and macro-adaptations. The short-term adjustments teachers make during their lessons are called micro-adaptations. Usually these emerge from observations and subjective judgements (Schrader, 2013)—which are among implicit forms of diagnosis—because teachers have to analyze the learning conditions, the learning success as well as the learning difficulties of their students throughout the implementation of a lesson

(Schrader & Helmke, 2001). This is different when making far-reaching, long-term decisions, so called macro-adaptations, for an entire lesson or teaching unit. Here, teachers have enough time to conduct an explicit diagnosis of their students' learning conditions (ibid.).

So far has been discussed that in order to adjust a lesson to the individual students' needs teachers need to be competent in diagnosis their students' learning levels as well as in adaptive planning and teaching. In addition, teachers must be able to clarify the mathematical content. This is necessary to get an overview of what the students should learn during the lesson or unit and to already get an idea of the possible learning difficulties. Teachers also need to know how to identify the necessary subject-related learning conditions of the lesson to be planned, because these are the aspects that have to be looked at during the diagnosis. Figure 6.1 shows a theoretical modeling of the modification process of a lesson to the students' learning conditions. Of course the second step—planning a lesson—need not be executed before the diagnosis. Especially experienced teachers might be able to skip it. However, it is presumed that it is easier for prospective teachers to diagnose the needed learning conditions for a particular lesson that has already been planned than for a vaguely envisaged learning process.

The principles for the steps discussed above are all taught at the University of Oldenburg, where this study took place—this includes principles for teaching mathematics in general, but not for adaptive teaching specifically. But does this mean that prospective teachers automatically succeed when they try to implement these theoretically developed principles during their practical phases? According to Patry (2014) this is very unlikely. He states that scientific theories are usually broad and therefore rarely concrete. This means, there exists a gap between theoretical principles and practical implementation. This gap must be closed by the prospective teachers. Furthermore, they also need to pursue several goals at once, which are addressed in different theories, and they must revert to multiple of their own beliefs. In addition, acting adequately is very specific to each individual situation. In other words: Prospective teachers must react appropriately to a specific given situation, while they pursue a variety of goals. Thus, multiple action-guiding beliefs are activated in their minds, which they must cope, while they can only resort to very few theoretically developed principles that must be adapted to the specific situation. This means that a direct translation into an action is not possible without additional effort.

To sum up, prospective teachers are supposed to be able to diagnose the learning level of their students and react to it accordingly by the end of their education. To do so they need to possess a variety of competences, such as diagnostic or adaptive teaching competences. The problem is that it is not sufficient, if they only know the theoretical principles of these aspects. These theoretical considerations lead to the main concern of this study: How do prospective teachers implement diagnosis and adaptive teaching in their practical phases?

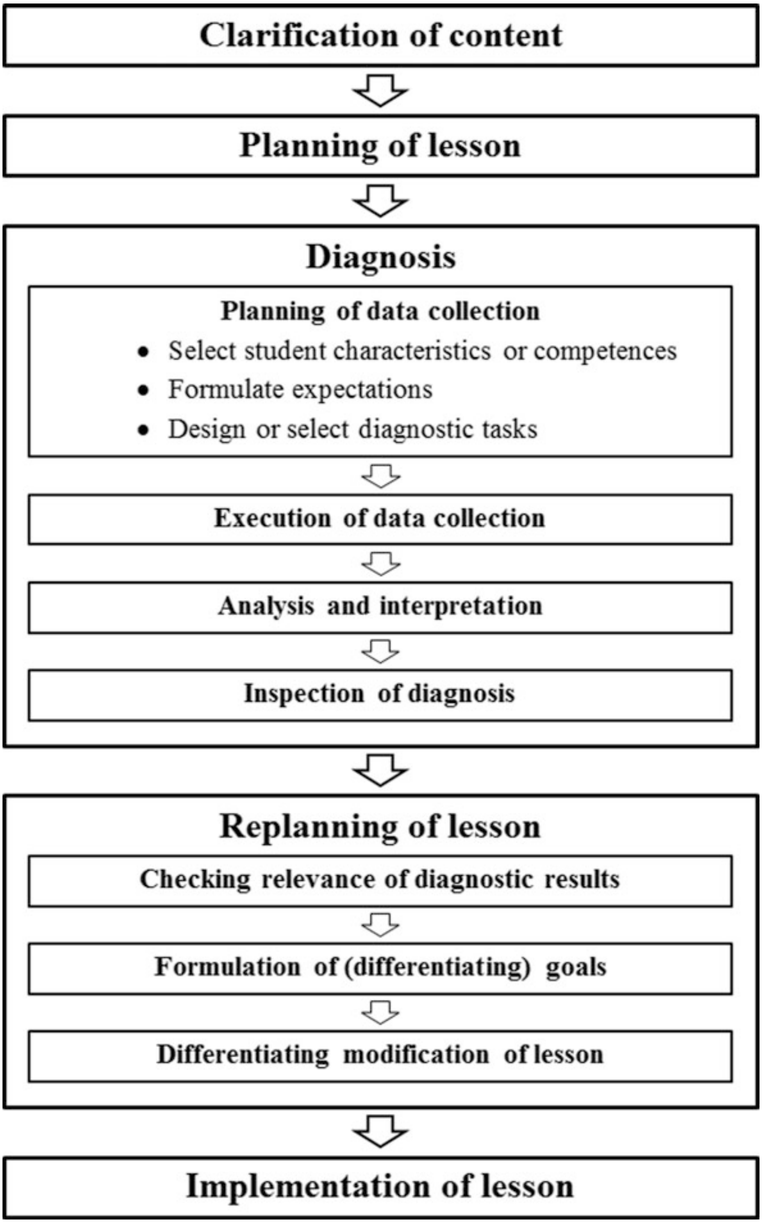

Fig. 6.1 Theoretical modeling of the modification process of a lesson to the students' learning conditions

6.3 Research Design

The presented research questions already indicate that the pursued issue is the *comprehension* of the individual actions and argumentations of prospective teachers when adjusting a lesson to their students' learning conditions. Therefore, an empirical-qualitative research approach was used (Mayring, 2014). The research of the implementation of adaptive teaching in practical phases by prospective mathematics teachers has to date not occurred (e.g. König et al., 2015). Hence, it was appropriate to use an explorative research design, which allows to develop new hypotheses in a relatively unexplored area, or to establish theoretical or conceptual requirements, so it is possible to formulate initial hypotheses (Bortz & Döhring, 2006). Below will be described what the participants of this study had to do and then the sample will be characterized.

The assignment addressed itself to prospective teachers, who were at that time about to start a practical phase at a secondary school, which lasted five to seven weeks. During this time they had one week to get to know the school, to observe some teachers in their classes and to decide in which classes they wanted to teach. In the second week they started to teach about one or two lessons per day. The prospective teachers worked on the following assignment in a class and a grade of their choice during the second or third week of their practical phase.

First the prospective teachers chose a specific mathematics lesson for this assignment. Then they began to plan it. During this step they were supposed to already think about the necessary subject-related learning conditions that the students needed to have in order to reach the goals of the lesson. Afterwards the prospective teachers designed a few diagnostic math problems to determine the learning level of their students in school. A few days before the implementation of the planned lesson they gave these diagnostic math problems to their students, which worked on them during a prior math lesson. After the prospective teachers collected the students' answers, they analyzed the students work and interpreted the results of the diagnosis. Then they were asked to modify their planned lesson with regard to the diagnostic results, if they thought this to be necessary. The last step was to implement the (possibly modified) lesson. During these steps the prospective teachers were not supervised and they were not allowed to accept any help from the experienced math teachers.

In addition, an open, partially standardized, guided interview was conducted after the implementation of the lesson, in which the prospective teachers' thoughts and decisions were put into focus. Here they talked about their interpretations of the students' learning level as well as their reasons for the chosen modifications. These interviews were videotaped and then transliterated by the author. Overall the following data was collected: the first teaching plan, the developed diagnostic tool including the students' work, the modified teaching plan and the interview transcripts. The research design and the research focus are shown in Fig. 6.2.

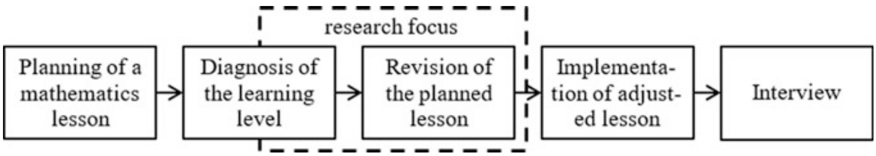

Fig. 6.2 Graphical representation of the empirical research design and the research focus

In February 2013, 15 prospective mathematics teachers participated in this study. Seven of them were female and eight were male. All prospective teachers studied mathematics to become a secondary school mathematics teacher. At the time twelve participants were in their seventh semester, three in their ninth (standard period of study in Germany: ten semesters). So, all of the prospective teachers had attended a lecture, which broached the issue of fundamental mathematics education. This included the justification and legitimation of mathematics as a part of the general education, reflections of the specifics of mathematical work, psychological principles of individual learning and social learning processes as well as consequences for the improvement of mathematical learning in the context of mathematics instruction. In addition, ten of them had gone to a lecture, which aimed to deepen their didactical understanding of either stochastics and analysis or geometry and algebra. Moreover, five prospective teachers already had participated in a seminar with an emphasis on diagnosis. Here they were taught how to develop diagnostic math problems and how to analyze students' answers. The fact that only five prospective teachers anticipated in this seminar led to the decision that all 15 prospective teachers had to attend a further seminar before entering the practical phase. In the course of the seminar they were shown criteria for "good" diagnostic math problems, they practiced developing such problems and how to analyze students' responses. However, the issue of adaptive teaching was not broached.

Still, it was—of course—possible that the prospective teachers identified the wrong learning conditions of the planned lesson or that they developed inadequate diagnostic math problems or that they analyzed the students' answers incorrectly or that they interpreted the results of the diagnosis wrong. This would be very unfortunate, but it does not affect the results of this study, because the focus is put on the decision process from the interpretation of the diagnostic results to the modifications of the planned lessons (see Fig. 6.2). At the end of the second step of the theoretical modeling of the modification process (see Fig. 6.1) the prospective teachers thought they had unearthed their students' learning conditions and this study wanted to understand what conclusions they drew from these. So at this point it does not matter, whether they identified the learning conditions correctly—of course for the implementation and the learning process of the students it makes a huge difference.

6.4 Analysis Method

Below the used methods of analysis are described. Here, the focus lies both on the approach of the collected data and the typification of arguments, because the used procedures were strongly adapted. However, the formation of the system of categories will not be depicted in detail, since Mayring's (2014) approach was implemented one-to-one.

6.4.1 Approach to the Collected Data

With a view to the collected data rose the question, how it could be compressed. This question came up because of two reasons: On the one hand, the data of the 15 prospective teachers was quite extensive. On the other hand, the participants of the study expressed many comments, which partly repeated themselves or were formulated in a different way—for example in the modified lesson plan and later during the interview. Here, it was the duty of the researcher to extract the statement that developed itself from these comments (Klein, 1980). The data was coded by the author of this contribution as well as another person with a mathematics education background.

For this purpose several procedures by Mayring (2014), which he proposed in the course of his *Qualitative Content Analysis*, were utilized. First of all a selection criterion was defined, which was determined by the theoretically derived subject of the creation of categories meaning the research questions of this study. This selection criterion allowed ignoring unimportant from the topic deviating text passages. The research questions suggested that only those statements were of interest, which addressed the results of the diagnosis, the prospective teachers' interpretations of the diagnosis results, the consequences for the planned lessons and the justifications for these consequences. All further comments were disregarded.

Hereupon the data set was worked through line by line (Mayring, 2014). As soon as a text passage complied with the selection criterion, which means that it could be assigned to one of the four described aspects above (result, interpretation, consequence, justification), it was color-coded and finally written out. Afterwards these comments were paraphrased. This included the elimination of all text components that lacked of content as well as the translation of all the remaining text components to a homogenous language level. For example, George's comment "However, it is also noticeable that a few students have problems to calculate the area of rectangles" was translated into the paraphrase "A few students have problems to calculate the area of rectangles". George expressed this in his second lesson plan. As he talked about the same diagnostic math problem during the interview, he mentioned "A few of them had difficulties with the calculations". This comment was also translated into the above paraphrase. Subsequently the originated paraphrases were reduced by combining those, which broached the same or at least a similar matter (Mayring, 2014). So from George's two comments, in which one was given in writing and the other one verbally, derived with this procedure the above paraphrase. According to Klein (1980) this paraphrase represents a statement, which George expressed using different comments —as shown above. All of the text passages that were extracted from the data set during the first cycle were treated in this way.

During the last step of the reduction of the collected data the statements of the prospective teachers were grouped according to the four aspects of the selection criterion, before another cycle of the data set was used to search for connections among these arguments. This enabled the graphic representation of which consequences deduced with which justification from which interpretation of which diagnosis result. Thus at this point so called argument trees (Klein, 1980) were utilized, which visualized not only the individual statements of the prospective teachers, but also how these statements formed an argument with one another (see Fig. 6.3). These argument trees made it possible to see all of the prospective teachers' diagnosis results, interpretations, consequences and justifications on a single page.

Afterwards an inductive system of categories was developed from the data set on hand. For this process further techniques of the *Qualitative Content Analysis* by Mayring (2014) were resorted to. Here especially his technique *Summarization and Inductive Category Formation* was used to form categories in the following three dimensions: the interpretation of a diagnosis result, the consequence for the planned lesson and the justification for this consequence. A full display of the taken steps during the development of the inductive category systems would go beyond the scope of this contribution, especially because the steps were implemented just like Mayring (2014) suggests them. However, Fig. 6.4. will give an overview of the analysis steps.

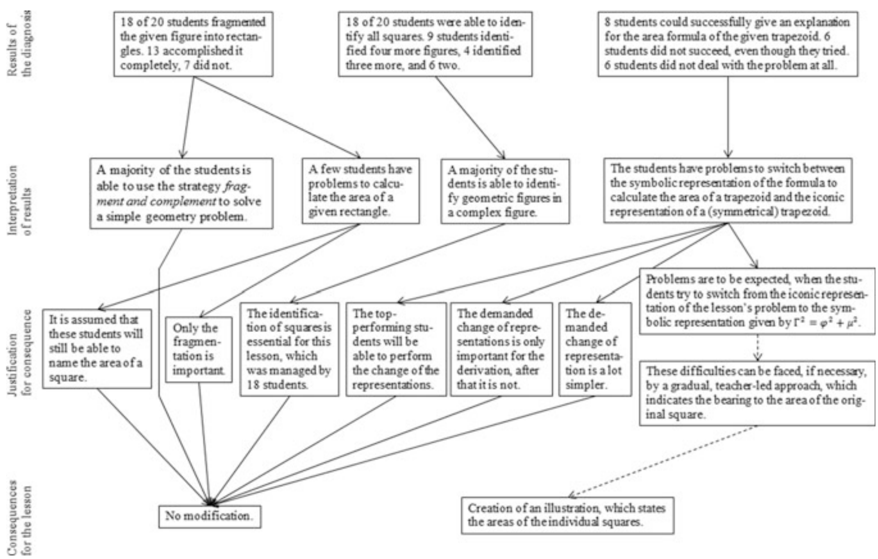

Fig. 6.3 Argument trees of George

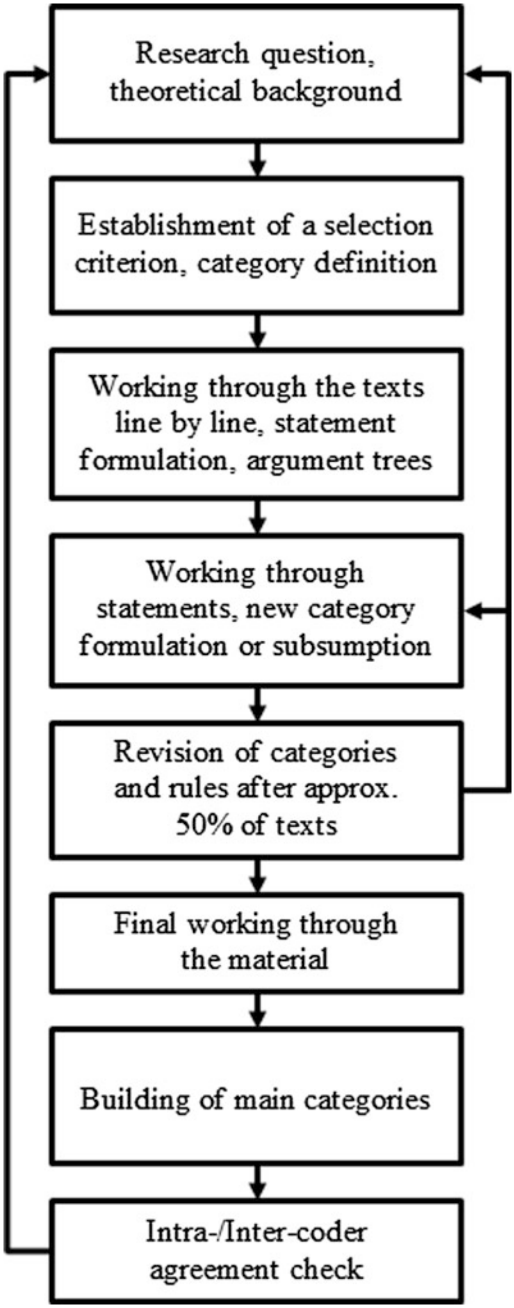

Fig. 6.4 Steps of inductive category development; based on Mayring (2014)

6.4.2 Typification

Below, how argument types were generated with the aid of the developed system of categories will be described. Overall this analysis procedure was oriented towards typification methods described by Kelle and Kluge (2010), but considerations of Mayring's (2014) *type-building content analysis* were partly taken into account as well. Kelle and Kluge (2010) formulate that a typology is the result of a grouping process. Thereby it is crucial that at the end internal homogeneity on the level of the type as well as external heterogeneity on the level of the typology is given. This means on the one hand that the individual components of a type should be quite similar, while on the other hand the types themselves should be very different from each other. Kelle and Kluge (2010) distinguish between one-dimensional typologies, which can be developed with regard to one single attribute, and multidimensional typologies, which are generated from a combination of attributes. In the second case the essential categories are combined and thereby an attribute space is created. This process can be illustrated clearly with the help of cross tables.

First of all, according to Mayring (2014) it is necessary to define the typification dimension plus the related specifications in order to work through the data set. In the study on hand, argument types were composed, which consist of a prospective teacher's interpretation of a diagnosis result, a consequence for the planned lesson and its justification. Consequently, the goal was not to assign a complete prospective teacher to one type of argument, but his or her arguments. So it was possible that the arguments of one prospective teacher would be allocated to multiple types. Thus, the typification dimension and its specifications were defined with the just mentioned aspects. A new cycle of the original data was not required, since the constructed argument trees already gave an excellent overview of all the existing arguments.

A typification process can be divided into four phases (Kelle & Kluge, 2010), which were all executed in the course of this study. First it was necessary to create relevant comparative dimensions, so that it was possible for categories to originate. These categories were needed both to identify similarities and differences between the arguments as well as to describe the developed types. This phase is similar to the above mentioned first step of Mayring (2014) and was already completed with the formation of the inductive category system.

During the second phase, the prospective teachers' arguments were grouped and empirical regularities analyzed. For this purpose the prospective teachers' statements were classified using the comparative dimension and the already developed categories. Via the use of multidimensional cross tables and the utilization of attribute spaces it was possible to determine all potentially appearing combinations of the categories as well as the actual frequency distribution of these combinations. The contrasting of arguments was also a part of this phase. This meant that arguments, which consisted of a certain combination of categories, were compared with one another to verify the above mentioned internal homogeneity of the originated types. In addition, it is essential to compare the types with each other to check if the external heterogeneity is fulfilled, because the diversity of the data set should be represented in the developed argument types.

The analysis of the content-related context was the focus of the third phase. The goal of the typification is not just to describe the appeared frequencies, but also to understand and explain the prospective teachers' arguments. Again, both the individual arguments within a type and the types themselves were compared and contrasted. The result of this analysis was that (a) arguments were moved to different types, because they were more similar to the arguments there, (b) peculiar arguments were for the time being ignored and later examined separately and (c) multiple types were combined, because they were similar to one another. This led to a reduction of the attribute space and hence to a decrease of the number of appearing combinations of attributes.

In the concluding fourth phase, the developed types of arguments were characterized on the basis their combinations of attributes as well as the identified and reconstructed content-related contexts. Kelle and Kluge (2010) note that many researchers would forget that this phase is an independent analysis step and yet the characterization is essential for the copious description of the individual types and for the further classification of other arguments. But one should also have in mind when describing similarities that the elements of a type are not identical. They are only similar. Both Mayring (2014) as well as Kelle and Kluge (2010) recommend to choose an illustrative prototype, which resembles the respective type especially.

6.5 Results of the Empirical-Qualitative Study

Below, the results of this study will be illustrated. First, the categories of two of the three dimensions—consequences and justifications—are explicated. Second, the eight identified argument types are depicted. Third, the empirical modeling of the modification process is described.

6.5.1 Consequences for the Lesson Planning

First of all, the consequences that prospective teachers deduced from their diagnosis for their planned lessons are presented. The analysis of the data set indicated the following five categories in the dimension *consequences for the planned lesson*:

1. no modifications
2. modifications of the subject-related content
3. modifications of a teaching step
4. adding support for or simplifying of a math problem
5. adding a learning objective

A total of 51 statements were classified in this dimension. Most of the time the prospective teachers came to the conclusion that the diagnosis results indicated that no modification was necessary. The second category includes modifications such as removing subject-related content from the current topic, illustrating the link between two mathematical concepts, establishing the relationship to everyday life or adding a revision, in which subject-related content from past topics is supposed to be reactivated in the students' minds. The consequences that were assigned to the third category refer to adding exercise sheets, changing the group classification or the educational reserve, or adding or changing the teaching step of securing the results. Modifications like adding solution cards, aid cards, written or oral hints, or diagrams, which are to support the solving process of math problems, fall into the fourth category. In addition, simplifying math problems is also a modification in terms of the fourth category. Only one statement was classified into the fifth category, but it is highly probable that the other prospective teachers pursued additional learning objectives with their modifications as well—however, they did not express this explicitly.

Overall, the data showed a broad scope of lesson modifications by the prospective teachers. It is possible to understand their planning decisions, if these modifications are linked to the correspondent justifications and the interpretations of the diagnosis results. Moreover, this could lead to further considerations regarding the difficulties, with which the prospective teachers are confronted, and which planning decisions are preferable or rather critical. For this purpose it is necessary to examine the justifications for the consequences first.

6.5.2 Justifications for the Consequences

The analysis of the data led to three different categories in the dimension *justification for the consequences*, whereby the third category also has seven subcategories (see Table 6.1):

1. no or little deficits resp. good planning
2. diagnosis results are irrelevant for the lesson
3. diagnosis results are relevant for the lesson

All in all, it was possible to assign 80 statements to the ten categories and subcategories. Statements, which addressed that (a) an aspect of the lesson was already well planned, (b) a problematic diagnostic task did not reveal any information about the students' learning level or (c) the students had the necessary competences available, were assigned to the first category. The prospective teachers' justifications, which were classified into the second category, broached the irrelevance of uncovered competences or deficits. These are, for example, only needed for one of many possible solution approaches or for the derivation of a theorem, but not its application. Furthermore, the prospective teachers argued that

Table 6.1 Subcategories of category 3 and their anchor examples

Subcategories of category 3	Anchor examples
3.1. Too much or too little was planned	"We will not get that far during the lesson."
3.2. Content has already been taught	"This topic has already been covered thoroughly. That is nothing new."
3.3. Links should be illustrated or established	"The difference between addition and multiplication of fractions should be clarified."
3.4. Math problem is too difficult	"The problems must be simplified. Finding the correct strategy should be easier and less open."
3.5. Prior knowledge should be reactivated	"Previous knowledge should be reactivated respectively recalled."
3.6. Partner or group work as a solution	"The students will solve the problems in groups, so they can supplement their knowledge."
3.7. Joint start resp. joint accomplishment of goals	"I want everyone to be on the same level."

either the knowledge gaps were uncovered accidently or the tested aspect was supposedly easier implemented in the planned lesson. If the prospective teachers stated that the availability or the absence of a diagnosed competence was problematic for the planned lesson, because it was needed, for instance, for the used worksheet, the assigned tasks or the application of a theorem, these statements were grouped into the third category. It also includes statements, which attributed the examined deficits gaps certain relevance, because the students would probably not have been able to understand the content of the lesson or to solve the given math problems due to their knowledge gaps.

Usually further statements, which justified the consequences more precisely, followed the conclusion of the diagnosis results' relevance. These statements were summarized into the seven subcategories of the third category. In the course of the first subcategories the prospective teachers concluded that they would be able to either cover more content than they had anticipated or less. The second subcategory comprised statements, which referred to the fact that dealing with a certain topic to the given time was not appropriate according to the core curriculum or that the topic was already broached extensively. An example for this subcategory is the justification that the planned lesson focused on the link between two mathematical concepts, so the prospective teacher had to act on the assumption that the students had comprehended these two concepts. The statements of the third subcategory addressed the necessity of clarifying or establishing a link or a transition between two mathematical expressions, representations or concepts. Other statements of this subcategory emphasize the need to broach the prior knowledge or the everyday experience of the students more intensively.

When the prospective teachers argued that the solution process of a math problem was at that time too difficult for the students and therefore the task had to be changed, so that, for instance, the strategy development is easier and less open, their statements were assigned to the fourth subcategory. The fifth subcategory contains justifications, which adverted to the necessity of reactivating prior knowledge in the students' minds. The statements of the sixth subcategory are again more multifaceted. Here, the prospective teachers brought forward the argument that the top-performing students would intercept the knowledge gaps by helping the under-performing ones due to the already implemented partner or group work. Further examples are the prospective teachers' assumptions that the students could solve the given tasks together, that the prior knowledge gaps could be closed or that the student could supplement their knowledge during the group work. All statements, which brought up the wish that all students should be at the same level or that they should have the correct solutions in their notebooks, were summarized into the seventh subcategory. The statements of this subcategory could also refer to the need that all students, and not only the top-performing ones, should accomplish certain goals of the lesson.

Consequently, similar to the consequences of the planned lessons, there was a grand variety of justifications for these consequences. As already mentioned the examination of the connection of all three dimensions—interpretation, consequence and justification—is important in order to comprehend the prospective teachers' planning decisions. Therefore these connections will first be illustrated in summary and then be more closely analyzed below.

6.5.3 Types of Arguments

A total of 104 arguments, each consisting of an interpretation of a diagnosis result, a consequence for the planned lesson as well as its justification, could be classified into eight different types (also see Fig. 6.5):

1. no modification due to problems with the diagnosis
2. no modification due to existing prior knowledge
3. no modification due to the irrelevance of the knowledge gap
4. no modification due to already planned group work
5. modification of the subject-related content to clarify a link
6. modification of the subject-related content to reactive prior knowledge
7. modification of a teaching step to establish similarities
8. simplification of math problems or adding support

In the course of the first argument type, the prospective teachers established that the negatively regarded results of the diagnosis had to be explained by problems with the diagnosis itself. Due to the fact that the prospective teachers were not able to determine their students' learning level, they did not modify the planned lesson at this specific point. The second type of argument occurred most often. A diagnostic task uncovered the availability of necessary, subject-related competences, so the prospective teachers did not make any modifications. The arguments of the third

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<tr><th colspan="2"></th><th colspan="5">Diagnosis</th></tr>
<tr><th colspan="2">Justifications</th><th>Problems with the diagnosis</th><th>Student knew or were able to do something.</th><th>Students were not able to do something.</th><th>Students were not aware of a link.</th><th>Students did not know something.</th></tr>
<tr><td rowspan="7">Consequences</td><td colspan="6"></td></tr>
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<tr><th>Consequences</th><th>Justifications</th><th>Problems with the diagnosis</th><th>Student knew or were able to do something.</th><th>Students were not able to do something.</th><th>Students were not aware of a link.</th><th>Students did not know something.</th></tr>
<tr><td rowspan="3">None</td><td>No deficits (determinable)</td><td>5 from 4</td><td>28 from 15</td><td></td><td></td><td></td></tr>
<tr><td>Results irrelevant; already good planning</td><td></td><td colspan="4">12 from 8</td></tr>
<tr><td>Results relevant, but group work is implemented</td><td></td><td colspan="3">16 from 9</td><td></td></tr>
<tr><td rowspan="2">Content</td><td>Results relevant, so clarification of links</td><td></td><td></td><td colspan="2">11 from 5</td><td></td></tr>
<tr><td>Results relevant, so reactivation of prior knowledge</td><td></td><td></td><td colspan="3">16 from 6</td></tr>
<tr><td>T. step</td><td>Results relevant, so common start/goal</td><td></td><td></td><td colspan="3">5 from 2</td></tr>
<tr><td>Support</td><td>Results relevant, tasks are too difficult</td><td></td><td></td><td colspan="3">11 from 6</td></tr>
</table>

Fig. 6.5 Overview of the identified types of argument (T. step = teaching step; entries in the cells indicate the number of arguments that were contributed by the number of prospective teachers, for instance, "12 from 8" means that twelve different arguments from eight different prospective teachers were assigned to the correspondent argument type)

type did also not lead to any modifications although the diagnosis detected knowledge gaps; however, these were appraised to be irrelevant due to several reasons. In the fourth type, the prospective teachers concluded that a small or a large part of the class did not possess the necessary, subject-related competences. Still, they did not modify the planned lesson, because the given tasks were to be solved in partner or group work, so the prospective teachers assumed that the knowledge gaps would be—in whatever form—intercepted by this teaching method.

The diagnosis of the fifth argument type referred to the problem that the students either did not know the link between two mathematical concepts or they confused them with one another. With the justification that the link between these concepts should be established or clarified, the prospective teachers modified the subject-related content of their lesson. The sixth argument type is characterized by a negatively interpreted diagnosis result as well as the conviction that the missing prior knowledge was already covered and therefore must be reactivated in the students' minds. Again, these considerations led to modifications of the subject-related content—usually broaching the prior knowledge in the course of a revision. Uncovered knowledge gaps were also the starting point of the seventh type of argument. Here, the prospective teachers took the decision to modify a teaching step of the planned lesson, such as additionally securing the results during the lesson, to ensure that all students accomplish a certain goal together. Both simplifying tasks and adding support for their solution process, for instance, by giving short oral or written hints, were the consequences of the eighth argument type to the absence of prior knowledge. The prospective teachers argued that the used math problems were too difficult for the students with regard to their learning level.

To this point the argument types were illustrated. Next, a part of the deeper analysis will be presented; whereby the focus is placed on the argument types 5 through 8. First, it can be observed that 13 of the 15 prospective teachers contributed at least one argument that was assigned to one of these four types. Hence, almost every prospective teacher actually decided to modify his or her lesson in some way. This is to be welcomed, since the reaction to an uncovered knowledge gap per se is something positive in general. It is satisfying that the prospective teachers recognize the need for action after they appraised the diagnosed deficits to be relevant. So in the case of the argument types 5 through 8 it is possible to speak of the implementation of adaptive teaching—at least to some extent.

If the arguments of the eighth type of argument, which address adding support for the solution process of exercises, are examined, one might think that the argument types 6 and 8 are quite similar, but this is not the case. The consequences of the sixth type wanted to intensively broach again the issue of basic concepts, such as relative frequencies, whereas the conclusion of argument type 8 aimed to remind the students of minor aspects. An example for the latter is the reminder of the scale factor, when the students were supposed to determine the equation of a parabola. The essential difference between the argument types 5 and 6 is that the prospective teachers whose arguments were assigned to type 5 did not only uncover and reactivate missing knowledge, but attributed it to the lack of knowledge of missing links or the confusion of concepts and reacted to this discovery.

Consequently, it can be positively mentioned that some of the prospective teachers actually distinguish between different kinds of mistakes and also react differently to those. If they diagnose the missing comprehension of a mathematical concept, they provide the needed knowledge instructively. However, if they come to the conclusion that their students confuse two concepts or are not aware of a link between them, they tackled these problems accordingly as well. Depending on the diagnosis result the prospective teachers recognize the need to build bridges (type 5), to close gaps (type 6), to establish a common ground (type 7) or to simplify or help with the given tasks (type 8).

Of course, there is always the danger to over- or underestimate the diagnosis results, especially the uncovered knowledge gaps, when modifying a planned lesson on the basis of diagnosis. This is due to the fact that the diagnosis is analyzed and interpreted by novices. Depending on how confident a prospective teacher feels relating to the own lesson planning as well as the content and the pedagogical content knowledge, the reaction to the results could be either insufficient or excessive. Both cases can be problematic. On the one hand, if the prospective teachers underestimate their students' knowledge gaps they might encounter the same problem like Bryan. His students were—according to his own statement—totally overstrained with the derivation of the formula for calculating angles between vectors in the three-dimensional space, because he underestimated the impact of their deficits. On the other hand, if the prospective teachers overestimate their student's knowledge gaps, they might have to make the same experiences as Paul. He had planned an extensive revision of the concepts of relative and absolute frequencies and had designed an exercise sheet as well. Later during the implementation of the lesson he realized that his students did not have deepened difficulties with the comprehension of the concepts themselves. They had only forgotten the word or the term for these concepts. In Bryan's case the students did not learn much during the lesson, while in Paul's case the lesson became less effective, because a lot of time was spent on an unnecessary revision.

6.5.4 Empirical Modeling of the Modification Process

A goal of the study on hand was also to empirically model the process, which leads from a diagnosis to adaptations of a lesson to the students' current learning level. Below, the used evaluation method are briefly illustrated, because due to the strong dependency of the exact approach on the developed system of categories as well as the identified types of arguments this illustration could not be given earlier. Afterwards, the results of these analyses are presented.

In the first step of the analysis the justifications of each argument type were examined more closely. During the process was checked whether these justifications were assigned to the same category or if they partly originated from different ones. In the first case the justification was converted into a polar question, which allowed the conclusion to the underlying justification. The second case was checked if the procedure of the first case was possible or if multiple polar questions had to be generated, so that the entire scope of the justification was still reflected. An example for the first case is the first argument type: *no modifications due to problems with the diagnosis*. The examination of the prospective teachers' justifications showed clearly that they asked themselves, whether their observed negative diagnosis results had to be attributed to problems with the diagnostic tasks or the implementation of the diagnosis. So, at this point the following polar question was developed: Were there any problems with the diagnosis? The third type of argument, *no modification due to the irrelevance of the knowledge gap*, is an example for the second case. Most of the given justifications addressed the irrelevance of a knowledge gap by expressing that they did not belong to the prior knowledge that was needed for the lesson. These justifications led to the polar question: Are the results relevant for the planned lesson? However, some prospective teachers argued that the planned lesson already intercepts the uncovered deficits, which meant that they were irrelevant due to another reason. Therefore a further polar question was necessary: Does the lesson already react to the results?

During the second step of the analysis the developed polar questions were brought into an appropriate and conclusive order. For instance, it would not have been reasonable that the prospective teachers first asked themselves if the implemented group work already reacts to a knowledge gap and then consider whether this knowledge gap has to be attributed to problems with the diagnosis.

In the third step an empirical model was generated, which illustrates the decision process of the prospective teachers from the interpretation of the diagnosis results to the modification of their planned lesson. It shows the polar questions that the prospective teachers asked themselves and also to what conclusion the affirmation or the negation of certain questions led. So, the result of the empirical modeling is a process model (see Fig. 6.6), which reflects the 104 arguments of the participants of this study. It consists of ten different ways, because the argument types 3 and 4 allowed respectively two action-guiding motives.

First the prospective teachers seemed to ask themselves if the results of a diagnostic task were to be regarded positive. Here, there is reason to presume that they compared the results of the diagnosis to their expected learning difficulties or the necessary, subject-related prior knowledge. If this question was affirmed by the prospective teachers they had to clarify if there were still some students, who did not have the needed competences—or at least parts of them—available. If this was not the case, the planned lesson was not modified due to the reason that no deficits have been discovered (cf. type 2). However, if there were students with some knowledge gaps, even though the diagnosis results were appraised to be positive,

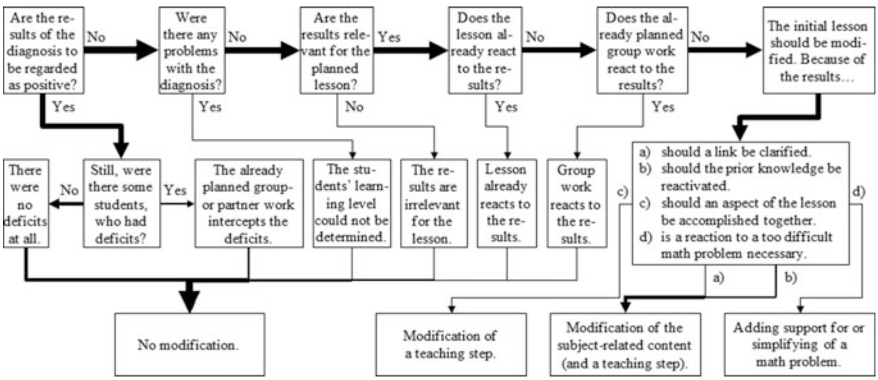

Fig. 6.6 Empirical modeling of the modification process; arrow thickness represents frequency

the planned lesson was nevertheless under no circumstances adapted. This decision was justified by stating that the already planned partner or group work would intercept these deficits (cf. type 4). All in all this implies that a positively appraised diagnosis result never led to a modification of the lesson, even if there were students, who did not possess the competences that were necessary to reach the lesson's goals.

In the case that the first question was answered in the negative—so the diagnosis results were not to be regarded as positive—the prospective teachers asked themselves, if the results were caused by problems with the implementation of the diagnosis or the used diagnostic tasks. If this was true, they concluded that the students' learning level could not be determined and therefore they were not able to adapt the lesson to their students' needs (cf. type 1). Could the negative diagnosis results not be attributed to problems with the diagnosis the prospective teachers contemplated, if the results were actually relevant for their planned lesson. If this was not the case the lesson was not modified either (cf. type 3). Were the uncovered knowledge gaps generally relevant for the lesson the prospective teachers considered, whether the lesson already intercepted these. An affirmation of this question did also not lead to an adaptation of the lesson (cf. type 3, again). The last question, whose affirmation resulted in the fact that no modifications were made, asked, if the already planned partner or group work could intercept the deficits of the students (cf. type 4).

If this was not true, the prospective teachers came to the conclusion that the initial lesson plan should be adapted on the basis of the diagnosis results. Depending on their motive, they (a) modified the subject-related content, (b) added support or simplified tasks or (c) adapted a teaching step. The wish to clarify or establish a link between two mathematical subjects (cf. type 5) or to reactivate prior knowledge (cf. type 6) always led to modifications of the subject-related content. If the prospective teachers wanted to accomplish goals or to start from the same initial point with the whole class, they adapted teaching steps (cf. type 7). Finally the awareness that the contemplated tasks were too difficult for the students resulted in adding support or simplifying tasks (cf. type 8).

Of course the model above neither claims that all students teachers have asked themselves all of these questions nor that they have done so in the suggested order. The goal was to develop a model that describes the appeared phenomena of this empirical-qualitative study. It makes it possible to recognize, for instance, how many polar questions have to be answered in a certain way so that the prospective teachers actually decided to modify their lessons on the basis of the diagnosis. Only when a diagnosis result was to be regarded positive, could not be attributed to problems with the diagnosis and was relevant for the planned lesson and when neither the planned lesson nor the planned partner or group work reacted to the results, the prospective teachers adapted their lessons to their students' needs. This phenomenon cannot be explained by the collected data, but many possible explanations are conceivable of which four will be addressed here.

1. The prospective teachers ponder thoroughly, whether an adaptation of the planned lesson would actually improve it. If they come to the conclusion that this is not the case, they look for an explanation or a justification with which they neglect the diagnosis results.
2. The prospective teachers concentrate on the (from their perspective) most essential prior knowledge gap. Other, less important gaps are, for instance, sourced out into the group work or described as irrelevant.
3. The prospective teachers do not know how they can or should react to the results of the diagnosis and therefore look for a justification, which explains why they do not react.
4. The quality of the prospective teachers' diagnoses is on such a low level that, on the one hand, there actually are problems with them and, on the other hand, they accidently uncover knowledge gaps, which are indeed irrelevant.

6.6 Conclusion and Outlook

Thus, the central results of this study were presented. The conclusion of this chapter begins with the explication of possible implications for the teacher education. Then an outlook will be given, which focuses on continuative research questions.

In summary, it was possible to show that the prospective teachers implement many of the single steps of the diagnosis and modification process to some extent well. However, it became also clear that the execution of the individual steps as well as the entirety of the process was partly problematic. The explanation approaches for the lack of reaction to some diagnosis results and the non-existent development of differentiating modifications, which were developed in the course of the empirical modeling of the modification process (see Fig. 6.6), give a first impression of the possible underlying difficulties. The question at this point is, whether the prospective teachers only lack practical experiences and exercises or indeed theoretical elements of knowledge as well. Either way, it therefore follows that first the individual steps of the modification process should be placed into focus. This means that both diagnosis and the differentiating, adaptive planning of lessons should be discussed and practiced separately, before the combination of these is tackled.

It is assumed that the theoretical level as well as the practical one is essential to learn the adaptive planning of lessons. On the one hand, various possibilities for differentiation and planning should be introduced in the theoretical part of the teacher education, whereupon their advantages and disadvantages should be discussed. On the other hand, diverse action alternatives should be talked about with the aid of concrete situations during the practical phases. The results of this study provide indications for this purpose. For example, the fourth type of argument, which does not modify the lesson due to already planned group work, leads to the opportunity to broach the issue of group work with the prospective teachers. When is this teaching method suitable? Which advantages does it possess? Which disadvantages or dangers does it implicate? What is the best way to implement it? A goal of group work is, for instance, to combine subject-related and social learning (Barzel, Büchter, & Leuders, 2011). In order for this to be achieved, it is necessary to lay the needed foundations. In particular the point of this teaching method is that different solution approaches are pursued by the individual members of a group and later discussed by the whole group. However, Barzel et al. (2011) do not explicitly mention that the purpose of group work is to have the top-performing students fill in the knowledge gaps of the weaker ones. In similar ways it should be discussed with the prospective teachers, in which cases it is appropriate to repeat already covered knowledge, to simplify math problems, to add support for the solution process of math problems or to clarify links between two mathematical concepts.

The following focuses on the outlook of this study. First of all it should be mentioned, that the collected data itself allows many continuative research questions. The initial lesson plan could be analyzed with regard to the used teaching methods, the selected math problems or other, similar aspects. Furthermore, it is possible to investigate, whether the participants of the study identified the necessary, subject-related learning conditions of their lesson correctly. The developed diagnostic tasks could also be a subject of an examination. Here could be checked, if the prospective teachers tested all of their identified learning conditions or to what extend the used math problems meet the criteria for diagnostic tasks (e.g. Dannenhauer, Debray, Kliemann, & Thien, 2008). In addition, the students' solutions of the diagnostic tasks are available. With these it would be possible to survey, whether the prospective teachers analyzed their data correctly.

The examined sample of 15 prospective teachers of the Carl von Ossietzky University of Oldenburg is in view of the preconditions, to which they resorted during their practical phase, relatively homogeneous because the department of mathematics education puts emphasis on diagnosis and improvement. Though the data indicated a few critical aspects during the implementation of the given task by the prospective teachers, it also showed a variety of positive issues, such as their competence-oriented perspective on the diagnosis results. But the question is which results would be received, if the research design was used to study prospective teachers from another university, which focuses on other issues within the teacher education. It is also conceivable that one would get very different results, if the prospective teachers' task is given to experienced teachers.

Moreover, all the aspects that have been revealed by the qualitative-explorative study at hand, whose surface has only just been scratched, should be researched more deeply. For instance, it was possible to unearth the prospective teachers' justifications for their selected consequences. But it was not discovered, for what reasons they dismissed alternative planning possibilities. Furthermore, the second type of argument, for example, which does not modify the planned lesson due to existing prior knowledge, raises the question, if the role of this available knowledge changes in the consciousness of the prospective teachers. Does its importance increase, because they know that they can rely on it or does it decrease, because they concentrate on the knowledge gaps instead?

To conclude, it should be noted that the design is not only suitable for research purposes, but also for the education of prospective teachers. As already mentioned above, at first it is important to address and practice all the single steps that are necessary to planning a lesson, which considers the students' learning conditions, individually. Afterwards prospective teachers could be asked to implement the task of the research design in their practical phase in order to try the entirety of the process. Combined with a close supervision, which proposes ideas for improvement from a mathematics education point of view, this could initiate effective learning processes.

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Part II
Use of Technologies and Tools

Chapter 7
Contemporary Framing of Technology in Mathematics Teaching

Rose Mary Zbiek

Abstract Many types of powerful digital technologies have been part of secondary school mathematics classrooms and mathematics teacher education in many places for decades. However, research on technology in teacher preparation continues to be sparse and challenging to synthesize. To organize and probe ideas, researchers and practitioners need better ways to frame their work. In this chapter, a blend of three conceptual tools is connected to existing literature to describe prospective secondary mathematics teachers' (PMSTs') professional growth in technology, content, and pedagogy in integrated and dynamic ways. The blending of Technological, Pedagogical, and Content Knowledge (TPACK); Mathematical Understanding for Secondary Teaching (MUST); and Play, Use, Recommend, Incorporate, and Assess (PURIA) perspectives underscores the complexity of learning to teach mathematics with technology.

Keywords Teacher knowledge · Technology use · Secondary mathematics
Mathematics teaching

7.1 Introduction

In research and practice, technology is both an object of learning and a pedagogical tool in the education of prospective secondary mathematics teachers (PSMTs) as PSMTs learn of technology and learn with technology. Diversity of technology leads to a body of literature that is challenging to synthesize in the interest of framing future studies and of informing practice. This chapter is not a research synthesis or another way to frame what PSMTs need to know and do with technology. The goal of this chapter is to integrate existing frameworks to better understand research and practice around technology as both object and tools in the

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_7

education of PSMTs. Underlying the integration is an assumption that PSMTs enter their preparation programs knowing something about each of technology, mathematics content, and pedagogy. Articulation and integration of the frameworks precedes a discussion of the implications of the amalgamated perspective.

7.2 Technology Opportunities and Options

In 2017, *technology* refers to an amazing array of products. Although it takes many forms, for the purposes of this chapter, "technology" refers to digital resources. Technology in mathematics education could be technology used for mathematical purposes, such as computer algebra systems, or for communication purposes, such as word processing packages and social media, or for other purposes, such as video analysis software that can be used by PSMTs to play and mark excerpts in the study of their classroom practice. Dick and Hollebrands (2011) refer to the first type of technology as "mathematical action technologies" (e.g., computer algebra system, dynamic geometry, graphics calculator, spreadsheet, online applets) and underscore, as do others, that not all of these products were developed initially for educational purposes. Bowers and Stephens (2011) refer to the second type of technology as "communication and visualization technology". PSMTs might enter teacher education programs with varying experience with one or more products in each of these general genres.

Technological tools exist in many different physical forms and can serve a variety of mathematical, pedagogical, or communicative purposes. Tools with very similar purposes can exist in different media. For example, graphing utilities can be found as phone applets, programs on laptop, and features of handheld calculators. Moreover, one tool might be used for different purposes. One example is the difference between a computer algebra system (CAS) used while solving a mathematics problem and the same CAS used to create a file in which a tutorial or assessment is embedded. As these nuances imply, one improvement in conducting, reporting, and synthesizing research and practice is to be attentive and clear about the technology being used and its mathematical, pedagogical, or communicative purpose.

7.3 Work of Educating Prospective Mathematics Teachers

As practitioners, mathematics educators prepare PSMTs to be the best possible teachers that they can be. As researchers, mathematics educators seek to understand not only how PSMTs develop their practice but also the nature of their knowledge, beliefs, identities, and other personal characteristics. PSMT preparation happens within the contexts of universities and schools and across mathematics content courses, pedagogy courses, and practical experiences in schools or in other educational venues. Accounts of technology use in secondary mathematics teacher education often fall in the important but sometimes challenging to defend overlap between a faculty member's research and the courses or programs in which the faculty member teaches.

In content courses, pedagogy courses, and practical experiences, discussions of technology can focus on PSMTs as learners with technology as the object of instruction or as individuals whose learning about other things is supported by technology. PSMT educators include a broad group of all who contribute to a teacher's development, such as, in a typical teacher education program, mathematics and statistics instructors, pedagogy course instructors, and field supervisors. The genres and specific pieces of technology PSMT educators choose to use and how they use those technologies can vary greatly. Pedagogy courses and practica also address the use of technology by the secondary school students whom the PSMTs instruct.

Although experiences are often spread across content courses, pedagogy courses, and practical venues, PSMTs must connect ideas across content, pedagogy, and practice to make sense of their preparation and to act on it in their own practice. For this reason, a body of technology-related literature, such as that referenced by Huang and Zbiek (2017), should be revisited in terms of what it reveals about how PSMTs develop understandings of content, pedagogy, and technology based upon what they know as they enter their teacher education programs—and how the PSMTs integrate new ideas and understandings.

Huang and Zbiek (2017) describe the process that led to the 18 articles they synthesize. Their multi-layered process for selection of the articles ensured that the cited studies were reported in detail in journal articles appearing in internationally circulated venues, were clear in their identification of technology, and were studies that focused specifically on prospective secondary mathematics teachers. The cited studies would ostensibly be useful in addressing fundamental questions that should be asked about the literature regarding technology and teacher preparation and how PSMTs develop understandings and improve their teaching practices. The task of posing such questions is problematic, however, in (at least) three ways. First, the very definition of *technology* is elusive. Second, mathematics teacher educators use a wide range of tools within the three contexts. Third, the secondary school student can be either an active user of the technology or a learner who benefits from the teacher's use of the tools.

A contrast of two of the 18 studies underscores the complexity of synthesizing the literature. For example, both Davis (2011) and Star and Strickland (2007) conducted their studies in the context of PSMTs developing their understanding of pedagogy. However, the two studies differed greatly in the technology used. Davis used computer algebra systems; Star and Strickland used video recordings of lessons. The two studies also differed in the aspects of pedagogy they targeted. Davis considered textbooks; Star and Strickland studied professional noticing. The mathematics content foci also differed. Davis' PSMTs considered the algebra and function strand across many lessons in a textbook; Star and Strickland's PSMTs viewed recorded lessons on angles, arc lengths, secants, and tangents. In Star and

Strickland's work, the secondary school students were not active users of the video. In Davis' work, secondary school students were expected to engage actively with the computer algebra systems.

As the comparison of Davis (2011) and Star and Strickland (2007) suggests, the literature on technology in secondary mathematics teacher education can be challenging to synthesize, but it can be synthesized. Huang and Zbiek (2017) chose to organize it around mathematics content courses, pedagogy courses, and practica. That parsing of the literature is useful in applying the results of the research to teacher education programs and course instruction. The same literature might be organized differently by attending less to the three typical venues of teacher education and more to PSMTs' learning, asking a question: *In general, of technology tools, mathematics content, and pedagogy, which is novel to PSMTs?* Asking this question synthesizes literature in a spirit of understanding and support of PSMTs continued development.

7.4 Probing the Literature About Technology and Teacher Preparation

A synthesis of literature used either to conceptualize new studies or to inform practice requires more than a collection of descriptive paragraphs of findings from individual studies. It requires having conceptual tools to frame and explain what existing studies offer. Because PSMTs are differently experienced with any one tool and its uses, and it is important to recognize what part of the experience is new to the PSMT, each of the following sections assumes that one of the three elements—technology, content, or pedagogy—is a novel piece for PSMTs. Each section then describes a conceptual tool that can be used to explore and describe in depth the subtleties and the nuances of the technology, the content, or the pedagogy. In a later section, the three conceptual tools are considered collectively.

7.4.1 When the Technology Is the Novelty

When digital technologies exploded in the late twentieth century, the tools were the novelty in classrooms for practicing teachers and also in teacher preparation. This is the setting, for example, during which technology became available for use in education. The Technological, Pedagogical, and Content Knowledge (TPCK or TPACK) framework (Mishra & Koehler, 2006) emerged as a useful conceptual tool to describe and support teacher development. Building from Shulman's (1986) seminal work around Pedagogy and Content Knowledge (PCK), Mishra and Koehler develop TPACK to capture the kinds of knowledge needed by a teacher to integrate technology into classroom practice. The framework includes

Technological Knowledge (TK), Technological Content Knowledge (TCK), and Technological, Pedagogical, and Content Knowledge (TPACK).

Bowers and Stephens (2011) call mathematics teacher educators to interpret TPACK as "an orientation that views technology as a critical tool for identifying mathematical relationships" (p. 290). The view of TPACK as orientation followed the prolonged attempts of these researchers to identify a set of skills and knowledge at the center of technology, content, and pedagogy that would be the skills and knowledge identified as TPACK. Bowers and Stephens began to ask a different question: "What factors *do* affect prospective teachers' development of a TPACK orientation?" (p. 290). The factors they identified included such things as a teaching style rich in "what if questions". Bowers and Stephens' identification of such factors led them to their conclusion that orientation rather than knowledge was critical. An improvement in discussions of technology as a novel agent in PSMT preparation is to recognize orientation as well as knowledge. Orientations, beliefs, and knowledge must be acknowledged and leveraged in teaching mathematics and in preparing PSMTs.

For situations in which technology is the novel element, mathematics teacher educators need to emphasize orientation as well as knowledge. Research that illuminates not only what orientations, knowledge, and beliefs PSMTs have but also how to address and leverage these things in teaching mathematics is critical. TPACK was explicitly explored in six of the 18 works cited by Huang and Zbiek (2017). The other 12 cited works were situated in content courses, pedagogy courses, and practical experiences but did not address exclusively, if at all, PSMTs' knowledge related to technology, pedagogy, and mathematics. For example, Davis (2015) considered how PSMTs read, evaluated, and adapted a textbook lesson that used computer algebra systems. In Davis' study, pedagogy was the novelty, while the technology and the mathematics content were more familiar.

7.4.2 When the Content Is the Novelty

Technology is not always that novel element for PSMTs, although it might be novel to the mathematics educators who work in PSMT preparation. Technology allows PSMTs to learn unfamiliar mathematics content and to learn new things about familiar mathematics content, making the mathematics content the novel element. Examples include such things as how PSMTs might understand and compose transformations of the plane differently after working with figures in dynamic geometry environments or with graphs in a Cartesian coordinate context. Another example might be how simulations and manipulations of samples in a dynamic statistics setting affects how PSMTs conceptualize and describe sampling distributions.

PSMTs learn not only new content but also have new opportunities to engage in mathematics. For example, PSMTs likely are familiar with exponential functions and their graphs. They might, however, not anticipate certain actions, such as

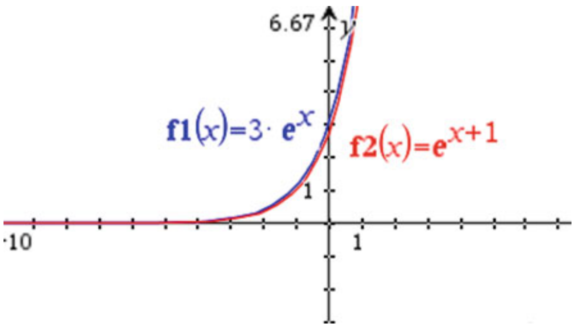

Fig. 7.1 Nearly the same graphs of $f(x) = 3e^x$ and $g(x) = e^{x+1}$

noticing that the graphs of $f(x) = 3e^x$ and $g(x) = e^{(x+1)}$ appear to be nearly identical, as in Fig. 7.1. This technology-based observation gives cause for a justification of whether these two graphs coincide. The observation also prompts the question of whether there are other pairs of functions of the form $f(x) = ae^x$ and $g(x) = e^{(x+b)}$ that have nearly the same graph. There is opportunity here to engage in the mathematical activity of noticing of the structure of mathematical systems. The structure of the symbolic representations is in contrast to the structure of transformations. Both graphs can be seen as transformations of the basic function, $p(x) = e^x$, with the non-trivial caveat that two transformations seem to map the basic function to the same graph—as shown in Fig. 7.1—but the transformations themselves are not equivalent. PSMTs engage in Mathematical Reasoning in terms of both Justifying/Proving and Reasoning When Conjecturing and Generalizing as PSMTs generalize their initial observations, test their claims, and symbolically verify their results.

As TPACK helps to sorts out different types of knowledge or orientations when technology is the new element, the conceptual tool does not allow nuances in mathematics content and action to be readily acknowledged. A framing different from TPACK—a tool that captures mathematical content, activities and the context of teaching—is needed. Mathematical Understanding for Secondary Teaching framework (MUST) (Heid & Wilson, 2015) is one such tool designed exclusively for mathematical knowledge for secondary mathematics education.

MUST is relevant to study and practice of secondary mathematics teacher education because it is based on the work of secondary school mathematics teachers. The emphasis on mathematics that is useful in teaching is a key point of the framework. Technology used for content development in teacher preparation serves well to extend and connect ideas that are common to school mathematics. The MUST framework captures the mathematics in Situations that were developed around incidents that happened in classroom settings and other venues of the daily work of teaching. The inspirational incident is what the MUST researchers called the Prompt. For example, consider the Prompt from the Division Involving Zero Situation, as shown in Fig. 7.2. The Situation is relevant to discussion of technology in secondary mathematics teacher preparation because not only might one

Fig. 7.2 Text for prompt from the division involving zero situation (adapted from Heid & Wilson, 2015, p. 95)

On the first day of class, preservice middle school teachers were asked to evaluate 2/0, 0/0, and 0/2 and to explain their answers.

There was some disagreement among their answers for 0/0 (potentially 0, 1, undefined, and impossible) and quite a bit of disagreement among their explanations:

- Because any number over 0 is undefined;
- Because you cannot divide by 0;
- Because 0 cannot be in the denominator;
- Because 0 divided by anything is 0; and
- Because a number divided by itself is 1.

use a variety of digital tools to explain the three indicated divisions in the Prompt but also one might encounter the situation within a classroom.

What might teachers use to address this Prompt? The answer to that question for a Prompt is found in the Foci. For example, two of the five foci for the Division Involving Zero Situation are the following:

- Mathematical Focus 2. One can find the value of whole number division expressions by finding either the number of objects in a group (a partitive view of division) or the number of groups (a quotitive view of division).
- Mathematical Focus 3. The mathematical meaning of ${}^{a}/_{b}$ (for real numbers a and b and sometimes, but not always, with $b \neq 0$) arises in several different mathematical settings, including slope of a line, direct proportion, Cartesian product, factor pairs, and area of rectangles. The meaning of ${}^{a}/_{b}$ for real numbers a and b should be consistent within any one mathematical setting.

The robustness of MUST relies on the quality of the Foci. The Foci for each Situation were created by a team of researchers from Penn State University and the University of Georgia and reviewed by another team of researchers from the two institutions. The Foci were also reviewed by groups of mathematics education researchers, of mathematicians, of mathematics teachers, and of mathematics supervisors at specified project meetings and public gatherings.

Qualitative analysis of hundreds of Foci produced in this way led to identification of three perspectives on mathematical understanding: Mathematical Proficiency, Mathematical Activity, and Mathematical Context of Teaching. The perspective of Mathematical Proficiency includes six aspects: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition, and historical and cultural knowledge. That is, Mathematical Proficiency was well captured by approximately six elements that echo the strands of mathematical proficiency in *Adding It Up* (National Research Council, 2001), with the addition of a strand of historical and cultural knowledge. The use of labeling from *Adding It Up* with slightly different descriptions was intentionally done both to allow for a connection between primary school and secondary school settings and to avoid offering a set of similar categories differently named.

The perspective of Mathematical Activity includes ideas offered under a variety of labels, such as mathematical practices (Common Core State Standards Initiative, 2010), process standards (National Council of Teachers of Mathematics, 2000), habits of mind (Cuoco, Goldenberg, & Mark, 1996), and specific areas such as mathematical modeling (Organisation for Economic Co-operation and Development, 2013). It extends to such things as symbolic insight (e.g., Bowers & Stephens, 2011). It includes Mathematical Noticing, Mathematical Reasoning, Mathematical Creating, and Integrating Strands of Mathematical Activity. Its first three areas can be further subdivided. For example, Mathematical Reasoning includes Justifying/Proving, Reasoning when Conjecturing and Generalizing, and Constraining and Extending.

Interestingly, in terms of technology in secondary mathematics teacher preparation, relatively little research is found in the first two MUST Perspectives. For example, Huang and Zbiek (2017) cite only two studies (Cory & Garofalo, 2011; Zengin & Tatar, 2015) that focus on Mathematical Proficiency. Both studies address conceptual understanding. A similar observation might be made about works related to Mathematical Activity. Notably, Huang and Zbiek cite only one study that addresses any form of Mathematical Activity. Zembat (2008) considers Mathematical Reasoning.

The small number of items related to Mathematical Proficiency and Mathematical Activity among the works cited by Huang and Zbiek (2017) might reflect the landscape of the 18 papers. Mathematical Context of Teaching, the third MUST perspective, is a view of the context for mathematical content in the MUST framework and might best be considered with Content issues. However, the details of the studies seem to fit better with pedagogy, for perhaps reasons that reveal both questions about research in the field and concerns about a dichotomy regarding technology use in secondary mathematics teacher preparation.

7.4.3 When the Pedagogy Is the Novelty

The main assumptions in Sects. 7.4.1 and 7.4.2 respectively, are that technology is the novel element in the teacher's practice and that mathematics is the novel element. These assumptions often work well in professional development with experienced mathematics teachers but they are not the totality of what is needed in work with PSMTs. PSMTs likely are digital natives who are familiar with school mathematics content and with a variety of mathematics, communication, or other technologies. For them, pedagogy is the new and intriguing piece.

As the third MUST perspective, Mathematical Context of Teaching considers mathematical work directly connected to the teaching of mathematics. The strands of this perspective are: Probe Mathematical Ideas, Access and Understand the Mathematical Thinking of Learners, Know and Use the Curriculum, Assess the Mathematical Knowledge of Learners, and Reflect on the Mathematics of Practice.

Works cited by Huang and Zbiek (2017) fell into only two of these categories. Two works addressed accessing the mathematical knowledge of learners (Akkoç, 2015; Santagata, Zannoni, & Stigler, 2007), and five of them explored accessing and understanding the mathematical thinking of learners (Hähkiöniemi & Leppäaho, 2011; Lee, 2005; Rhine, Harrington, & Olszewski, 2015; Star & Strickland, 2007; Wilson, Lee, & Hollebrands, 2011). The evidence suggests the need for mathematics teacher educators to help PSMTs to develop some aspects of mathematics understandings for secondary teaching (e.g., questioning, student thinking) and perhaps work with particular genres of technologies (e.g., video) of communication and collaboration technology. Using MUST to frame content issues within research and practice suggests there are aspects of mathematical understanding for teaching that need attention as they relate to pedagogy, especially in terms of technology.

It is important to keep in mind that in thinking about pedagogy the attention is not on the teachers (the PSMTs in this case) and their characteristics but on their teaching. An analysis of the current published studies about technology in teacher education from the sources used by Huang and Zbiek (2017) suggest that there is a growing body of literature about the use of digital video to capture, present, and study teaching.

In a synthesis of the research literature on technology, Zbiek and Hollebrands (2008) probe the literature about technology in the teaching and learning of mathematics to answer the question of how one learns to teach with technology—a question that essentially places pedagogy as the novel element in the teacher's practice. The curious answer to that question in a word is PURIA—a path to teaching with technology based in name and in spirit on Beaudin and Bowers' (1997) discussion of how teachers become proficient in using computer algebra systems. PURIA is an acronym representing modes (Play, Use, Recommend, Incorporate, Assess) through which Beaudin and Bower claimed teachers must pass in order to become highly proficient classroom technology users.

Although they located no studies that explicitly generated or tested the theory/framework at the time, Zbiek and Hollebrands (2008) argue that the literature to that date indicates that teachers do grow pedagogically (and technologically?) across these five realms. First, a person must Play with the technology and Use the technology for personal purposes. Although use of the word "Play" might give a different impression, the idea is that a person typically begins with open-ended opportunities to try what the technology can do.

Regarding mathematics technology, consider the introduction of computer algebra systems into a school. The person might attend a CAS workshop and be handed a CAS-capable calculator. The person's first instinct might be to Play with the device, figuring out how to turn it on, type some garble, enter an arbitrary function rule to see what a graph looks like. In these actions the person is not trying to do mathematical work, but rather attempting to see how the technology works and what it might do. Later that day, the person might then think about how the device might be used to solve a system of equations by graphing and then symbolically. Although he or she may fumble with keystrokes and menu options, the person is now working with a mathematical purpose and his or her intent it to Use the technology for personal mathematical purposes.

An example of the integration of a communication/collaboration tool might start with the mathematics teacher educator who Plays by drawing silly faces and saves and erases them when he or she first encounters an interactive white board. The next week, the mathematics teacher educator might Use it to draw and save a geometric diagram to show the class and have students mark as they work on proving a particular theorem. In this way, the mathematics teacher educator as the teacher of the lesson is making the interactive white board as a pedagogical tool for his or her own use but not yet making the interactive white board something to be learned by his or her PSMTs. In essence, the Play and Use phases seem indicative of opportunities for teachers (including teacher educators) to develop TK.

Following Play and Use, the person is prepared to Recommend use of the technology to others, Incorporate the technology into practice, *and* Assess students' use of the technology. These are the phases in which the person engages with others—and especially with students, as we might see by the continuations of the interactive whiteboard (IWB) and computer algebra system (CAS) examples in the next two paragraphs.

The individual who has begun to use CAS for his or her own purposes might next Recommend it to others. For example, the teacher might give it to a small group of students so they can learn how to produce solutions to the systems on their way to answering the question: "How many solutions can a system of two linear equations in two unknowns have?" The teacher would observe what happens and have some idea of how CAS was helpful—or not—for the students. A next move might be for the teacher to Incorporate the technology into a lesson on the number of solutions of a system so that all students might use it, and then the teacher might use it in other lessons as she or he Incorporates the technology into her or his practice. With reflection on these classroom experiences to Assess the technology's use and potential, he or she might conclude that the use is productive and then refine how he or she employs it in similar lessons.

The mathematics teacher educator who has started to use an interactive white board (IWB) as an instructor might next Recommend its use to a small group who is preparing a presentation for their class. The Incorporate move might then be the mathematics teacher educator offering a lesson in which all PSMTs or groups in the class have to use the IWB as part of their work, perhaps in lieu of a non-interactive presentation tool (e.g., PowerPoint or Prezi projected on a standard screen). The Assess piece might come with reflection upon whether and how the PSMTs used the IWB to enhance their presentations and to engage their classmates in the conversation.

Two points need to be made about PURIA before a discussion of how it might be integrated with MUST and TPACK. First, the framework as proposed by Beaudin and Bower (1997) has not been tested as a model of learning. Such empirical verification, though desirable, would be a long-term, demanding research effort that would likely involve more than one study. Zbiek and Hollebrands (2008) examined existing literature and noted how the compiled findings of the literature, at that time, supported PURIA as a framework for how teachers—including PSMTs—move from gaining initial knowledge of technology to developing classroom practice and pedagogy around the technology. Implicit in the following section is the observation that recent research in technology in teacher education also supports the PURIA framework.

7.5 Conceptual Tools to Inform Practice and Inspire Research

The blending of TPACK, MUST, and PURIA underscores the complexity of learning to teach mathematics with technology. The extent to which particular mathematics, specific technology, or pedagogy is the most novel and critical element depends on the individual PSMT.

To illustrate how the three conceptual tools are useful in understanding how PSMTs develop as teachers of mathematics with technology, consider a report by Bowers and Stephens (2011). The researchers argue that four of the resulting categories (TK, TCK, TPK, and TPACK) could be conceived as levels. Although one might be skeptical about whether these four TPACK orientation categories truly are levels in the classic sense of a level theory, connecting Bowers and Stephens' work to MUST and PURIA is enlightening, and perhaps fortifies their argument that the four categories are indeed levels.

Bowers and Stephens' bar graph in Fig. 7.3 illustrates the number of PSMTs that the researchers coded as being in each of four TPACK orientations. Although the numbers are small, the data invites the question of not only why are more of the PSMTs seeming yet to reach TPACK—the unspoken highest level—but also what might be the trajectory of their learning. The PSMTs arguably had some course experience with technology, so the claim here is not that this is a natural

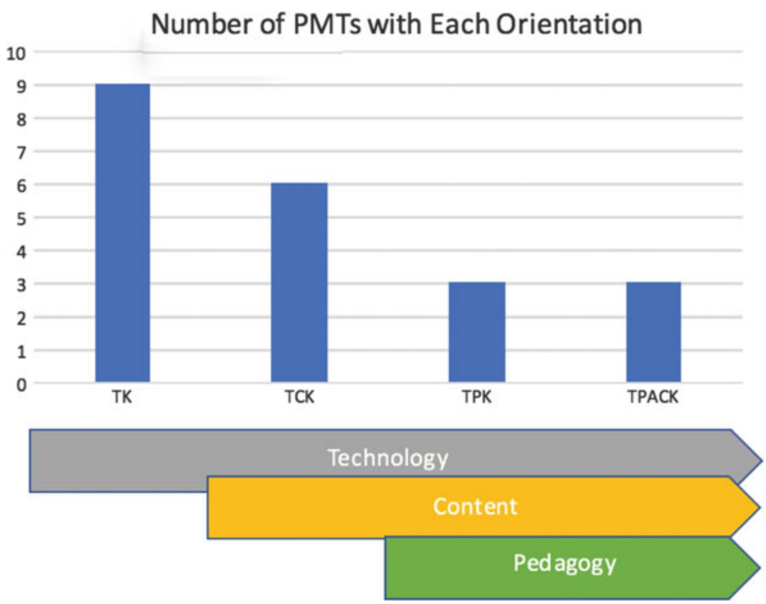

Fig. 7.3 Bowers and Stephens' (2011) coding results with banners representing focus on technology, content, and pedagogy

progression but rather it is a progression that agrees with multiple conceptual tools. A first look at the data suggests the relative prominence of technology, the moderate role of content, and the lesser role of pedagogy, in terms of what dominates the PSMTs' orientations. The horizontal arrows in Fig. 7.3 visually convey these relative roles.

Suppose the Technology, Content, and Pedagogy labels in Fig. 7.3 are replaced by "Play with Technology," "Use as Tool," and "Recommend/Implement," respectively, as shown in Fig. 7.4. This move, which might seem arbitrary at first, yields a revelation. Technology was present in all of the orientations—perhaps due to choices that Bowers and Stephens made. Replacing "Technology" with "Play with Technology" means the longest horizontal arrow could represent the development of technology orientations experienced through their teacher preparation program or by virtue of being digital natives. "Use as Tool," which replaces "Content," could be the use of tool to learn new mathematics or to understand familiar technology better. It also might be to learn about teaching and pedagogy through technology. Perhaps the orientation towards content follows from attention to PSMTs experience with technology as Use as Tool. The replacement of "Pedagogy" with "Recommend/Implement" might suggest that pedagogy-related orientations build on technology, which is part of all four orientations. In addition, the smaller number of PSMTs coded with TPK than coded with TCK might be a sign that Technology and Content orientations naturally and/or more productively precede Pedagogy orientations.

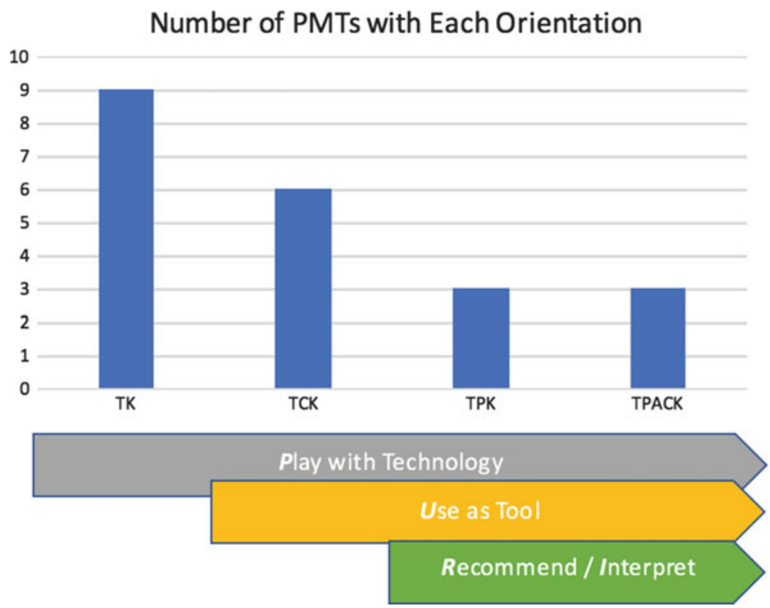

Fig. 7.4 Bowers and Stephens' (2011) coding results matched with PURIA modes

Researchers and practitioners interested in the development of PSMTs have provided some pools of related literature among a collection of studies that are largely descriptive of PSMTs and disparate in terms of the types of technology used. Examples of this tendency of existing literature are the studies cited by Huang and Zbiek (2017), as well as the earlier studies considered by Zbiek and Hollebrands (2008). Interesting work yet to be done, in addition to connecting the pieces of the current literature landscape, could undertake the overarching question of along what trajectory(-ies) do the technology, content, and pedagogy understandings and orientations of PSMTs develop and co-evolve. The claims stated in the previous paragraph are tentative. They raise a number of questions and suggest a number of subsequent studies aimed at exploring further how prospective teachers develop knowledge and orientations and integrate technology into their emerging classroom practices. Yet, the way in which components of the TPACK framework and those of the PURIA framework align provide support for both frameworks as feasible ways to look at PSMT development.

The reader might be wondering how MUST then fits into the framework picture with PURIA and TPACK. First, MUST as a way to refine how we look at Mathematical Proficiency and Mathematical Activity can be used to determine the extent to which technology is used for particular mathematical purposes. MUST's Mathematical Context of Teaching perspective touches on skills and actions often associated with pedagogy. For example, student thinking, school curriculum, and assessment—which correspond to Access and Understand the Mathematical Thinking of Learners, Know and Use the Curriculum, and Assess the Mathematical Knowledge of Learners—are common topics in many pedagogy courses. The implementation of these moves often fit within education but the corresponding mathematical understandings needed to execute these things are often left unaddressed in both content and pedagogy courses. They perhaps surface in practica as PSMTs must react to student thinking and assessment in authentic, open-ended ways. Use of MUST could help to focus both technology use and research questions about technology use in mathematics pedagogy courses and practical experiences.

Another point about MUST is that if technology is used for all of the elements of Mathematical Proficiency and Mathematical Activity, it seems that mathematics technology has the potential to be used for more than answer checking or the execution of basic procedures (a criticism often offered regarding the use of technology in mathematics instruction). Attention to the Mathematical Context of Teaching seems to allow for the blended use of mathematics tools and communication/collaboration tools. Focusing mathematics education research on both the type of mathematical work in which people engage and the nuances of pedagogy might be a productive way to simulate key aspects of the related work of teaching in pedagogy courses and of studying and improving practice in practica.

The final point regards connecting Figs. 7.3 and 7.4. The TPACK elements shown in Fig. 7.3 and the PURIA aspects shown in Fig. 7.4 are not necessarily mathematics-specific. While MUST aspects of mathematical understanding do not appear explicitly in Figs. 7.3 and 7.4, these figures are consistent with the idea of

PSMTs as learners coming to terms with particular technologies as tools for doing mathematics—and perhaps as tools for teaching mathematics—in the spirit of what Guin and Trouche (1999) name *instrumental genesis*. Instrumental genesis suggests that the PSMTs would have their ideas within each MUST perspective influenced by what technology offers and that they would begin to tamper with their tools to adjust the technology to their mathematical and pedagogical purposes. Integrating MUST, PURIA, and TPACK to the extent that PSMTs draw fluidly upon ideas within each of technology, content, and pedagogy and across these areas, using technology as a tool for both mathematics and pedagogy and to do so in reflective practice might be the ideal for which secondary mathematics teacher education research could reveal insights and secondary mathematics education practice could aspire to achieve and disseminate.

7.6 Implications

The extent to which different conceptual tools such as MUST, PURIA, and TPACK co-inform the work of teaching mathematics with technology in mutually complementary ways is an indicator of how robust they might be as tools to conceptualize and report research and inform and improve practice. Past and current literatures suggest that technology work with PSMTs happen around content courses, pedagogy courses, and practical experiences. Bowers and Stephens' (2011) data provide a rough snapshot of where a group of PSMTs are in their developing not simply knowledge but orientations towards technology use in the teaching of mathematics. Evidence from this chapter's compilation of empirical findings, literature, and theory—and perhaps the author's and reader's practice—indicate that informal experiences with technology (***P***lay) and then work with technology to do mathematics (***U***se) precede use of the technology with others in small ways (***R***ecommend) and then in major ways (***I***ncorporate). The observation suggests that PSMTs should encounter multiple forms of technology in all venues of their preparation, including mathematics courses, pedagogy courses, and practica and have time to work with these tools and provide time for PSMTs to develop the technology as their own tools and incorporate them into their daily work. Technology needs to be incorporated in teacher preparation in comprehensive ways for learning and for teaching. Importantly, mathematics teacher educators cannot assume that technology—or mathematics content, or pedagogy is the novel element for PSMTs—an assumption that was valid and necessary at the time that TPACK first emerged. The observations about the extent to which MUST's Mathematical Context of Teaching overlaps with pedagogy are indicators that more could be done in terms of how technology might serve to help PSMTs weave together their understandings of mathematics and technology.

The seemingly low number of studies located by Huang and Zbiek (2017), which amounted to less than two articles per year, raises several questions. First, where is the research on technology in secondary mathematics teacher preparation?

Is it not being submitted? Not being accepted? Or, is it shared more readily in venues other than journal articles? International norms and experiences with journal editing suggest each of these three questions raises a serious issue. Now is the time to contribute to the ongoing investigation of PSMTs' knowledge and skill in and across content, pedagogy, and technology and to the needed exploration of how PSMTs develop practice that incorporates technology as a full partner with content and pedagogy.

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Chapter 8
Prospective Mathematics Teachers' Perspectives on Technology

Mar Moreno and Salvador Llinares

Abstract This chapter examined prospective secondary mathematics teachers' perspectives on the role that technological resources play in supporting students' learning. In particular, we study prospective teachers' pedagogical reasoning in order to understand their decisions about the use of technology and their effects on students' mathematics learning. We analysed prospective secondary teachers' lesson plans on teaching mathematics through problem solving by integrating technology. Prospective secondary mathematics teachers' perspectives on the use of technology for supporting students' mathematical learning varied in two dimensions: (i) how technological resources are used, and (ii) what mathematical activity that prospective teachers should present to support students' learning. These dimensions are related to the idea of instrumental integration that is used to describe how teachers organize the conditions for instrumental genesis. We identified three ways of integrating technological resources.

Keywords Technology · Instrumentalisation · Mathematical activity
Teachers' perspective

8.1 Introduction

This research focuses on prospective secondary mathematics teachers' learning when using and integrating technology to support students' mathematics learning and reasoning (Goos, 2008; Niess, 2005; Tondeur, van Braak, Sang, Voogt, Fisser, & Ottenbreit-Leftwich, 2012; Wilson, Lee, & Hollebrands, 2011). Reviews on technological pedagogical content knowledge (Voogt, Fisser, Pareja Roblin,

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_8

Tondeur, & Van Braak, 2012) show that pedagogical beliefs affect how teachers integrate technology. Within teacher education contexts, it is necessary to study prospective teachers' pedagogical reasoning in order to understand their decisions about the use of technology, and how prospective teachers' technological reasoning affects their decision making while using technology (Yigit, 2014). Technology, in this study, refers to the use of applets and dynamic geometry software to design dynamic representations of tasks. Using technology resources in teaching is related to the increasing emphasis on how prospective teachers can learn to engage students in meaningful mathematics tasks using technological tools (Stohl, 2005). Technology resources are tools that can help prospective teachers enact their perspectives on teaching and learning in lesson planning. This can be done if prospective teachers unpack mathematical contents into their constituent parts to define learning goals in their lessons (Morris, Hiebert, & Spitzert, 2009).

During lesson planning, prospective teachers' pedagogical reasoning can come up within the context of learning how to use technology resources to engage students in meaningful mathematical tasks. When prospective teachers are doing lesson planning, they should make decisions about how to use technological resources and have to determine the nature of the problem solving activity they aim to achieve. Lesson planning, as an activity in teacher education programs, involves a psychological process in which prospective teachers visualize the future, inventories means and ends, anticipate students' strategies, and constructs a framework to guide their future actions (Santos-Trigo & Camacho-Machín, 2009; Schoenfeld, 2011), and also a phenomenological approach in which they tell us what they plan to do. In the activity of lesson planning, prospective teachers should design instructional activities to address different mathematical contents by aligning instructional activities with learning goals, anticipating students' responses, thinking about assessment tasks to determine if students understand the learning concepts.

The use of technology for teaching through problem solving, underlines some aspects of the mathematical activity such as visualization, representations, formulation and conjectures, and generalization (Moreno-Armella & Santos-Trigo, 2016) that should be taken into account in lesson planning. These aspects are different from the mathematical activity generated on "paper and pencil" problem solving (Santos-Trigo, 2007; Santos-Trigo & Camacho-Machín, 2009). The transformation of mathematical problems that aims at creating learning opportunities for students to learn mathematics is a context in which the prospective teachers' approach toward technology appears. For this reason, lesson planning is an adequate context to study prospective teachers' pedagogical reasoning and how they learn to teach (Morris et al., 2009).

Therefore, the goal of this research is to identify prospective secondary mathematics teachers' perspectives on the role that technological resources play in supporting students' learning, when they planning a lesson that integrates technology through problem solving.

8.2 Theoretical Framework

Research on prospective teachers' learning attempts to explain how they acquire knowledge, beliefs, values and attitudes of their profession. Nowadays, the attempts to introduce technological resources in mathematics teaching raise new challenges for teachers and teaching (Goos et al., 2010). Using technology in teaching can imply using new kinds of mathematical tasks, modifying the nature of mathematical activities in classroom based on a set of pedagogical principles. From a teacher learning's perspective, the way in which prospective teachers learn to integrate technological resources in mathematics teaching could be mediated by their beliefs about the nature of mathematics, mathematics learning and mathematics teaching. So, learning to teach mathematics when digital tools are presented should make prospective teachers rethink the nature of the mathematical activity during problem solving and they should reflect on the role played by the teacher. In this study, we bring together two aspects of work that address how prospective teachers learn to integrate technological resources in mathematics teaching. The first one focuses on how prospective teachers' perspective can condition their learning to teach. The second focuses on the process of how prospective teachers organize the conditions for instrumental genesis of the technology (Chai, Koh, & Tsai, 2013).

From this perspective, learning to use technological resources in mathematics teaching may show different "prospective secondary teachers' perspectives" about teaching and learning. These perspectives could be considered as cognitive references through which prospective teachers learn to make decisions on teaching (Simon & Tzur, 1999). Simon and colleagues (Simon & Tzur, 1999; Tzur, Simon, Heinz, & Kinzel, 2001) conceptualise the expression "teachers' perspective" as a structure of pedagogical conceptions—knowledge and beliefs, which are responsible for organizing some aspects of their practice. Teachers' perspectives influence their learning and their cognitive references to make sense of learning contexts. In our study, we focused on the perspectives underlying prospective mathematics teachers' activity in lesson planning. For designing activities that integrate technological resources in their lesson planning, prospective teachers need to anticipate information about students' understanding. When prospective teachers anticipate students' answers, they might adjust learning opportunities. Regarding prospective teachers' activities in lesson planning to introduce technological resources, we consider that it is possible to identify aspects related to traditional, perception-based, and conception-based perspectives characterized in a different context (Tzur et al., 2001). Tzur et al. (2001) point out that from a traditional perspective teaching could be characterized by teachers' attempt to transmit particular mathematical ideas to students. While from a conception-based perspective, teachers attempt to orchestrate conditions that engage students in actively seeing and connecting those ideas, seeing mathematics as a web of conceptions that students abstract through reflection (Olive, Makar, Hoyos, Kor, Kosheleva, & Sträßer, 2010).

Secondly, prospective teachers' who are learning to use technology to support student's mathematical understanding and to develop problem solving skills could be placed in the intersection of research on how prospective teachers organize the conditions for instrumental genesis of the technology proposed to the students and the extent to which mathematics learning is fostered through instrumental genesis. In this study, instrumental genesis is understood to be the shaping of thinking by the tool in the construction of mental schemes and instrumentalisation as analogous to activities that involve the shaping of the tool by users (Goos et al., 2010; Healy & Lagrange, 2010).

The way in which an artefact becomes part of an instrument in the hands of a student is called instrumental genesis (Drijvers, Kieran, & Mariotti, 2010). In this case, the way in which prospective teachers design students' learning opportunities by integrating technological resources could support or not students' instrumental genesis. The role played by the prospective teachers' lesson plan in sharpening the instrumental genesis (in its double role of instrumentation as the way the applets—as an example of artefact—affect students' behaviour and thinking, and instrumentalisation concerns the way the students' thinking affects the use of applets) will define these students' learning opportunities. Since instrumental genesis consists in developing students' cognitive schemes and techniques, prospective teachers' perspective on the nature of mathematical knowledge and the role of technological resources in the teaching and learning of mathematics, reflected in lesson planning, will define opportunities to interrelate technical and conceptual elements during problem solving. Furthermore, when prospective teachers anticipate key moments in problem solving situations in which students interrelate technical and conceptual elements, they could define the institutional conditions to support the enhancement of instrumental genesis. The way in which prospective teachers consider the interrelation between technical and conceptual elements, in their lesson plans, the interaction between the techniques involved in using the applets—as an artefact—and the students' mathematical thinking becomes apparent. Additionally, when prospective teachers had to think about key moments in problem solving situations to orchestrate students' collective instrumentation, they had to anticipate ways of didactic configurations (additional tasks, type of questions, and so) considering the various stages of a mathematical situation. These aspects define the ways prospective teachers could orchestrate students' collective instrumentation (Bueno-Ravel & Gueudet, 2009).

8.3 Method

8.3.1 Participants and Context

The participants were 25 prospective secondary school mathematics teachers enrolled in a course on mathematics teaching in a postgraduate teacher education program. The prospective teachers were graduates in mathematics, engineering, and —computer sciences. They had different levels of knowledge about the use of technology as resources for teaching.

The postgraduate program granted them the qualifications required to teach mathematics in Secondary Education and included courses of mathematics education, mathematics, pedagogical studies—psychological and sociological studies—and eight weeks of teaching practices in secondary school classrooms. The mathematics education subjects represented 30% of the program's workload. Courses in mathematics education are designed to provide prospective teachers with the knowledge of teaching and learning mathematics.

The course on mathematics teaching and technology (a mathematics education course) lasted 50 h (four hours per week for 13 weeks). In this course, prospective mathematics teachers analysed curricular standards, tasks and lessons from mathematics textbooks, they also had the opportunity to explore applets for teaching mathematics, discussed class-teaching situations (teaching cases) in which technology was integrated and analysed the consequences on students' mathematical activity when technology was integrated into mathematics teaching which focused on problem solving. Geogebra was a technological resource introduced during some of these sessions. In these sessions, prospective teachers engaged in exploring different mathematics contents with applets to understand the opportunities and constraints that could be likely to create whilst using technology in mathematics teaching and learning. When they had to plan a lesson, using technology that focused on problem solving, they needed to understand how technology resources offered opportunities and constraints to students' learning. Prospective teachers read and discuss several research papers related to mathematics teaching and technology (Santos-Trigo & Camacho-Machín, 2009; Stein, Engle, Smith, & Hughes, 2008).

8.3.2 Instrument

As an assessment task at the end of the course, every prospective secondary mathematics teacher was asked to select a problem from a secondary mathematics textbook and modify it to plan a lesson focused on problem solving and integrating the use of technology. Prospective teachers had to modify the problem to create opportunities that favoured students' instrumental genesis to support aspects of the mathematical activity such as making and proving conjectures, using multiple representations, facilitating experimentation and particularization, generating connections and generalization. Prospective teachers are required to use some technological resources (applets or dynamic geometry) in their lesson plans to support students' mathematical activity. They had to anticipate students' answers. For this purpose, prospective teachers had to highlight the learning goals of the lesson, and solve the problem. Prospective teachers used the following template:

1. Anticipate ways in which students could solve the problem to examine if they were aligned with the achievement of the goals.

2. Identify features of mathematical activity (specialize? particularize, making conjectures and testing conjectures, ways of communicating, etc.) and possible evidence of students' learning.
3. Anticipate key moments in the resolution process to pose new challenges to students. Prospective teachers had to anticipate mathematical processes, which could be enacted during problem resolution, to identify the strengths and limitations involved in using the various representations and consequently plan how to encourage students to formulate and pursue questions in an attempt to establish mathematical relations.
4. Anticipate which students' answers could reflect different understanding and provide comments on the type of help to students, and indicate additional tasks and intentional and systematic organization of the various artefacts in guiding students' instrumental geneses, through instrumental orchestration.

8.3.3 Analysis

We analysed the lesson plans by attending to: (1) learning goals defined by the mathematical activity, which prospective teachers expected to develop, (2) how technology was used, and (3) how students' instrumental genesis was considered, including their arguments for using technology and the implications of its use.

The problem in the lesson plan was classified with regard to its cognitive demand, as high-level or low-level in relation to the mathematical activity that prospective teachers were expected to generate. Problems were classified with a high level cognitive demand when the questions required students to make connections between multiple representations engaged students in the conceptual ideas underlying the procedures, provided a context to go from specification—to generalization (Stein, Grover, & Henningsen, 1996). This type of problem could require that students experiment to make a conjecture and prove it. In this case, prospective teachers used the problem to support the students' reflections about relations between different mathematics concepts and representation registers. Problems used in this type of lesson plans and how they were described allowed students to set goals and engage in activities to solve them. We infer from these features a conception-based perspective in which mathematics is "thought as a web of conceptions that humans abstract through reflection" (Tzur et al., 2001). This approach underlines the interrelation between technical and conceptual elements as evidence of instrumental genesis defining the teacher's intention to support the interactions between the students and the artefact with a particular learning goal in mind. Prospective teachers who designed this type of lesson underscored the closely related co-emergence of the technical and conceptual aspects during the problem solving.

On the other hand, a problem in the lesson plan was classified with low-level cognitive demand when it only required students to reproduce previously learned facts, using a procedure to calculate, without providing any explanation. This approach defines the use of technological resources as a tool to only "display" the mathematical subject matter. This perspective does not take into account the students' understanding neither the potential of different technological tools, like dragging or visualizing relations between different types of registers (analytical-algebraic and geometrical).

Furthermore, the way in which prospective secondary teachers used technology affordances, like dragging objects, using sliders and quantify parameters, informed us about their ideas on how to promote students' mathematical activity and the role played by technological resources (that is to say, how the genesis of instrumentalisation is handled by prospective teachers in lesson planning). We focused on the arguments given by prospective teachers to justify the role of technological resources, during lesson planning, on problem solving in relation to students' learning (the relation between tools and mathematics learning). That is to say, how prospective teachers considered the use of digital resources with a mathematical intention (the instrumentation). We compared the descriptions of how prospective teachers proposed to use technological resources, during the lesson, to identify the reasons for using a given technological resource. This focus allowed us to infer the relation between the technological resources introduced during problem solving, the learning objective defined, and how they anticipated the students' answers.

Finally, prospective teachers' pedagogical reasoning in lesson planning were compared in an attempt to identify the differences and similarities of possible pattern groups in the data provided by prospective teachers.

8.4 Results

Based on these lesson plans, we identified three groups of prospective teachers taking into account two dimensions to characterize their perspectives. The first dimension is related to the way prospective secondary teachers considered the mathematical activity when students are engaged in problem solving using technology: ways of supporting mathematical relations, mathematical properties that could be emphasised using technology, using particular cases to make conjectures and so on. The second dimension is linked to how technology is used. That is to say, how the genesis of instrumentalisation is handled by prospective teachers in justifying the lesson plan. In other words, how prospective teachers orient students towards the use of an artefact (instrumentalisation) and towards the problem solving (the instrumentation). The way in which the use of technological resources was planned allows us to relate prospective teachers' reasoning based on the nature of the mathematical activity proposed to students. We present below three cases to show the different perspectives of prospective teachers.

G1—Technological resources to "display"

In the first group (n = 13 prospective teachers), the use of technological resources in the lesson plan of problem solving was anecdotic. Prospective teachers in this group used the technology resource only to present some aspects of the problem without being related to the nature of mathematical activity that could be generated. The prospective teachers used the technological resource only as a tool for illustrating the problem but not for reasoning with it. For example Jesus, one of the prospective teachers in this group, planned to use the technological resources to "illustrate" the topic that had previously been introduced in his explanations. Jesus' problem was suitable for a class of 14–15 year-old students. His goal was to "illustrate" how to calculate areas and perimeters of a 2D-shape (Fig. 8.1) (Calculate the area in which a tied horse could eat grass if there were two stakes that conditioned the horse's movements).

Jesus planned to use Geogebra to draw the geometrical figure that defined the horse' grazing field and to verify the calculations previously made by hand. For this prospective secondary teacher, the use of technology did not influence the nature of the student's mathematical activity and went on to solve the problem without technology. He justified his lesson plan by defining technology as an "illustrating and proving" tool.

Jesus stated:

> As we can see, the maximum area in which the horse could move along is delimited by these two sectors of circles, the pink one with radius equals seven and the blues one with radius equals two. Therefore, the total amount of grass that the horse is likely to eat is the sum of the interior circular sectors. Using algebra, the areas of each sector would be the pink sector area = (3/4) * π * (7^2) and the blue sector area = (1/4) * π * (2^2), and the total area would be the sum of the two sectors' area, approximately, 118.59 m^2. If we use dynamic software like Geogebra, it is easy to see and verify that the result is the same! (Added emphasis)

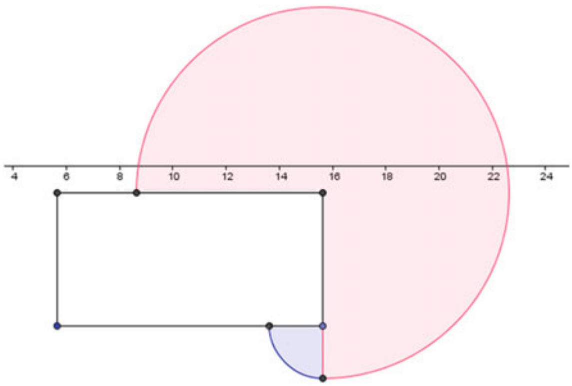


Fig. 8.1 Representation of Jesus' horse problem

When Jesus anticipated students' responses to exemplify difficulties in achieving the goals, he identified technical and procedural difficulties without indicating other high-level mathematical activities such as conjecturing, testing, particularizing and generalization:

> May be students have difficulties imagining the conditions of situations – how to go from one circumference to another, or how to consider the relation between the wall and the rope, … We could try to unlock these difficulties by posing questions like…:
>
> 1. Imagine that you are tied to a rope and you try to turn the corner, what happens?
> 2. What is the radius of the small circumference?
>
> For a possible generalization:
>
> 1. Would it always be the same if I put the horse on any other vertex?

To generate learning opportunities for students, prospective teachers in this group used the problems in their lesson plans without recognizing the potential of technological resources to modify students' mathematical activity. This feature makes transparent the potential of technological resources to visually represent geometrical invariants amidst simultaneous variations induced by, for example, dragging activities. So, for these prospective teachers, it was not possible to consider the utilities of Geogebra in interrelating the hypothetical mathematical conceptions that could have been developed (the question in the problems could be solved without the use of the technological resources). Consequently, it was not possible to talk of instrumental genesis. For example, Jesus focused on procedural aspects to calculate the areas and Geogebra was a tool used to validate the results previously obtained by a "paper and pencil solution":

> The purpose of this activity is to correctly represent and calculate the areas. The action followed by the teacher was to guide them to discover the steps that should be followed. This strategy is the most optimal. Thus, in situations that require such representations and calculations, students will know how to proceed. Even to use Geogebra to validate results.

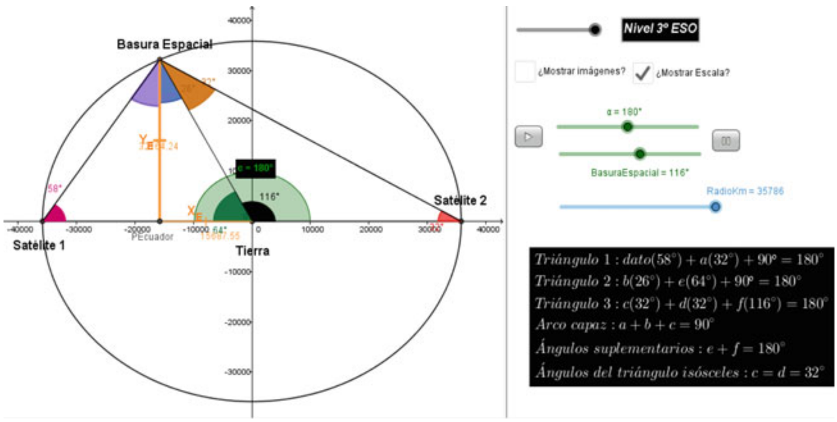
G2—Initiating the design of learning opportunities to support instrumental genesis

A second group of prospective teachers (n = 6 prospective teachers) planned to use technological resources to create learning opportunities for students to generate a mathematical activity that focused on the variability and relations between representation modes. These prospective teachers used sliders and dragging object as a means to discover mathematical relations. The problem used as a key element in the lesson plan generated a context in which the students' instrumental action would favour students' reflection about the relation between the action and the conceptual elements involved. These prospective teachers took advantage of potential offered by technological tools and provided the context for students to experiment and be able to relate solutions to different modes of representation or discover properties.

For example, David presented a modelling problem from an applet with Geogebra in which students (14–15 years old) had to connect the description of the situation with the use of an applet (Fig. 8.2) in a trigonometric lesson (isoptic, set of all the points from which a segment AB is seen under a given angle).

David initially proposed solving the problem without technology, and using technology to validate the calculations

> (With Geogebra, using the algebraic menu to get the values. Check the values obtained in the previous section with Geogebra.)



(With Geogebra, use the algebraic menu to get the values)

1. *Check that the values you got in the previous section correspond to what Geogebra shows?*
   How much are a (purple angle), b (blue angle), and c (brown angle)? And its sum?
2. *Change the values of α and find the value for which the problem is as simple as possible (it is not worth the Space Debris to be on top of one of the satellites)*
   a) *How much are angles a (purple angle), b (blue angle) and c (brown angle)? And its sum?*
   b) *How much is now Ye? And Xe?*
   c) *How much is the difference between α angle and the Space Debris angle?*
3. *Leaving α fixed, change the Space Debris angle*
   a) *Is there any other value for which you get the same result as in the previous case? What is the difference between these two angles?*
   b) *Is there any other value of the Space Debris angle for which you get the same Xe (ignoring the sign)? What is the relationship between these angles?*
   c) *And for Ye angle? What is the relationship between those angles?*

Fig. 8.2 Some questions in the David's problem to support the experimentation and the connection between representations, using sliders to make conjecture

He proposed to use the applet to generate a learning context to go beyond calculating. Using the Geogebra menu, he represented geometrical invariants from simultaneous variations induced by dragging. This prospective teacher introduced conditions in his lesson plan to generate opportunities to generate a "more or less stable sequence of interaction between the user and the artefact with a particular goal" (Drijvers, Kieran, & Mariotti, 2010, p. 109). In this case: modify values and notice the relation between new values. The goal of this sequence of questions in the lesson plan is: to identify invariants in the situation as a way of making mathematical conceptions emerge (in this example the mathematical notion of isoptic curve: for a given curve C, consider the locus of point P from where the tangents from P to C meet at a fixed given angle). The goal of the instrumented action scheme is to make the student notice the relation between the variability of parameters in the situation and the pattern that emerges from the mathematical conception in organizing this situation. The applet is designed to facilitate that student observe the connections between the graphical representation of the situation and the analytical expressions of the mathematical equations. Furthermore, the prospective teacher uses sliders to create a context to conjecture new relations between given values. This pedagogical use of sliders added a new aspect to the student's mathematical activity, conjecturing relationships between variables to modify the given values. The use of the applet create new learning situations for students enhancing mathematics activities as conjecturing relations between the given values that are not presented when the problem is enacted without technology.

However, when prospective teachers in this group anticipated students' responses, they only considered a procedural perspective of the students' mathematical activity. For example, when David anticipated students' answers he focused his attention only on identifying the equations, on the difficulties in solving systems of equations, and in handling the applet. This prospective teacher indicated the following as possible difficulties:

* Set the equation of the first triangle (data + a + 90° = 180°)
* Set the equation of the second triangle (b + 90° + e = 180°)
* Identify the congruence of angles in isosceles triangle (c = d)
* Set the equation of arc capable (a + b + c = 90°)

That is to say, while David could conceptualize a teaching situation through problem solving with the support of an applet, favouring certain mathematical processes as conjecturing, noticing the invariable in the situation, and setting connections between representations, he was only able to anticipate difficulties in identifying equations, in solving systems of equations and technical difficulties in handling the applet. When he indicated the student's difficulties, he focused on procedural elements but not on conceptual elements. In this case, the prospective teacher does not rely on the capacities of technological resources to generate learning opportunities in relation to the meaning of the capable arc and the properties of the angles inscribed on a circumference spanning the same arc.

This is apparent in the prospective teacher's behaviour: based on the different perspectives of his lesson plan, he anticipated students' mathematical thinking to be independent from the mathematical knowledge considered in the lesson. In this context, for example, Lourdes, a prospective teacher modified a problem of first and second grade equations to introduce Geogebra to facilitate the connections between different solutions. The problem is addressed to define a difference variable from the experimentation about particular cases

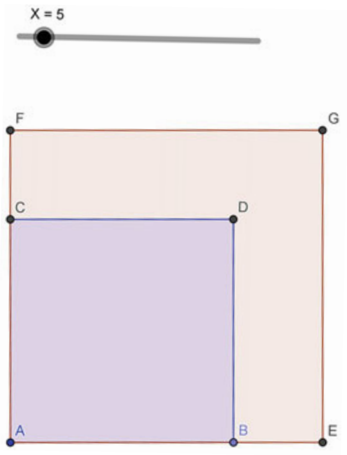
- Calculate the length of the side of a square, if by increasing its length by two centimetres, its area increases by 24 cm^2
- Construct the difference function of areas, represent it and obtain the solution
- Relate the dynamic model to the graphical representation of the function.

She guided the construction of the square and proposed to use sliders to approach the resolution of the problem (Fig. 8.3).

Lourdes use the technological resources to link different representation modes,

> We can take advantage of the potential of this program [Geogebra] to link the algebraic expression of the area difference function and the equation corresponding to the problem, as well as to establish connections between different resolutions.

This prospective teacher's approach to students' mathematical activities allows for the possibility of establishing connections between algebra and geometry.



Step 5: Create the difference variable
Create the variable dif= polygon2-plygon1, which gives us the difference between the areas of the enlarged square and the original square.
Step 6: We move the slider until we find the solution to the problem, which will be the value that the slider takes when, dif = 24. The solution is X = 5
Therefore, the side of the original square measures 5 cm and its area is 25 cm^2, while the side of the enlarged square measures 7 cm and its area is 49 cm^2

Fig. 8.3 Part of the Lourdes' square problem

In addition, it creates opportunities for students to guess the difference between areas by increasing the side length of the square. However, when she anticipated students' activity and the possible difficulties that students could face, her focus concentrated on the procedural aspects, not clearly explaining how to establish the relationship between the representation registers and the properties of the area function:

> Some students may correctly perform the resolution using the dynamic model, but do not reach the same solution from the graphical representation of the function of the difference of areas. Students may find the cut-off point of the graph of the function with the vertical line x = 24, instead of the point of intersection with the horizontal line y = 24. As the point of intersection is (24,100), the student would say that the side of the initial square measures 100 cm, which is the ordinate of that point. This indicates that the student has a good understanding of the geometric elements, but not the concept of function or the graphical representation of functions.
>
> Therefore, it interchanges the meaning of the coordinates of the points in the represented graph. In this situation, I would pose the following question:
>
> What do the points on the graph represent?
>
> I would ask him about the meaning of different particular points, so that he would arrive at the general idea. Then I would ask him for the meaning of the abscissa point 24, so that he would realize his error.
>
> Finally, I would ask: What point will give us the answer to the problem?

This prospective teacher focused exclusively on the meaning of ordered pairs did not take advantage of the potential of the relationship between the geometric screen, the graphical representation of the area function and the possibility of generating a table of values. This potential of the technological resource would have helped students to deduce the functional relation and the effect of the change of the value of the variables in the area of the square.

Prospective teachers in this group plan a lesson in which to integrate the potential of technological resources to favour the student's instrumental genesis. The problems used and their justifications of how to modify the cognitive demand of the problem are aimed at developing schemes and techniques. In particular, they were able to generate learning opportunities to identify invariant organization in a given situation. For that reason, they consider the conditions required for students to generate sequence of interactions using applets with a particular goal. With these characteristics of the lesson plans, prospective teachers support the co-emerge of technical and conceptual aspects; orchestrate conditions to engage students in seeing patterns and connecting ideas. However, this focus on the instrumental genesis in the lesson plan disappears when anticipating students' strategies and difficulties. When focusing on student's mathematical activity prospective teachers' perspective shifts to students' abilities to execute mathematical procedures. This difference between the perspective on the lesson plan and on student's activity reflects a dichotomy in how prospective teachers learn to integrate the technological resources when learning to teach.

G3—Integrating an epistemological stance about mathematical knowledge and students' mathematical activity

Finally, there is a third group of prospective teachers (n = 6 prospective teachers) that integrate their epistemological stance about mathematical knowledge in the lesson plan and how to anticipate students' mathematical activity. This approach showed an integrated perspective on the way of approaching mathematical activities with the support of technological resources and the cognitive stance on students' learning. For instance, Pablo a prospective teacher in this group chose a problem, in his lesson plan (for 14–15 years old students), which consisted of sub-problems from a particular case to general case:

> Calculate the length of the median in an equilateral triangle and the radius of inscribed and circumscribed circles to the triangle. Starting with an equilateral triangle (the length of the side is $10\sqrt{3}$ cm) and then with an isosceles triangle with the length of the different side is 12 cm.

Pablo's lesson plan is based on generating a mathematical situation, to support secondary school students in identifying the properties of mathematical objects and anticipate definitions. Pablo posed the problem, identified key moments of the resolution, which could be useful in getting over students' obstacles and difficulties, to generalize properties (Santos-Trigo & Camacho-Machín, 2009). For example, when Pablo anticipated the students' answers, he pointed out that some students would think that the property for the equilateral triangles works for all triangles. He identified relations between the mathematical contents and limitations of the properties that could be mobilised in resolving the problem (Fig. 8.4):

> In the hypothetical situation, a secondary school student would use the property, which is valid for equilateral triangles, with isosceles triangles, "**I will ask him to argue his answer and after then, he should make the construction**. For me, **it is a key moment for generating conflict and contradiction**. If **I supposed that the student shows me a good construction, which works, I would vary the length of one side and things began to fall down**."

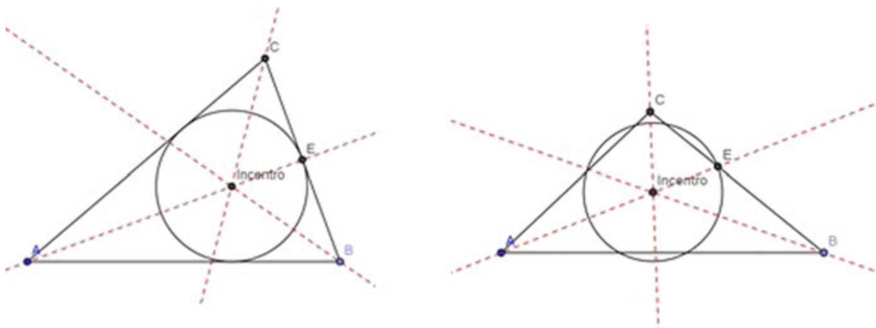


Fig. 8.4 Particular constructions that show limitations of the properties

Pablo anticipated possible challenges and difficulties for secondary students. He knew the difference between a geometrical construction and a simple representation or drawing. Dynamic software like Geogebra offers the possibility of taking advantage of its dynamism to enhance mathematical activity. For this reason, it is very important to consider the justification and argumentation that demonstrate what someone is exposing. Pablo indicated in his report that it is necessary that students realize that there are properties that only met some types of triangles. Pablo argued that point using the following figures constructed with Geogebra. Pablo displayed this fact modifying the lengths of the triangle sides.

For Pablo, using dynamic software could be suitable for learning mathematics and problem solving and let secondary students establish the differences between what it is a simple drawing and a "geometrical construction of mathematical objects". That fact underlines the role of technology in helping secondary school students to go more in depth into the knowledge of geometrical thinking (in this particular case, providing sense of the idea of geometric construction). It is exemplified by Pablo, in the particular case of an isosceles triangle whose construction coincides with the equilateral triangle when Pablo modifies the length of the side AC from $10\sqrt{3}$ to 11, it shows that the construction is not correct and students will have to look for other properties in constructing the circumscribed circle.

The characteristic of this lesson plan is that Pablo uses the technological resource to generate situations in which students can reflect on many particular cases to abstract the mathematical conception. The possibility of generating cognitive conflict when the generalization of a mathematical relation is not fulfilled is considered as a context that supports the student's reflection. In Pablo's lesson plan, Geogebra supports the generalization processes from sets of particular cases. Furthermore, when Pablo anticipates students' answers to the problem, he considers the potential of the dragging tool in Geogebra to represent geometrical invariants to induce visually the abstraction of the mathematical conceptions.

8.5 Discussion and Conclusion

This study examines how prospective secondary school mathematics teachers use technology resources, like applets and software of dynamic geometry as Geogebra and technological affordances as dragging objects, to quantify parameters and use sliders to support students' mathematics learning. The study uses Simon and Tzur's (1999) theoretical construct "teacher's perspective" to focus on how technology is used in a lesson plan and documented different ways in which prospective teachers use technology. These prospective teachers' perspectives are cognitive references through which they make decisions on teaching and it allows us to relate their epistemological stance about school mathematics (what type of mathematics activity could be supported) and what is the focus of students' mathematics learning.

We use two dimensions to define prospective teachers' perspectives. These dimensions take into account the nature of the student's mathematical activity that could be supported by the technological resources and how the prospective teachers plan to use the technological resources in problem solving. These dimensions are related to the idea of instrumental integration used to describe "how the teacher organize the conditions for instrumental genesis of the technology proposed to students and to what extent (s) he fosters mathematics learning through instrumental genesis" (Goos et al., 2010). We have characterized three ways in which technological resources, when integrated in prospective teachers' perspectives, show how they plan a lesson and anticipate students' mathematical activity.

Some prospective teachers pay more attention to the results than to the process of solution, and attach less importance to the students' mathematical activity such as conjecturing, proving, arguing, and connecting different representation modes. These prospective teachers turned technological resources into an end for itself and its use was anecdotic throughout the development of the mathematical activity and problem solution. On the other hand, some prospective teachers organised their lesson plan considering the mathematical activity generated by taking advantage of the potentials of the technological resource and by identifying key moments during problem solution. The prospective teachers' lesson plan is based on dragging objects, quantifying parameters and using sliders to support students' mathematical activity as conjecturing and testing, identifying properties and so on. For example, the identification of key moments in the problem solving process allowed prospective teachers to focus on the study of particular cases as an initial step in the search of properties, facilitating the connection between ways of representations and looking for problem alternative solution. However, other prospective teachers transformed problems in their lesson plans by using dynamic representations of problems to support mathematical relations, but when they anticipated students' mathematical activity they only took into account procedural mathematical aspects. These prospective teachers could be considered as those who have not yet established a bridge between the discipline's epistemological stance and the students' cognitive dimensions in learning to use the technological resources to support the mathematical learning in problem solving context (Santos-Trigo, Moreno-Armella, & Camacho-Machin, 2016).

These results suggest that learning to integrate technology in mathematics teaching aimed at promoting the development of the mathematical activity is a complex process. The different ways in which prospective teachers may consider technology as a pedagogical resource to support students' learning provide means of tracing learning trajectories of how prospective teachers learn about mathematics teaching (Stohl, 2005). Furthermore, we argue that the variability in which prospective teachers thought about technology and the role played by technology in problem solving could also be explained by the prospective teachers' beliefs about learning (Lin, 2008), and the nature of the mathematical task. This last issue emphasizes the need to carry out more research on the relations between knowledge, beliefs, and nature of the task in the lesson about how to use technology to support students' mathematics learning.

Acknowledgements We acknowledge the support received from the Spanish Projects: I+D+i, EDU2011-29328 and EDU2014-54526-R from the Minister of Sciences and Innovation, Spain.

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Chapter 9
Preservice Mathematics Teachers' Effective Use of Technology: Analyzing the Cognitive Demands of Technology-Based Instructional Activities

Ahmet Oguz Akcay and Melissa D. Boston

Abstract This study examined pre-service teachers' (PST) ability to integrate technology into instructional activities in ways that support students' mathematical thinking and reasoning, using the Instructional Quality Assessment to assess the cognitive demand of: (a) instructional tasks, (b) description of how tasks would be implemented or were implemented during the lesson, and (c) level of response expected from or produced by students. Results show that PSTs designed technology-based instructional activities with high-level cognitive demands and aimed to maintain high-level implementation and student response. Results suggest that focusing on cognitive demands of tasks and implementation may be productive for supporting PSTs to incorporate technology in ways that enhance students' mathematical learning.

Keywords Pre-service teachers · Mathematics · Technology · Cognitive demands Instructional activities

9.1 Introduction

Technology has been used for decades in K-12 teaching and learning environments and higher education. In mathematics education, the use of instructional technology has great potential. Technology can increase the quality of mathematical investigations, portray meaningful mathematics ideas to students and teachers from

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_9

multiple perspectives, and change traditional ways of doing mathematics (NCTM, 2000, 2014). In fact, the National Council of Teachers of Mathematics (NCTM) asserts in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014): "*An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking*" (p. 78).

The United States Department of Education's (USDE) *Preparing Tomorrows Teachers to Teach with Technology* (PT3) program has supported 441 grants since 1999 (see http://www2.ed.gov/programs/teachtech/index.html), and provided millions of dollars to educational institutions to better prepare in-service teachers and pre-service teachers (PSTs) to integrate technology effectively in education (Polly, Mims, Shepherd, & Inan, 2010). National organizations and educational researchers agree that technology must be a central focus of courses preparing PSTs for the classroom (International Society for Technology in Education, 2000; Niess et al., 2009; Thompson, Schmidt, & Davis, 2003; USDE, 2010). Many teacher education programs offer technology courses for PSTs to improve their skills in integrating technology (Polly et al., 2010); however, these courses often feature only basic technology skills (Kay, 2006). Research also indicates that, while PSTs can improve their technology integration skills from field experiences and student teaching (Chen, 2010), PSTs often incorporate technology into the field or student teaching classroom in superficial ways (e.g., presentation technology) that do not serve to enhance students' learning (Johnston, 2012). In mathematics classrooms, this would include using technology for presentation, computation, or in other such ways as an *amplifier* (e.g., using technology to support previous ways of teaching mathematics), but not using technology as a *reorganizer* to significantly restructure students' engagement with or learning of mathematics (Sherman, 2011, 2014). In fact, studies show that mathematics PSTs use technology most frequently as a presentation or demonstration tool and least frequently to support students' reasoning and problem solving (Johnston, 2012).

The purpose of this study is to explore pre-service teachers' ability to design mathematics lesson activities that integrate technology in ways that support students' learning of mathematics. The PSTs in this study all participated in mathematics methods courses that emphasized the level of cognitive demand of mathematical tasks and task implementation (e.g., Stein, Smith, Henningsen, & Silver, 2009; Stein, Grover, & Henningsen, 1996) in planning, teaching, and reflecting on mathematics instruction. In this study, we analyze the cognitive demands of technology-based instructional activities created and/or used by pre-service teachers as a measure of the effectiveness of the activities in supporting students' learning of mathematics. In the next section, we describe how cognitive demand serves as a framework for this study and provides a pedagogical structure to support PSTs' integration of technology into instructional activities in the specific content area of mathematics. While our work centers exclusively on PSTs in mathematics, we posit that a cognitive demand perspective can be generalized beyond PSTs and beyond mathematics to support and assess PSTs' and inservice teachers' use of technology to enhance students' learning in other content areas, as well.

9.2 Theoretical Background

In preparing PSTs to use technology, teacher educators must consider how to enhance PSTs' knowledge and practices in using technology, content, and pedagogy concurrently. Specifically, we must prepare prospective teachers to use technology in ways that support students' learning in specific content areas. Decades of research in mathematics education have identified the importance of the cognitive demand of instructional tasks and task implementation in supporting students' learning of mathematics (e.g., listed chronologically: Doyle, 1988; Stein et al., 1996; Boaler & Staples, 2008; Boston & Wilhelm, 2015). Hence, in this study, we utilize research on the impact of cognitive demand on students' learning of mathematics as the underlying theoretical basis for analyzing PSTs' effective use of technology to teach mathematics.

9.2.1 Cognitive Demand Perspective

Drawing on the work of Stein and colleagues (1996, 2009), we define a *mathematical task* as a single complex problem or set of problems that focuses students' attention on a particular mathematical idea, and we define *cognitive demand* as the level and type of thinking required to solve a task. Stein and colleagues delineate four levels of cognitive demand in tasks and task implementation: (1) memorization or stating previously learned facts or definitions; (2) procedures without connections to meaning or understanding (e.g., applying rote computations or algorithms); (3) procedures with connections to meaning and sense-making; and (4) doing mathematics (e.g., mathematical problem solving). Tasks at the level of memorization and procedures without connections place only low-level cognitive demands on students' thinking; tasks classified as procedures with connections and doing mathematics engage students in high-level thinking and reasoning.

Empirical research consistently indicates a positive association between students' engagement with high cognitive demand tasks and higher student achievement and learning in mathematics (Boaler & Staples, 2008; Boston & Smith, 2009; Stein & Lane, 1996; Tarr et al., 2008). Similarly, mathematics curricula intentionally designed to contain high-level tasks have been shown to increase students' mathematical achievement, reasoning, and problem-solving at the elementary level (e.g., Schoenfeld, 2002; Sztajn, Confrey, Wilson, & Edgington, 2012), middle school level (e.g., Cai, Wang, Moyer, Wang, & Nie, 2011; Post et al., 2008), and high school level (e.g., Grouws et al., 2013; Schoen, Cebulla, Finn, & Fi, 2003).

This body of evidence supports the importance of the cognitive demand of mathematical tasks in providing students opportunities to learn mathematics. Equally important is the implementation of those tasks throughout the instructional episode. Stein and colleagues' work (Stein et al., 1996, 2009) indicated that the cognitive demands of tasks can be altered throughout a lesson, and they developed the Mathematical Tasks Framework (MTF) to identify key points at which these alterations are likely to occur. From an analysis of over 300 mathematics lessons, Stein and colleagues convey in the MTF that task demands are frequently altered from: (1) the task as written in the textbook or curricula; to (2) the task as set-up or introduced by the teacher; to (3) the task as implemented by the teacher and students throughout the lesson. Lessons in which high cognitive demands were maintained throughout the lesson were associated with the greatest gains in students' learning (Stein & Lane, 1996).

Drawing on this work, we consider the cognitive demands of instructional tasks and task implementation in technology-based instructional activities created by PSTs as an indicator of PSTs' ability to effectively use technology to enhance students' learning of mathematics. Stein and colleagues' levels of cognitive demand and Mathematical Tasks Framework (Stein et al., 1996, 2009) served as a central frame: (1) in the teacher preparation courses (in elementary, middle level, and secondary mathematics) taken by PSTs; (2) in our analysis of PSTs' use of technology in ways that support students' learning of mathematics; and (3) in the research tool (Boston, 2012) used to analyze technology activities created by PSTs.

Supporting PSTs to select and implement tasks with high cognitive demands in a specific content area (mathematics) requires enhancing their pedagogical content knowledge (Ball, Thames, & Phelps, 2008; Shulman, 1986). Supporting PSTs to select and implement technology-based instructional activities that support students' learning in a specific content area requires enhancing their *technological* pedagogical content knowledge (Koehler & Mishra, 2009), discussed in the following section.

9.2.2 PSTs' Knowledge of Technology, Pedagogy and Content

Technological pedagogical content knowledge (TPACK) (Koehler & Mishra, 2009) stems from the idea of pedagogical content knowledge (PCK) originally described by Shulman (1986). At the intersection of pedagogy and content, PCK describes teachers' knowledge of pedagogical strategies and students' ways of thinking and reasoning specific to the discipline. In mathematics, Ball et al. (2008) delineate additional components of content knowledge (common content knowledge, specialized content knowledge, and horizon content knowledge) and pedagogical content knowledge (knowledge of content and students, knowledge of content and curriculum, and knowledge of content and teaching) to highlight the domains of

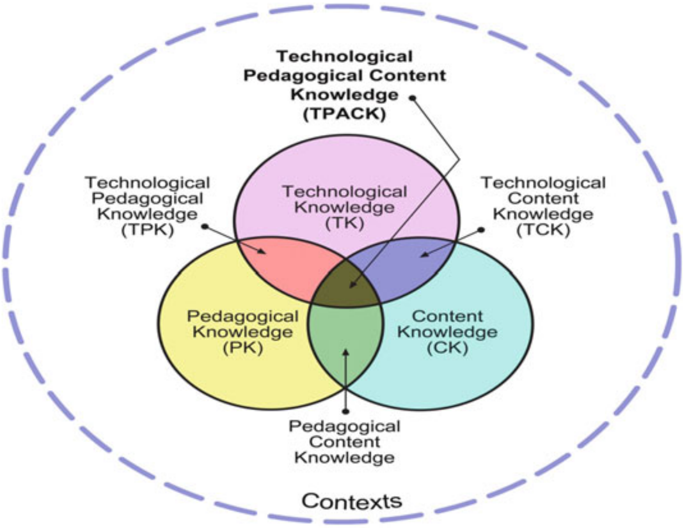


Fig. 9.1 TPACK model. Retrieved from http://mkoehler.educ.msu.edu/tpack/what-is-tpack/

mathematical knowledge for teaching (MKT). Shulman's general PCK model and Ball's specific domains of MKT elevate the importance of teachers' knowledge of pedagogical strategies capable of supporting students' learning within a specific discipline. The TPACK model (Fig. 9.1) adds knowledge of technology to the key components of PCK. The center of the model represents teachers' knowledge of technological tools that could be used to support discipline-specific pedagogical strategies and students' learning.

Grandgenett (2008) determined that mathematics teachers with a strong background in TPACK positively impacted students' learning of mathematics. Research has also identified relationships between PSTs' training and preparation to use technology and their actual use of technology (computer and Internet) in the classroom (U.S. Department of Education, 2010). Hence, teacher education programs should provide PSTs with opportunities to build instructional technology skills, in addition to content knowledge and pedagogical practices (Koehler & Mishra, 2009).

In this investigation, we take the stance that pre-service mathematics teacher education should develop PSTs' knowledge of specific pedagogical strategies empirically shown to support students' learning of mathematics. Such strategies (e.g., questioning techniques, use of cognitively demanding tasks) have been identified as "non-negotiable," effective mathematics teaching practices (NCTM, 2014) and thus represent essential components of PSTs' pedagogical content knowledge (PCK). By adding technology to these components of PCK, we define TPACK specific to mathematics teaching and learning as PSTs' knowledge of instructional strategies incorporating technology *that serve to support students' learning of mathematics*. Specifically, we consider the level of cognitive demand of tasks and task implementation in technology-based instructional activities as an indicator of PSTs' effective use of technology to support students' learning of mathematics.

9.2.3 Prior Research on Technology and Cognitive Demands

Previous research has examined the level of cognitive demand of technology-based instructional activities. Sherman (2011) grouped teachers' use of technology into two main categories: (1) an amplifier, where "technology allows for more efficient execution of by-hand procedures," and (2) a reorganizer, where technology "has the potential to change the cognitive focus of the task, for example, by giving students access to mathematical concepts, representations, or behaviors that might otherwise be difficult or impossible" (p. 121). Sherman observed and interviewed four secondary mathematics teachers (three high school, one 6th grade; all third-year teachers) and collected samples of students' work. All four teachers graduated from a teacher preparation program strongly focused on selecting and implementing cognitively challenging instructional tasks. Sherman found that 23 of 48 (47.9%) tasks using technology at set-up (i.e., in the beginning or "launch" of the lesson) had high-level cognitive demands. However, only 7 of 56 tasks (12.5%) using technology in implementation (i.e., lesson activities) maintained high-level demands throughout the lesson. Low-level mathematical tasks were often associated with technology serving as an amplifier, which generally "had little or no influence on the cognitive demand of the task" (p. 292) and did not induce thinking or reasoning on the part of the students. Using technology as a reorganizer, or as both reorganizer and amplifier within the same implementation, was associated with students' engagement in cognitively demanding mathematical work and thinking.

Johnston (2012) collected mathematics lesson plans and reflective documents from 35 PSTs. Johnson categorized technology lesson plans into four types: display (21; 64.3%), student exploration (7; 17.9%), review and practice (6; 14.3%), and productivity (1; 3.6%). Johnson reports that 15 of the 35 (42.9%) pre-service elementary teachers selected to use the SmartBoard, which was the most popular choice of technology in their lesson plans. Hence, the majority of PSTs in Johnson's study planned lessons that used technology in limited ways and did not engage students in high-level thinking and reasoning (e.g., using technology for display or review and practice). Johnson's framework attends to how technology was being used *by the teacher*, rather than the impact of technology on *students' learning*. Similarly, recent work by Hollebrands, McCulloch and Lee (2016) considers how technology is positioned in PST's lesson plans (e.g., for mathematics versus for display) and whether the use of technology is teacher-centered or student-centered.

In this investigation, we highlight the use of technology to support students' learning of mathematics using the lens of cognitive demand. By considering the cognitive demands in a technology-based instructional activity, we switch the focus of technology use from the *role of the teacher* to the *impact on the student*. Hence, this study adds to previous research by considering how PSTs use technology *to support students' learning of mathematics*. In this way, this study sits at the intersection of PCK and effective use of instructional technology; in other words, in the TPACK space. This study addressed the following research question:

> When pre-service mathematics teachers create technology-based instructional activities during a mathematics methods course and/or for use during student teaching, what are the cognitive demands of: a) the instructional tasks; b) the implementation of the instructional tasks; and c) the level of students' response?

In the next section, we describe the context and methodology of the study.

9.3 Method

The purpose of this study is to explore how pre-service teachers design mathematics lesson activities that integrate technology. Specifically, we analyzed the level of cognitive demands of technology-based instructional activities: (1) created by PSTs for an assignment in their elementary, middle level, and/or secondary mathematics methods course, and (2) created and used by secondary mathematics PSTs during student teaching. While this monograph has secondary PSTs as its focus, we include data on elementary and middle level PSTs for comparisons.

9.3.1 Participants

Participants in this study were PSTs (n = 80) in the elementary (grades PK-4; n = 41), Middle Level (grades 4–8; n = 17), or Secondary Mathematics (grades 7–12; n = 22) certification programs during the 2014–2015 school year in a mid-size private university in the northeast U.S. Most participants (74/80; 92.5%) were undergraduates in the first semester of their senior year in Fall 2014 and engaged in student teaching in Spring 2015, though 6 participants (7.5%) were in post-baccalaureate programs obtaining their initial teaching certification and engaged in student teaching in Fall 2014 or Spring 2015. All PSTs in this study completed a mathematics methods course appropriate to their certification level in Fall 2014 or Spring 2015, taught by the second author and/or another mathematics education faculty member at the University. The first author described the study to PSTs in person during class sessions of the mathematics methods courses. The course instructors did not know who decided to participate in the study until after course grades had been finalized.

9.3.2 Context of the Methods Course and Technology Activities

Each mathematics methods course was organized around: (1) a developmental, social-constructivist view of teaching and learning mathematics, using texts such as, "Elementary and Middle School Mathematics: Teaching Developmentally" (Van de Walle, Karp, & Bay-Williams, 2007); and (2) a task-focused approach to lessons planning (Smith, Stein, Arbaugh, Brown, & Mossgrove, 2004), mathematical discussions (Smith & Stein, 2011), and reflection (Stein et al., 2009). As such, each course emphasized selecting and implementing cognitively challenging mathematical tasks. PSTs in each course engaged in a "Task Sort" activity (Smith et al., 2004) to learn to identify the cognitive demands of mathematical tasks. PSTs also analyzed curricula, justified the level of cognitive demand of tasks used in their own lesson plans, and had frequent opportunities to explain why tasks used in class had high-level demands. PSTs also analyzed narrative and video cases of mathematics instruction, samples of student work, peer teaching during the course, and their own teaching of mathematics lessons (in K-12 classrooms and to their peers). In these analyses, PSTs considered whether (and how) high-level cognitive demands were maintained throughout the lesson.

Each methods course includes a Technology Assignment, where PSTs design technology-based instructional activities. In the elementary mathematics methods course, PSTs investigated the teaching and learning of mathematics in grades PK-4. PSTs produce lesson plans for whole number operations, fractions, and geometry at specified grade levels. For the Technology Assignment, PSTs created technology activities for grades PK-4 in the mathematical topics of algebraic thinking and statistics (data or probability). In the middle level methods course, pair/groups of 2–3 PSTs planned units of instruction (e.g., series of consecutive lessons around a specific mathematical topic), to engage middle school students in learning mathematics. The Technology Assignment required middle-level PSTs to design technology-based instructional activities aligned with the mathematical content of their unit plans. In the secondary mathematics methods course, PSTs investigated curricula and planned lessons in algebra, geometry, trigonometry, statistics, or calculus. Each PST created three lesson plans: (1) algebra, (2) geometry, and (3) any higher-level mathematics course. For the Technology Assignment, PSTs planned technology activities that addressed important mathematical content at the high school level (e.g., algebra and beyond).

In each course, the Technology Assignment occurred toward the end of the semester. While the Technology Assignment required PSTs to use technology to teach specific mathematical content (aligned with the PSTs' certification level), the Technology Assignment did not explicitly indicate that PSTs should select or create a cognitively challenging instructional task. PSTs were expected to *find* a technology resource (e.g., an applet, graphing calculator application, or web-based game, software, or animation) and *create* an instructional activity using that resource. For the Technology Assignment, participants provided screenshots or copies of the task, descriptions of how they would use the task in an instructional activity to teach mathematics, and their expectations for students.

During student teaching, PSTs completed a "Showcase Portfolio" containing at least three examples of their impact on students' learning. Each example contains artifacts from a lesson or series of lessons (e.g., lesson plans, copies of instructional tasks, assessments) and samples of students' work (with students' names removed). Secondary mathematics PSTs were required to include a technology-based instructional activity in their Showcase Portfolio. As described in the next section, the Technology Assignments and Showcase Portfolios served as data in this study.

9.3.3 Data and Coding

Data include 68 instructional activities from Technology Assignments in the methods courses (41 elementary level, 19 middle level, and 8 secondary level mathematics) and 14 instructional activities from secondary mathematics PSTs' Showcase Portfolios. The Instructional Quality Assessment (IQA) (Boston, 2012) was used to assess the cognitive demand of each technology activity, based on three indicators:

(a) *Potential of the Task*: the level of cognitive demand of the instructional task portrayed by the technology resource, as displayed on screen, including any directions included within the task display itself, but not including any additional directions or instructional activities for using the task created or described by the PST;
(b) *(Described) Implementation*: the level of cognitive processes elicited by the instructional activities surrounding the technology resource, including additional directions or instructional activities created and/or described by the PST. In the Technology Assignments, the description of how the task would be implemented in the lesson activity (*Described Implementation*); in the Showcase Portfolios, how the task was actually implemented in the student-teaching classroom (*Implementation*);
(c) *(Expected) Student-Response*: the elaborateness of student-responses (or products) required by the task display, task directions, and any additional instructional activities created and/or described by the PST. In the Technology Assignments, the level of response expected from students (*Expected Student Response*); in the Showcase Portfolios, the level of response actually produced by students (*Student Response*) in the student-teaching classroom (based on student-work samples).

The rubrics for *Potential of the Task* and (*Expected*) *Student Response* are provided in Fig. 9.2. The *Implementation* (or *Described Implementation*) rubric (summarized in Fig. 9.3) parallels *Potential of the Task*, with "lesson activity" replacing "task" in each score level and the verb changing to past tense for Showcase Portfolio activities (since the lesson activity was actually implemented in

	Potential of the Task	***(Expected) Student Response***
4	The task has the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as (from Stein et al. 2009): • Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); or • Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts. The task must explicitly prompt for evidence of students' reasoning and understanding. For example, the task MAY require students to: • solve a genuine, challenging problem for which students' reasoning is evident in their work on the task; • develop an explanation for why formulas or procedures work; • identify patterns;…justify generalizations based on these patterns;…	The (expected) student response provides evidence of students' mathematical thinking and reasoning (such as multiple representations or strategies, diagrams, etc.) AND an explanation is explicitly required.
3	The task has the potential to engage students in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the task does not warrant a "4" because: • the task does not explicitly prompt for evidence of students' reasoning and understanding. • students may need to identify patterns but are not pressed to form or justify generalizations; • students may be asked to use multiple strategies or representations but the task does not explicitly prompt students to develop connections between them;…	The (expected) student response provides evidence of students' mathematical thinking and reasoning (such as multiple representations or strategies, diagrams, etc.) BUT no explanation is required.
2	The potential of the task is limited to engaging students in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task…. The task does not require students to make connections to the concepts or meaning underlying the procedure being used… (e.g., practicing a computational algorithm).	The (expected) student response is a computation or procedure,…or procedural explanation such as "Show your work." Students *are not required to* demonstrate connections to mathematical concepts in their response to the task, even if task itself provided opportunities for connections.
1	The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions…	Students (*are asked to)* provide brief numerical or one-word answers (e.g., fill in blanks, provide only the result or answer).

Fig. 9.2 Coding rubrics for potential of the task and (expected) student response

Implementation of the Task (Boston, 2012)	
4	Students engage in using complex and non-algorithmic thinking or by exploring and understanding the nature of mathematical concepts, procedures, and/or relationships.*
3	Students engage in complex thinking or in creating meaning for mathematical procedures and concepts BUT the problems, concepts, or procedures do not require the extent of complex thinking as a "4"; OR The "potential of the task" was rated as a 4 but students only moderately engage with the high-level demands of the task.*
2	Students engage with the task at a procedural level. Students apply a demonstrated or prescribed procedure. Students may be required to show or state the steps of their procedure, but are not required to explain or support their ideas. Students focus on correctly executing a procedure to obtain a correct answer.
1	Students engage with the task at a memorization level. Students are required to recall facts, formulas, or rules (e.g., students provide answers only). OR The task requires no mathematical activity.
N/A	Reason:
*Refer to descriptors in *Potential of the Task* rubric.	

Fig. 9.3 Summary of coding rubric for (described) implementation

the student-teaching classroom). A score of 0 indicates that the Technology Assignment or Showcase Portfolio activity *did not* include a mathematical task, require students' engagement with a mathematical task, and/or expect a student response (e.g., technology used for presentation or display). Scores of 1–2 indicate low-level cognitive demands (e.g., memorization, rote procedures or procedures without connections) and scores of 3–4 represent high-level cognitive demands (e.g., procedures with connections to meaning and sense-making, and doing mathematics or problem-solving).

In this study, the first author scored all data and the second author scored 20% of randomly selected tasks from the 68 method course activities and from the 14 Showcase portfolio activities to determine reliability. Exact-point agreement between the two authors was good to excellent: 94% for *Potential of the Task*, 82% for the *Implementation* and *Described Implementation*, and 88% for the *Students Response* and *Expected Student Response*. All coding questions were discussed between raters until reaching consensus.

Figure 9.4 provides examples of technology tasks (as they appear on screen) at each level of the *Potential of the Task* rubric. The "Parts of a Circle" task in Fig. 9.4a receives a score of 1 for *Potential of the Task*, because students only need to recognize a vocabulary term to solve the task correctly. The task does not require students to draw a diagram, perform a procedure, or explain the result.

The "Plotting Points" example in Fig. 9.4b is a game activity in which students control a bug on a coordinate plane starting at (0, 0), with the goal of finding specified coordinates from all four quadrants using left, right, below, and above arrows. *The Potential of the Task* receives a score of 2, because identifying points on a coordinate grid requires a standard procedure.

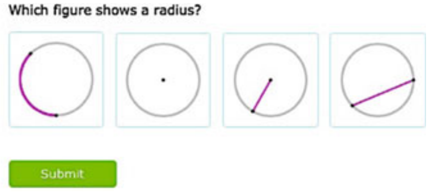
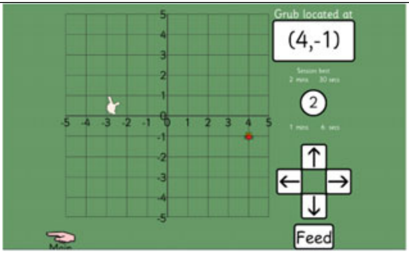
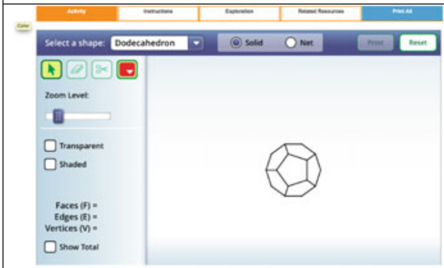
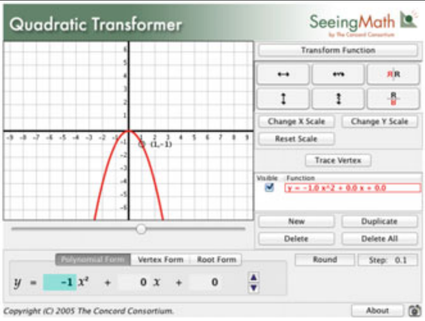
	**(a) Parts of a Circle** **Potential of the Task: 1** **Retrieved from https://www.ixl.com/math/geometry/parts-of-a-circle**
	**(b) Plotting Points** **Potential of the Task: 2** **Retrieved from http://resources.oswego.org/games/BillyBug2/bug2.html**
	**(c) Nets of 3-D Shapes** **Potential of the Task: 3** **Retrieved from http://illuminations.nctm.org/Activity.aspx?id=3521**
	**(d) Quadratic Transformer** **Potential of the Task: 4** **Retrieved from http://seeingmath.concord.org/resources_files/QuadraticGeneral.html**

Fig. 9.4 Examples of score levels for potential of the task

The "Nets of 3-D shapes" activity in Fig. 9.4c requires students to explore 3-D shapes (cubes, tetrahedrons, or dodecahedrons) and how the shapes relate to 2-D nets. Students explore faces, edges, and vertices of shapes to understand the 3-D forms. The task directions ("For any polyhedron, what is the relationship between the number of faces, vertices, and edges?") provide opportunities for students to identify mathematical relationships, but do not require explanations or justifications; hence the *Potential of the Task* scores 3.

The aim of the Quadratic Transformer activity in Fig. 9.4d is to help students to make sense of the parent graph and transformations of quadratic functions by considering connections between the graphic and symbolic representations. The task directions state, "How does the number you chose for the coefficient of x^2 (the letter a) change the shape of a parabola? Write your conclusions and explain your reasoning." This activity requires an explanation of the effect of changing a in $f(x) = ax^2 + bx + c$, and the values of b and c with the location of the vertex. *Potential of the Task* scores a 4, because the task explicitly asks students to explain their reasoning.

While *Potential of the Task* scores indicate the cognitive demand of the technology activity as it appears on screen or in print, the (*Described*) *Implementation* and (*Expected*) *Student Response* are both scored on how the instructional activity was designed and/or conducted by the PST. For example, the *Expected Student Response* of the original "Nets of 3-D Shapes" task (Fig. 9.4c) scores a 3, because student are required to find relationship between faces, edges, and vertices (corners) of each shape. Students must look for patterns and relationships, and they are required to produce more than a rote procedure or calculation (score 2). However, they are not asked to produce an explanation or generalization for why the relationships occur (score 4). If the PST included directions in the Technology Assignment or Showcase Portfolio that explicitly prompted students to explain the relationship between the faces, edges and vertices of each shape or why those relationships occur, the *Expected Student Response* would be 4. Conversely, if the teacher's directions only asked students, "Find the number of faces, vertices, and edges of each shape," instead of asking about the relationships between them, *Expected Student Response* would score a 2. Similarly, the "Plotting Points" task in Fig. 9.4b scores a 2 for *Potential of the Task*, since the purpose of the activity is plotting points on a coordinate graph. However, if the task was used as a tool to promote thinking and reasoning, the *Described Implementation* could score a 3 or 4.

In summary, each Technology Assignment received three IQA scores: *Potential of the Task*, *Described Implementation*, and *Expected Student Response*. Similarly, Showcase Portfolios (accompanied by student work samples) from PSTs' student teaching classrooms were scored for *Potential of the Task*, *Implementation*, and *Student Response*. Note that each technology activity in the student teaching Showcase Portfolio contained at least 4 samples of students' work. A set of student work was scored by considering the level of implementation and student responses provided by the majority of students.

9.3.4 Analyses

In this report, we present overall data on all 80 PSTs in the study. Data from PSTs in the elementary and middle level programs are included to provide points of comparison with data from secondary mathematics PSTs. Means, medians, percentages and the frequency of technology activities at each IQA score level are reported in frequency tables, in order to describe the level of cognitive demand of PSTs' technology activities, allow for comparisons between rubrics, and to provide an indication of PSTs' ability to plan lessons that incorporate technology in ways that supports students' learning. A one-way analysis of variance (ANOVA) was used to evaluate differences between PSTs at different certification levels for each of the IQA rubrics. Non-parametric t-tests for independent samples were used to make comparisons between the technology activities designed for the method class and the technology activities used during student teaching, as an indication of whether ideas from the methods class would be evident in PSTs' classroom practices during student teaching.

9.4 Results

9.4.1 Technology Assignments

Table 9.1 provides the results for PSTs' "Technology Assignments" overall and delineated by certification level. Overall results indicate that PSTs overwhelmingly selected instructional tasks with high-level cognitive demands (scores of 3 or 4 for *Potential of the Task*; 52/68; 76.5%), created instructional activities to engage students in cognitively challenging mathematical work and thinking (scores of 3 or 4 for *Described Implementation*; 57/68; 83.8%), and expected high-level student-responses and products (scores of 3 or 4 for *Expected Student Response*; 55/68; 80.9%). In five technology activities, PSTs created high-level instructional activities (e.g., score of 3 or 4 for *Described Implementation*) from tasks that originally (in print or on screen) had low-level cognitive demands. PST modified five tasks with low-level demands into technology activities that required high-level cognitive demands for *Expected Student Response;* however, in two cases, scores declined from high-level *Potential of the Task* to low-level *Expected Student Response* (resulting in a net change of 3). Overall scores of 4 indicate that 17.5% of tasks, 30.8% of instructional activities, and 32.4% of expected student responses required students to explain their mathematical thinking and reasoning.

Table 9.1 IQA scores for technology activities by certification level

IQA rubric	Mean	Median	Number (%) at each score level			
			1	2	3	4
Elementary (PK-4) (n = 41)						
Potential of the task	3.15	3	0 (0%)	2 (5%)	31 (75.5%)	8 (19.5%)
Described implementation	3.30	3	0 (0%)	2 (5%)	25 (61%)	14 (34%)
Expected student responses	3.22	3	1 (2.5%)	3 (7.5%)	23 (56%)	14 (34%)
Middle level (grades 4–8) (n = 19)						
Potential of the task	2.58	2	0 (0%)	12 (63.1%)	3 (15.8%)	4 (21.1%)
Described implementation	2.74	3	0 (0%)	9 (47.3%)	6 (31.6%)	4 (21.1%)
Expected student responses	2.63	3	3 (15.8%)	6 (31.6%)	5 (26.3%)	5 (26.3%)
Secondary (grades 7–12) (n = 8)						
Potential of the task	2.75	3	0 (0%)	2 (25%)	6 (75%)	0 (0%)
Described implementation	3.38	3	0 (0%)	0 (0%)	5 (62.5%)	3 (37.5%)
Expected student responses	3.38	3	0 (0%)	0 (0%)	5 (62.5%)	3 (37.5%)
Overall (n = 68)						
Potential of the task	3.00	3	0 (0%)	16 (23.5%)	40 (59%)	12 (17.5%)
Described implementation	3.18	3	0 (0%)	11 (16.2%)	36 (53%)	21 (30.8%)
Expected student responses	3.10	3	4 (5.9%)	9 (13.2%)	33 (48.5%)	22 (32.4%)

Within each certification level, data indicate a greater number of high-level scores and/or scores of 4 for *Described Implementation* or *Expected Student Response* compared to *Potential of the Task*. For example, for secondary mathematics PSTs, we see a shift from no tasks scoring a 4 for *Potential of the Task* to three lesson activities scoring a 4 for *Described Implementation*. Similarly, PSTs designed lesson activities with high-level demands (e.g. score of 3 or 4 for *Described Implementation* and *Expected Student Response*) from two tasks that began with low-level demands (e.g., score of 2 for *Potential of the Task*).

A one-way analysis of variance (ANOVA) indicated a statistically significant differences between certification levels for each of the IQA rubrics: *Potential of the Task* ($F[2,65] = 6.695$; $p = .002$), *Described Implementation* ($F[2,65] = 5.593$; $p = .006$), and *Expected Student Response* ($F[2,65] = 4.013$; $p = .023$). Follow-up tests identified significant difference between middle level and elementary level, with the middle level means significantly lower than elementary level means for all IQA rubrics. No significant differences were found between secondary level versus middle level or secondary level versus elementary level for any of the IQA rubrics.

9.4.2 Showcase Portfolios

Table 9.2 provides data for the 14 technology-based instructional activities in the Showcase Portfolios from secondary mathematics PSTs. The majority of *Tasks* (13/14; 93%), *Implementation* (11/14; 78.6%), and *Student Responses* (10/14; 71.4%) had high-level cognitive demands. However, three (21.4%) technology activities beginning with high-level tasks (e.g., *Potential of the Task* score of 3 or 4) declined in cognitive demand for *Implementation* and/or *Student Response*.

An independent samples t-test indicated a statistically significant difference between activities in the Technology Assignments and Showcase Portfolios for only *Potential of the Task* (t [80] = −2.095; p = .039). The mean score of the tasks for the Technology Assignments from the methods courses (M = 2.96, SD = .66) is significantly lower than the mean score of the tasks in the Showcase Portfolios (M = 3.36, SD = .63). The test was not significant for *Implementation* (t [80] = .742; p = .461) or *Student Response* (t[80] = .857; p = .394).

9.5 Discussion

In this study, we examined technology-based instructional activities created by PSTs for a Technology Assignment in their mathematics methods courses or for their Showcase Portfolios during student teaching.

9.5.1 Technology Assignments

9.5.1.1 Elementary PSTs

Almost all elementary level PSTs selected or designed technology-based instructional activities with cognitively demanding tasks (95%), implementation (95%), and expected student responses (90%). Hence, elementary level PSTs demonstrated the ability to effectively integrate technology in ways that support students' learning

Table 9.2 IQA scores for showcase portfolios (n = 14)

IQA rubric	Mean	Median	Number (%) at each score level			
			1	2	3	4
Potential of the task	3.36	3	0 (0%)	1 (7%)	7 (50%)	6 (43%)
Implementation	3.00	3	0 (0%)	3 (21.4%)	8 (57.2%)	3 (21.4%)
Student responses	2.86	3	0 (0%)	4 (28.6%)	8 (57.2%)	2 (14.2%)

of mathematics. Elementary PSTs' success at designing technology activities with high cognitive demands might have been impacted by the methods course instructors' frequent use of the National Council of Teachers of Mathematics (NCTM) Illuminations site and the National Library of Virtual Manipulatives (NLVM)] site during the course to support PSTs' own learning of mathematics and as resources PSTs might use to support their students' learning of mathematics. The nature of the web-based virtual manipulatives, tools, and resources featured on these websites may have provided PSTs with an easily accessible source of high-level technology-based tasks. Even so, elementary PSTs in the study demonstrated the ability to: (1) identify high-level tasks provided by the websites, and (2) design high-level instructional activities using the technology resources provided by the websites. Hence, elementary PSTs in this study identified resources and designed activities that would provide students in elementary schools opportunities to actively engage with mathematics using technology.

9.5.1.2 Middle Level PSTs

While the majority of middle level PSTs selected technology tasks with low-level cognitive demands (63.1%), the majority were then able to design technology-based instructional activities that could engage students in high-level thinking and reasoning (52.7%). The *Potential of the Task* median score of 2 for middle level PSTs is the only median score below a 3 across all IQA rubrics and certification areas. Interestingly, middle level PSTs posted higher mean scores for *Described Implementation* and *Expected Student Response* than *Potential of the Task*, indicating that middle level PSTs designed instructional activities and expected responses that increased the demands of the original tasks in print or on screen. In many large-scale studies of mathematics teachers' use of tasks during instruction, tasks typically have higher cognitive demands than implementation and discussion (Boston & Wilhelm, 2015). In other words, teachers do not often design or implement instructional activities with higher demands than the original tasks themselves. Middle level PSTs may have selected fewer high-demand tasks at the outset because of the context of the Technology Assignment given in the middle level methods course. While the directions were the same as those given to elementary and secondary PSTs, middle level PSTs had to choose technology tasks that promoted students' learning of a specific mathematical topic aligned to a unit of instruction they were planning in the course. For this reason, middle level PSTs may have searched for technology tasks based on mathematical topics (rather than level of cognitive demand) and then adapted those tasks to create instructional activities with higher-level demands.

9.5.1.3 Secondary Mathematics PSTs

Secondary level PSTs selected technology resources with high-level cognitive demands (75%) and designed technology-based instructional activities that maintained or/and increased the level of cognitive demands of these tasks (e.g., 100% of *Described Implementation* and *Expected Student Response* at a score of 3 or 4). Hence, they successfully demonstrated the ability to design technology-based instructional activities and to adapt low-demand tasks into high-demand instructional activities. This ability may be due to the fact that secondary level PSTs took methods courses with a focus on cognitive demands during two consecutive semesters, and this experience may have helped them to design technology-based instructional activities with high-demand mathematics tasks.

9.5.1.4 Overall Results

Overall, many of the technology-based tasks selected by PSTs scored high-level (52/68; 76.5%). PSTs planned instructional activities integrating technology-based tasks where students would have opportunities to engage in complex thinking and reasoning (score 3) and also provide explanations and justifications (score 4). This is important because setting up a lesson with high-level cognitive demand tasks is key to encouraging students' thinking and reasoning throughout the lesson (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Stein et al., 1996). Research on mathematics teachers' use of high-demand tasks rarely shows an increase from task potential to task implementation, and studies using the IQA also find that scores of 4 occur in very low percentages across all IQA rubrics (Boston & Wilhelm, 2015). In this study, the overall means for *Described Implementation* and *Expected Student Response* were higher than the overall mean for *Potential of the Task* for all grade levels, and approximately 30% of *Described Implementation* and *Expected Student Response* scored at level 4. These results indicate that PSTs designed activities that would enhance students' opportunities for thinking and reasoning beyond what was present in the original tasks (in print or on screen), and frequently required students to provide explanations and justifications.

For example, Fig. 9.5 illustrates a Technology Assignment submitted by a secondary mathematics PST in which the cognitive demands of the original task were increased from a score of 3 for *Potential of the Task* to a score of 4 for *Described Implementation* and *Expected Student Response*. The task requires students to use Algebra tiles to solve linear equations. The PST selected the applet from the NCTM Illuminations website (http://illuminations.nctm.org/activity.aspx?id=3482), stating, "this Internet applet is great for students because it gives them a chance to use technology for a mathematical concept instead of using pencil and paper, and allows the students to visually see what they are doing to solve an equation."

The directions provided by the website include: "Build your model. Solve the equation." The website also provides a list of what students can do with applet:

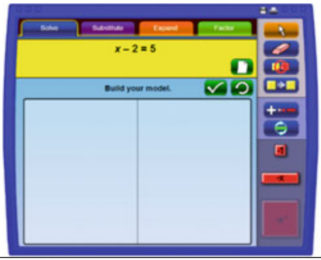
	1. Start with an equation.
	2. Use the pointer tool and place the correct pieces in the workspace. After you build the model of the given problem, check your answer to move on to the next step. Only tile type, tile quantity, and workspace area are checked, not the way in which tiles are arranged.
	3. Try eliminating the necessary tiles to create zero pairs. Remember, what you do to one side, you must do to the other side!
	4. After you solve the problem, check your answer.
5. Practice: Solve the following equations using the Algebra tiles: a) 4x – 1 = 2x + 3 c) 4x – 3 = 5 b) 2x + 2 = 4 d) 5x – 5 = 4x + 2	

Fig. 9.5 Example of a high-level task and instructional activity

"Use tiles to represent variables and constants, learn how to represent and solve algebra problem. Solve equations, substitute in variable expressions, and expand and factor. Flip tiles, remove zero pairs, copy and arrange, and make your way toward a better understanding of algebra." The *Potential of the Task* scores a 3, since the task has the potential to engage students in creating meaning for mathematical concepts and procedures by asking students to use a representation and build a model for solving equations. In the Technology Assignment, the PST described the implementation of the activity step by step (see Fig. 9.5). Furthermore, the PST planned to ask three additional questions:

(1) What is the goal for solving equations?
(2) How do the Algebra tiles allow you to better visualize the concept of zero pairs?
(3) Explain the phrase “whatever you do to one side, you must do the exact same thing to the other side”?

Hence, the *Described Implementation* score is 4, because students are required to explain and understand the process of solving linear equations using multiple strategies and making connections between representations. *Expected Student Response* also scores 4, because students are explicitly asked to explain their mathematical thinking and reasoning. The cognitive demands of the original task were high level (score of 3), and the *Described Implementation* and *Expected Student Response* in the PSTs’ instructional activity maintained and increased the high-level demands to a score of 4.

While ANOVA tests identified significant differences between elementary and middle level PSTs on all three rubrics, with elementary PSTs outperforming middle-level PSTs, no significant differences existed between secondary mathematics PSTs and either elementary or middle level PSTs. Interestingly, the TPCK model might suggest that secondary mathematics PSTs would outperform their counterparts due to greater content knowledge and technological content knowledge, as secondary mathematics PSTs would have encountered more mathematics and perhaps utilized more technology as learners of mathematics. We contend that similarities in pedagogical knowledge and PCK amongst participants in the study may have supported PSTs at all levels to design technology activities that support students’ mathematical thinking and reasoning. Specifically, we consider PSTs’ experiences during methods courses in attending to the cognitive demands of mathematical tasks and designing lessons to foster students’ high-level engagement as paramount in enabling PSTs at all certification levels to select tasks and design lesson activities in ways that promote students’ learning of mathematics.

9.5.2 Showcase Portfolios

In secondary mathematics PSTs’ Showcase Portfolios, almost all tasks (13/14; 93%) scored 3 or 4, indicating that PSTs selected tasks for technology-based instructional activities that could provide students opportunities to engage in complex mathematical thinking and reasoning, use multiple representations, and provide explanations (score of 4; 43%). It is highly likely that PSTs selected their best activities for the Showcase Portfolio, because Showcase Portfolios are used by the University teacher educators to evaluate PSTs' performance during student teaching. Even so, the results provide evidence that during student teaching, secondary mathematics PSTs could identify technology-based tasks with high-level cognitive demands and could design and implement instructional activities that maintained students' high-level thinking and reasoning. In other words, the ideas and instructional strategies prevalent in the methods courses were also evident in instructional materials created by secondary mathematics PSTS' for their student teaching classroom.

In the Showcase Portfolios, a few tasks rated as high-level for *Potential of the Task* received a score of 2 for *Implementation* and/or *Student Response*. Of 13 tasks with the potential to elicit high-level thinking and reasoning, 11 maintained high-level thinking during *Implementation* and 10 maintained high-level demands in *Students' Responses*. In these few cases where PSTs did not maintain high-level cognitive demands and/or students' responses, the student-work samples included in the Showcase Portfolio displayed only computations or procedures (score of 2). Similarly, of six tasks with the potential to elicit a score of 4, only three were maintained for *Implementation* and two were maintained for *Students' Responses*. In these cases, even though the original task explicitly asked for an explanation (score of 4), students did not actually provide explanations in their written work. Instead, students may have used multiple representations or engaged in problem-solving without explaining or justifying their work (score of 3), or they may have only demonstrated procedures or computations (score of 2). Hence, PSTs experienced some difficulty maintaining high-level cognitive demands when technology activities were actually implemented with students in the classroom. However, the percentages of instructional activities maintained at a high-level and at a score of 4 exceed the findings of previous research (e.g., Boston & Wilhelm, 2015; Johnston, 2012; Sherman, 2014). For example, mathematics teachers observed by Sherman (2014) and the lesson plans of PSTs analyzed by Johnston (2012) identified technology tasks and technology-based lessons with low cognitive demands in far greater percentages than identified in this study.

9.6 Conclusion

In this study, PSTs designed technology-based mathematics instructional activities with high-level cognitive demands and frequently maintained or increased the cognitive demands of the original tasks. We hypothesize that PSTs' overall success in selecting and designing cognitively demanding technology-based instructional activities is due to a strong focus on the cognitive demands of tasks and task implementation throughout each of the methods courses. The coherent focus on cognitive demands in planning, implementing, and reflecting on mathematics instruction throughout the methods courses may have enabled PSTs to design and implement technology-based instructional activities that could support students' learning of mathematics.

Limitations to this study include the fact that PSTs' instructional activities were being evaluated for a course assignment or for their student teaching, and may not reflect the nature of instructional activities the PSTs would create on their own. Also, while we could examine PSTs' *ability* to design technology-based instructional activities that supported students' learning, we could not make generalizations about PSTs' typical practice over time. Additional work is needed to examine the types of technology-based instructional activities PSTs use to teach mathematics over extended periods of time and the impact of these activities on students' learning.

Given these limitations, the results of this study remain important because they suggest that focusing on the cognitive demands of instructional tasks may be a productive pathway for supporting PSTs to use technology in ways that enhance students' learning in a specific content area. While our focus was PSTs in mathematics, the results generalize to other content areas and from pre-service teacher education to inservice teacher professional development. By connecting technology to effective pedagogy (e.g., maintaining high-level cognitive demands) in a specific content area, our approach engages all facets of PSTs' (or teachers') TPCK in effectively integrating technology to support students' learning.

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Part III
Mathematics for Teaching

Chapter 10
Prospective Mathematics Teacher Argumentation While Interpreting Classroom Incidents

Despina Potari and Giorgos Psycharis

Abstract This paper aims to analyze the structure and quality of prospective mathematics teachers' (PMTs)' argumentation when identifying and interpreting critical incidents from their initial field experiences. We use Toulmin's model and recent elaborations of it to analyze the discussions that took place at the university where PMTs reflected on their recent classroom experiences. Our aim is to identify the structure of the argumentation and characterize the emerging warrants, backings, and rebuttals. Results indicate different argumentation structures and types of warrants, backings, and rebuttals in the process of PMTs' interpretations of students' mathematical activity. We discuss these findings from the perspective of noticing to identify shifts at the level of PMTs' interpretations.

Keywords Teacher argumentation · Argumentation structures · Warrants
Noticing · Critical incidents

10.1 Introduction

Current approaches in research in mathematics teacher education acknowledge the importance of noticing as a construct to study what and how prospective mathematics teachers (PMTs) attend to when observing, analyzing, and interpreting teaching (Scherer & Steinbring, 2006). Noticing has been considered as a complex action that involves teachers in identifying what is significant in a classroom interaction, interpreting this noteworthy incident on the basis of their knowledge and experiences, and linking these with broader principles of teaching and learning (van Es & Sherin, 2010). Moreover, at the level of teacher education and in

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_10

collaborative contexts, interpreting teaching phenomena is a joint action that involves the development of claims, conjectures, and arguments. Studying teachers' argumentation is a means of understanding the resources upon which teachers base their interpretations. As regards PMTs, a challenge is to gain insight into the nature and structure of argumentation in relation to their multiple experiences from school, teacher education courses, and field experiences. Steele (2005) points out that pedagogical argumentation has a diverse and fragile knowledge base and thus, "It is difficult to access and agree upon fundamental elements of a pedagogical argument" (p. 296). He also considers teaching as an interpretive act that is highly contextualized and dependent on teachers' experiences in the educational culture. Therefore, we consider pedagogical argumentation as a lens to have access to PMTs' interpretive acts of teaching and to the sources upon which they base their interpretations.

In this paper, we focus on PMTs' argumentation in the process of selecting and interpreting critical classroom incidents as part of their fieldwork activities. The study took place in the context of a 14-week undergraduate mathematics education course with the philosophy of linking theory-driven instruction on the teaching and learning of secondary school math with actual mathematics teaching in classroom settings. In a recent paper based on this context (Potari & Psycharis, Submitted), we used critical incidents as a structure to facilitate PMTs' reflection and study their conceptualizations of mathematics teaching and learning. The analysis showed PMTs' shifts from observing teaching to questioning aspects of it and conceiving it in a relational way. It also brought to the fore a richness of arguments developed by PMTs as they supported their claims or challenged their peers' interpretations. In this paper, we use Toulmin's model of argumentation and recent elaborations of it to analyze the quality of PMTs' argumentation and its development while identifying and interpreting critical incidents. The research questions are:

- What is the structure of PMTs' argumentation while interpreting classroom incidents?
- What are the sources upon which PMTs base their interpretations of critical incidents?
- How do the PMTs' interpretations evolve in the context of the course?

10.2 Theoretical Framework

10.2.1 Teacher Noticing and Critical Incidents

Noticing has been introduced to mathematics teacher education to study shifts in the structure of teachers' attention and, through this, address different levels of awareness in mathematics and in mathematics teaching (Mason, 2002). In resonance with a number of current research approaches (c.f., Jansen & Spitzer, 2009;

Scherer & Steinbring, 2006), noticing is an activity involving description, analysis, and interpretation of teaching practice. According to van Es and Sherin (2002), noticing is a more complicated action than just observing teaching. Rather, it requires teachers to identify significant teaching and learning incidents, to interpret them on the basis of their knowledge and experiences, and to link these with broader principles of teaching and learning. Further, van Es and Sherin also highlighted the importance of interpreting classroom interactions as a way of informing teachers' pedagogical decisions. Therefore, promoting teachers' noticing includes, "to first notice what is significant in a classroom interaction, then interpret that event, and then use those interpretations to inform pedagogical decisions" (p. 575).

Researchers have been concerned about the introduction of sufficient structures for making the act of teacher noticing more concrete. Critical incidents are an example of a structured framework for reflection on classroom episodes. A critical incident can be considered as an everyday classroom event that has significance for the teachers, makes them question their practice, and seems to provide an entry for their better understanding of teaching-learning situations (Hole & McEntee, 1999). Critical incidents have been used in mathematics education for analytical and developmental purposes (Goodell, 2006; Skott, 2001). For example, Skott (2001) used the term "critical incidents of practice" to describe moments of a teacher's decision-making in which multiple and possibly conflicting motives of their activity evolved that challenged the teacher's own school mathematics images and provided learning opportunities for students. Goodell (2006) used critical incidents to promote PMTs' noticing, as well as to address her own development as a mathematics teacher educator. The issues raised by PMTs in her study included: teaching and classroom management; student factors; relationships with colleagues, parents and students; and school policies and procedures. She also identified that PMTs fruitfully addressed important aspects of mathematics teaching that supported students' understanding. Similar to Goodell's study, our research has a developmental character as it aims to address the quality of PMTs' argumentation and its development while identifying and interpreting critical incidents. While Goodell investigates PMTs' learning through the analysis of their written reports of critical incidents, we focus on PMTs' discussions in the context of a teacher education course while sharing their reflections with their peers and the teacher educator. In this paper, our focus is on PMTs' interpretation of their selected classroom incidents and in particular on their argumentation when analyzing and interpreting them.

10.2.2 Teacher Argumentation and Toulmin's Model

In the mathematics education field, teacher argumentation has been studied in the context of the classroom (Knipping & Reid, 2015), in teacher education programs (Metaxas, Potari, & Zachariades, 2016), as well as in teachers' responses in hypothetical scenarios (Nardi, Biza, & Zachariades, 2012). Toulmin's theory has

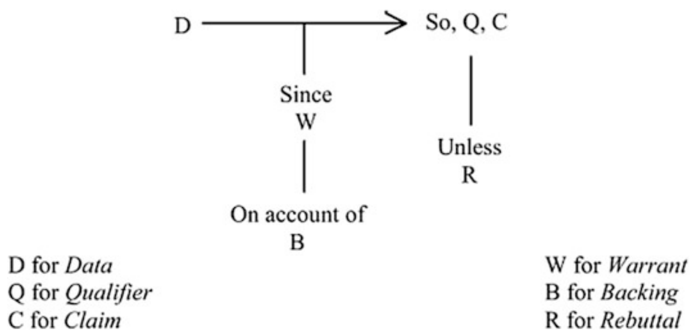


Fig. 10.1 A representation of Toulmin's model

been the basis of most studies in the analysis of teacher argumentation and in particular its model for the layout of arguments (Toulmin, 1958). His model (see Fig. 10.1) consists of six basic elements: the claim (C) is the position or claim being argued for; the data (D) are the foundation or supporting evidence on which the argument is based; the warrant (W) is a general rule of inference that authorizes the step from the data to the claim; the backing (B) supports the legitimacy of the warrant; the modal qualifier (Q) represents the degree of force or strength that the data confer on a claim in virtue of the warrant; and the rebuttal (R) consists of exceptions to the applicability of the warrant.

Toulmin's model has often been combined with other frameworks so as to address not only the structure, but also the quality of argumentation. For example, Metaxas et al. (2016) used argumentation schemes to analyze the internal coherence of mathematics teachers' arguments in the context of a master's course, while Nardi et al. (2012) adopted Freeman's framework to identify the different types of warrants mathematics teachers use to support their claims when they interpret hypothetical classroom scenarios. Nardi et al. (2012) address the quality of teachers' argumentation by placing their arguments in relation to teacher considerations and priorities—pedagogical, curricular, professional, and personal. In particular, they distinguish seven types of warrants in teachers' arguments: a priori-epistemological and a priori-pedagogical (based on mathematical or pedagogical principles); empirical-professional and empirical-personal (based on their teaching or learning experiences); institutional-epistemological and institutional-curricular (based on practices for the mathematics community or on curriculum and textbook recommendations/requirements); and evaluative (based on a personally held view/value/belief). This categorization helped Nardi et al. (2012) to analyze teachers' warrants and to identify the sources on which they based their arguments. In our study, we adopted the same categorization to analyze PMTs' warrants, backings, and rebuttals as we considered all indicators of the sources of PMTs' argumentation. Since our study refers to PMTs, we considered their professional experiences stemming from their fieldwork practices and other personal teaching experiences they had.

Moreover, Knipping and Reid (2015) distinguished local from global arguments to study proving processes in the mathematics classroom. Local arguments represent a step of an argument that can be analyzed by Toulmin's model. Global arguments lay out the structure of interconnected local arguments indicating the structure of an argumentation process as a whole. Knipping and Reid also identified different types of global argumentation structures (e.g., source-structure, spiral-structure) as constructs to address differences in the argumentative process. To explain these differences, they considered the nature of local arguments that make up global structures. While both source and spiral structures have several similar characteristic features (e.g., parallel arguments, argumentation steps with more than one datum, refutations), they differ in the way these features appear in the global structure. Spiral structures involve parallel arguments leading directly to the final conclusion, whereas in the source structure the parallel arguments lead to different data that provide a basis for another argumentation step supporting the conclusion. Furthermore, in the spiral structure more refutations are used to oppose the main claim. These structures emerged from the study of proving processes in upper secondary mathematical classrooms. We adopt this framework to study PMTs' argumentation in the teacher education context.

Taking into account the theoretical constructs discussed above, in our study, we use Toulmin's model, the classification of the warrants proposed by Nardi et al. (2012) and the structures developed by Knipping and Reid (2015) to analyze PMTs' interpretations of critical incidents they identified when reflecting on lessons observed and/or taught. Toulmin's model provides a structure to analyze PMTs' local arguments and relates to our first research question. Knipping and Reid's elaborations allow us to compare different argumentative processes to address potential shifts in PMTs' interpretations of classroom phenomena. This helps us to answer our first and third research questions. Nardi et al.'s approach helps us to characterize the sources of warrants, backings, and rebuttals and address our second and third research questions. The combined use of these approaches offers us a tool to address the quality of PMTs' argumentation.

10.3 Methodology

10.3.1 Context of the Study and Participants

Prior to enrolling in the course which provided the context of the present study, PMTs had a background of undertaking at least four other mathematics education courses as a part of their teacher education program at the university. In parallel to their university studies, most PMTs were helping school students on a private basis with their math homework. The course aim was to engage PMTs in critical consideration of aspects of mathematics teaching as these emerged from the complexity of teaching practice in schools. Every second week (for the entire semester) PMTs were asked to participate in a number of field activities in secondary schools (over six field activity-weeks) while each week following the activities in schools included a 3-h meeting at the university. We note that in Greece students enter secondary education (Grades 7–12) at the age of 13 after six years of primary education. PMTs' field activities consisted of observing other teachers' mathematics teaching for 6 h in total (first three field activity-weeks) and designing and teaching lessons for the whole class for three teaching hours (fourth, fifth, and sixth field activity-weeks).

During their fieldwork in schools, the 22 PMTs (9 males, 13 females) who served as participants in this study, were asked to: (a) identify the specific content of a lesson in the curriculum and to trace it throughout the different grades; (b) look for possible research evidence related to potential students' difficulties; (c) make a lesson plan describing the main tasks and their rationale; (d) keep systematic notes from and/or record the lessons; (e) select critical incidents and provide a reflective account on the basis of justifying their selection, interpreting them, and proposing potential teaching actions. The PMTs were divided into pairs and carried out collaboratively the field activities under the supervision of eight mentors (postgraduate students of mathematics education). The mentors accompanied the PMTs to schools and supported them in their fieldwork activities by providing feedback on the PMTs' designs and discussing with them events from the lessons. Before the beginning of the course, the group of mentors met twice with the teacher educators to discuss the course philosophy and the PMTs' responsibilities. The mentors had access to the course materials and participated in the university meetings.

Instructional practice in the university sessions aimed to support PMTs' reflection on their recent field experiences and link emergent issues with existing mathematics education research in order to develop deeper levels of awareness. Typical activities in which the PMTs were engaged in the university meetings were: (a) to present critical events they had identified in their observations and in the analysis of their own teaching; (b) to discuss and question emerging issues; (c) to present their analyses of transcriptions of events with the aim of interpreting their criticality and linking them to their research readings; and finally, (d) to propose alternative teaching actions.

The teacher educator (first author) facilitated the discussion, but also challenged the PMTs to justify their selection of critical incidents, provide evidence of their claims, make interpretations, and describe their potential teaching decisions. The teacher educator also enriched the discussion by offering research—informed input. The second author participated in the university meetings as a participant observer and provided input during the discussions with the aim of promoting PMTs' reflections. The researchers took into account ethical issues related to PMTs' consent to participate in the research study. To avoid biases related to the dual role of teacher educator (as teacher and researcher), the second author had the main role in the data collection and discussed with her conflicting issues emerging in data analysis.

10.3.2 Research Design and Data Collection

Noticing critical incidents from mathematics classrooms and questioning aspects of mathematics teaching was a rather new practice for PMTs. It was supported through the discussions in the university meetings and the field activities. We considered critical incidents as a methodological tool for triggering PMTs' reflection on teaching practice. In the first two university meetings, the teacher educator introduced PMTs to the idea of critical incidents through (a) a brief presentation of Goodell's (2006) study (including the meaning of critical incidents, the classification of them, and examples from PMTs' written reports) and (b) analysis of lesson transcripts to identify critical incidents and discuss/justify in the class their criticality. In all subsequent meetings, the pairs of PMTs were asked to select and present in the next session a critical incident that they had experienced during their fieldwork activities.

We collected the data for this study over the entire semester, which consisted of: (a) PMTs' personal portfolios, including their written accounts of critical incidents and material related to the design, implementation, and presentation of the field activities in the classroom (e.g., worksheets, lesson plans, presentation files); (b) video recordings of all meetings at the university (eight in total); and (c) researchers' field notes. We base the present paper on the analysis of the transcripts of the university meetings.

10.3.3 Data Analysis

Under a grounded theory perspective and open coding (Strauss & Corbin, 1998), we identified four themes of critical incidents discussed in the meetings (i.e. students' activity, epistemological issues, lesson planning and classroom management, and wider contextual and social factors). Within each theme, we conducted a fine-grained analysis of the data in terms of the three dimensions of van Es and Sherin's (2002) description of teachers' noticing (i.e. what they observed, how they interpreted it, and how this informed their pedagogical decisions). Next, we used our combined theoretical approach described above to analyze the PMTs' argumentations developed when interpreting critical incidents related to each theme. In particular, for each theme we identified the claims that the PMTs made while reflecting on classroom observations and their own teaching, the data on which they based their claims, the warrants and backings they used to support them, and the rebuttals they provided when the validity of the conclusions was under question. Then, we focused on the interrelationships among the arguments related to the specific theme throughout the university meetings. Our purpose in this part of the analysis was to describe argumentation structures and to trace their progressive development in order to identify shifts in PMTs' interpretations. Finally, we analyzed the warrants, backings, and rebuttals to identify the sources on which PMTs based their arguments following Nardi et al.'s (2012) classification. In this paper, our focus is on PMTs' argumentation concerning the theme "students' activity" and in particular the construction of mathematical meaning that appeared to be dominant in what PMTs' noticed.

10.4 Results

From the initial sessions, PMTs' noticing of students' activity focused primarily on students' difficulties. However, our grounded analysis of the discussions in the meetings revealed that towards the end of the course, PMTs' selection and interpretation of critical incidents regarding students' mathematical learning was progressively enriched. In particular, the PMTs' interpretations were justified by relating students' activity to a multiplicity of factors acting as warrants for their claims. For example, PMTs interrelated teachers' actions, the nature of tasks, the classroom communication, and the use of language with students' mathematical learning (e.g., classroom interaction, norms). They also offered backings based on research in mathematics education, as this was targeted in the course. For example, PMTs appeared to: identify different forms of mathematical thinking and understanding (e.g., formal versus informal, procedural versus conceptual, understanding versus memorization); value students' mathematical ideas; consider epistemological aspects underlying students' learning; and appreciate the role of affective issues in the process of learning mathematics.

To illustrate the above findings, we analyzed transcripts of discussions of nearly the same length related to the theme of students' activity in two university meetings. The first meeting took place at the beginning of the course, after PMTs' initial experiences with classroom observations, and the second towards the end of it, after the completion of PMTs' own teaching. Our focus is on the structure of PMTs' argumentation and the resources upon which they based their arguments to highlight and trace the quality of their interpretations throughout the course.

10.4.1 Argumentation in the Third University Meeting

The structure of argumentation

The teacher educator encouraged PMTs to report on critical incidents they had identified during their first classroom observation. A main issue discussed in this meeting was the construction of mathematical meaning. Initially, the focus was on students' mistakes emerging primarily from their difficulties with connecting algorithmic procedures to the underlying concepts and properties. For example, PMT's discussed different arithmetic and algebraic mistakes in the meeting. The teacher educator challenged the PMTs to interpret why these mistakes appeared and what they revealed about the students' conceptual understanding. At that point, the PMTs identified a number of factors they considered relevant for explaining students' difficulties, such as the language and the use of symbols, the classroom norms, and the students' lack of motivation.

The main claim (C1) developed in the discussion was that students face difficulties in moving beyond a surface understanding to a deeper conceptualization of the underlying concepts and properties. PMTs used a number of different data sources upon which they based their claim. These sources came from their classroom observations and concerned students' difficulty in transforming a fraction to an equivalent one, using the algebraic properties to solve a first-degree equation, or simplifying an arithmetic or algebraic expression.

For example, the data (D1) a PMT reported was about a classroom interaction between two students concerning the transformation of the fraction $\frac{7}{5}$ to its equivalent with 30 as the denominator. The first student completed the transformation by multiplying both terms of the fraction by 6. Then, the second student wondered why their classmate had not used a technique taught traditionally in the Greek primary schools. According to this technique, the quotient of the division of the least common multiple of the denominators by the denominator of each fraction is placed over each numerator and then it is multiplied by both nominator and denominator for each fraction. The students usually follow the procedure without understanding why they do this. The prospective teacher interpreted the phenomenon by considering this technique as a "picture" in the student's mind that might provide a barrier to conceptual understanding: "The second student seems to have clear in their mind a picture without knowing why this method works, the essence of the method" (Kostas) (W1).

Another PMT (Petros) brought data (D2) from his fieldwork observation regarding how the teacher managed a similar situation in the context of algebra. Instead of stressing the rule "change side, change sign" commonly used in solving algebraic equations, the schoolteacher emphasized the properties involved in the solution process. Petros found this approach fascinating as it was beyond his own experiences.

A third PMT, Orestis, brought the data (D3) related to students' difficulties with simplifying arithmetic or algebraic expressions and referred to his observation in an 8th grade classroom:

> While solving an equation of two fractional expressions with numerical denominators, the students transformed them into equivalent fractions but they did not change the nominators. Then, they equated the two nominators without understanding their mistake. (Orestis)

After encouragement from the teacher educator ("What does it mean for you that the students use techniques without understanding? How do you explain this?"), the PMTs started to provide justifications for their arguments. Orestis interpreted students' difficulty with conceptualizing mathematical ideas with the warrant that in school textbooks mathematics loses its meaning:

> We use terms or expressions that have nothing to do with mathematics. For instance, the Rule of Three,[1] central in school textbooks at primary level, is a technique, rather than a mathematical method. (W2)

Another PMT, Leonidas, offered a new warrant by referring to the different meaning of symbols in arithmetic and algebraic expressions in the school textbooks. He mentioned that:

> In some cases, 3½ is a mixed number, while $3x$ where x is $\frac{1}{2}$ is a product. These two different meanings of similar representations come one after the other in the textbook. (W3)

In a subsequent phase, PMTs' attention moved to what students bring into the lesson and how this influences teaching. The new claim (C2) they were formulating concerned the important role of students' contributions to the lesson. The argumentation was enriched by other data coming from classroom observations.

For example, the PMT Irene described a critical incident as it emerged in a 9th grade classroom during a geometry lesson on congruency of triangles concerning the fact that some students brought pieces of knowledge that had not yet been taught in the classroom:

> One student mentioned the term 'adjacent angles' that had not been taught. The classroom teacher responded by saying, "We have not said anything yet about adjacent angles here," and she continued the lesson. I initially thought that the student might have read it in the textbook. But the word 'adjacent' is rather difficult for students to remember even at the upper secondary school level. Finally, I think that this knowledge came from private lessons. Sometimes this knowledge does not empower the students as I had expected from giving private lessons myself. In contrast, it constrains the classroom interaction as I see it now from the classroom teacher's perspective.

Other PMTs brought similar examples from their classroom observations. For instance, Marina mentioned a case where the teacher introduced the concept of angles in the 7th grade, but the students referred to its measure that they had encountered in primary school: "When the teacher asked, 'What is an angle?,' one student said, 'degrees.' The teacher commented that, 'We have not discussed that yet.'"

In the above two extracts, Irene and Marina bring new data (D4 and D5) to support the claim. Irene offers also as a warrant that the students have private lessons (W4) while the discussion follows Marina's observation that, "The pupils have already met the same concepts in primary school" (W5). In this way, PMTs started to identify factors related to curriculum and to wider cultural context that interfere with teachers' attempts to promote conceptual understanding. In their attempts to support the warrant W4, PMTs offered the following backings: "They [school students] take private tuition because parents do not have the knowledge or the time to help their children with their homework" (B1); "The requirements of

[1]The Rule of Three is a mechanical method for solving proportions. Briefly, it says that if we know three numbers a, b, and c, and want to find d such that a/b = c/d then d = cb/a. Algebraically, one can multiply the equation (proportion) by bd, giving ad = bc and then divide by a.

school mathematics are increasing so students need help in order to be successful" (B2); "The national examinations are rather demanding" (B3); and "The students need more individualized teaching" (B4).

In the realm of this discussion, Orestis expressed a rebuttal by questioning the tendency to support students to become good at mathematics through continuous guidance:

> It is not necessary for every child to be successful in mathematics. So, close guidance does not allow students to take decisions for their future according to their interests. (R1)

Anta referred to her own experience with her parents who had always helped her at home "although they had worked all day" (R2).

Towards the end of the meeting, the PMTs brought new data from their classroom observation about students' algebraic mistakes coming back to the initial claim (C1):

> In the 7^{th} grade classroom that I observed, the teacher assigned the task to explore if $2^2 \cdot 3 \cdot 5^2$ is a multiple of 90 expressed in the form $2 \cdot 3^2 \cdot 5$. The students wanted to make the calculations first and then check if $2^2 \cdot 3 \cdot 5^2$ is a multiple of 90. When the teacher asked them if $547 \cdot 90$ is a multiple of 90, they did not respond. They wanted to do the calculations again. (Thenia) (D6)

> The teacher gave the task of simplifying the expression $2 + 4(2x + 1)$ and one student wrote $4 \cdot 3x$. Although the teacher reminded the students about the distributive law, some of them provided a wrong answer. (Thenia, D7)

Here, the PMTs started to identify elements of students' mathematical thinking by offering as a warrant that, "The students conceive the distributive rule visually as a picture in their minds and use it without understanding its meaning" (Anta, W6). They also started to provide justifications with reference to the classroom norms established by the teacher. For instance, Irene interpreted D7 by arguing that:

> The student might be embarrassed to ask again although she had not understood. The teacher's authority could possibly be an obstacle. Thus, the student pretended that she had understood. This is what we used to do as students. (W7)

Orestis offered a new rebuttal (R3) to Irene's warrant by interpreting the incident through taking into account students' motives, "The students might prefer to be out of the classroom and playing, but they are obliged to respect the rules and pretend that they understand."

Focusing on the interrelationships of arguments, we recognized a number of argumentation steps based on different data sources (D1, D2, D3, D6 and D7) that appeared at the beginning and at the end of the discussion. These steps indicated the existence of parallel arguments supporting the claim C1 concerning students' difficulty in developing mathematical meaning in algebra. For the claim C2, new data sources appeared (D4, D5) that provided the basis of new argumentation steps. Warrants and backings (W1–W7, B1–B4) were used to support both claims, while emerging rebuttals (R1, R2, and R3) challenged claims and warrants. We could possibly argue that the argumentation structure that emerged in PMTs' initial attempt to address students' construction of mathematical meaning followed

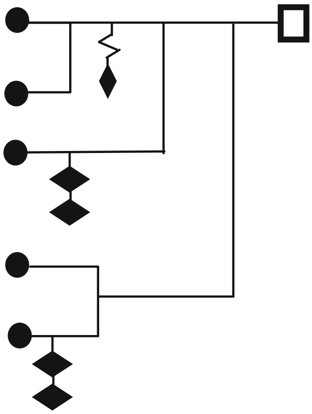
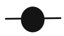
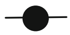


Fig. 10.2 Argumentation structure of C1 (3rd university meeting)

Fig. 10.3 Symbols used in the argumentation structure diagrams

	Data or Claims
	Warrants or Backings
	Target Claims
	Intermediate Claims
	Refutations

characteristic features of what Knipping and Reid (2015) named as source-structure, where the emphasis is more on collecting information (data and conclusions) than on the connections between the different argumentation steps (see Fig. 10.2 the argumentation structure of the claim C1, and Fig. 10.3 for an explanation of the symbols used in the diagrams).

The sources of PMTs' interpretations

In this part of the discussion, the PMTs grounded their warrants, backings, and rebuttals mainly on their broader views about teaching and learning (evaluative), as well as on recommendations of the curriculum and the textbooks (institutional-curricular). PMTs' personal experiences as learners (empirical-personal), held pedagogical principles (a priori-pedagogical), and practices of the mathematics community (institutional-epistemological) emerged as sources in PMTs argumentation. In Table 10.1, we summarize our classification according to the framework of Nardi et al. (2012).

Table 10.1 Classification of warrants, backings, and rebuttals in the third meeting

A priori-pedagogical	B4
Institutional-epistemological	W2
Institutional-curricular	W3, W4, W5
Empirical-personal	W7, R2
Evaluative	W1, B1, B2, B3, R1, W6

10.4.2 Argumentation in the Eighth University Meeting

The structure of argumentation

At this meeting, the PMTs presented critical incidents selected from their own teaching and provided transcripts from the classroom interaction related to the incidents. This process had started in the sixth meeting. Here, the focus of the discussion was on a critical incident reported by two PMTs, Anna, and Marina. The incident that constituted the data (D1) for the subsequent argumentative process concerned the difficulty that a student had with linking the algebraic identity $(a+b)^2 = a^2 + 2ab + b^2$ and its geometrical representation. The students had been given a model representing the design of a square shaped house with side $(a + b)$ divided into rooms with areas a^2, ab, ab, and b^2. Both Anna and Marina referred to a case of a student that although he was encouraged by the teacher to work with the geometrical model and then to recognize the algebraic identity, he only recalled the algebraic formula that he had already known. The main claim (C1) in the discussion was the students' difficulty with connecting different representations of mathematical knowledge, such as algebraic and geometrical.

Different PMTs expressed their interpretations about this incident. Marina reflected on her own experience as a school student to interpret the student's reaction:

> Actually, the student offered a safe answer! I also used to do the same as a student at school. When the teacher asked me something that I did not know, I provided a formula I could relate to the question. This is what the student did here.

In her comment, Marina refuted the initial claim (R1) and provided as a warrant that, "the student offered a safe answer" (W1). She further supported her warrant by implicitly referring to existing norms in the classroom where a student feels obliged to give an answer to any question (B1). She brought data from her own experience as a learner (D2).

Sofia offered as a warrant the students' difficulty with applying their prior formal knowledge to an open task (W2) and brought new data from similar cases that she had met in her classroom teaching to back it up, "We met similar incidents many times in our teaching (D3). Often students' prior knowledge was an obstacle to engaging them in an activity" (B2). Then, Irene and Anta refuted the initial claim by arguing that the student might have been able to make the connections very fast (R2, R3). Anna provided further evidence from her interaction with the student and offered data from another incident, "In a similar task of connecting the identity of

$(a+b)^3$ with the volume of a solid, the student made the same mistake" (D4). Similarly, Marina brought more data from the student's attempts to link the areas of the decomposed rectangles and squares to the algebraic identity (D5).

In the following excerpt, Leonidas supported the main claim by referring to the student's use of language in the excerpt provided by Anna and Marina, "The student used the word 'solution' to refer to the algebraic identity" (D6). He offered as a warrant that the student, "cannot see the equivalence of the two parts of the identity, but he considers it as a procedure that needs to be followed" (W3). In this phase of the discussion, the PMTs provided multiple warrants. These warrants referred to classroom norms (Efi and Leonidas offered as a warrant that the student provided an algebraic answer because the lesson was on algebra and not on geometry, W4 and W5), the curriculum (four PMTs offered as a warrant that the curriculum did not connect algebra and geometry, W6–W9), and students' attitudes (preference to algebra over geometry, W10). Irene brought another warrant, which recognized that linking algebra and geometry was not a simple task even for PMTs: "This connection is too difficult for the students" (W11), "It is difficult even for us to see how a geometrical situation can be expressed by algebraic symbols and operations" (B3). Here, the warrant was backed by PMTs' similar difficulties as learners at the university. In this case, Irene's experience at the university operated as a new source of data (D7). Later in the discussion, after the challenge from the teacher educator ("Can you interpret this incident based on what you have learned in mathematics education courses?"), PMTs enriched their interpretation of this specific incident by bringing data from research and theory of mathematics education (D8). Irene offered as a warrant that, "The students are used to applying the mathematical content to exercises" (W12). She supported it further by offering a backing that included the qualifier "I think" (Q1) and it was influenced by her research readings in the course. She said, "I think that this has to do with the didactic contract and the social norms of the classroom" (B4). Alexandros refuted W12, saying that, "The children are more creative than adults" (R4) and offered a warrant for this: "For the kids to use models to form algebraic relations is like playing a game, so they are successful" (W13). He referred to his experience at the university (D9) and used this as data to back W13: "We [as university students] tend to follow complicated solutions. Our minds are not used to seeing the simple solution" (B5). Irene and Anta referred to the critical role of representations in the construction of mathematical meaning in algebra (W14, W15), while Kostas referred to geometry and the dominant role of prototypical figures to further support Irene and Anta's argument (B6). Kostas also offered another warrant for students' difficulty with conceptualizing the meaning of the equal sign in algebra (W16).

A second critical incident that Anna and Marina brought concerned once again a student's difficulty with conceptualizing the identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ in a similar geometrical context involving the decomposition of the volume of a cube with side (a + b) into other solids. The main problem that the PMTs addressed was that the student could not relate the concept of volume with the space occupied by the solid. The PMTs offered new data from their teaching (D10) and the formulated claim was that conceptual understanding is a complex process (C2). The discussion initially centered on the teacher's role and classroom management issues. Then the students' activity was once more the main focus. Anna and Marina brought data from the transcript of their lessons about two unexpected incidents. In the first one, Dina, a student who had appeared not to participate in the interaction between Dimitris (a competent student) and the teacher (D11), provided the correct answer. Anna offered as a warrant with a qualifier, "probably" (Q2), that "Dina was thinking differently from Dimitris. Probably, she was not constrained by the existing formal knowledge, thus she had an open mind without reproducing mechanically taught methods" (W17). Irene further supported this warrant: "Her thinking does not follow a specific channel. Students like this can approach mathematical concepts in a more global way and see the meaning behind them" (B7). Irene also added an affective dimension to explain Dina's response, "As she was not interacting with the teacher at that time, she was not as anxious to provide a correct answer as Dimitris was, so she felt free to express her thinking" (W18).

In the second incident, the students easily recognized the algebraic identity (square difference) by transforming area manipulatives provided by the PMTs (D12). Concerning this incident, the PMTs offered warrants to support their expectation that the task would be difficult for the students and indicated the validity of the claim C2. Therefore, Anna argued that, "Even my friends could not visualize the identity" (W19), while Marina admitted that, "I had no idea how to rearrange the tiles to form a rectangle" (W20). Backings to these warrants were based on PMTs learning experiences at school and at university. As Anta put it, "Our thinking is highly constrained by the formal knowledge at school and at university so that we cannot think in a simpler way" (B8). Irene offered another backing by arguing that, "The children engage easily in playing with bricks, puzzles, and constructions and use their imagination. But we are far away from this" (B9).

In terms of the argumentation structure, PMTs made two main claims (C1, C2), as well as a number of warrants and backings in parallel argumentation steps that led to the support of the main claims based on different sources of data (D1–D12). In this process, we also observed the presence of refutations (R1, R2, R3, R4) in the argumentation structure, claims that were supported by a multiplicity of warrants (W1–W20) and backings (B1–B9), as well as the use of qualifiers (Q1, Q2) in PMTs' attempts to consider the claims from different viewpoints. This structure has similar features as the spiral argumentation structure of Knipping and Reid (2015). This is because it involves parallel arguments that could stand alone leading to the final claims, warrants, and backings that adequately justify the claims, and refutations of the main claims (See the argumentation structure of C1 in Fig. 10.4).

The sources of PMTs' interpretations

In this part of the discussion, the PMTs grounded their warrants, backings, and rebuttals mainly on the theory and research on mathematics education (a priori-pedagogical) their personal experiences as learners at school and at the university (empirical-personal), and their broader views about teaching and learning (evaluative). They also based their interpretations on their current teaching

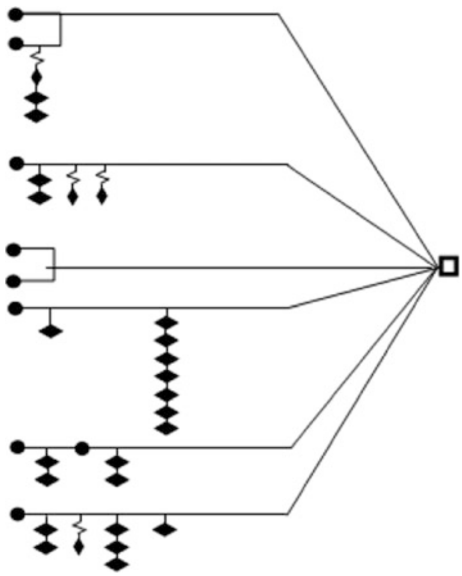


Fig. 10.4 Argumentation structure of C1 (8th university meeting)

Table 10.2 Classification of warrants, backings, and rebuttals in the eighth meeting

A priori-pedagogical	W3, W11, B4, W13, W14, W15, W16, B6
Institutional-curricular	W6, W7, W8, W9
Empirical-professional	W2, B2, W4, W5
Empirical-personal	W1, B1, B3, B5, W19, W20, B8, B9, R1
Evaluative	R2, R3, W10, W12, R4, W17, B7, W18

experiences (empirical-professional), as well as on curriculum and textbook recommendations (institutional-curricular). In Table 10.2, we summarize our classification according to the framework of Nardi et al. (2012).

Warrants, backings, and rebuttals are based on a multiplicity of sources. The co-existence of a priori-pedagogical, empirical-personal, and evaluative types indicated that PMTs had started to interpret students' activity in different ways. For example, they combined research findings and theories they had encountered at the university with their personal experiences as learners and with their broad views about teaching and learning. These connections indicated a reflective stance towards classroom phenomena and a development of awareness of the complex interrelationships that underlie these phenomena.

10.4.3 Comparing Argumentation in the Two Meetings

The analysis of the discussions in the two university meetings showed shifts in the process of interpreting students' activity through different argumentation structures and types of warrants, backings, and rebuttals. While observing other teachers' teaching PMTs' argumentation was based on different data they brought from their observations with a small number of warrants, backings, and rebuttals. The emphasis then was on collecting data and conclusions without making connections between them and warrants or backings. In contrast, the PMTs' argumentation based on their actual teaching was enriched by a large number of warrants, backings, and rebuttals, as well as by qualifiers. In this case, the emphasis was on the connections between data and conclusions based on justified arguments that could support the final claims in different independent ways.

Through the analysis of the types of warrants, backings, and rebuttals, we identified a balanced distribution of them in the categories of Nardi et al. (2012). Evaluative arguments, evident in both university meetings, indicated the role of personal views and beliefs about learning and teaching mathematics in the process of interpreting classroom incidents and noticing in general. However, the presence of a priori pedagogical arguments in the second case revealed the research-based character of arguments that strongly related to the aims of the course and the PMTs' experiences in their university studies. At the same time, the reflective stance promoted in the course seemed to play a unifying role between the experiences that PMTs brought from research, personal learning histories and views, and current teaching practices.

10.5 Conclusions

For interpreting students' activity, the PMTs used different sources of data based on their prior school experiences, current university studies, and fieldwork. In particular, they made links between students' conceptualizations and their own experiences as learners at school and university and they looked for evidence in their classroom observations and teaching. The analysis of the discussions about students' activity in the two university meetings showed different argumentation structures in terms of the use of warrants, backings, and rebuttals and their interrelations. The structure that emerged from the analysis of PMTs' reflections on their classroom observations (third university meeting) involved parallel arguments, warrants, and backings without rich connections between them. Conversely, towards the end of the course while PMTs reflected on their own teaching (eighth university meeting) their argumentation was based on argumentation steps consisting of a large number of warrants, backings, and rebuttals that targeted the final claim. Using Knipping and Reid's (2015) framework in a teacher education context, we identified similar argumentation structures in the pedagogical discourse.

However, in both structures that we identified, the argumentation steps seemed to lead directly to the main claim. Thus, in our case the difference between the two structures related to the number of warrants, backings, and rebuttals that were richer in the second case. This indicated that the PMTs had developed deeper and justified interpretations.

Using Nardi et al.'s (2012) approach in analyzing PMTs' warrants, backings, and rebuttals, we identified a multiplicity of sources on which PMTs grounded their interpretations. Towards the end of the course, it was more evident that PMTs had developed justifications balancing sources from theory and research in mathematics education, from personal views about learning and teaching mathematics, and from their experiences as learners and teachers.

The differences between PMTs' interpretations of incidents selected when observing other teachers' teaching and their own teaching can be explained from different points of view. First, existing research (e.g., Stockero, 2008) shows that PMTs' experiences in analyzing other teachers' lessons can enhance deeper levels of reflection on their own teaching. Another explanation could be that towards the end of the course, PMTs started to have examples from their own teaching as a basis for reflection. This experience seemed to have facilitated their progress in realizing interrelationships between teaching and learning. Also, the use of critical incidents as a teacher education strategy in the course seemed to have supported PMTs' in reconstructing prior experiences about teaching and learning mathematics in the light of new experiences in the teacher education context.

Our study offers an analytical framework (argumentation structures and classification of warrants and backings) that can contribute to the field of research in teachers' noticing. First, by analyzing the argumentation structures, we relate noticing to PMTs' justification of their claims. According to Mason (2002), this is an indication of their awareness of mathematics teaching that constitutes an important aspect of noticing. Second, our approach allows researchers to address the sources upon which PMTs base their interpretations. Although existing research has emphasized the important role that PMT's prior learning experience, beliefs and orientations play in noticing (Ding & Dominguez, 2016), our study provides a lens to analyze how PMTs' experiences (personal learning, fieldwork, university courses) influence the process of noticing. Finally, our analysis of the types of warrants and backings in PMTs' argumentation makes it possible to address the developmental trajectory of noticing. According to van Es (2011), teachers' transition to higher levels of "how they notice" is characterized by their ability to make connections between events and principles of teaching and learning and to propose alternative pedagogical solutions based on their interpretations. Our findings show that PMTs reached higher levels of noticing indicated by their making of connections between events and research on teaching and learning. Therefore, the analysis of the warrants and backings allows us to identify the shifts in PMTs' interpretations. Further research is needed in order to address the potential of this approach to mathematics teachers' noticing.

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Chapter 11
Designing a Competence-Based Entry Course for Prospective Secondary Mathematics Teachers

Fou-Lai Lin, Kai-Lin Yang and Yu-Ping Chang

Abstract The goal of this study aims at introducing an entry course of a 3-year sequential courses module for a secondary mathematics teacher education program in Taiwan. This module is a reformed teacher education curriculum planned for Prospective Secondary Mathematics Teachers (PSMTs) to learn how to teach with the field-study approach. The field-study approach provides abundant opportunities for PSMTs to cultivate their competencies in teaching. In this chapter, we take the first year course to deliberate why the Psychology of Mathematics Learning is selected as an entry course for the teacher education program and how it works. Considering the importance to raise PSMTs' awareness of students' mathematical thinking and to cultivate their competencies of sensitizing students' mathematical thinking, and ultimately to bear the competencies as the habitus in their future teaching professional, the mission of the course focuses on PSMTs' learning of understanding students' mathematical thinking through the process of cyclic learning. The quality of PSMTs dynamic learning in the field study can be evaluated by their study work. This chapter provides one example of PSMTs' survey study in one complete learning cycle, and summarizes several criteria of evaluating how PSMTs conduct a study to understand students' mathematical thinking in a holistic perspective.

Keywords Teacher education · Competence-based entry course
Prospective mathematics teacher · Field-study approach

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_11

11.1 Background

11.1.1 Emergent Teacher Education Issues

The goals of teacher education program diverse among countries and societies and change in recent decades. For example, there are goals focusing on teachers perceiving knowledge and the proficiency of teaching skills (e.g. Cochran, DeRuiter, & King, 1993; Shulman, 1986), and goals focusing on their thinking or reasoning process when learning (e.g. Putnam & Borko, 2000; Shulman, 1987). Analogizing the continual problem of in-service teachers' professional development (Cooney, 2001), i.e. the sustainable effectiveness of professional development program (Zehetmeier & Krainer, 2011), the continuity of *what* teachers learned in teacher education program to their future teaching professional, is a potential yet challenging issue (Lin & Rowland, 2016) for teacher education.

Zeichner (1983) traces back to the philosophical and epistemological perspectives and lists four commonly seen paradigms of teacher education through orienting prospective teachers in viewing teaching contents as *received* or they are *reflexive* (active) participants, and in viewing teacher education and schooling contexts as *problematic* or *certain*. One of the paradigm, *inquiry-oriented* teacher education, points out the important variations about teaching methods and teaching contexts, and therefore the development of teaching in teacher education should not only emphasize on the knowledge or skills but the competencies in teaching. Being aware of oneself teaching with active reflection and viewing teaching contexts as dynamic and varied become crucial.

The abstraction of mathematics makes teaching more complex than other subjects. For example, students' mathematical misconceptions, learning difficulties, anxiety in mathematics, etc. make the teaching contexts even challenging for teachers to deal with. To cultivate a mathematics teacher to be *reflexive* on mathematics contents and consider the pedagogical situation as *problematic* rather than *certain*, the *inquiry-oriented* paradigm seems to be valuable to inducing more interactions between teacher and students.

11.1.2 Micro Reform of Teacher Education Curriculum

Teaching is usually viewed as delivering knowledge to students (e.g. Hill, Rowan, & Ball, 2005), and a teacher's subject-matter content knowledge or former learning experiences are therefore considered as significant and influential factors in his/her teaching (Darling-Hammond, 2006). The international comparative study of primary and secondary mathematics teacher education, the Teacher Education and Development Study in Mathematics (TEDS-M), provides an outlook on prospective mathematics teachers' knowledge in mathematics and its pedagogy (Blömeke, Hsieh, Kaier, & Schmidt, 2014). Though the prospective mathematics teachers in

Taiwan showed their outstanding performances in this comparative study, beginning mathematics teachers often claimed that they cannot directly use what they have learned in university to their teaching practice in their early career when they came back university to attend seminars (group discussions, 2013, 2014, 2015, 2016). Therefore, it pushes us to inspect how the teacher education programs influence prospective mathematics teachers learning how to teach. Conventionally, the courses of the teacher education program at the National Taiwan Normal University (NTNU) are provided with a structure of 9 *course units* (incl. required and optional) in three years, from the sophomore to the senior year, for prospective secondary mathematics teachers' (PSMT) professional development. Such teacher education program is even unique in Taiwan.

In considering the *lasting learning efficacy* for prospective mathematics teachers and the specific *needs* of mathematics teachers in Taiwan, one reformed curriculum is designed as a 3-year course module to strengthen mainly the PSMTs' *competencies in how to teach mathematics* rather than to increase mostly their mathematical knowledge and teaching skills. This course module is proposed with the conjecture that *once prospective teachers are equipped with the competencies, they can deal with the cognitive and affective issues of teaching and learning mathematics, especially students' mathematical thinking, in all mathematical topics*. The designed course module is composed of 6 course units: the Psychology of Mathematics Learning (I & II) for the first two semesters, the Instructional Materials and Methods for Mathematics and the Study of the Instructional Materials and Methods for Mathematics for the third and fourth semesters, and the Mathematics Teaching Practicum (I & II) for the last two semesters. Only the Psychology of Mathematics Learning (entry course; 2 course units) is the optional course for PSMTs in this course module. All these units were scheduled sequentially for the PSMTs at NTNU from years 2013 to 2016. It is noted that the entry course of the first year was piloted one year in 2013.

11.1.3 Competencies for Prospective Mathematics Teachers to Learn

After reflecting upon what teachers should learn for a changing world (Darling-Hammond & Bransford, 2005), we consider that future mathematics teachers need (1) to understand *students' learning processes and needs*, (2) to understand *curriculum contents and goals*, (3) to understand *teaching skills*, and (4) to develop *productive disposition of teaching and learning*. Hence, it can make the effort to improve the teaching effectively and efficaciously. Furthermore, we consider that the primitive image (of learning) influences a lot on a learner's following learning, the entry course of this module then plays a crucial role to cultivate a teacher's profession in the beginning of their teacher education learning. The inquiry-oriented teacher education provides abundant opportunities for prospective teachers to develop themselves as an action researcher (Corey, 1953), as an innovator (Joyce, 1969), as an inquirer (Bagenstos, 1975), as a self-monitoring teacher (Elliot, 1976), as a participant observer (Salzillo & van Fleet, 1977), etc. in teacher education program. However, not all prospective mathematics teachers orient themselves toward the above mentioned roles. How to design an entry course for the inquiry-oriented teacher education for prospective mathematics teachers learning *how to teach* is still an unsolved issue. In the following sections, we first present how we design the entry course for PSMTs' effective learning in this module by describing its *goals*, *learning approach*, *learning contents and resources*, and *learning cycle*. Second, we provide one example of PSMTs' field study with a complete learning cycle. Last, we conclude with the criteria of evaluating PSMTs' learning with field-study approach to understanding students' mathematical thinking.

11.2 The Entry Course Design

Taking the importance of primitive image in learning and the habitus of teaching into consideration, we believe that the opportunity to experience students' various learning in mathematics may provide prospective mathematics teachers rich viewpoints on teaching in reality. Even though these PSMTs performed at the 87.59th percentile of the national college entrance examination in 2013, they have almost no experiences of students' misconceptions or learning difficulties in learning mathematics. Therefore, raising their awareness of students' mathematical thinking (Llinares & Krainer, 2006) becomes a priority of the entry course design. In addition, the cultivation of the cognitive *sensitivity to students* (Jaworski, 1994) may provide a more active function in predicting students' performances before a teacher makes a pedagogical decision. Consequently, *understanding students' mathematical thinking* is then the focal competence we hope prospective mathematics teachers can develop. Hence, in the entry course, we take the prior mission to develop the PSMTs' sensitivity to students' cognitive thinking in mathematics by supporting them various mathematical contents, theories (incl. plentiful empirical studies) and opportunities to practice with peers, and provide them guidance from experienced mathematics teacher educator-researchers (MTE-R), i.e. the instructors of the course. In order to record the performances of PSMTs, all the lessons of the interactions between instructors and PSMTs were videotaped and PSMTs' assignments, written reflections on the course, and attitudes (interviews and questionnaire survey) were collected as either digital files or written papers for tracing their development longitudinally.

11.2.1 The Goals of the Course

The entry course is a *competence-based* design which aims mainly to develop PSMTs' competencies of pedagogical reasoning in students' hypothetical learning trajectory (Simon, 1995), i.e. understanding students' mathematical thinking, and at the same time to consider strengthening their positive beliefs in mathematical pedagogy. To reach this aim, the course assigns tasks to PSMTs to conduct 2–3 field studies in each semester, with the guidance from the instructors. The assignments of these field studies are to provide opportunities for PSMTs (1) to comprehend students' mathematical thinking via studying their cognitive and affective performances in learning mathematics, i.e. the learning of knowledge, skills, (2) to cultivate their experiences of exploration and practice in comprehending students' mathematical thinking via the actions of *recognizing*, *analyzing* and *interpreting* students' performances, and (3) to develop a positive belief of their learning processes of this course as a foundation of teaching and learning mathematics.

In addition, this course positions PSMTs as *creators* (higher level) or *re-producers* (basic requirement) rather than *receivers* of knowledge (Ebby, 2000; Hiebert, Marris, & Glass, 2003; Stein, Engle, Smith, & Hughes, 2008) in the process of conducting field studies. Moreover, PSMTs learn not only through theories, but also through practice and reflection on theories, and through collaborative learning with others (Korthagen, Loughran, & Russell, 2006), i.e. their peers and the instructors.

In brief, in designing the course, three learning goals are set for PSMTs to achieve: (1) learn different methods to explore students' mathematical thinking, (2) have the (takeaway) competencies of recognizing, analyzing and interpreting students' mathematical thinking after the entry course learning, and (3) accumulate generic examples of students' representative performances through continuous practice of field study and constant reflection on and dialogue with the theories (esp. the national survey data).

11.2.2 The Design Principles

Taking the consideration of long-term influences of the entry course on PSMTs' following courses of the teacher education program and their teaching professional, two principles are seriously concerned in designing and conducting the entry course. One is that the course should convince PSMTs the usefulness of the course through their *enactment* (Clarke & Hollingsworth, 2002) of the field trial practice, i.e. enhance PSMTs' engagement in the course. The other is that the course should help PSMTs to reflect in action/enactment and reflect on action/enactment, through the process of interacting (incl. the preparation of field trial and the report presentation) between theories and practices, i.e. be a *reflective practitioner* (Clarke & Hollingsworth, 2002; Schön, 1983, 1987).

11.2.3 The Field-Study Approach to Learning How to Teach

Thinking through the possible variations of teacher education, Kansanen (2006) takes the interactions between two significant factors into account: the *ways of organizing activities* (inductive vs. deductive) and the *pedagogical thinking* (intuitive vs. rational), to structure the possible teacher education approaches. He discerns four different approaches of teacher education program from this structure: (1) school-based, (2) research-based, (3) experiential and personal, and (4) case approach and problem-based. Considering our rationale in cultivating mathematics teachers, the pedagogical thinking needs more rational decision rather than intuitive judgement. The two approaches "case approach and problem-based" which is inductively organizing activities and applies rational pedagogical thinking, and "research-based approach" which is deductively organizing activities and applies rational pedagogical thinking, are closed to the process of inquiry-oriented teaching (see Sect. 11.1.1).

In teacher education program, the two approaches can provide rich opportunities for prospective mathematics teachers to practice their knowledge and skills, and be aware of the lack of pedagogical competencies in teacher education program. In order to fulfill the rich learning of *inquiry-oriented teaching*, we therefore device an approach called *field-study approach* which integrates the problem-based and case approach and research-based approach. The field-study approach seeks to make teacher education to be practical in linking theories and practices.

To concrete the essential features of the course in preparing mathematics teachers, we adapted the idea of learning by *doing* from Schoenfeld (1996), emphasizing on the importance of a situation that can increase PSMTs' *authentic appreciation for*, and *understanding of* the content being learned (Barab & Duffy, 2000), as the fundamental consideration of creating a learning environment for PSMTs. In this learning environment, PSMTs are motivated to conduct field studies, either interview or survey, on students' mathematics learning. The research methods regarding interview skills and survey are delivering as the additional learning contents for PSMTs.

To start a field study, they should choose one topic they are interested with and start to pose their research questions regarding to its pedagogical issues in mathematics, especially the cognitive perspectives. By conducting studies, they need to strive to connect the relationship between the learned theoretical knowledge and the practical challenge they meet, which is not easily to achieve if they only view learning how to teach is to absorb theoretical knowledge in the course. During the process of conducting studies, they have to continue *raising conjectures* underlying subjects' performances to adjust their interview questions or to interpret the survey results that went far from their predictions. Therefore, in conducting their studies, they need to treat the subjects as the ones they are going to learn with. To conclude one round of the field study, they are required to report the complete study to their peers. Sharing the results with their peers drives them to re-organize the study and the data. During this process, their peers are not only listeners but also criticizers. This mentioned procedure to work with field-study can be viewed as one learning cycle (see Sect. 11.2.5).

11.2.4 The Learning Contents and Resources

School mathematics is no doubt a good start for prospective mathematics teachers to learn how to teach (Cooney, 2001; Davis, 1999; Gerdes, 1998; Presmeg, 1998) for that the contents of perceived predetermined mathematics is familiar to the prospective mathematics teachers and can help them acquire pedagogical skills from their former learning experiences quickly. Therefore, the contents of the entry course focus on the range of school mathematics especially the secondary school mathematics.

According to the five strands of mathematical proficiency raised by Kilpatrick, Swafford, and Findell (2001): (1) *conceptual understanding*, (2) *procedural fluency*, (3) *strategic competence*, (4) *adaptive reasoning*, and (5) *productive disposition*, we consider that these strands of proficiency can have parallel contribution to prospective mathematics teachers' learning. In order to discriminate the learning contents and inclination to learn how to teach for PSMTs, we therefore adapt them into four essential contents as part of our content structure and the productive disposition is intended to embed in the content structure to raise their engagement and attitudes towards the course (see Lin, Yang, Chang, & Hsu, 2014). Moreover, many studies of students' problematic mathematics learning show the close connection to students' intuition (e.g. Fischbein, 1982) and can be found in almost every mathematical topics, we then include it as one necessary content in the content structure for PSMTs to learn.

After reflecting on the theories and research of teaching and learning mathematics, five imperative categories of learning contents are considered necessity for PSMTs to learn in this entry course. When giving the introduction of these categories, different mathematical content topics are selected as examples to introduce. They are: (1) mathematics intuitive rules: more A more B and same A same B (Stavy & Tirosh, 1996, 2000; Tirosh & Stavy, 1999), (2) mathematics conceptual understanding, (3) students' procedural knowledge, (4) students' strategies and thinking of problem solving, and (5) reasoning and argumentation. The field studies are therefore conducted based on each of these five content issues in two semesters.

Furthermore, the national survey studies on Taiwanese school students' (esp. those from the secondary schools) *conceptual understanding* in various mathematical topics from serial projects during 1980s and 2000s, are selected according to students' common performances to provide for PSMTs as their learning resources. The topics include, for example, the *ratio and proportion* from ages 13 to 15, the *fraction* from ages 11 to 14, and the *algebraic operations*, the *algebraic argumentation*, the *geometric shapes*, of adolescents etc. (see Lin, 1988, 1991). Though these empirical results released decades ago, they play an important role to provide a holistic view of local students' cognitive development in mathematics systematically and as well referential models for conducting field studies. In addition to the national survey studies, the classical literatures of several classifications are also provided to them to consolidate their theoretical knowledge. These classifications include the individual's development (e.g. stages of cognitive development, and scientific concept development), understanding (e.g. instrumental and relational understanding, and understanding (the process of) understanding mathematics), concepts and knowledge (e.g. conceptual and procedural knowledge, concept image, and concept definition), thinking (mathematical intuition, geometrical reasoning, and algebraic thinking), and problem solving. With these resources, they can meet closer to students' thinking and have a dialogue when they conducting a field study and analyzing and interpreting their results.

11.2.5 The Learning Cycle

A cyclic *learning cycle* is intentionally designed for PSMTs to experience in order to cultivate their learning habitus via several rounds of practice with the learning cycle on the instructors' assigned missions. Since this chapter discuss the first year of the course in which the practice is set for PSMTs to conduct the field research after learning the theories and the empirical study results in the beginning of the course, the PSMTs have to practice how to conduct research. Before they start their practice, they are asked to group 2–3 peers by themselves. This group is a community of practice with the same research interests. In this way, the PSMTs within the group should communicate and discuss to each other to have the consensus of the research topic through several rounds of discussion. Then, they have to present and share their research plan to the whole class which is viewed as a social learning community and the community needs to provide the critical suggestions to the corresponding plan. In brief, each group has two rounds of practice, either quantitatively or qualitatively and they have to first choose the study topic and discuss their conjectures and possible results of students' mathematical thinking with their group peers and then discuss with the whole class. The instructors will provide interventions on and supports to their presentation of each group. Additionally, the instructors have their rich background of research experiences and acquaintance with theories. Then the PSMTs start to design their study and implement it in real situation, i.e. interviewing students or conducting a survey in classes. Finally, they have to reflect on the data they collected and report it to the whole class. In their report, they need to have a dialogue with the theories and the whole class.

In brief, this cyclic learning cycle can be summarized in three phases (Fig. 11.1): (1) *Initiation*: conjecturing the possible performances of students and making confirmation with theories, and peers and instructors, (2) *Conduct*: finalizing the design of study and implementing it, and (3) *Reflection*: reporting the results and leading the communication within the learning community.

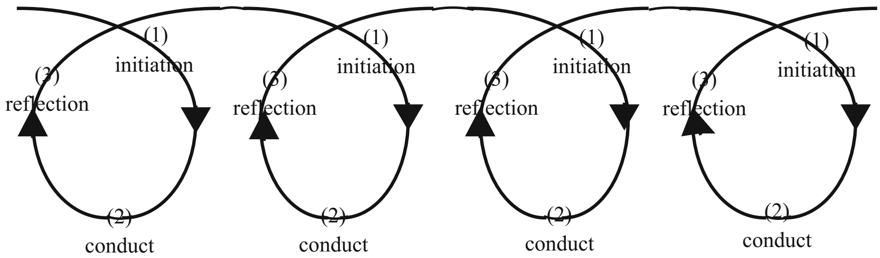

Fig. 11.1 The field study-based cyclic learning cycle

11.3 Influences of the Entry Course: The Example of One Field Study

This entry course is designed not for temporal learning but for a long-term consideration of professional development, therefore, the evaluation of its effectiveness cannot ignore the PSMTs' performances in learning chronologically of the module courses. To consider the connection among the entry course, the course module, and teachers' future teaching, we consider *four crucial stages* to examine the effect on PSMTs' learning of the entry course: (1) the *immediate performances* in the entry course, i.e. PSMTs' performances in each research practice, which are viewed as the preliminary learning effect of the entry course, (2) the *transitional performances* in the courses of the second and third year in linking theories and their teaching practices, (3) the performances in their *practicum* as the extended effect of the entry course and the course module, and (4) the performances of their teaching practice in their beginning *career*, which can be examined in their future professional development, as the lasting effect of the entry course and the course module.

The PSMTs' practice in field study can mostly contribute to mathematics educators to evaluate their *immediate performances*. Moreover, conducting one field study needs to process one round of *learning cycle*. Therefore, we choose two PSMTs' field study work in one of their research practice as an example to provide a broad view of how one learning cycle functions and present their preliminary learning effect by the entry course. Below shows one group's immediate performances of conducting the study in surveying junior high school (grades 7–9) students' fractional and proportional notions. The group is composed of Jay and Zed. Their performances in the study are discussed in one learning cycle of three phases: (1) discuss the emergence of the study, especially how they *generated* the research questions, and how they *designed and revised* the study, (2) *conduct* the survey, and (3) *reflect* on their findings.

11.3.1 Phase 1: Design and Revise the Study

Starting from setting research aim and questions. In selecting mathematics contents for the field study, Jay and Zed *first* considered negative numbers, fractions, and proportions which are three unfamiliar notions/topics for 7th graders learning mathematics. However, they considered fractions and proportions are two topics closely connected to daily life. Moreover, they were eager to know *whether the intervention of guiding students to think fraction and proportion can enhance their learning* (Zed's written report, 2013). Therefore, they selected fraction and proportion as contents to design two test sheets: one with guidance and the other without, of the same content. Both test sheets are composed of 4 sets of serial questions which are revised from items of the national survey study (i.e. the learning resources provided in the entry course), and intended to induce students' understanding and learning difficulties in fraction and proportions based on their unjustified conjectures.

Revising the design. Before they conducted their survey, they discuss their designed questions one by one within the learning community (including the course instructor) to review whether their design can connect their (unjustified) conjectures of students' possible performances/outcomes and the theoretical knowledge. The resources of the national survey results on students' thinking of fractions and proportions play an important role during the discussion. Moreover, since the serial questions of their original design were intentionally designed to guide students to answer/solve, the logic of the serial questions also discussed a lot. After the discussion, Zed noted the reason why they revised the numerals from their first serial questions regarding proportion in his report, "*the first and second questions are based on the conjecture that students can use folding* (halving) *and iterating strategies* (according to the results of national study, see Table 11.1), *however, in the* (our) *original design the iterating strategy is the only used strategy, therefore the numbers are revised to* ...". From Zed's record, it is obvious that the discussion within the learning community helped them to revise their original test sheets with the assistance of empirical and theoretical supports.

11.3.2 Phase 2: Conduct the Survey

In order to investigate their research, Jay and Zed conducted their survey with 4 different classes including 7th, 8th, and 9th graders, total 127 students (13–15 yrs) in the same school. After collecting students' responses from the survey, they analyzed the collected data and finalized their report with findings and their reflections on the whole survey (see Sect. 11.3.3. Phase 3).

This field study was the second study assignment given to the PSMTs in the entry course and was followed the introduction of one national survey study of adolescents' proportional thinking in Taiwan. In their first study, Jay and Zed chose

Table 11.1 Students' hierarchical levels of proportion and the corresponding features

Levels	Difficulty (%)	Characteristics	Correctness (%)	Item#
I	65–85	No needs for ratio. The easy multiple concepts of 2 times, 3 times, half, etc.	73	1(1) i
			77	1(1) ii
			66	3(1)
II	51–60	Easy ratio. Applying the method of halving and iterating, such as 2:3, 2:5, etc.	60	1(2) i
			59	1(2) ii
			51	3(2)
III	34–50	Needs for ratio. Involving the calculation of fraction or (finite) decimals	50	4
			35	5(1)
			41	8(5)
IV	19–32	Needs for proportion expression. The relation among variates is more significant than finding a ratio	24	3(3)
			19	5(2)

to interview students' intuition in geometry content. Therefore, they planned to set a challenge for themselves in the second study by applying a different method. Though the method of their survey study was not rigor enough, especially the implementation process and also the variates control (e.g. the arrangement of students' in responding two versions of test sheet: with/without intervention), it is not the main issue to be concerned in the entry course. Therefore, the instructor spent less time on discussing the rigor of their experiment design. They were asked to spend more time on the contents of their research design and the interpretation of their results. However, the situation in conducting an interview will be completely different from a survey study. The interview skills and further questions to probe students' mathematical thinking are concerned as a key competence to evaluate. In the last section of this chapter, we summarize the criteria of how we evaluate PSMTs' *interview* ability in their study of investigating students' mathematical thinking.

11.3.3 Phase 3: Reflect on the Study

The PSMTs were asked to report their study in the final step of one learning cycle (initiation-conduct-reflection) and hand in a written report of the whole study. Their reports were guided with a format which was provided as one course material. The structure of this format is composed of five categories:

1. The background of the study: title of the study, the involving mathematical topics and contents, and key words of the study

2. Involved design materials: questioning design (for interview study) or questionnaire design (for survey study), predictions of students' responses and interpretations, analysis of students' outcomes or individual student's responses
3. Step 1 of design: PSMTs' (learning) expectations of the study, predictions of students' difficulties in understanding and learning within this topic, the conjectures of possible factors prompting those difficulties, and the goal of the design
4. Step 2 of design: according to students' difficulties listed in step 1, what kind of questions or problems cannot be completed by students themselves? What if students are stuck with your question, what kinds of further questions or sub-questions can you give? Is there any other related questions or problems can be used to check students' difficulties? Do the above mentioned questions correspond to your goal of the design? If not, what are you going to adjust, e.g. goal of design, missions, tasks, further questions or sub-questions?
5. Analysis and interpretation of the interview or survey results.

Therefore, PSMTs' reflections on their study can be guided with the orientation from the structure of this report format.

Connecting the study to learned theoretical knowledge. In Jay and Zed's exploratory study, they presented their ability in connecting research results to what they learned in the entry course in two perspectives. First, they interpreted the survey results with similar representation of the national survey study (course materials), for example, they analyzed students' answers by categorizing their solution strategies, e.g. halving and iterating, adding up, subtraction, etc., item by item. The various solution strategies are introduced in the entry course with examples. Second, they interpreted the survey results with the former national survey results, i.e. use *proportional thinking level* in Table 11.1 developed/used in the national survey study (Lin, Kuo, & Lin, 1985) as theoretical knowledge to interpret students' problematic items.

Reflecting on the unexpected results rather than the holistic study. Jay and Zed spent many spaces in presenting the distributions of students' outcomes in their presentation. They pointed out that in their study, students treat the serial questions of one set as disjointed questions to solve which was unexpected to them. Their reflection was guided by students' performances rather than their set target. That is to say, they did not reflect on their research between their research goal (i.e. whether the intervention of guiding students to think fraction and proportion can enhance their performances) and the findings. Though after the discussion of the learning community, Zed recorded this point in his report, he mentioned that they (he and Jay) recognized that their goal is to investigate whether the intervention of guiding students to think helps them to solve fractional and proportional problems, he still could not answer it for the restriction of their study. Though they found unexpected findings in their study, they still could not guarantee whether their guiding design had no influence on students' answers. Therefore, he proposed an alternative solution to consider to devise the further instructional research for this unexpected result.

11.4 Final Remarks

When analyzing and categorizing all participating PSMTs' works regarding to conducting field studies (research practice), it is found that field-study approach brought challenges to them and was not easy to tackle with. In preparing the plans of field study, most of their research designs, conjectures on subjects' performances, findings, and reflections on the study are *not* well-organized *nor* innovative, even though the formats of reports, related theoretical knowledge, and instruction have been already provided and instructed to them before they started their field studies.

To provide a general view on how we systematically analyze all PSMTs' immediate performances in conducting an interview study of understanding students' mathematical thinking, we crystalize the criteria of evaluating their works and summarize them in brief according to our reviews on their work.

1. The designs of PSMTs' study can be categorize to three: (1) the imitations of the learning resources (see Sect. 11.2.4), (2) the revisions of the learning resources, and (3) the inventions of a new study. Their designs show their ambition to conduct a study. However, these three categories cannot guarantee the quality of their field study.
2. The variety of conjectures of students' anticipated thinking regarding the mathematics contents raised by PSMTs contributes to their research design, depth of interview, and interpretation of results.
3. The interview results expose different treatments of further questions: (1) the *unexpected results*: interviewers stop questioning for that (a) they cannot raise any conjecture of the dilemma or (b) they try as many conjectures as they can but in vein; (2) the *expected results*: (a) interviewers satisfied with interviewees' immediate response without any further conjectures, (b) interviewers try to solve the dilemma by interacting several rounds of conjecturing-questioning with interviewees till they reach the expected responses.
4. The quality of reports can be generally categorized in three levels: (1) merely describing the phenomenon, i.e. the description and record of the study processes or results; (2) analyzing students' mathematical thinking underlying their outcomes or performances; and (3) reflecting on the findings and interpreting students' mathematical thinking according to the literatures or the results of the national survey.

Moreover, the quantitative evidences from PSMTs' perspectives (Lin et al., 2014) show that this entry course provide opportunities for them to understand students' mathematical thinking, to cultivate the competencies of exploration and practice, and to develop positive beliefs in learning how to teach mathematics. The influences of this field-study approach, for PSMTs to learning how to teach, present its potentials for inquiry-based teacher education. Last, this entry course and the criteria of evaluating PSMTs' learning can contribute mathematics teacher educators a model to deliver the complex course and to evaluate how PSMTs understand students' mathematical thinking.

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Chapter 12
Nurturing Knowledge of Mathematical Modeling for Teaching

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Abstract In this paper we will report results of our efforts towards providing mathematical modeling experiences for a cohort of prospective secondary teachers. Using a Teacher Development Experiment design, we traced the impact of these experiences on teachers' reported efficacy towards and knowledge about utilizing modeling in their teaching. Analysis of results indicate that although teachers maintained modeling to be an important skill to be developed in instruction, absence of extensive experiences with mathematical modeling in the course of their own mathematical preparation served as a barrier to their ability to access specific pedagogical actions they believed could be used in their instruction.

Keywords Mathematical modeling · Prospective secondary mathematics teachers

12.1 Introduction

The demand that secondary mathematics teachers will infuse mathematical modeling, as a content standard, in curriculum is now paramount in the educational reform efforts in the US as pioneered by the Common Core State Standards of Mathematics (CCSSM, 2010). Although the Standards document does not provide detailed description of what this particular strand might entail or how teachers may nurture the desired capacities among students they do offer topical examples of tasks that students should be able to do. There is some evidence that due to absence

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_12

of information teachers maintain misconceptions regarding the nature of mathematical modeling as well as its associated pedagogies (Gould, 2013). Mathematics teacher educators are naturally charged with the task of preparing teachers to realize effective ways that mathematical modeling could be implemented in classrooms. Due to paucity of reports (Cai et al., 2014), it remains unclear what type of experiences might need to be designed and utilized in advancing teaching knowledge in this area. The international community of researchers on teaching and learning of mathematical modeling have stressed the need to explore models and programs that might assist teachers to meet implementation challenges (Doerr & Lesh, 2011), and to document potential impact of these efforts on teachers (Cai et al., 2014). The current report aimed to address these two areas. Three questions guided our data collection and analysis:

(1) What knowledge bases related to mathematical modeling do prospective secondary mathematics teachers bring to teacher education?
(2) How do prospective teachers characterize mathematical modeling process?
(3) What is the impact of a unit of instruction on prospective secondary mathematics teachers' knowledge about teaching and learning of mathematical modeling in schools?

12.2 Context and Background

There is consensus within the research community that effective teaching of mathematics hinges upon the teachers' Mathematical Knowledge for Teaching (MKT) characterized by various scholars to encompass a range of domains among many include a deep understanding of the subject matter, curriculum and curricular resources, children' thinking and instructional tools that facilitate development of mathematical cognition among learners (Ball & Bass, 2000; Rowland & Ruthven, 2011). Development of these domains of knowledge demands extensive time and exposure to effective pedagogies modeled by both professors of content and pedagogy and best supported when efforts are collaborative (Anhalt & Cortez, 2016). At the secondary teacher preparation level, however, common national trend continues to be the one in which the prospective teachers complete their coursework in the disciplinary department and then engage in the study of pedagogy through courses housed in the college of education (APLU, 2017; Hawkins, Stancavage, & Dossey, 1998). Despite widespread, and longstanding lack of satisfaction with this divide and the negative consequences it has on teachers' development (Lingefjärd, 2007) breaking this mold has been difficult to achieve. Due to this, methods courses designed for teachers continue to serve as prime places where teachers are granted opportunities to develop deeper understanding of mathematical concepts but to also learn techniques for effective practice (Monk, 1994). Addressing new competencies identified by the Common Core Standards of Mathematics resides, for the most part, within the prevue of methods courses. Such is the case at our institution.

The research we report here is a part of an extensive program evaluation in which we study growth and development of secondary mathematics teachers as they complete the requirements for obtaining a teaching certificate. The goal of the larger study is to trace ways in which program courses, collectively, influence the candidates' practices during their induction years. The current effort was a design experiment aimed to explore how a course on methods of teaching secondary school might be manipulated to accommodate for development of teachers' MKT specific to mathematical modeling.

Mathematical modeling as a disciplinary arena has its own intricate cognitive demands (Doerr & Lesh, 2011). Skills needed for successful mathematical modeling are different from other types of mathematical work since modeling demands a great deal of decision making and reflection on the part of the modeler (Gould, 2013). Instruction aimed at improving modeling cognition among learners would need to be sensitive to the unique and complex demands of modeling process. Current conceptualization of mathematical knowledge for teaching has not yet unpacked these particular demands and ways in which teachers' knowledge may be enhanced in this area. Due to this, our work while guided by theory surrounding dimensions of mathematical knowledge for teaching and how they may be supported in teacher preparation, aimed to provide an understanding of the type of knowledge needed for teaching mathematical modeling.

The methods course that served as the site for the current study is the second of a sequence of two courses on methods of teaching high school mathematics. The focus of the first course—STEM Math Method I—is on teaching of Algebra, Calculus and number theory concepts. Using Principles of Inquiry Based Learning and Teaching (Artigue & Blomhøj, 2013) the course draws attention to connections between student thinking and instruction. In this course teaching candidates are also introduced to the state and national curricular standards and expected to design lessons and units of instruction that meet them.

For the purpose of the redesign, we set the overarching theme of the second methods course to address reasoning and sense making (NCTM, 2000). Activities were selected and/or developed to help the candidates develop an understanding of how to assist school learners grow in their reasoning skills by drawing on various concepts from targeted content areas such as Geometry, Measurement, Probability, Statistic and Discrete Mathematics.[1] Mathematical modeling strand was to be addressed explicitly during three weeks of instruction. While some learning objectives were considered based on the literature on teachers' need pertaining to learning about mathematical modeling (Doerr & Lesh, 2011), our final plans resided in data obtained from the candidates using an extensive survey of knowledge, and the specific needs they identified or appeared to be paramount in their

[1]The program requires the teacher candidates to take a course in Technology in STEM. An additional course on Assessment is also required. These two courses cater to all teacher candidates in mathematics and science education.

responses. The survey was administered at the beginning of academic semester and prior to development of the unit of instruction.

12.3 Participants

The participants consisted of 11 prospective secondary mathematics teachers enrolled in a 7–12 teacher preparation program at a large research institution.[2] The participants had completed all prerequisite coursework towards a major in mathematics (including but not limited to Calculus I, II and III; Geometry; Linear Algebra; Abstract Algebra I, II; Real Analysis I, II; Discrete Mathematical Models; Foundations of Higher Math; History of Mathematics) and were enrolled in the second course of a sequence of two methods courses on teaching mathematics in secondary schools. All participants were scheduled to commence their student teaching phase during the subsequent academic semester. As enrolled in the methods course all participants were also completing a field-experience that required them to spend 10 h a week in schools for 16 weeks. The field experience is divided into two eight weeks long parts where each part is completed at either a middle school or a high school. Collectively, these two distinct experiences provide teachers opportunities to observe 7–12 grades learners. During each part of the field experiences, the candidates worked with two different teacher mentors, observe lessons, work with individual or small group of learners, as assigned by the teachers.

12.4 Methodology

This study followed a Teacher Development Experiment (TDE) (Simon, 2000) methodology. A TDE draws on the principles of constructivist teaching experiment (Steffe, 1991) and capitalizes on improving how individuals learn mathematics through a cycle of interaction and reflection by a researcher-teacher. Because learning to teach mathematics involves more than learning mathematics content, Simon (2000) describes an alternate view of a teaching experiment. A TDE is concerned with how teachers develop in mathematical pedagogy in addition to understanding mathematics content. The TDE process is similar to that of a teaching experiment. The researcher-teacher interacts with teachers directly with the goal of furthering their development in specific ways, reflects on and analyzes what happened to determine any changes that may need to be made for the next cycle of interaction.

[2]In the American school system, 6–8th grades is labeled as middle or junior high school whereas 9–12th grades is considered high school. The 7–12 teacher preparation program licenses candidates to teach in these grade levels.

In light of this methodology and following its guiding principles, our investigation consisted of three phases. First, a survey of knowledge of mathematical modeling for teaching was designed and administered to collect base line data on the candidates' assumptions about teaching mathematical modeling in schools. Second, in light of the survey results and identification of areas that seemingly needed development, a unit of instruction on mathematical modeling for teaching was conceptualized and implemented during three course sessions (approximately 8 h). A post implementation survey was administered on the last session of the academic semester to trace impact of the experiences provided for teacher candidates as expressed by the participants. These are described in detail below.

12.5 Survey

Drawing from the literature on Teacher Knowledge (Rowland & Ruthven, 2011), the survey consisted of 20 items and addressed four areas (see Appendix). The first part of the survey collected biographical information from the candidates, the range of mathematics courses they had completed, and their assessment of courses in which they believed they had gained mathematical modeling experiences as learners.

Four items addressed the participants' claimed level of confidence with mathematical modeling, teaching it, and their assessment of the importance of modeling for school learners. The decision to consider these various dimensions of teachers affective and cognitive domains pertaining to teaching and learning of mathematical modeling was due to two primary reasons. First, previous research has already established that prospective teachers enter their teacher education program overly confident about their ability to teach mathematical modeling (Darling-Hammond, 2010). Second, researchers have also posited that a high level of confidence in their ability to teach impede prospective teachers from tending closely to the teaching approaches promoted in their education coursework. We had hypothesized that an increased knowledge of mathematical modeling and its complexity the prospective teachers might become more sensitive to their own facility with mathematical modeling and teaching it. Additionally, the prospective teacher participants were asked to identify how frequently they had observed mathematical modeling implemented in classrooms.

The third part of the survey consisted of open response items that collected data on the candidates' description of mathematical modeling, similarities and differences they observed between modeling tasks and other kinds of activities used in classrooms, and processes they associated with mathematical modeling. The participants were asked to provide illustrative examples in each part.

The last portion of the survey aimed to obtain specific data on the participants' ability to identify suitable examples of modeling tasks to be used with middle and high school students as well as how they envisioned gauging learners' progress when engaged in such tasks (Watson & Mason, 2005). We had anticipated that candidates' responses to the last set of questions would allow us to more carefully detail their knowledge related to modeling based curriculum and instruction.

The same survey was administered again during the last session of the academic semester and upon conclusion of the experimental unit of instruction. The post-implementation survey included two additional questions. These two questions attempted to capture the participants' assessment of they believed they had gained from the course experiences. The goal was to gauge the success of our initiatives/ choices as judged by the prospective teachers themselves.

12.6 Course Design

In light of the results of the survey, we set four goals for our work with the participants so to help them: (1) develop a deeper understanding of the mathematical modelling process and its intricacies, (2) discriminate between mathematical modelling as a process and solving routine application problems, (3) learn about suitable resources that could be used for simulating modelling tasks, and (4) understand how student learning could be gauged using the modelling cycle (Blum & Borromeo Ferri, 2009) as a platform for assessment. The candidates were introduced to the modeling cycle during the first day of implementing the modeling unit. The modeling cycle was revisited throughout the range of activities the candidates completed in class.

Each course session was divided into two parts. During the first part of the session, the candidates worked on one or two modeling tasks, compared and constructed their answers, and tried to refine their solutions (see Table 12.1 for

Table 12.1 Sample tasks and resources

Task and recourses	Objectives
Cost Problem Jensen (2007). Mathematical Modelling in Danish Schools What is the cost of me?	Precision and accuracy Full Modeling Cycle
Additional tasks and recourses Modeling with Olympic Running Record Selena Oswalt's Thesis "Mathematical modeling in the high school classroom" COMAP Voting tendency problem from the book "Mathematics methods and modeling for Today's Mathematics Classroom" (p. 106) Simulations Dan Meyers' three act teaching: https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/edit#gid=0 Math Modelling Lessons. Indiana University: http://www.indiana.edu/~iucme/mathmodeling/lessons.htm NRICH Mathematics: https://nrich.maths.org/5741	

examples of tasks used). Emphasis was placed on identifying variables, parameters and how these variables and parameters shaped the mathematization of contexts leading to a construction of a tentative model. In particular, candidates were asked to consider how different mathematical tools and concepts could be used in the model building stage. This portion of the session provided us the opportunity to examine the candidates' mathematical artifacts and make suggestions about how they may refine their models. These discussions allowed us to introduce different mathematical tools the participants may not had considered, either independently or collectively, which could have been used to construct more robust models (Approximately 3½ h in total).

The second part of the session was devoted to deliberations on how the same tasks could be implemented in schools, the type of scaffolding that may need to be provided, or ways in which new ideas could be introduced in tandem. The participants were then introduced to specific resources they could use and available simulations they could access and utilize in instruction to ground learners' activities (see Table 12.1 for a sample of resources introduced and used). Candidates were encouraged to identify particular skills they could target with the use of specific tasks they examined (approximately 3 h).

12.7 Data Analysis

Data analysis followed two phases. First phase involved coding the candidates' responses to the pre-implementation survey. Descriptive statistics regarding the outcomes of the multiple-choice items were produced. Analysis of open response items followed a grounded theory model (Glaser, 2003). All responses were studied first. Major themes that emerged were catalogues and labeled respectively. A second review of the open response items was completed using the emergent themes as analytical codes. This analysis informed the content and structure of the unit of instruction.

Phase II of the analysis involved comparing the pre-post survey results. As a starting point in coding the post implementation data the analytic framework developed in Phase I was used to complete a sentence-by-sentence coding of the open response items. New coding categories were developed and noted to capture the range of responses the candidates provided. Lastly, similar to phase I, descriptive statistics regarding the outcomes of the multiple-choice items were produced. Comparison of pre-post results then followed. Findings from the analysis are detailed in the next section.

12.8 Findings

12.8.1 Phase I: What Teachers Claimed They Knew?

As described above, in an attempt to be responsive to the candidates' particular needs when designing modeling experiences for them they were asked to complete a survey in which they provided data on their knowledge of mathematical modeling. Results indicated that although the participants seemed aware of the differences between modeling activities and other types of tasks used in instruction, they were less clear about unique features of modeling as a process, ways to gauge student learning, or anchoring design of modeling tasks in 7–12 curriculum as described below.

All 11 participants reported having had experience with mathematical modeling in their Discrete Mathematics course. A half of the candidates reported having had adequate exposure to modeling experiences in their calculus, differential equations, and probability and statistics courses. Because of these experiences they felt confident in their ability as mathematical modeler though unsure of how to teach it. Only 3 (27%) candidates reported having observed classroom teachers who served as their school based mentors implement mathematical modeling activities.

In describing mathematical modeling and its process, 7 (64%) candidates characterized it as using mathematics to represent and analyze real world situations. The participants' responses however varied according to the amount of detail they chose to include in outlining their thinking. For example, one candidate wrote "*mathematical modeling is the cyclic process of taking real world problem, quantifying them mathematically, and refining and improving the mathematics used to describe the problem*", while another one described mathematics modeling as "*the way that math concepts can be used to represent and analyze real-world situations*". Three (27%) participants perceived mathematical modeling as using manipulative, simulations or world problem to demonstrate a mathematical concept.

In explaining specific actions associated with modeling two common themes emerged. One group equated mathematical modeling process with problem solving (i.e. "*read the information given to you. Write out your givens. Analyze what the goal is.*") The second type of description concerned data modeling with a focus on statistical context. None of the candidates' referenced defining variables, setting parameters, building a mathematical representation of the situation, interpreting and refining the model (Blum & Leiβ, 2007) as part of the modeling process.

On the follow up question that asked the candidates to report how they would assess school learners' mathematical modeling progress, all but one participant offered general descriptions that did not tend to unique features of mathematical modeling. Further, to illustrate differences between modeling tasks and other types of mathematical activities, candidates relied on phrases such as "*open ended*", "*multiple approaches*", "*multiple solutions*" to describe their thinking. Although only one candidate provided illustrative examples to distinguish modeling tasks from other types of activities, the participants' overall descriptions indicated that they were aware of the differences though unsure about how to articulate them.

12.8.2 Phase III: What Was Gained?

In tracing the impact of the course experiences on teacher candidates, we focused our analysis on comparing their responses to a common question on pre and post implementation survey. These are described below. Note that when including illustrative examples from the candidates' responses we opted to consider comments from the same individuals whose answers were also depicted in Phase I results section of this report.

Descriptions of mathematical modeling and its process: On both surveys the candidates were asked to describe mathematical modeling and what they perceived as specific processes involved in this sort of mathematical work. In the post implementation survey, nine (82%) participants described mathematical modeling as using mathematics to represent, analyze and solve real-world problems. Comparing these responses to those on pre-implementation survey, their remarks were more reflective of the nature of mathematical modeling as a process (Blum & Leiß, 2006). Most frequently cited description of modeling included "*creating a useful model to represent an event of the real world, whether it be a symbolic equation or a working* '*machine*'." "*Mathematical modeling should incorporate interpretation from the real world to the math world, and reinterpreted back to the real world after the model carries out it course.*" "*Mathematical modeling is cyclic process, so it does not stop after one loop.*"

In the post implementation survey, when asked to outline actions that may be involved in mathematical modeling process, eight (72%) candidates noted specific cognitive actions (making sense of the situation, identifying/defining variables, making assumptions, using mathematics to build a model, interpreting the model, and revisiting the initial model). Compared to their responses to the same question on the pre-implementation survey, these descriptions more closely match stages depicted in the modeling cycle. Common responses cited a sequence of actions, "*interpret problem → make assumptions → make quantitative model → find solution → validate model/solution → revisit any previous steps as needed.*" 2 (18%) of the candidates associated modeling with using manipulative or simulation to demonstrate mathematical concepts.

Examples generating: In the post implementation survey, when asked to illustrate the differences between modeling activities and other types of mathematical activities, phrases such as "open ended", "multiple representations", "real world connection", "multiple approaches", "a variety of directions", "multiple entry and exit points", "minimal constraints", "opportunity to define relevant variables", and "revisiting solutions" were referenced by all 11 candidates. Ten (91%) of the candidates referenced "minima constraints", "revisiting solutions", and "define relevant variables" to describe modeling tasks, which were not present in their

Table 12.2 Types of modeling tasks generated by participants

	Exercises	Building physical models or simulations	Data collection and analysis	Applications resembling textbook contexts	Modeling tasks matching the descriptions offered in the literature	Optimization tasks
Middle School (pre)	0	2	5	2	0	2
Middle School (post)	0	1	5	0	3	2
High School (pre)	0	0	3	5	1	2
High school (post)	0	0	3	1	4	3

responses in the pre-implementation survey. Table 12.2 offers a summary of the types of examples of tasks the candidates listed as appropriate to be used in Middle and High school grade bands as noted on pre and post implementation surveys. Results indicate a shift in the quality of knowledge regarding modeling as a process and the types of tasks that could motivate and elicit cognition.

The shift in responses is promising. As evident a larger number of tasks that matched criteria for modeling situations as defined in the literature were identified on the post implementation survey. More specifically, the contexts that the candidates referenced drew on various content areas. Indeed, a majority of the contexts they referenced typified content of a course in Discrete Mathematics. The candidates also noted the activities used in the methods course as illustrative examples of modeling tasks to be used with learners.

Assessing modeling progress: Compared to pre-implementation survey on which none of the candidates appeared to have had a platform for gauging learners' modeling progress on the post implementation survey 7 (64%) of the participants offered specific plans relying on the language of modeling cycle for identifying specific behaviors they would seek out. An illustrative example of commonly noted description included, "*I would assess students based on the assumptions they made and how those assumptions lead the students to their answer.*" Another participant expressed "*the process is where the assessment lives. It is not about the answer; it is about how well they can work through the process to find an answer they feel is acceptable*."

Self efficacy and beliefs of learning from the course: Figs. 12.1 and 12.2 summarize the candidates' responses to the two additional questions listed on the post implementation survey. The first question asked the participants to rate their

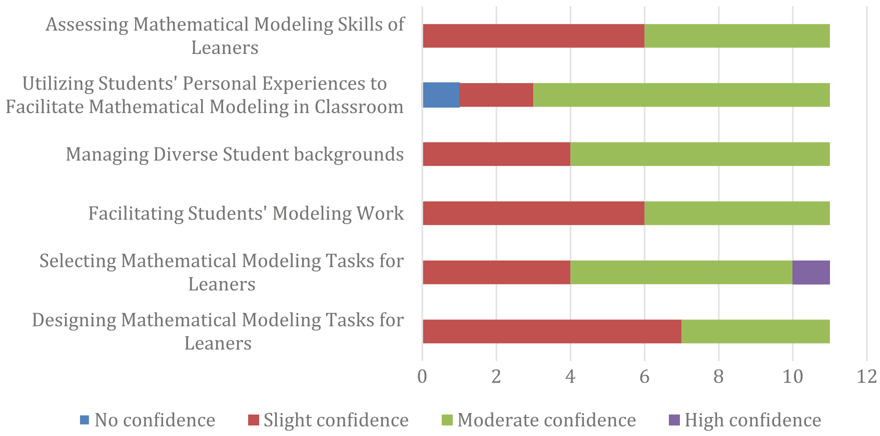

Fig. 12.1 Participants' confidence level in enacting mathematical modeling instruction practices

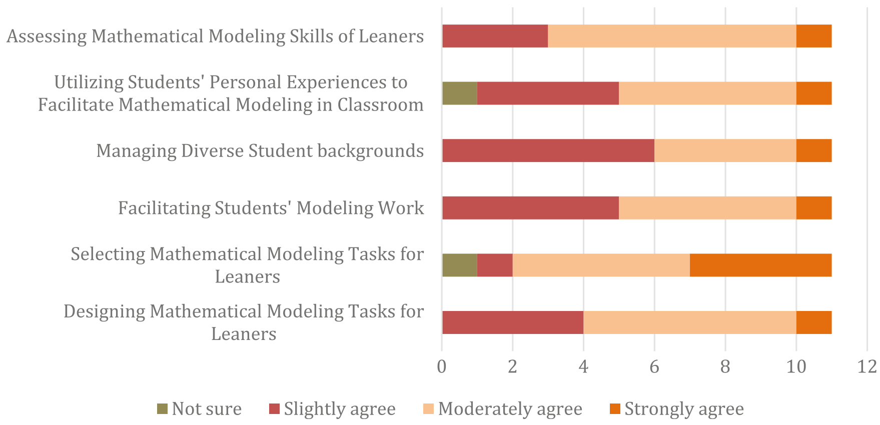

Fig. 12.2 Participants' perception of learning gains from the course

confidence in enacting each of 6 practices central to mathematical modeling instruction. The second question asked them to rate their learning in each of these domains as the result of course experiences.

In response to the first question nearly all candidates felt slight to moderate levels of confidence in their ability to navigate demands of a modeling based curriculum. Moderate confidence was most prominent in the areas of "selecting mathematical tasks for learners," "managing diverse student backgrounds" and "utilizing students' personal experiences." The candidates felt most vulnerable when assessing learners' skills, facilitating students' modelling work and designing modelling tasks.

In response to the second question, seven (64%) candidates stated that compared to the beginning of the semester they felt more knowledgeable about designing mathematical modeling tasks for learners; nine (82%) candidates moderately or strongly agreed that compared to the beginning of the semester, they felt more knowledgeable about selecting mathematical modeling tasks for learners. Eight (73%) participants moderately or strongly agreed that compared to the beginning of the semester, they felt more knowledgeable about how to assess mathematical modeling skills of learners; Six (55%) students moderately or strongly agreed with both the statements that they felt more knowledgeable about facilitating students' modeling work and utilizing students' personal experiences to facilitate mathematical modeling in classroom. Five (45%) candidates claimed that compared to the beginning of the semester they felt more knowledgeable about managing diverse student backgrounds.

The results indicate that despite their positive reports the candidates remained insecure about managing various aspects of instruction in presence of a modelling curriculum.

12.9 Discussion

Previously Gould (2013) in her survey of approximately 270 teachers across the US reported fragile understanding of mathematical modeling among K–12 teachers. She offered that a majority of the teacher participants in her study perceived mathematical modeling as "solving application problems," "using manipulatives to illustrate models," and "encouraging multiple representations of concepts." Teachers in her study also felt confident in their ability to teach mathematical modeling. Findings of our research concur with some of Gould's findings.

A majority of the participants in our study did perceive mathematical modeling as the process of using mathematics to solve real world based tasks and their understanding of the complexities associated with teaching it increased. The candidates also believed modeling cognition to be difficult to nurture (Lingefjärd, 2007). Because of this, compared to teaching other content areas, they felt less efficacious in helping children develop proficiency in the area. Two particular challenges they articulated included how to effectively build on the learners' extra mathematical knowledge when engaging them in model building process as well as managing diverse student backgrounds. These issues have not yet been adequately addressed in the literature.

Analysis of the post implementation survey data revealed that although course experiences did not have any significant impact on the teachers' sense of efficacy towards teaching mathematical modelling, their description of the modelling process, knowledge of task design and resources available to use, along with ways they could assess and monitor student progress towards establishing more sophisticated mathematical models increased. Candidates felt vulnerable in gauging their own instructional interventions in the course of learners' modeling process. This is not surprising since development of knowledge of effective scaffolding techniques has been identified as a particularly complex one to acquire (Blum, 2011) and one that demands time and practice to mature.

Data also indicated that the candidates exhibited care when identifying modeling activities suitable for the middle grades learners. Examples of instructional modeling tasks they provided contained greater detail. The candidates were more precise when describing specific mathematical skills that could be taught or reinforced with using the activities they had proposed. We note also that in providing illustrative examples of modeling tasks the participants made specific references to the activities they had completed in the methods course. This point certainly supports Niss, Blum, and Galbraith (2007) position that educational programs designed for teachers must utilize modeling experiences that parallel those expected of them to teach. We further highlight that the candidates' past mathematical experiences exerted tremendous influence on the types of modelling situations they were able to reference. For instance, Discrete analysis course which had been identified as one in which they had gained greatest exposure to mathematical modelling is one of the most referenced context.

It would be ambitious of us to make serious claims to effectiveness of our choices on advancing teachers' mathematical modeling skills or their knowledge about teaching it to children since the development of both domains demands extensive time for reflection and contemplation, sustained exposure and experience, and a practice of skills in an authentic way (Blum, 2011). It is reasonable however, to argue that the course managed to provide the candidates with a language through which they could articulate ideas about mathematical modeling and its form and content. Even so, acquiring the language provides some evidence of learning. Because of this we posit that while inclusion of experiences we designed appeared to have familiarized the participants with some key issues and methodologies, they were not by any means sufficient to have helped them reach a level of proficiency that would need to be in place for effective implementation. Significant need exists for additional scholarly reports on existing efforts aimed at improving modeling specific pedagogical capacities among teachers.

Appendix: Survey

1. Degree Program

    A. B.S.Ed.
    B. M.Ed.
    C. M.A.
    D. M.S.
    E. Ph.D.

2. Area of specialization
   A. Mathematics
   B. Science
   C. Engineering
   D. Instructional Technology
3. Years of experience
   A. None
   B. 1–3
   C. 4–10
   D. >10
4. Mathematics courses you have completed in your program of study:
   A. Calculus I
   B. Calculus II
   C. Linear Algebra and Matrix Theory
   D. Discrete Mathematics
   E. Differential Equations
   F. Engineering Mathematics
   G. Computing and programming
   H. Probability and Stats
   I. Geometry
   J. College Algebra
   K. History of Mathematics
   L. Pre-calculus
   M. Calculus based Physics
5. In which required courses for your program of study did you learn about modeling? (The extent in which you believe you learned about how to use mathematics to solve real life problems in each of the following courses (if taken)).

	A great deal	Enough to learn	Some exposure	None
Calculus I	○	○	○	○
Calculus II	○	○	○	○
Linear Algebra And Matrix Theory	○	○	○	○
Discrete Mathematics	○	○	○	○
Differential Equations	○	○	○	○
Engineering Mathematics	○	○	○	○
Computing and Programming	○	○	○	○
Probability and Stats	○	○	○	○
Geometry	○	○	○	○
College Algebra	○	○	○	○

(continued)

(continued)

	A great deal	Enough to learn	Some exposure	None
History of Mathematics	○	○	○	○
Pre-calculus	○	○	○	○
Calculus based Physics	○	○	○	○

6. What does mathematical modeling, or modeling problems based on how it is used in your discipline, mean to you?
7. How is mathematical modeling (or modeling tasks) different from other types of activities (tasks) commonly used in classrooms (this may include exercises or application type problems)? Give comparative examples to illustrate the difference.
8. Describe what specific actions may be involved in the (mathematical) modeling process? That is, are there specific stages involved in mathematical modeling that may not be involved in other forms of mathematical activity? You may draw a diagram to illustrate your thinking. Describe what specific actions may be involved in the (mathematical) modeling process? That is, are there specific stages involved in mathematical modeling that may not be involved in other forms of mathematical activity? You may draw a diagram to illustrate your thinking.
9. How do you envision using the process you described above to be used when assessing students' mathematical modeling progress?
10. Give an example of a modeling task suitable for elementary school students. Explain why you consider this task suitable for children in that grade band.
11. Give an example of a modeling task suitable for middle school students. Explain why you consider this task suitable for children in that grade band.
12. Give an example of a modeling task suitable for high school students. Explain why you consider this task suitable for children in that grade band.
13. How experienced are you with doing mathematical modeling?

    A. Very experienced (have had more than one course emphasizing mathematical modeling AND/OR have done modeling in my previous career)
    B. Fairly experienced (have had experiences it in different classes I have taken)
    C. Not experienced (I have rarely experienced modeling in my education)
    D. Not sure (I don't know what mathematical modeling is)

14. How experienced are you with mathematical modeling as a teacher?

    A. Very experienced (I feel quite confident about teaching modeling as a content and helping learners acquire needed skills)
    B. Fairly experienced (I think I know how to use modeling to teach different concept)
    C. Not experienced (I have never tried teaching modeling)
    D. Not sure (I don't know what mathematical modeling is)

15. How often are modeling tasks used in classrooms you have observed or taught?

    A. Daily
    B. Weekly
    C. Monthly
    D. 3–4 times a year

16. How do you feel a modeling approach to teaching might compare to other teaching approaches?

    A. More challenging
    B. About the same
    C. Less challenging

17. How important do you believe development of modeling skills might be to school learners?

    A. Very important
    B. Somewhat important
    C. Not particularly important

18. Consider the following events. First, decide whether the context presents a situation for mathematical modeling. Then decide where the context might be suitable to be implemented for school learners' explorations. If you believe the context is not appropriate OR that it may be suitable to be used in different subjects please provide an explanation for your choice.

    A. *Design a parking lot for a retail store.*
    Does the context present a situation for mathematical modeling?

    - Yes, it is a modeling context
    - No, it is not a modeling context
    - I don't know

    B. *A physicist is studying properties of light. She wants to understand the path of a ray of light as it travels through the air into a smooth lake, particularly at the interface of the two different media.*
    Does the context present a situation for mathematical modeling?

    - Yes, it is a modeling context
    - No, it is not a modeling context
    - I don't know

C. *Harry Smith is determined to write the next best-selling book.*
Does the context present a situation for mathematical modeling?

- Yes, it is a modeling context
- No, it is not a modeling context
- I don't know

D. *A couple is trying to decide whether to buy or rent a house.*
Does the context present a situation for mathematical modeling?

- Yes, it is a modeling context
- No, it is not a modeling context
- I don't know

19. The scenarios below is vaguely stated. First state a problem worth studying. Then identify what variables affect the behavior in the problem as you stated it and state, in order of importance, at least four of the those most important.

A. *A car rental company with distributorships in two different locations caters to travel agents who arrange tourist activities in both cities. Consequently, a tourist may choose to rent a car in one city and drop it off in the second city. Tourists can begin their itinerary in either city. The company wants to determine how much they should charge for the "drop-off" convenience.*

- Problem to explore
- Variables
- Order of importance

B. *A proportion of population of an island travels abroad and returns to island infected with a disease.*

- Problem to explore
- Variables
- Order of importance

20. Consider two species whose survival depends upon their mutual cooperation. An example would be a species of bee that feeds primarily on the nectar of one plant species and simultaneously pollinates that plant. Letting a_n and b_n represent the bee and plant population levels, respectively, after n days, we have the following model:
$a_{n+1} = a_n - k_1 a_n + k_2 a_n b_n$
$b_{n+1} = b_n - k_3 b_n + k_4 a_n b_n$
Where the k_i are positive constant.

- Discuss the meaning of each k_i in terms of mutual cooperation.
- What assumptions are being made about growth of each species in the absence of cooperation?

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Chapter 13
Future Teachers' Use of Multiplication and Fractions When Expressing Proportional Relationships

İbrahim Burak Ölmez

Abstract The purpose of this study was to investigate how six future middle grades mathematics teachers used explicit, quantitative definitions for multiplication and for fractions when reasoning about proportional relationships. The future teachers were recruited from a preparation program in the United States based on their performance on a fractions survey. The data collection consisted of 1-hour semi-structured interviews with each future teacher. An explanatory case study was used to make comparisons across the future teachers. Results revealed that explicit use of the quantitative definition of multiplication is a helpful organizing tool for future teachers to generate and explain equations for proportional relationships.

Keywords Definition of multiplication · Definition for fractions
Future middle grades teachers · Proportional relationships · Algebraic equations

13.1 Introduction

One of the most central and difficult concepts of elementary and secondary mathematics education is ratios and proportional relationships (e.g., Kilpatrick, Swafford, & Findell, 2001; Lamon, 2007; National Council of Teachers of Mathematics, 2000). Lesh, Post, and Behr (1988) considered ratios and proportional relationships to be the capstone of elementary mathematics and the cornerstone of high school mathematics. Vergnaud (1983, 1988) described these relationships as part of the multiplicative conceptual field—a web of interrelated ideas including multiplication, division, fractions, and more. The National Mathematics Advisory Panel (2008) stated that the interrelated ideas of ratios, proportional relationships, and fractions are foundational for algebra.

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_13

Past research on teachers' understandings of the multiplicative conceptual field has reported that in-service and future teachers have trouble when explaining the product of two fractions or decimals embedded in problem situations despite their correct computation of the algorithms (e.g., Ball, Lubienski, & Mewborn, 2001; Izsák, 2008; Tirosh & Graeber, 1990). Past research has also acknowledged that many teachers in the U.S. struggle explaining division (i.e., multiplication with an unknown factor) when it is embedded in problem situations although they can determine the quotient of two fractions or decimals (e.g., Armstrong & Bezuk, 1995; Jansen & Hohensee, 2016).

The relatively few studies that have examined how in-service and future teachers reason about ratios and proportional relationships have demonstrated that teachers have many of the same difficulties reported in the much larger literature on students' reasoning about proportional relationships. In one of these studies, middle grades teachers were found to show poor performance on test items that their students are expected to solve (Post, Harel, Behr, & Lesh, 1991). In another study, Harel and Behr (1995) reported that many teachers were not able to solve the problems involving proportional relationships correctly. Rather, these teachers guessed at operations by performing each of these operations on the quantities until reaching out a reasonable solution or they searched for particular words (i.e., key words) in the problems to decide which operation to use. In addition to these findings, past research has documented that teachers can have a hard time distinguishing missing-value problems that describe directly proportional relationships from ones that do not (e.g., Cramer, Post, & Currier, 1993; Fisher, 1988; Izsák & Jacobson, 2017) and can struggle conceiving a ratio as a measure of a physical attribute, such as steepness (e.g., Simon & Blume, 1994).

The purpose of this study was to examine how six future middle grades mathematics teachers used explicit, quantitative definitions for multiplication and for fractions to develop equations that relate quantities in a proportional relationship. Both definitions were introduced in content courses that the future teachers were completing as part of a preparation program in the United States. The following research question was addressed in this study:

- How do future middle grades mathematics teachers reason with quantitative definitions for multiplication and for fractions when solving proportion problems?

The study makes two contributions. First, existing studies have consistently acknowledged that solving proportions involving whole-number multiples is easier than solving proportions involving fraction multiples (e.g., Kaput & West, 1994; Karplus, Pulos, & Stage, 1983), but no studies have examined how explicit, quantitative definitions for multiplication and for fractions can support and constrain reasoning about proportional relationships. Second, the present study demonstrates that the quantitative definition of multiplication is accessible to future teachers in terms of generating and explaining appropriate equations that involve proportional relationships.

13.2 Theoretical Framework

The framework for this study is based on the quantitative definition of multiplication explicated by Beckmann and Izsák (2015) as in Fig. 13.1. In the equation M · N = P, the multiplier, M, is interpreted as the number of groups; the multiplicand, N, is the number of units in one group; and the product, P, is the number of units in M groups.

This study is also based on a definition for fractions that is consistent with the one found in the Common Core State Standards in the United States (CCSS) (Common Core State Standards Initiative, 2010). A two-part definition for a fraction is as follows:

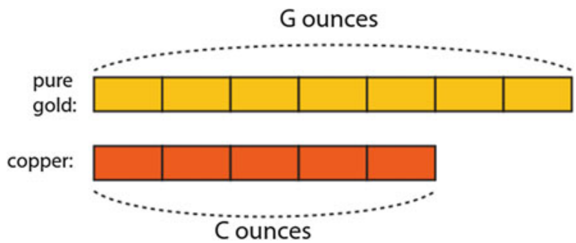
(a) $1/b$ is the quantity formed by one part when a unit amount (or whole) is divided into b equal parts; each part is $1/b$ of the unit amount.
(b) a/b is the quantity formed by a parts of size $1/b$ of the unit amount.

Beckmann and Izsák (2015) explained how maintaining distinct roles played by the multiplier and multiplicand in Fig. 13.1 leads to two distinct perspectives on proportional relationships, one termed the multiple batches perspective, which has been studied widely, and the other termed the variable parts perspective, which has been largely overlooked in mathematics education research.

For the present study, we focused on the variable parts perspective, which we illustrate with the Jewelry Problem. The variable parts perspective, combined with a drawn model called a strip diagram (Fig. 13.2) supports at least two different solutions (Beckmann, Izsák, & Ölmez, 2015).

Fig. 13.1 A quantitative definition of multiplication

Fig. 13.2 A strip diagram for the Jewelry Problem



Jewelry Problem: A company makes jewelry using gold and copper. The company uses different weights of gold and copper on different days, but always in the same 7–5 ratio. Let G and C be some unspecified number of ounces of gold and copper the company will use that same day. Please use a strip diagram to help you explain the relationship between G and C.

One method future teachers can use to develop equations relating G ounces of gold and C ounces of copper is the "how much in one part" method (Fig. 13.3). With this method, future teachers can view the total amount of gold as one group consisting of seven parts and the total amount of copper as one group consisting of five parts. Here the number of parts is fixed and, by convention, all parts contain the same number of ounces. At the same time, the number of ounces in each of the 12 parts can vary with the total amount of jewelry gold being made. Future teachers can solve the problem by determining the number of ounces in one part, $C/5$ oz, and by using the quantitative definition of multiplication to generate the following equation:

$$7\,(\text{groups}) \cdot C/5\,(\text{ounces in one group}) = G(\text{ounces in 7 groups})$$

A second method future teachers can use is the "how many total amounts" method (Fig. 13.4). With this method, future teachers can treat the copper strip as 1 group of C ounces. By asking how many groups of five parts are in seven parts and applying the definition for fractions to the copper strip as unit amount or whole, future teachers can see that the gold strip consists of 7 parts each containing the same number of ounces as 1/5 of the copper strip, and the five parts copper strip fits into the seven parts gold strip 7/5 times. Thus, future teachers can conclude that the amount of gold, the G ounces, is 7/5 groups, and use the definition of multiplication to generate the following equation:

$$7/5\,(\text{groups}) \cdot C/5(\text{ounces in one group}) = G(\text{ounces in 7/5 groups})$$

Past research on understanding teachers' solutions to problems similar to the Jewelry Problem has found that teachers tend to resort to cross-multiplication as a rote computation algorithm without reasoning about the quantities and guess at

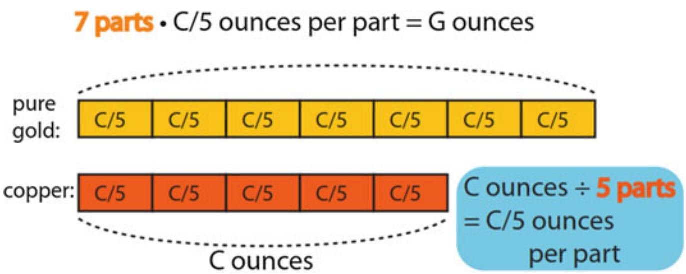


Fig. 13.3 "How much in one part" method (Variable Parts Perspective)

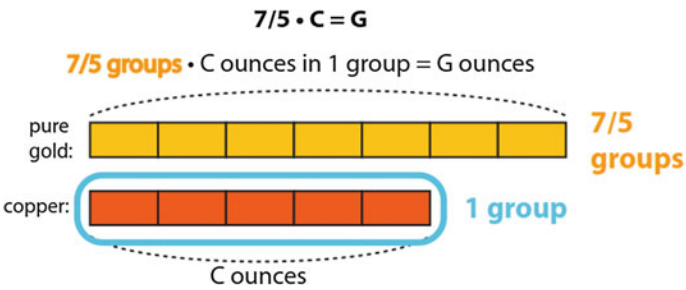


Fig. 13.4 "How many total amounts" method (Variable Parts Perspective)

operations (e.g., Fisher, 1988; Harel & Behr, 1995); have difficulty coordinating two proportionally related quantities (e.g., Orrill & Brown, 2012); and rely on additive relationships rather than multiplicative ones when reasoning about these quantities (e.g., Ölmez, 2016). Moreover, current recommendations on middle grades students suggest developing their conceptual understanding for solving these problems before teaching cross-multiplication as an algorithm (Siegler et al., 2010) and exposing these students to solve such problems using math drawings like strip diagrams and double number lines (CCSS, 2010). To apply the current recommendations for middle grades students, future middle grades teachers, as prospective teachers of these students, should have capacities to reason about proportionally-related quantities and connect their capacities within the multiplicative conceptual field. Therefore, the theoretical framework of this study is consistent with the recommendations of *The Mathematical Education of Teaching II* for middle grades teachers (Conference Board of the Mathematical Sciences, 2012, p. 39).

13.3 Methods

Data for this paper come from a larger on-going study of future middle grades (grades 4–8) mathematics teachers' multiplicative reasoning. At the time of the study, the six future middle grades teachers were enrolled in a teacher education program at a large university in the Southern United States. The future teachers had already taken a first semester calculus course required by the program. As part of the program, they also took a content course on number and operations in Fall 2014 and a content course on algebra in Spring 2015. The sequence of instruction in the number and operations course were as follows: numbers, the base-ten system, the definition of fractions, equivalent fractions, comparing fractions, fraction addition and subtraction, the definition of multiplication, properties of multiplication, applying properties of multiplication, fraction multiplication, division of whole numbers, fraction division, and connecting division with fractions. The main focus in this course was to develop a foundation for future teachers to use the quantitative definitions for multiplication and for fractions. In addition, the sequence of instruction in the algebra course followed identification of proportional relationships, the multiple batches perspective, the variable parts perspective, developing equations in two variables and developing equations of lines that come through the origin. The main focus in this course was to introduce the two perspectives on proportional relationships and strip diagrams.

A project team member was the instructor for both courses. Both content courses were taught from Mathematics for Elementary Teachers with Activities (Beckmann, 2014). Each course met for three 50-min sessions each week for 16 weeks. Most class sessions started with about 30 min of small group work on a set of problems focused on a particular topic. During whole-class discussion, the future teachers shared their solutions to problems that involved proportional relationships and asked each other questions. The instructor wrapped up the discussion by highlighting the key ideas that emerged during the conversation.

The project team selected six future teachers from a class of 22 future middle grades mathematics teachers who took the number and operations course in Fall 2014. The project team recruited future teachers who were mathematically diverse based on their performance on a survey (Bradshaw, Izsák, Templin, & Jacobson, 2014) that assessed facility with multiplication and division of fractions in terms of measured quantities. An explanatory case study was used because a case study is appropriate when the aim is to investigate causal relationships (Yin, 1993). Another project team member conducted six semi-structured hour-long cognitive interviews with each future teacher during the two semesters in 2014–2015. During the interviews, future teachers solved paper-and-pencil tasks similar to, but not the same as, those used in their course work. They were each interviewed twice during the numbers and operations course in Fall 2014 and four times during the algebra course in Spring 2015. All interviews were videotaped and transcribed verbatim for analysis.

Data for the present study consisted of future teachers' interview videos, audio transcripts, and a scanned copy of each future teacher's written work for two tasks that were given during the fourth interview, which was conducted through the middle of the algebra course. Main goals of the previous three interviews were to examine the extent to which future teachers identify groups, number of units in each group, and product amount in their definitions of multiplication, and distinguish distinct types of division in their definitions. The two interview tasks of the fourth interview were as follows:

Task 1 "How do you interpret the meaning of $1/6 \cdot X$?"
Task 2 Jewelry Problem (the same Jewelry Problem discussed above)

At the time of the fourth interview, future teachers had studied the quantitative definitions for multiplication and for fractions as described above during the numbers and operations course and had received initial instruction on ratios and proportional relationships during the algebra course, but they had not yet had instruction in developing equations in two variables, such as $5/7 \cdot G = C$. Thus, the interview was designed to probe students' initial capacities to reason about the quantitative definitions for multiplication and fractions when developing equations for proportional relationships before instruction.

The project team reviewed the data multiple times by placing interview transcripts side-by-side with the videos, and examined future teachers' words, gestures, and inscriptions for evidence of their thinking processes. The future teachers' initial responses to the tasks, not follow-up questions, were analyzed to obtain their ideas and ways of reasoning that they felt most comfortable with and confident in. To analyze their responses, detailed summaries describing each future teacher's reasoning on each task were written. The team then analyzed each detailed summary to identify emerging themes such as ideas, concepts, and ways of reasoning that the future teacher demonstrated as they worked on a task. Specifically, the team focused on the future teachers' use of key resources such as the definitions for multiplication and for fractions, the use of the variable parts perspective, and the use of strip diagrams. As more passes were taken through the data, it became increasingly clear that there was a substantial diversity between future teachers' use of the definition of multiplication, and their ability to develop appropriate equations for the Jewelry Problem.

13.4 Results

Table 13.1 provides a summary of the performance of all six future teachers in terms of their use of the quantitative definitions for multiplication and for fractions while working on Task 1 and Task 2, and their ability to generate correct equations in Task 2. According to the table, the future teachers who consistently used the definition of multiplication during Task 1 and Task 2 (i.e., consistent with instruction), also generated correct equations for the proportional relationship in Task 2. On the other hand, the future teachers who did not make explicit use of the definition of multiplication appeared to have a hard time developing correct equations.

Table 13.1 A summary table presenting the future teachers' performance

Names[a]	Definition of multiplication in Task 1	Definition of fractions in Task 1 or Task 2	Definition of multiplication equations in Task 2	Generating correct in Task 2
Alice	No	Not enough evidence	No	No
Jeff	No	Yes	No	No
Linda	Yes	Not enough evidence	Yes	Yes
Claire	Yes	Yes	Yes	Yes
Diana	No	Yes	No	No
Kelly	Yes	Yes	Yes	Yes

[a]All names are pseudonyms

13.4.1 Future Teachers' Performance on Task 1

In response to Task 1, half of the future teachers (3 out of 6) did not use the quantitative definition of multiplication consistently (see Table 13.1). Although Alice used the definition of multiplication, her use was not appropriate and consistent across the interview because her referent units lacked the precision with which units were discussed in class. Instead of using the word "number" and an explicit referent unit for each term in her equation, she used the phrase "how many" and left out referent units for *X* and for the product (Fig. 13.5a). To express her thinking in this task, she drew the strip diagram in Fig. 13.5b, but she interpreted the 1 group as 1 part of the whole strip instead of 6 parts of the whole strip.

Alice: So, 1/6 is our number of groups, and *X* would be the size of each group. So, we're pretty much… *X would be like contained in the 1/6* when you… if that makes sense. So, just multiplying 1/6 · *X*, which is how many we have in 1 group, would equal how many we get in all of the groups of 1/6 of *X* size.

Interviewer: Could you give a context or a drawing to sort of also communicate that thinking?

Alice: I guess I would start with this, and we could cut it into 6 parts. So, one of them would be 1/6, and that'll be the size of the group (draws Fig. 13.5b).

Interviewer: Maybe I didn't hear it. This whole strip from here to here (points to each end of the strip in Fig. 13.5b), what does this represent?

Alice: It's just 6. I guess 6 parts. So, this is like one whole, 6 outta 6 (writes 6/6), so one of these would be 1/6, and that's what this is, too (writes 1/6 in the 1-part strip in Fig. 13.5b). And so, *this is 1 group* (points to the 1-part strip). And there's like… I guess *X* is… there's like *X is a number of some sort*. So, say *it's like 2*, there's 2 parts in each of these little things (writes 2 in five parts of the 6-part strip in Fig. 13.5b). *It's hard to like come up with a drawing*. *So, we start with our how many groups, and then the size of the group would be like 2*. So, we have… would have 1/6 · 2 equals how many would be in all of the groups, so how many would be in this (points to the 1-part strip in Fig. 13.5b).

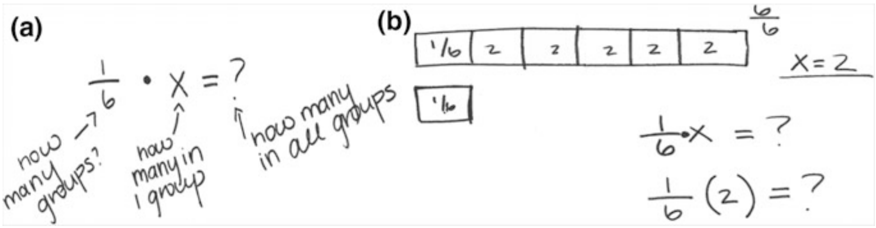


Fig. 13.5 **a** Alice's equation in Task 1. **b** Alice's representation of her equation

The data demonstrate that Alice labeled some parts of her strip diagram with 1/6 and others with 2 as a specific value for *X*. After completing her strip diagram drawing, she also acknowledged that for her to explain the definition of the expression with a drawing was challenging (even though drawings were used regularly in the content courses). Alice's association of 2 with each part rather than the entire strip indicated her incorrect coordination between units and groups.

Moreover, Jeff and Diana did not interpret 1/6 · *X* using the definition of multiplication in an appropriate way because they could not interpret 1/6 as the number of groups. Jeff switched the order of the given multiplication, turning 1/6 from the multiplier position into the multiplicand, as indicated in the strip diagram he drew (Fig. 13.6a, b):

Jeff: So right here (points to Fig. 13.6a), I drew a strip or a bar, and divided it into 6 equal parts, which gave me 1/6. And so, I see *1/6 times X as 1/6 times a certain amount of groups of 1/6. So, if it was X equals 2, I would have 2 groups of 1/6. Or if X was 6, I would have 6 groups of 1/6*, which would give us the 1.

Interviewer: If you had, say, 7 groups?

Jeff: I would just do the 1/6 seven times, and see that it was 7/6 (draws Fig. 13.6b).

Interviewer: Where do you see the *X* in your diagram?

Jeff: Each part. So, 1, 2, 3, 4, 5, 6, 7 (counts each circle in Fig. 13.6b).

Interviewer: Each one of those circles is an *X*? Am I understanding you correctly?

Jeff: Okay. So, I see the *X* as, I shouldn't have drawn all the circles (points to Fig. 13.6b), I see the *X* as the whole 7 pieces of 1/6, because if we're saying *X* equals 7, so we have *7 groups of 1/6*.

Interviewer: I think you said it, but can you tell me again… when you look at this expression (points to Task 1), 1/6 · *X*, and can you just interpret it in terms of the meaning of multiplication from class?

Jeff: Okay. So, I see it as 1/6 times a certain number of groups of 1/6. So here (points to Fig. 13.6b) *X* is 7, so it is 7 groups of 1/6.

When Jeff discussed "do[ing] 1/6 seven times," he used 7 as the number of groups or multiplier, not 1/6, indicating his switch of the order of the multiplier and

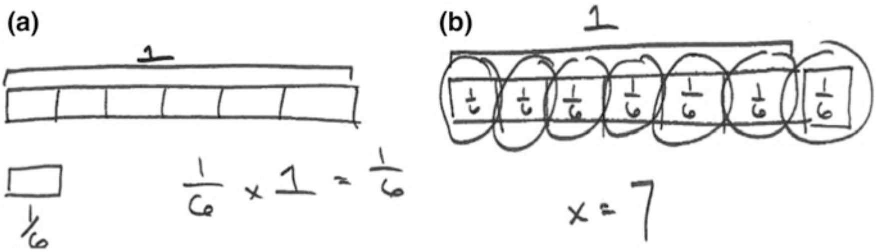


Fig. 13.6 **a** Jeff's view of the whole strip. **b** Jeff switches the order of 1/6 · *X*

multiplicand. Although Jeff could not use the definition of multiplication consistently as discussed above, his consideration of 7/6 as "7 groups of 1/6" indicated his use of the definition for fractions. In contrast to Jeff and Diana's misinterpretation of $1/6 \cdot X$, they used the definition of multiplication as "4 groups with X in each group" when they were asked to interpret $4 \cdot X$. When the interviewer asked Jeff whether his interpretations of $4 \cdot X$ and $1/6 \cdot X$ are same or not, Jeff seemed to be aware of his different interpretations as follows:

Interviewer: Do you think that you're using one interpretation of multiplication for the original $1/6 \cdot X$ problem and the new problem $4 \cdot X$, or do you think that you're using sort of different ways of thinking about multiplication for each example?

Jeff: This one (points to $4 \cdot X$), I'm using the definition of multiplication that I'm most common to in the class. This one (points to $1/6 \cdot X$), I didn't use that.

Interviewer: Why?

Jeff: I think just initially I switched the order, because it was hard for me to see 1/6 as a number of groups… if that makes sense.

Jeff acknowledged that for him it was difficult to imagine 1/6 as the number of groups in his drawing. That it is difficult for him to think of 1/6 as the number of groups indicated his struggle of coordinating units and groups.

Of the future teachers who were able to use the quantitative definition of multiplication consistently (3 out of 6), all three maintained the distinct roles for the multiplier and for the multiplicand (see Table 13.1). Although Linda initially seemed to interpret $1/6 \cdot X$ in a way different from class instruction by saying "1/6 is the size of the group and X is the whole in the group," later in the interview she clarified that she meant "number of groups" when she said "size of the group."

Interviewer: How do you interpret the meaning of $1/6 \cdot X$?

Linda: When I see $1/6 \cdot X$, I think of X being the whole and then dividing that into 6 parts, and then taking one of those parts. And so, that would be $1/6 \cdot X$ where this is X (points to the whole drawing in Fig. 13.7).

Interviewer: How would you use the meaning of multiplication that you've been discussing in class to interpret that statement?

Linda: So, *1/6 would be like the size of the group and X would be like a… the whole in the group*. So, the whole would be X or 6/6, and then we're taking 1/6 of that whole.

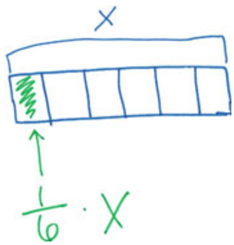


Fig. 13.7 Linda's interpretation of 1/6

The data make clear that she considered the unit for 1/6 (i.e., multiplier) as *X* and she saw that *X* (i.e., multiplicand) as 1 whole group, indicating her coordination of units and groups.

Furthermore, Claire used the definition of multiplication in an explicit and consistent way by coordinating units and groups appropriately in the expression of 1/6 · *X* (Fig. 13.8).

Claire: The way I interpret it (points to 1/6 · *X*) is 1/6 · *X* is 1/6 groups of *X*. Should I show you an example of what… like with a drawing?

Interviewer: That was my next question.

Claire: All right. Let's say this drawing (draws Fig. 13.8) is *X*, we see that there's 1, 2, 3, 4, 5, 6 groups that make the whole. So, this is our *X* (points to the whole drawing in Fig. 13.8). I'll use a different color. And, this (circles one part of the whole drawing with six parts in Fig. 13.8) is one of 6 parts of *X* or 1/6 of *X*. So, this (points to the circled part in Fig. 13.8) is *1/6 of group of the whole X*.

Similarly, Kelly identified the referent units for each term, coordinated units and groups in her use of the definition of multiplication for 1/6 · *X*, and also used the definition for fractions (Fig. 13.9).

Kelly: We have *1/6th of a group*. And *in that group, it's size X*. And then the product will be the total size of 1/6th of a group.

Interviewer: Could you give like a context and or drawing to explain that?

Kelly: Okay (draws Fig. 13.9). One group, *the size of one group is X*, so there is *X* in one group but we want to know how much or how many of X is in one 1/6th of a group. So, if I take this one group, divide it into 6, each part is one part of 6 total parts, *each part of 1/6th in size*. And if I just want to look at this 1/6th part of my one group, there will be $1/6^{th}$ of *X* inside it.

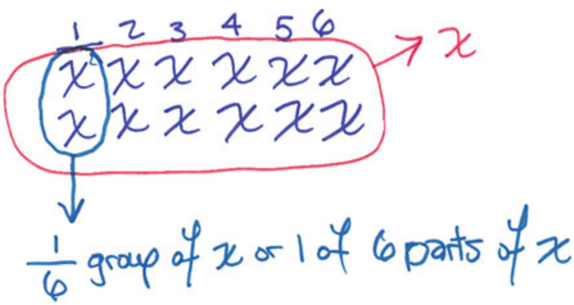


Fig. 13.8 Claire's interpretation of 1/6

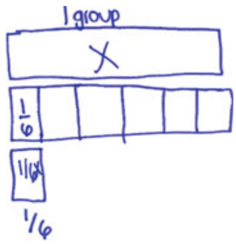


Fig. 13.9 Kelly's interpretation of the whole strip

The data indicate that Kelly and Claire could coordinate the whole strip as the multiplicand and the unit amount for the fraction 1/6 as multiplier. Their identification of the units in the expression $1/6 \cdot X$ and their association of the entire strip as 1 group in their drawings indicated that they could coordinate units and groups appropriately, at least for this task.

13.4.2 Future Teachers' Performance on Task 2

In response to Task 2, which asks future teachers to use a strip diagram to explain the relationship between the amounts of gold and copper that are in 7–5 ratio, Alice, Jeff, and Diana did not generate an appropriate equation involving G and C (see Table 13.1). In several cases, these future teachers' interpretation of the equal sign was inconsistent with normative usage, which in this task would equate the number of ounces of gold and of copper. From the data, it is not clear if the future teachers might have employed more normative usage of the equal sign in another situation or if their interpretation of the equal sign caused problems when they tried to apply the definition of multiplication in this task.

Alice did not form an equation that related the ounces of gold and copper appropriately, despite writing separate equations with whole number multipliers $5 \cdot C/5 = ?$ for copper and $7 \cdot G/7 = ?$ for gold. When she was reminded to produce an equation that involves the ounces of gold and copper, she wrote $5\ C = 7\ G$ as follows:

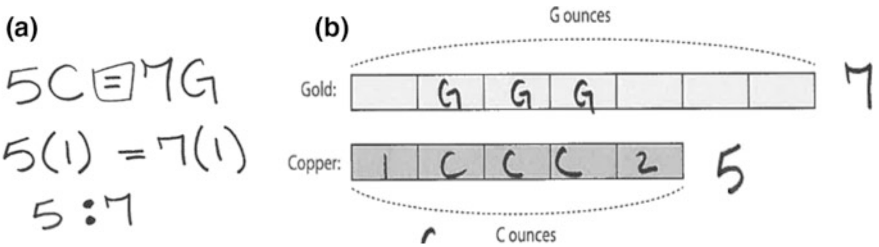


Fig. 13.10 **a** Alice's equation in Task 2. **b** Alice's interpretation of the strip diagram

Interviewer: Could you produce an equation that had *G* and *C* in it?

Alice: My equation, I guess, would be *5 parts copper equals 7 parts gold* (writes the equation in Fig. 13.10a).

Interviewer: How are you thinking about that?

Alice: I guess for *C* would be how many parts are in each of these? So, if like 5… if our size of the parts is 1, we would still be within that 5 to 7 ratio. So *not necessarily that these are equal, but they're proportionate*.

Interviewer: Yeah, I was going to ask you how you were thinking about *the equal sign*. So, you're saying it's indicating that they're proportionate.

Alice: Yes, *because they aren't equal*. Because 5 and 7 aren't equal, but … *they have this relationship of 5 to 7*. Then, I go back to think that *C* ounces and *G* ounces aren't going to be the same. But when I think about … each of these as like a *G* and each of these as *C*, I was like … thinking about those as size of the parts then your proportional relationship would be the same (puts *G* and *C* into each part in Fig. 13.10b). But I don't know how to think of it as like how *G* and *C* are related.

Alice appeared to understand that *G* and *C* are in a 7 to 5 ratio, but she used the equal sign to indicate certain amounts of gold and copper were associated with one another, not to indicate that numbers of ounces of gold and copper would be the same. In addition, she did not use the definition of multiplication to generate an appropriate equation.

Moreover, Jeff did not succeed in generating an appropriate equation either. He interpreted *G* as 7 oz and *C* as 5 oz in the given task, and he developed an incorrect equation as 7 + 5 = 12 oz (Fig. 13.11).

Jeff: I'm thinking about setting up another equation (other than his current Eq. 7 + 5 = 12 oz), but I'm having a hard time figuring out the parts of the equation.

Interviewer: How are you interpreting the diagram that we provided (refers to the given strip diagram in Task 2)? Walk me through kind of how you're interpreting it.

Jeff: I'm seeing the gold (points to *G* in the given diagram) as 7 oz. And then this *C* ounces as 5.

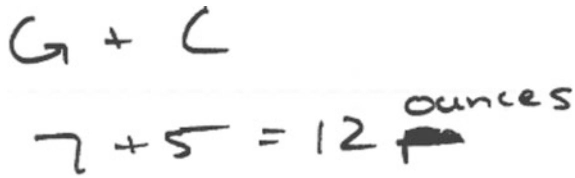


Fig. 13.11 Jeff's equation in Task 2

Although Jeff did not feel comfortable with his Eq. 7 + 5 = 12 oz and searched for another equation, he acknowledged that he could not figure out the specific terms that would form that equation. Jeff's interpretation of *G* ounces and *C* ounces as numeric values instead of variables (i.e., conflation of ounces and parts) indicated that he was not coordinating units and groups appropriately. In addition, he did not use the definition of multiplication to generate an appropriate equation that relate ounces of gold and copper.

Diana produced incorrect equations as $C = 5 \cdot 1/7$ and $G = 7 \cdot 1/5$ with whole number multipliers, and she interpreted the equations in terms of the definition for fractions (i.e., "5 parts each of size 1/7 of *G* ounces" and "7 parts each of size 1/5 of *C* ounces"), even though she was consistently reminded to use the definition of multiplication.

Diana: Okay. So, I guess the relationship would be for every 7 oz of gold there are 5 oz of copper or for every 7, I guess, *G* ounces there are 5 *C* ounces. Or, you could say the *G ounces is 7 parts, each a size 1/5 of the C ounces* or vice versa would be *C* ounces is equal to *5 parts, each of size 1/7 of the G ounces*.

Interviewer: So, what would you write for that? For either one of those possibilities?

Diana: So, I guess *G* would be equal to *7 parts (I'd) say times 1/5 of C ounces* and then the other one would be *C* is equal to *5 parts each of size 1/7 of G ounces* (writes $C = 5 \cdot 1/7$ and $G = 7 \cdot 1/5$ in Fig. 13.12).

Interviewer: Why are you writing the *C* ounces and the *G* ounces sort of down below (points to her equations in Fig. 13.12)?

Diana: I guess just showing *what 1/5 and 1/7 represents*. So, like you see like the number problem, but you see what each number means below it.

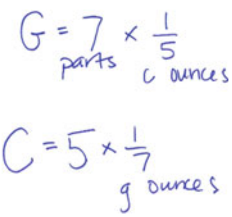


Fig. 13.12 Diana's equations in Task 2

Diana's only use of the definition for fractions and her lack of reliance on use of the definition of multiplication did not lead to generation of an appropriate equation in this task. Moreover, her notations of "*C* ounces" and "*G* ounces" underneath her equations $C = 5 \cdot 1/7$ and $G = 7 \cdot 1/5$ in Fig. 13.12 and her explanation for her use of such notations above provided evidence that she might interpret the equal sign as association between the ounces of gold and copper instead of equality of these ounces.

On the other hand, Linda, Claire, and Kelly, who demonstrated solid understanding of the definition of multiplication in Task 1, continued using this definition and succeeded in generating a correct equation in Task 2 (see Table 13.1). They all produced an equation with a fraction multiplier, such as $C = 5/7 \cdot G$, by perceiving 5/7 as "the number of groups" in their strip diagram drawings. Linda, immediately, indicated the relationship between the ounces of gold and copper as "copper is 5/7 of gold" and "gold is 7/5 of copper," and she generated correctly $C = 5/7 \cdot G$ as follows:

Linda: So, copper is 5/7 of gold or gold is 7/5 of copper (points to the given strip diagram drawing in Task 2). Copper equals $5/7 \cdot G$ where *G is the whole and 5/7 is the size of the group* (writes $C = 5/7 \cdot G$).

Interviewer: How do you interpret the equal sign? You wrote an equal sign right there (refers to her equation $C = 5/7 \cdot G$). When you read that, what are you thinking?

Linda: Copper equals 5/7 of gold… The number of ounces copper has. Yeah, copper has 5/7 the number of ounces as gold.

The data show that Linda interpreted the equal sign as having the same number of ounces of gold and copper rather than an association between them. Her use of the definition of multiplication as "5/7 is the size of the group" (i.e., "the number of groups" for her) and "*G* is the whole" appeared to regulate her thinking in generating the appropriate equation.

Furthermore, Kelly generated two correct equations of $C = 5 \cdot G/7$ and $C = 5/7 \cdot G$ and presented them in one equation as $5 \cdot G/7 = 5/7 \cdot G$. She used the definition of multiplication in both of her equations as "5 groups" and "*G*/7 in each group" for $C = 5 \cdot G/7$, and "5/7 groups" and "*G* in one group" for $C = 5/7 \cdot G$. Her use of the definition of multiplication seemed to help her develop appropriate equations.

Kelly: 5 groups, *G*/7 in each group (refers to the left hand side of her equation in Fig. 13.13), so it's like 5/7ths *G* of like, in terms of the gold is in copper.

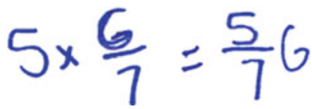
Interviewer: The left side and the right side are a little different, the way you've written them. (refers to her equation in Fig. 13.13)

Kelly: I just combined them (refers to both sides of her equation in Fig. 13.13).

Interviewer: Could you explain the meaning of multiplication in each of those cases?

Kelly: *There are 5 groups and each of the groups has G/7 in them* (points to the left hand side in Fig. 13.13) … For this 5/7th *G* (points to the right hand side in Fig. 13.13), *5/7ths is my group* because 7/7th would be one group of gold. And the copper is 5/7th group, so we're looking at 5/7th of *G*, which is 7/7th or a whole.

Fig. 13.13 Kelly's equation in Task 2



Finally, Claire kept the language used in definitions for multiplication and for fractions distinct. She interpreted $5/7 \cdot G = C$ from both definition of fractions (i.e., "5 parts each size 1/7 of the whole amount of gold") and the definition of multiplication (i.e., "5/7 groups times the amount of ounces in 1 group is equal to the amount of copper in 5/7 groups of gold").

Interviewer: How would you explain the meaning of the equal sign (refers to her Eq. $5/7\ G = C$ in Fig. 13.14b)?

Claire: That means they're equivalent or the same amount of ounces, because the gold has like 7 parts versus the copper has 5 parts. But *if we take 5/7 of that gold, we're going to have 5 of the 7 parts, which is equivalent to the parts of copper* (points to the strip diagram accompanying Task 2). So, 5/7 of the amount of gold is equivalent or the same amount of ounces as the total amount of copper we have.

Interviewer: Could you interpret the left hand side of this (points to her Eq. $5/7\ G = C$ in Fig. 13.14b) for me using the meaning of multiplication?

Claire: I know two meanings, so… like I know… *the fraction meaning*, and I also know *the meaning of the multiplication*. And you're talking about multiplication, right?

Interviewer: Tell me about both.

Claire: Well, with the fraction meaning 5/7 that means we have… the whole unit is 7 parts and the amount of parts we have is 5 of that… those 7 parts or 5 of the whole unit. So it's *5 parts each size 1/7*. And with the multiplication… So, 5/7 groups times the amount of gold that we have is the amount of copper we have. So, it's like *5/7 groups times gold, which is the amount of ounces in 1 group or the amount of gold, is equal to copper, which is total for the 5/7 groups of gold*.

Claire exhibited solid performance based on an appropriate interpretation of the equal sign, the definitions for multiplication and for fractions in addition to her generation of single, correct equation. The given strip diagram appeared to support her performance for perceiving the relationships between the ounces of gold and copper.

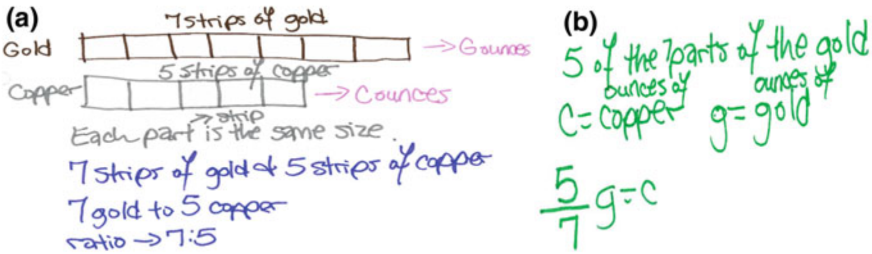


Fig. 13.14 **a** Claire's strip diagram in Task 2. **b** Claire's equation in Task 2

13.5 Conclusion and Discussion

The purpose of the present study was to examine how six future middle grades mathematics teachers used quantitative definitions for multiplication and for fractions when developing and explaining equations for proportional relationships. The results revealed that future teachers' use of explicit, quantitative definition of multiplication in a consistent way co-occurred with their success when generating and explaining equations that involve proportional relationships. In other words, future teachers' explicit use of the definition of multiplication facilitated their ability to develop equations and to reason about proportional relationships. All three future teachers (i.e., Linda, Claire, and Kelly), who demonstrated an explicit and consistent use of the definition of multiplication, seemed to use this definition as an organizing tool to regulate their reasoning process while working on the given tasks. Their use of the definition as an organizing tool apparently enabled them to coordinate units and groups appropriately, to keep distinct wording between the definitions for multiplication and for fractions, and to generate and explain appropriate equations for proportional relationships. In addition, their interpretation of the equal sign was consistent with normative usage, which is to equate the number of ounces of gold and of copper. Furthermore, when multipliers were placed as fractions in the tasks, these future teachers were able to identify the multiplier as the number of groups by viewing how many groups (or parts) of one strip were in another strip in their drawings.

On the other hand, the remaining three future teachers (i.e., Alice, Jeff, and Diana), who did not explicitly express the quantitative definition of multiplication, had difficulties in generating equations and in reasoning about proportional relationships. These future teachers experienced consistent difficulties distinguishing the different roles played by the multiplier and multiplicand, and coordinating units and groups appropriately. Specifically, they had trouble identifying the multiplier as the number of groups when multipliers were fractions, and this even caused some of them to switch the order of multiplier and multiplicand so that the multipliers were whole numbers. Moreover, the placement of fractions as multipliers in the tasks caused these teachers to misuse phrasing from the definition for fractions when applying the definition of multiplication. Thus, lack of the explicit use of the definition of multiplication in a consistent way apparently contributed to these future teachers' blended wording from both definitions rather than keeping them distinct, and to struggle with reasoning about proportional relationships. Moreover, these future teachers' interpretations of the equal sign were inconsistent with normative usage, at least for Task 2.

Given that developing algebraic equations is difficult and this study took place before instruction on equations, it is important that three of the six future teachers generated and explained appropriate equations that involve a proportional relationship by using the definition of multiplication. Therefore, the results of this study suggest that explicit use of the quantitative definition of multiplication is helpful organizing tool for future teachers in terms of developing equations and reasoning about proportional relationships. Furthermore, past research has documented teachers' difficulties with particular topics such as fraction multiplication and multiplication with an unknown factor (e.g., Armstrong & Bezuk, 1995; Izsák, 2008). Regarding that past research has not examined the extent to which teachers' facilities with multiplication and fractions can support and constrain their generation of equations and reasoning about proportional relationships, the present study concludes that using the quantitative definition of multiplication to construct viable arguments related to proportional relationships and equations is accessible to future middle grades teachers. Robust arguments about proportional relationships and equations depend in turn on the ability to keep distinct the definitions for multiplication and for fractions. In addition to stating the quantitative definition of multiplication verbally, future teachers also need to visualize how many parts of one strip are nested in another strip by viewing the relationships between the multiplier and multiplicand in the strip diagrams.

As the main implication of this study, future teachers in middle grades programs should be given opportunities to develop capacities for reasoning with quantitative definitions for multiplication and for fractions across problem situations. Future studies should continue, with larger samples and tasks, to evaluate the effects of using explicit, quantitative definitions for multiplication and for fractions on both teachers and students' generation of equations and their reasoning about proportional relationships. Additional research is also needed to compare the effects of using these definitions for future middle grades mathematics teachers in developing and explaining algebraic equations before and after class instruction.

Acknowledgements I would like to thank Andrew Izsák and Sybilla Beckmann for helpful comments on earlier drafts of this monograph. This research was supported by the National Science Foundation under Grant No. DRL-1420307. The opinions expressed are those of the author and do not necessarily reflect the views of the NSF.

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Chapter 14
Marking Mathematics Exams. A Tool for Secondary Teacher Education

Alberto Arnal-Bailera, Eva Cid, José M. Muñoz-Escolano and Antonio M. Oller-Marcén

Abstract The marking of exams is a usual tool in the teaching and learning processes in mathematics. Nevertheless, there is (at least in Spain) a lack of systematic education for prospective secondary teachers on this topic. In this study, we analyze how 58 prospective secondary school teachers mark answers to mathematics written exams. In particular, we observe differences between graduates in mathematics and graduates in engineering according to their marking practices. Furthermore, we design and implement a first cycle of Action Research involving a sequence of activities focused on different aspects of the marking process. This sequence of activities helps prospective teachers to realize that marking exams is a complex activity that requires reflection and specific education.

Keywords Prospective secondary teachers · Marking practices
Assessment · Mathematical test

14.1 Introduction

In the learning and teaching events, assessment plays a fundamental role because it is the only way to know if the students have learned what they have been taught and if they are prepared for the society requirements (Rico, 2006). Even if there are

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_14

some other instruments to assess students' learning, written tests or examinations are still widely used by secondary school mathematics teachers (McMillan, Myran, & Workman, 2002; Senk, Beckmann, & Thompson, 1997). It is not usual to include the correction of mathematics exams in the teacher education process, although this task is carried out by nearly all of the mathematics teachers. Thus, prospective teachers are educated through informal conversations, debates with other students or practicing teachers, reading about other teachers' practices or reflecting on their own experience (Sowder, 2007). As a consequence of this lack of systematic education, unexpected and undesirable phenomena arise when different correctors mark the same sets of exams involving constructed-response tasks (Gairín-Sallán, Muñoz-Escolano, & Oller-Marcén, 2012, 2013).

Hence, the following research questions arise:

- How do prospective secondary mathematics teachers mark written exams?
- Is it possible to design and implement a sequence of activities that promotes reflection on marking practices?

14.2 Theoretical Framework

Grading or marking practices of students' mathematical procedures are part of evaluation and assessment practices. Evaluation and assessment in mathematics education are studied from many different points of view (Romberg, 1992). Even if there are some other instruments to assess the students' learning, written tests or exams are still widely used by secondary school and university mathematics teachers (McMillan et al., 2002; Rochera, Remensal, & Barberá, 2002).

There are studies about different aspects of how teachers mark written exercises in mathematics (Hungwe & Nyikahadzoi, 2002). Some researchers study different factors involved in marking mathematics exams. These factors are related to the knowledge, conceptions and beliefs about mathematics of the correctors and to the tasks and the specific answers of the students. Hence, up to six factors that influence in the corrections are presented in the research of Wang and Cai (2006) and Meier, Rich, and Cady (2006). These are the teaching experience of the corrector, the educational level where this experience has been gained, the mathematical knowledge of the corrector, his beliefs about teaching and learning mathematics, the nature of the task, and the answers of the students (arising bigger differences when mathematical errors are shown).

In fact, Morgan, Tsatsaroni, and Lerman (2002) consider different positions adopted by teachers when marking exams. These positions are based on the different resources that teachers bring to bear when they read students' work. Morgan and Watson (2002) identify, among others, the following resources: teachers' personal knowledge of mathematics, teachers' beliefs about the nature of mathematics, teachers' expectations about how mathematical knowledge can be communicated, teachers' experience and expectations of students and classrooms, and teachers' cultural backgrounds. Note that these resources mostly involve personal, social, and cultural features of the teachers. This points out the *interpretative nature of the assessment*. In this regard, Sakonidis and Klothou (2007, p. 153) state that "teachers proceed to assessments subjectively" while observing that teachers mainly rely on unofficial personally constructed discourse when assessing written works in mathematics.

On the other hand, recent research on mathematics teacher education has shown that it is very important that prospective teachers acquire the skills of identifying students' strategies and procedures, interpreting their understanding and deciding how to respond accordingly to the students' understanding (Sherin, Jacobs, & Philipp, 2010). This will help prospective teachers to develop their competence as future mathematics teachers (professional noticing). There exist several teaching proposals aimed to promote professional noticing among prospective secondary school teachers. For instance, Sánchez-Matamoros, Fernández, and Llinares (2015) design and implement a sequence (focused on the concept of derivative) in which prospective teachers give interpretations of written solutions to problems involving the derivative concept before and after participating in a teacher education module.

Regarding the mathematical knowledge for teaching (MKT), Shulman (1986) distinguished between content knowledge, pedagogical content knowledge and curricula knowledge. Later Ball, Thames, and Phelps (2008) refined Shulman's theory identifying the six categories shown in Fig. 14.1.

Fernández, Callejo, and Márquez (2014) in a context of prospective primary school teachers education conducted an activity involving grading answers of students and point out the formative interest of these kind of activities. Nevertheless, there are not many works which study the formative use of grading

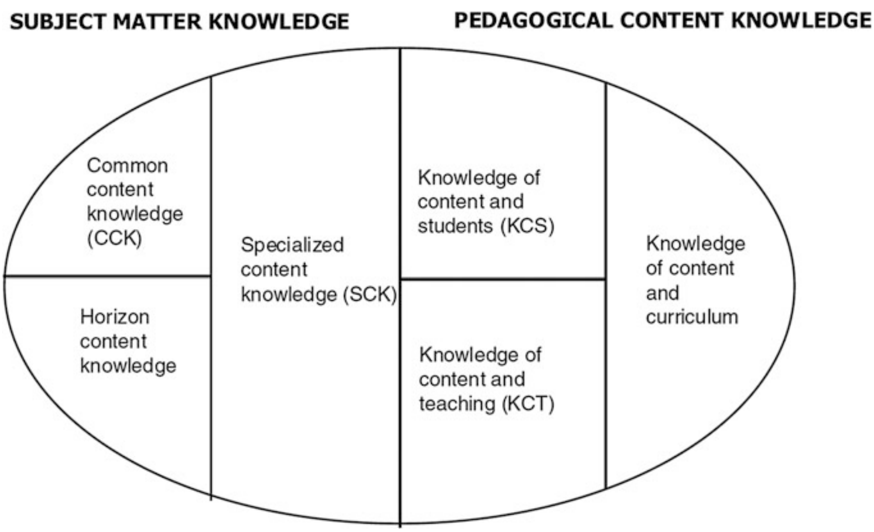


Fig. 14.1 Ball, Thames, and Phelps (2008)

and marking answers of secondary students for prospective teachers. In the MKT frame, Morris, Hiebert, and Spietzer (2009) developed an intervention with pre-service teachers involving tasks such as anticipating an ideal student response, evaluating correct and incorrect answers and analyzing a classroom session. From our point of view, most of these tasks play an important role in order to educate prospective teachers to reflectively mark exams. Specifically, the aforementioned works mainly focus on specialized content knowledge. Nevertheless, the marking of mathematics exams implies certain knowledge of content and curriculum as well as of horizon content knowledge. For instance, the relevance of errors is related to the type (arithmetic, algebraic, etc.) and hierarchy (main, auxiliary, etc.) of the task where they appear (Gairín-Sallán et al., 2012, 2013). Moreover, the hierarchy of a task heavily depends on the main learning objective that is being evaluated in a particular exercise.

The awareness of the subjectivity underlying the practices of marking in mathematics as well as the interpretation and appraisement of the students' errors constitute key knowledge for the future mathematics teachers. They will bring into play this knowledge when they establish marking criteria or rubrics in the process of marking actual student's productions.

14.3 Objectives

The main objectives of our study are related to the research questions stated in the introduction. In particular, Objective 1 tries to answer the first question while Objective 2 gives a first step in the answer of the second question:

- Objective 1: To analyze how prospective secondary school teachers mark answers to mathematics written examinations.
- Objective 2: To design and implement a sequence of activities focused on different aspects of the marking of mathematics examinations.

The first objective can be specified in the following way:

- Objective 1.1: To describe how prospective secondary school teachers mark correct answers that use different solving methods, analyzing the influence of the solving method on their marks.
- Objective 1.2: To describe how prospective secondary school teachers mark answers containing different types of errors in different types of tasks, analyzing the influence of errors and tasks on their marks.
- Objective 1.3: To analyze the influence of the prior qualifications of the prospective secondary school teachers on their marks.

The second objective involves the design and implementation of a sequence of activities with prospective secondary school teachers. The different skills that we want our prospective teachers to acquire are:

- Skill 2.1. To notice how different methods can be used to solve the same task in secondary school mathematics and to anticipate them in order to plan their marking.
- Skill 2.2. To identify errors in the students' answers, interpreting and assessing them with regard to the learning goals of the task and to the part of the task where these errors appear.
- Skill 2.3. To reflect on the role that the elaboration and application of marking schemes has on their future profession.

14.4 Method

To achieve these objectives, we carried out our work in two stages. The first stage corresponds to Objective 1 and it is of exploratory and descriptive nature. We designed a questionnaire where prospective teachers were asked to mark (from 0 to 10 points) ten different answers of secondary school students to the same problem about the computation of critical points of a function and to provide reasons for their marks. This type of problem, involving rational functions, is very common in the Spanish university entrance exams (Nortes & Nortes, 2010; Zamora-Pérez, 2014). Hence, the given function was $f(x) = x^2/(4 - x)$.

In order to design the different incorrect answers of the questionnaire, some of the authors performed a detailed analysis of around 400 Spanish university entrance exams (Gairín-Sallán et al., 2012, 2013), where they identified the most common errors made by the students on this type of tasks finding them consistent with previous results (Nortes & Nortes, 2010). In particular, they identified errors regarding algebraic manipulation, applying differentiation rules or showing the lack of proper knowledge of a method. On the other hand, the correct answers involved different methods of resolution, more or less standard according to their appearance in Spanish textbooks.

We show in Table 14.1 a description of the 10 answers with information about the correctness of the numerical result and about the type of error present in each of them, if any.

The sample for the first stage was formed by the 58 prospective teachers (34 women and 24 men) enrolled in the Master's degree on the teaching of secondary school mathematics at the university of Zaragoza during the academic years 2012–13, 2013–14, 2014–15 and 2015–16. Therefore, it was a convenience sampling (Flick, 2009), and it was stratified according to the prior education of its members (29 graduates in Mathematics, 23 graduates in Engineering and 6 graduates in Sciences). Some of them (36) had previous teaching experience in non-formal contexts (mainly in individual or group private classes). The mean age was 30 years. There was a high variability (7.6) due to the fact that most of them were recent graduates but we find a relevant group of people (16 prospective teachers)

Table 14.1 Main details of the 10 answers

Answer	Numerical result	Error
A1	Incorrect	On algebraic manipulations
A2	Incorrect	On differentiation techniques
A3	None	Incomplete method
A4	Correct	None
A5	Correct	None
A6	Correct	None
A7	Incorrect	Incorrect method
A8	Incorrect	On differentiation techniques
A9	Correct	On differentiation techniques
A10	Incorrect	On algebraic manipulations

aged over 35. Most of these elder people have been working in different sectors but need this master's degree in order to improve their professional status.

The collected data during the first stage were analyzed using descriptive and inferential statistic tools with the help of the SPSS and R packages.

The second stage corresponds to Objective 2. In particular, after the first stage, we design a sequence of six activities with prospective secondary school teachers corresponding to six sessions of about one and a half hour and related to the marking of mathematics exams. Throughout the sequence, different aspects of the theoretical constructs previously described are addressed. Now, we briefly describe each activity.

- Activity 1: Filling in the initial questionnaire.
- Activity 2: Discussing on the variability of marks. Resources Positions.
- Activity 3: Analyzing textbooks and curricula.
- Activity 4: Working with actual wrong answers.
- Activity 5: Revisiting the results of the questionnaire.
- Activity 6: Elaborating marking criteria.

Activity 1 is an introductory activity and its main goal is to engage the prospective teachers in an actual in-service task. Moreover, the obtained data are extensively used and revisited in many of the forthcoming activities. Activities 2, 5 and 6 are designed with a view to showing the different positions adopted and resources used by correctors. This also makes explicit the interpretative nature of the assessment. Professional noticing is partially addressed on Activities 2, 4 and 6. In particular, prospective teachers have to identify different students' strategies and procedures (when dealing with correct answers) and to interpret their understanding (when dealing with incorrect answers). Finally, Activities 3 and 6 cover some aspects of the MKT model such as knowledge of content and curriculum (in Activity 3) and specialized content knowledge (in Activity 6). Throughout the sequence, prospective teachers work in mixed groups of three people (mathematicians and engineers) and whole class discussion takes place at the end of each activity.

The second stage was carried out only during the academic years 2014–15 and 2015–16 with 18 and 14 prospective teachers, respectively. It took place in a 3 ECTS credits[1] course of the Master's Degree (Assessment, innovation and educational research in mathematics) which was delivered during the second term of the academic year. This course deals with the interplay between teaching practice, innovation, research, etc., it has a mainly practical nature and partially underpins the Master's Thesis. These prospective teachers have had very little experience with actual students at the moment of our experimentation. In the first term the prospective teachers receive pedagogical and psychological courses but just a few lectures on mathematics education while the most relevant internship period at the high school takes place at the end of the second term.

The collected data during the second stage were analyzed using content analysis techniques (Krippendorf, 2013). The main source of data was the students' written productions, obtained during the implementation of the sequence. Observational techniques were also applied by the researchers conducting the sequence (Postic & De Ketele, 1988). While one of them acted as a teacher, the other played the role of external observer. At the end of each session, the teacher wrote down a daily log which was later compared with the external observer notes in order to validate the data and the actual implementation of the course.

The methodological framework used is an action-research design (Zeichner, 2001), which reflects about educational practices and pursues the improvement of the quality of the educational processes (McNiff, 2013). In particular, we carried out a *classroom action research* with a *practical purpose* (Kemmis, McTaggart, & Nixon, 2014). We followed the classical approach consisting of a sequence of four phases (planning, action, observation and reflection) which, eventually, might be repeated cyclically (Lewin, 1946). Stage one is partly used to identify the problem. During stage two we conducted a first cycle of an action-research project. The planning phase corresponds to the design of the previously described activities. The implementation of the sequence, that also included the data collection, corresponds to the action phase. Finally, the analysis and discussion of the results correspond to the observation and reflection phases.

14.5 Results

14.5.1 Results for the First Stage

In Table 14.2, we present the main descriptive statistics arising from the collected data.

[1]ECTS stands for European Credit Transfer and Accumulation System. 3 ECTS credits are usually equivalent to 75 h of total student workload.

Table 14.2 Descriptive statistics about the marks given to initial questionnaire

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Mean	3.99	3.22	1.35	8.78	8.29	9.66	2.78	5.77	6.50	4.04
S.D.	1.73	1.99	1.11	1.75	2.05	0.75	1.77	1.87	1.68	1.97
Minimum	0.25	0.00	0.00	3.50	2.00	6.00	0.00	1.00	3.00	0.00
Maximum	8.00	8.00	4.00	10.0	10.0	10.0	7.00	9.00	10.0	9.00
Q1–Q3	2	2	1.38	1.88	3	0.5	2	2	2.19	2
Median	4	3	1	9.5	9	10	2.75	6	6.5	4

We observe a high influence of the solving method over the assigned marks in the case of correct answers (A4, A5 and A6). In particular, the means are significantly different (at 95%) being higher when the method is more standard (Arnal-Bailera, Muñoz-Escolano, & Oller-Marcén, 2016). When the answers only contain algebraic or differentiation errors (A1, A2, A8, A9 and A10), some other factors seemed to be relevant. For instance, when the final numerical result coincided with the correct one (A9) or the student explained the steps followed (A8), the mark was significantly higher. On the other hand, if the answer was poorly presented (A2), marks were lower. Finally, errors showing the lack of knowledge of a method (A3 and A7) received the lowest marks.

We also observe a high global variability with standard deviation greater than 1 in all but one of the answers of the questionnaire and range greater than 6 in all but two (A3 and A6). This variability is lower when the answer is correct using a very standard method (A6) or it shows the lack of knowledge of a method (A3 and A7). On the other hand, variability increases in the presence of errors or non-standard methods.

Regarding the three strata of the sample, we observed clear differences, even though some of them are not statistically significant due to the small size of the sample. For instance, mathematicians showed a higher dispersion in their marks than engineers or scientists. The cases in which the differences are statistically significant (at 90%) seem to point out that, compared to mathematicians, engineers give lower marks and emphasize the right numerical solution.

If we compare (Table 14.3) the group of mathematicians and the group of engineers (the main strata of the sample), we notice that engineers seem to give higher or equal (statistically speaking) marks to those answers whose numerical result is correct (A4, A5 and A6), even if they contain mistakes (A9). Nevertheless, in those answers having incorrect numerical result and containing algebraic (A1 and A10) or differentiation errors (A2 and A8), engineers were stricter and gave lower marks. In the remaining cases, the marks were similar.

Table 14.3 Means of marks according to prior discipline

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Mathematicians	4.32	3.60	1.19	8.88	7.84	9.65	2.74	5.90	6.05	4.34
Engineers	3.43	2.68	1.67	8.97	9.05	9.66	2.73	5.55	7.23	3.73

Table 14.4 Means of marks according to gender

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Women	4.24	3.44	1.23	8.49	7.83	9.43	2.87	6.02	6.40	4.38
Men	3.64	2.91	1.52	9.20	8.94	9.98	2.65	5.42	6.64	3.56

According to gender (Table 14.4), we observed that women gave lower or equal (statistically speaking) marks to correct answers (A4, A5 and A6). In the answers with errors, women gave higher marks, except to A3 and A9, but these differences are not statistically significant, probably due to the small size of the sample. All of these facts seem to point out that, compared to men; women tend to be more exigent with correct answers and more indulgent with answers that contain errors.

With respect to the age of our prospective teachers, we found no statistical correlation with the marks given to any of the answers. Nevertheless, if we consider the prior (in non-formal education contexts) teaching experience we find that the answers A5 and A9 have statistically significant slight negative correlations with the given marks. It means that more experienced prospective teachers give lower marks when the method is non-standard, even if the answer is correct (A5) and when there is an error in the process, even if the numerical result is correct (A9).

14.5.2 Results for the Second Stage

After the analysis of the data obtained in the first stage and based on previous related research studies by different authors, we designed and implemented a sequence of activities that we now present:

- Activity 1: Filling in the initial questionnaire.
- Activity 2: Discussing on the variability of marks.
- Activity 3: Analyzing textbooks and curricula.
- Activity 4: Working with actual wrong answers.
- Activity 5: Revisiting the results of the questionnaire.
- Activity 6: Elaborating marking criteria.

Now, we present an account of the implementation of the sequence focusing on its impact on the prospective teachers.

14.5.2.1 Filling in the Initial Questionnaire

In the first session, the prospective teachers filled in the questionnaire described in the Method section. They were asked to mark ten different answers during a period of 90 min. They were told to act as if they were practicing teachers marking

University entrance exams. In addition, they were not allowed to use any kind of support like textbooks, internet, etc.

The prospective teachers approached the activity with interest, as an actual in-service activity. Some questions spontaneously arose about the existence of common marking criteria in university entrance examinations or about the expected solving methods. The questions were not answered by the researcher/teacher because their answers were going to be subject of the forthcoming discussion.

14.5.2.2 Discussing on the Variability of Marks

The main goal of the second session was to show that marking exams, even in mathematics, has an important subjective component. After a discussion about 'acceptable' variability, the researcher showed the results of the questionnaire to the prospective teachers with the objective to point out interesting phenomena. These phenomena were finally exemplified using actual instances of marked problems taken from university entrance examinations.

The discussion about 'acceptable' variability started by asking the prospective teachers about the different statistics that they know to measure variability and their convenience in this context. Prospective teachers spontaneously mention (in order of appearance) the standard deviation, the range, the number of different values (in the case of categorical variables) and the interquartile range. In addition, some inappropriate answers (mean, mode, etc.) appeared, showing a lack of statistical knowledge. Throughout the process, the researcher made suggestions, pointed out mistakes, and specified the meanings and uses of the different mentioned concepts.

Once the adequate statistics were chosen, the discussion continued and the prospective teachers were asked what actual numerical values would be considered as reasonable in this context and about their expected values in each of the med exercises. The chosen statistics and their values considered reasonable were: the standard deviation (smaller than 1.25), the range (smaller or equal than 4) and the number of different grades[2] (smaller or equal than 2). The actual values (see Table 14.2) were, in general, much higher than those considered reasonable. Even if the expected values varied among the prospective teachers, most of them were smaller than those considered reasonable. Thus, our prospective teachers were, in general, 'optimistic'.

When they were faced with the actual results, prospective teachers were surprised about the high variability of marks (see, for example, the production of prospective teacher AG in Fig. 14.2).

They did not seem to expect the wide diversity of marking criteria. In the words of prospective teacher PL: "... *I deduce that each of us was expecting different aspects from the resolution of the exercise: some wanted to see if the student knew*

[2]The scores were grouped into grades in the following way, according to the usual Spanish grading system: A = [9, 10]; B = [7, 9); C = [5, 7); D = [3, 5); E = [0, 3).

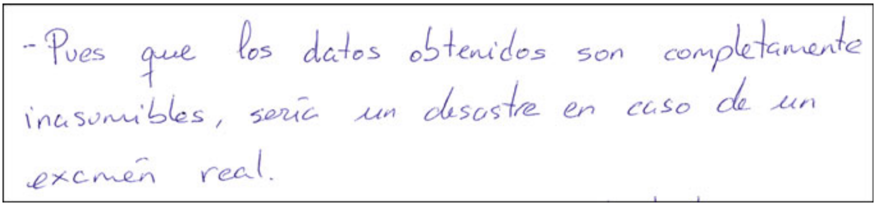


Fig. 14.2 Surprise of prospective teacher AG about variability of marks ("*The obtained data are completely unacceptable; it would be a disaster in the event of an actual exam.*")

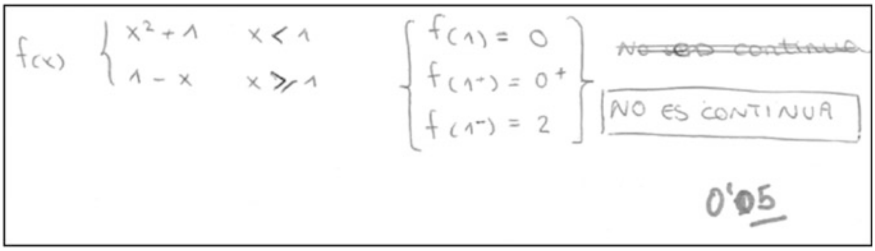


Fig. 14.3 Strong penalty due only to non-standard notation in an actual university entrance examination answer

how to derive, others if he knew what maxima and minima are, others wanted the explanation of the procedure, etc.".

Finally, this activity concluded with the analysis of the actual instances of marked problems taken from University entrance examinations, exemplifying different phenomena. For instance, in Fig. 14.3, the solver received 0.05 points out of 1 in an answer that is essentially correct in spite of its evidently non-standard notation.

This final discussion contributed to make even more evident the necessity of educating prospective teachers in marking and the elaboration of clear marking criteria. Moreover, once they become aware of this fact, they easily get involved in the rest of activities.

14.5.2.3 Analyzing Textbooks and Curricula

The main goal of the third session was to make explicit the fact that the same mathematical problem may have different correct solution methods. The prospective teachers analyze textbooks and the Spanish official curriculum to be aware of the different correct solving methods for the task that appear on the initial questionnaire (A4, A5 and A6 answers).

In this session, the prospective teachers worked both individually and in groups of three. The groups were formed by the researchers in order to have in the same group at least one mathematician and one engineer. Moreover, in order to promote discussion, we tried to have in each group prospective teachers whose marks to the answers A4, A5 and A6 were clearly different.

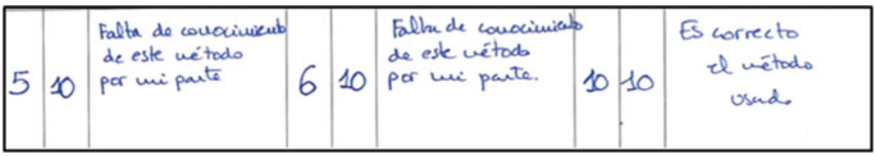
The first set of tasks in this activity involved mathematical questions about the computation of minima and maxima of a real function. A second set of tasks were focused on the analysis of the official curricular documents and its transposition into four textbooks. The last part of the activity sharpened the re-interpretation of answers A4, A5 and A6. In light of this, our prospective teachers were asked to mark again these three answers, both individually and as a group.

The first set of tasks asked about the description and application of different methods to compute the relative extrema of a real function. Most of the prospective teachers showed difficulties applying the method that involves the second derivative; they could not manage the case when the first and the second derivative are simultaneously zero at the given point. Some of them were surprised by their lack of knowledge in a content considered by them as a 'basic' part of a 'very standard' method.

The second set of tasks helped the prospective teachers to notice that there is not a unique solving method in school mathematics. This idea was supported by the Spanish curriculum, which does not mention a single method, and by the selected textbooks, which present more than one. Once the analysis was performed, prospective teachers EH and MS admitted that they had learned various solving methods apart from the second derivative one, overcoming their initial lack of knowledge. In Fig. 14.4, for instance, we show the original and revised marks of prospective teacher EH and his explanations about the changes.

The last part of the activity consisted of two tasks about interpreting the correct answers and the individual and collegiate re-marking of them. Most of the prospective teachers moved from the simple identification of errors (or of the absence of a correct method) to a deeper analysis and interpretation of the answers from a wider point of view. Thus, prospective teachers who initially considered answer A5 as an incorrect procedure in activity 1; found it mathematically correct after carrying out this activity. In fact, contrary to his initial opinion, prospective teacher AG pointed out that knowledge and comprehension showed by answer A5 is deeper than those in A4 and A6.

There is an almost unanimous agreement on marking every correct answer with 10 points. Many of the prospective teachers justify their previous marks either on a



5	10	Falta de conocimiento de este método por mi parte	6	10	Falta de conocimiento de este método por mi parte.	10	10	Es correcto el método usado

Fig. 14.4 After the textbooks analysis, EH changes his marks to A4 and A5 and explains the reasons of the low marks given in initial questionnaire: "Lack of knowledge of this method for my part"

lack of mathematical knowledge about other methods (Fig. 14.4) or on their expectations about the use of a particular method involving the second derivative. In this respect, prospective teacher BA points out that she was 'mentally blinded' by the second derivative method because this was the method she was expecting, 'it is correctly solved, even if he is not using the method I was waiting for' said about A5. The prospective teacher MS remarked an interesting fact: 'I think the method used by the corrector pushes upwards the marking if the student uses the same method or downwards if he uses a different one' (about A5).

Finally, it should be noted that a few prospective teachers pointed out that the context where the marking practice takes place plays an important role and somehow determines the way correctors give their marks. Thus, they considered that, acting as university entrance examiners they would have awarded the highest mark to the three answers, while acting as regular class teachers they would not have. The reasons given are related to rewarding more sophisticated methods in terms of the use of more 'advanced' concepts.

14.5.2.4 Working with Actual Wrong Answers

The researcher presented actual answers of secondary students to different problems containing errors so that prospective teachers identified and found reasons for them. The main goal of this activity was to emphasize the importance of the task where the error appears with respect to the whole solving process.

In the first part of this activity, the researcher provided the prospective teachers with actual answers of university entrance exams containing errors together with the official marking criteria. For each of them, there is a group discussion to identify the errors, to interpret and explain and to grade them according to the official criteria. The prospective teachers remark that the official criteria are, at least, incomplete, ambiguous and they do not provide enough information in order to mark the answers adequately (see Fig. 14.5).

During the discussion, it is observed that this ambiguity implies that the use of the given criteria would lead to different marks to the same answer. The comparison of the different marks often leads to sometimes lively debates regarding aspects like the distinction between 'minor' and 'serious' errors. Finally, prospective teachers

A2. b) Estudiar la continuidad de $f(x)=\begin{cases} x^2+1 & x<1 \\ 1-x & x\geq 1 \end{cases}$ (1 punto)

b) El cálculo de los límites laterales se valorará con 0.75 puntos

Fig. 14.5 Example of incomplete criterion: "Computation of one-sided limits is worth 0.75 points" realize that the criteria should address the question of the relevance of a particular error in order to assess its importance with respect to the whole solving process.

After this discussion, the researcher presents some marking guidelines to design marking criteria. These guidelines require a clear statement of the main assessment goals of the problem. After that, the different tasks involved in a solution must be classified according to their relevance in the whole process and to their proximity to the assessment goals. Errors appearing in more relevant or closer to a main goal tasks must receive higher penalties.

14.5.2.5 Revisiting the Results of the Questionnaire

The main goal of the fifth session is to apply the given marking guidelines to set new and common marking criteria to the problem of finding the relative extrema of a function presented in the questionnaire of Activity 1. In order to do so, we promote the prospective teachers' reflection on their own previous criteria, on interpreting secondary school students' answers where the errors were found and on the nature and the relevance of the task where the error appears with respect to the whole problem. The researcher found relative agreement about the application of the marking criteria. The prospective teachers are able to set sub-objectives and sub-tasks identifying different types of tasks according to their relevance in the given problem.

During this session, a quantitative analysis of the qualifications given by the prospective teachers is performed. Some of the results explained in the first part of this chapter (see Table 14.2) are shown to the prospective teachers, in particular those related to the answers containing errors. Moreover, a qualitative analysis of their comments in the questionnaire of Activity 1 was presented to the prospective teachers. For instance, regarding answers A2, A8 and A9 (which contain the same type of error) the researcher presented a summary of different prospective teachers' justifications of their marks. These justifications were categorized by the researcher as follows: expected method, correctness of result, clearness of answer, use of explanations and corrector's misconceptions. This leads to a discussion in which the prospective teachers reflect about the motivations behind their previously given marks and they compare them with the ones they would give if they were asked to do so again.

In Fig. 14.6, we can read a quite common example of the interpretation of a secondary school student answer to A10. After the discussion carried in activity 5, most prospective teachers refined their initial explanation to "He is wrong doing operations with polynomials within the derivation of a quotient".

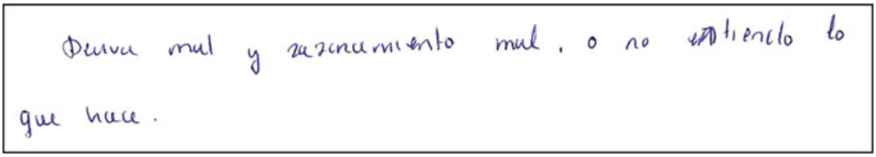


Fig. 14.6 Example of initial interpretation by prospective teacher BR: "He differentiates badly and reasons badly, or I do not understand what he is doing"

14.5.2.6 Elaborating Marking Criteria

The main goal of this last session was to design marking criteria, using the previously introduced marking guidelines, for a problem in a very different context and to use them to mark the answers given to this problem by seven different secondary school students.

In particular, this new problem (Fig. 14.7) involved the computation of areas at 8th grade (13–14 years old). Some of the seven answers were correct, while the others contain different types of errors in different tasks.

In the first part of this activity, the prospective teachers were asked to solve the problem in small groups (2 or 3 people) using two different methods. In fact, some groups spontaneously provided up to four different solving methods.

After that, the prospective teachers had to design marking criteria for this problem. They had to apply the guidelines that were discussed during Activity 3. These criteria were very homogeneous among the different groups and followed

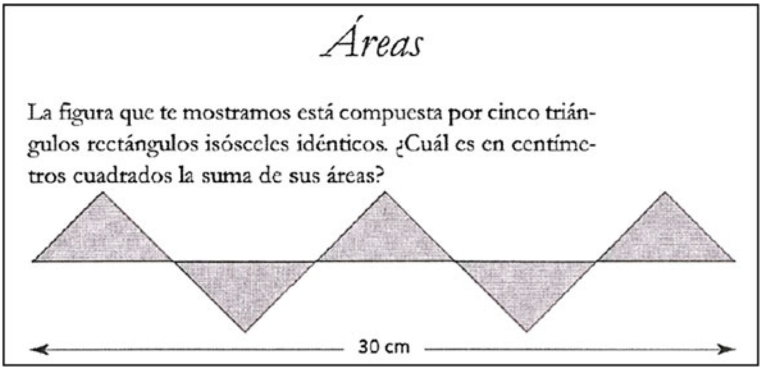


Fig. 14.7 Problem "What is the area of the shaded region?"

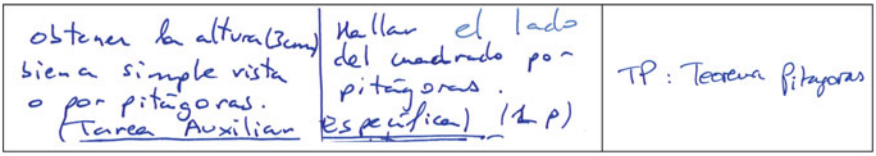


Fig. 14.8 Different views of the Pythagorean theorem, as an auxiliary task on the left ("Tarea Auxiliar Específica") and as a main content on the right ("T[area] P[rincipal]")

quite faithfully the guidelines. The main disagreements arose regarding the relevance of the Pythagorean Theorem within the whole solving process and also regarding the relevance of the use of justifications and explanations by the student. For instance, some groups do not mention the Pythagorean Theorem when designing their criteria; some consider it an auxiliary task and others consider it as one of the main contents involved in the solution of the problem (Fig. 14.8).

Then, the researcher provided the prospective teachers with seven answers to this problem in order to mark them using their own recently designed criteria. In most of the answers, the variability was clearly lower than in Activity 1. Only two out of seven answers presented slightly high variability. One of them presented a correct general procedure but it used a flawed system of equations and contained algebraic errors. The other (with a higher variability) was a conceptually incorrect answer containing both correct and nonsense steps.

In Fig. 14.9, we present three different marks to the same answer to the problem shown in Fig. 14.7. They range between 1.5 and 6 points out of 10. The answer only presents two correct features: the computation of the hypotenuse of each shaded triangle (30/5 = 6) and the statement of the formula for the area of a triangle. The three presented examples identify these correct features; however the rest of the answer is a non-sense sequence of computations that is rather hard to interpret.

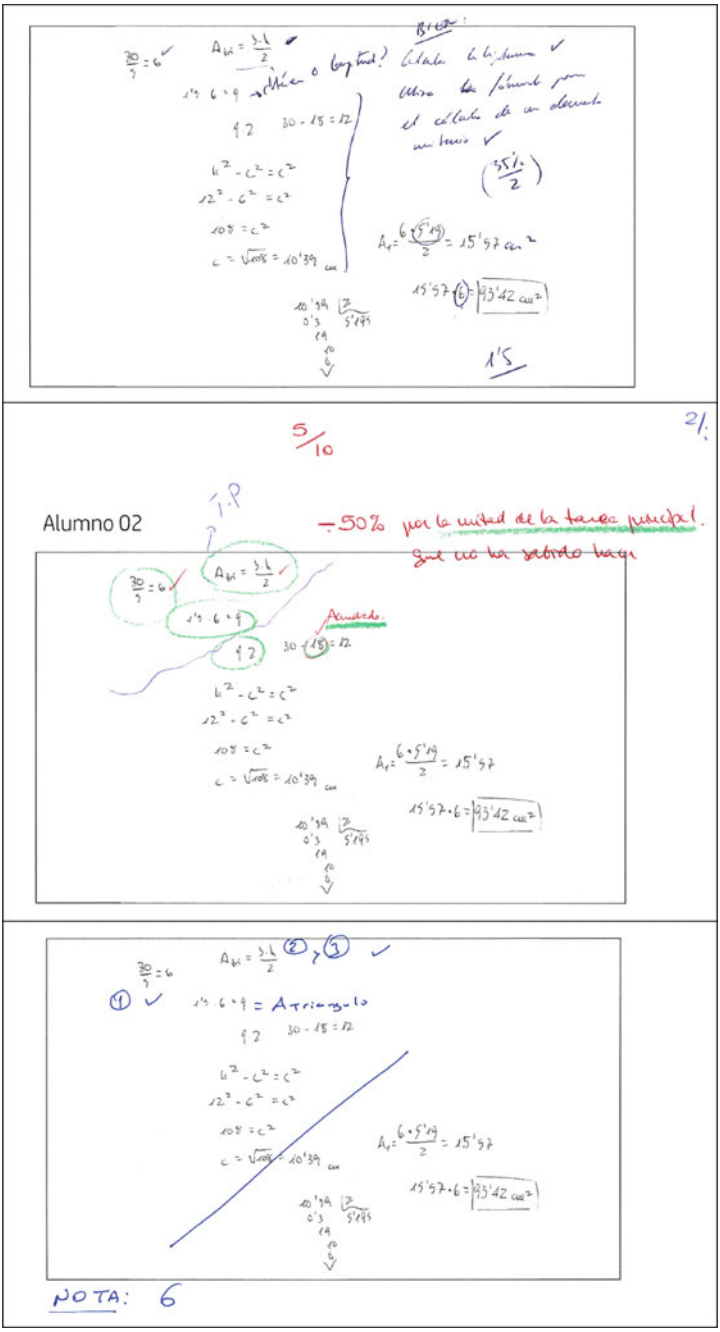


Fig. 14.9 Different marks to the same incorrect answer

14.6 Discussion and Conclusion

Our results regarding objective 1 point out that the variability among the different correctors is influenced by their familiarity with the solving method or by the existence of errors in the answers. The prior qualifications of the correctors also determined differences in the way they give their marks. If we assume a relation between prior qualification and mathematical knowledge, then these observations coincide with results from Wang and Cai (2006) and Meier et al. (2006).

Morgan and Watson (2002) pointed out that the personal knowledge of mathematics is an important resource for teachers when marking exams. In our case, when marking correct answers, this is made clear in the fact that A5 receives significantly lower marks than the other correct answers (A4 and A6) because this method is not usually presented in the classroom, and this was the first time our prospective teachers faced it. Nevertheless, in the case of engineers their beliefs about the nature of mathematics, (mainly seen as a tool for solving problems) reduces the gap between A5 and the other two because the result is correct regardless the method.

When marking answers containing errors, we observed that the prospective teachers do not seem to take into account the type and hierarchy of the task where they appear (Gairín-Sallán et al., 2012, 2013). For instance, answers A2, A8, and A9 contained errors on differentiation techniques and received very different marks. Some other factors related to the expectations about the communication of mathematical knowledge (Morgan & Watson, 2002) like the explanations given by the solver or the clearness of the answer, seem to be relevant.

Our results show that more experienced prospective teachers give lower marks when the method is non-standard, even if the answer is correct (A5) and when there is an error in the process, even if the numerical result is correct (A9). In this respect, experienced prospective teachers behave very much like practicing secondary school teachers (Arnal-Bailera et al., 2016). An interesting difference between prospective teachers and practicing teachers arises in A4. Prospective teachers usually give lower marks than practicing teachers, possibly because they are not familiar with the method used in A4, which although is often showed in textbooks, is not the most commonly learnt and used by the students.

Regarding objective 2, we have designed and successfully implemented a sequence of activities regarding the marking of written exams that is part of a first cycle of action-research. For the moment, this first cycle seems to be useful providing prospective teachers with resources other than unofficial personal discourses (Sakonidis & Klothou, 2007) and developing the skills mentioned in the ‘Objectives’ section, as we have seen throughout this chapter.

As the prospective teachers have progressed through the activities, they have become more skilled in understanding the interpretative nature of the assessment, in identifying students’ strategies and procedures or in establishing the mathematical sub-concepts of learning goals.

When the prospective teachers face the fact that different correctors assign different marks to the same answers and when they are asked to justify their choices, they become aware of the interpretative nature of the assessment (Morgan & Watson, 2002). During our sequence, this mainly took place in Activities 2, 5, and 6. For instance, in Activity 6, the corrections to the answer that still contained a high variability (recall Fig. 14.9) heavily rely on the interpretation given by the corrector to the student's steps. Thus, the different positions adopted by teachers (Morgan et al., 2002) arise explaining the observed variability.

Identifying students' strategies and procedures is mainly promoted during our sequence in Activities 2, 4, and 6. For instance, in Activity 2, the prospective teachers observed the treatment given by actual correctors to different correct but unexpected answers in the context of university entrance examination. On the other hand, in Activity 4, prospective teachers faced different incorrect answers, and they had to identify and explain the errors in order to grade them according to different criteria. This skill contributes to the development of some aspects of professional noticing as pointed out by (Sherin et al., 2010). In particular, our prospective teachers noticed that errors must not only be identified but also interpreted in the process of marking exams according to different factors.

The implemented sequence develops several domains of the MKT model (Ball et al., 2008). The main are knowledge of content and students, specialized content knowledge, and knowledge of content and curriculum. In Activity 3, where the prospective teachers analyzed textbooks and curricula, they observe that, even if textbooks present different solving methods for the problem under consideration, curricula do not establish any of them as "official". This activity contributes to the development of knowledge of content and curriculum. Elaborating marking criteria, as performed in Activity 6, requires the establishment of the mathematical sub-concepts of learning goals. This promotes the development of specialized content knowledge as pointed out by Morris et al. (2009).

One of the main features of action-research is its iterative character. After the reflection phase, we have identified some developments and changes that might improve the teacher education module for future implementations. For example, Activity 6 involved the marking of a task with a higher cognitive demand (Smith & Stein, 1998) than that of the tasks coming from the Spanish university entrance examinations which were previously used throughout the sequence. As a consequence, the prospective teachers found difficulties applying the marking guidelines from Activity 3. The straightforward way to avoid these difficulties would be to change the task in Activity 6, presenting one with a similar cognitive demand as the previous one. However, this observation points out the possibility of improving the sequence by the inclusion of modelling and problem solving tasks. Another possible way to improve the sequence would be to include tasks containing errors of more than one type; i.e., to include tasks combining errors regarding algebraic manipulation, applying differentiation rules or showing the lack of proper knowledge of a method. These answers will be more difficult to analyze by the prospective teachers but they are closer to the actual answers of students that they will face in their future professional life.

The teacher education module that we have designed could be combined with other works related to professional noticing. In particular, the work by Sánchez-Matamoros et al. (2015) seems to be especially appropriate because it also deals with the concept of derivative, which is one of the main mathematical concepts involved in the problem appearing in our questionnaire.

In conclusion, prospective teachers learned that marking exams is a complex activity that requires reflection and specific education. For instance, prospective teacher SM states that: "*[…] there is no training about marking. It would be necessary that teachers receive some courses […]*". Finally, all the prospective teachers actively incorporated this knowledge to their professional practice when they elaborated on a lesson plan for the teaching of a secondary school mathematics topic at the end of the Master's degree.

Acknowledgements This work was carried out by the research group 'S36_17D-Investigación en Educación Matemática' funded by the Aragón Government and the European Social Fund. It was also partially funded by the Spanish Economy and Competitiveness Ministry (Project EDU2015-65378-P).

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Part IV
Professional Identity and Disposition

Chapter 15
Prospective Mathematics Teachers' Professional Identity

Márcia Cristina de Costa Trindade Cyrino

Abstract This study aims to understand how the examination of a multimedia case featuring a mathematics teacher's practice supported prospective mathematics teachers in the construction of their professional identity. The data analysis focused on the meaning negotiation processes occurring during the examination of the multimedia case and written reflections from prospective teachers, concerning the following dimensions: the beliefs, self-image, professional knowledge, vulnerability and the sense of agency. The results show that the prospective teachers had the opportunity to: share their repertoires, discuss their written productions, (re)consider their performance in the student teaching, report and discuss an "ambitious" pedagogical practice, establish connections between observations and empirical interpretations and a broader theoretical background, recognize vulnerabilities, and seek a sense of agency. The examination of multimedia enabled the development of an investigative attitude towards pedagogical practice and the (re) signification of their future professional practice.

Keywords Teachers' professional identity · Preservice mathematics teachers education · Multimedia case

15.1 Introduction

The preparation of secondary prospective mathematics teachers is a challenge that involves aspects other than building essential skills for professional performance. This knowledge is only one piece of the puzzle in the constitution and development

This project is supported by the National Council for Scientific and Technological Development—CNPq and the Araucária Foundation.

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_15

of a professional teacher identity (Connelly & Clandinin, 1999; Lasky, 2005; Oliveira & Cyrino, 2011). It is considered fundamental that the preservice mathematics teacher education programs take into account the need to develop the reflective ability of prospective teachers contributing to their graduation as responsible, autonomous and ethically demanding professionals, able to effectively reflect on their pedagogical practice.

In Brazil, researchers involved in teacher education programs have been working to understand and promote learning opportunities and establish the professional identity of mathematics teachers. The "Study and Research Group on the Education of Teachers Who Teach Mathematics"—Gepefopem, based at the University of Londrina (UEL), has investigated, in the last twelve years, perspectives of the preservice mathematics teacher's education in order to identify factors that may intervene in the process of constructing the identity of these professionals. In this research project, it was investigated how the analysis of a multimedia case featuring one mathematics teacher's practice supported prospective mathematics teachers in the construction of their professional identity.

This case consists of classroom videos associated with other elements, such as lesson plans, interviews with teachers, written productions of students, problem-solving questions, and texts, which can be accessed electronically in an online platform (upon login and password). For each phase of the class, questions are asked, which challenge the prospective teacher to examine specific aspects of the teacher's action.

15.2 Theoretical Background

The construction of a professional identity is a complex process that includes the personal, professional, intellectual, moral and political aspects of the groups to which the subjects are involved (Beijaard, Meijer, & Verloop, 2004; Connelly & Clandinin, 1999; Cyrino, 2016; Day, 1999; Kelchtermans, 2005, 2009; Lasky, 2005; Oliveira, 2004; Oliveira & Cyrino, 2011; Ponte & Chapman, 2008). It consists not only of what others think or say about us, although that is also part of our way of living, but also of how we see ourselves and our capacity to reflect on our experience. According to Wenger (1998) "identity in practice is defined socially not merely because it is reified in a social discourse of the self and of social categories, but also because it is produced as a lived experience of participation in specific communities" (p. 151).

The way teachers typify themselves as teachers and on what others mirror back to the teacher is not neutral; instead, it expresses their orientations, their likes and their values about themselves and their future professional practice. The teacher's professional knowledge, although personal, becomes public in the act of teaching and may or may not be legitimized by the school community. Thus, it is important that in the pre-service teacher education, time and space are offered that envision such reflections and discussions, so they can revisit their knowledge, views and expectations, and become aware of their learning and political commitment as prospective educators.

In this research project, the teacher's professional identity[1] is perceive as a set of interconnected beliefs to self-knowledge and to knowledge about his/her work, associated with autonomy (*vulnerability* and *sense of agency*) and political commitment.

The prospective teacher, when starting her education as a teacher, brings a set of *beliefs* about her future profession. The set of beliefs that prospective teachers have of themselves, of their future profession, of what it means to be an "excellent teacher" and the type of teachers they want to be, among other things, are interconnected and affect the *knowledge* they develop about their work (Shulman, 1986; Ball, 1990; Ball & Bass, 2002; Ball, Thames, & Phelps, 2008; Silverman & Thompson, 2008) as well as with aspects outside the school that influence their practices in classroom. Discussing these beliefs can contribute to their professional self-image (Kelchtermans, 2009), necessary for dealing with professional situations inside and outside the classroom. Working with pre-service teachers' self-image, self-esteem, motivation to work, knowledge of their duties, and prospect for the future is important during teacher preparation (Kelchtermans, 2009).

This set of beliefs, knowledge and professional self-image, built on the relationship between theory and practice, is associated with the *development of autonomy* and *political commitment*. The teaching action is not only related to values, educational standards or the knowledge and beliefs of individual teachers. There are political relationships, not always explicit, that permeate teachers' relationships with the students. Such political relationships are the school context, the educational organization, and the educational public policies. As a rule, teachers do not have full control of their working conditions. Instead, they have only limited control in relation to education, the context, the curriculum, being subject to the decisions and rules established by others. For the construction and development of the teacher's professional identity to take place, an opening in the education process is necessary to encourage the development of *autonomy* (*vulnerability* and *sense of agency*). In this sense, teachers' preservice education should encourage the rise of *vulnerabilities* and the appropriation of values and norms of the profession (Lasky, 2005). Following this author, vulnerability is understand as

> a multidimensional, multifaceted emotional experience that individuals can feel in an array of contexts. It is a fluid state of being that can be influenced by the way people perceive their present situation as it interacts with their identity, beliefs, values, and sense of competence. It is a fluctuating state of being, with critical incidents acting as triggers to intensify or in other ways change a person's existing state of vulnerability. (Lasky, 2005, p. 901)

[1]We do not consider professional identity as static, this will ignore or denie its dynamic and biographical nature, at a certain moment in time. Instead, we consider professional identity as a result of an ongoing process.

Oliveira and Cyrino (2011) highlight the fact that this vulnerability is not something that weakens and paralyzes,

> […] but one that allows us to hold back for some more or less long and more or less frequent moments, our certainties and convictions, which makes us question ourselves. It is also vulnerability in the sense of exposing ourselves to others and, consequently, becoming "the target of criticism and argumentations". (Oliveira & Cyrino, 2011, p. 112)

This is the kind of vulnerability that allows prospective teachers, during the education process, to recognize their difficulties and limitations; to deal with the resulting conflicts and dilemmas related to the teaching practice and to recognize mistakes as mutual learning opportunities. To prevent this vulnerability from turning into weaknesses it is necessary to implement actions that, by means of instituted spaces, promote the reconsideration of their practices and beliefs and offer them opportunities to develop knowledge in order to overcome their difficulties and limitations. These spaces must help develop the future teacher's capacity to face any vulnerability and develop the *sense of agency* (Oliveira & Cyrino, 2011; Eteläpelto, Vähäsantanen, Hökkä, & Paloniemi, 2013).

Within the social perspective of learning, the notion of agency allows to highlight that "rather than an individual acting in isolation, the agent is viewed as an irreducible aggregate of individual (individuals) together with mediation means such as language, technology (…) or policy mandates" (Lasky, 2005, p. 902). This way, it makes sense to talk about a 'mediated agency'. It is important that during preservice mathematics teachers' education, tasks are proposed (in a disciplinary context or not) in which prospective teachers have the opportunity to reflect on and interpret the social requisites and norms of their future practice, as well as act upon different contexts in which these practices operate (regardless of the embarrassments), and, consequently, develop a sense of agency as they position themselves and develop autonomy, "by taking into account their perspectives, knowledge, and potentials" (Oliveira & Cyrino, 2011, p. 114).

For Wenger (1998), an identity is developed in social contexts through the ways their members negotiate meanings. An identity is "a way of talking about how learning changes who we are and creates personal histories of becoming in the context of our communities" (Wenger, 1998, p. 5). Learning is understood as a social practice which takes place in the context of our routine experiences with the world in a process of meaning negotiation. Meaning negotiation presupposes an interaction between two other processes: the participation process and the reification process.

Sfard and Prusak (2005) consider identity as a discursive practice, building up a collection of stories about the person and relating them to mathematics learning. The authors use identity as an analytical tool to investigate learning. Producing stories is a natural way for people to make sense of situations in which they are involved; it is a way to express or to shape their experiences (Connelly & Clandinin, 1999; Elbaz-Luwisch, 2002; Kelchtermans, 2009). In this direction, it is believed that working with narratives during pre-service mathematics teachers' education may be a fruitful option, enabling them to demonstrate their learning and their self-image, i.e., the way they see themselves as prospective teachers. Talking about themselves or explaining their reflections about themselves is an essential aspect for developing their professional identity (Graven, 2011). The continuous feedback provided by trainers and colleagues, in an interrogative, questioning, and encouraging style, encourages the development of a more problematized writing, centred on the person/author of the reflection (Oliveira & Cyrino, 2011).

The way someone understands themselve and others is analogous with what is disclosed in the texts dealing with themselve and others. This understanding undergoes a direct influence from contexts in which they perform the production and interpretation of their experiences (Larrosa, 2009). The narrative allows the author to choose the themes and terminology to be used to explain their reflections. Thus, the narratives are a source of valuable information in the preparation and data analysis process, which enable the teacher educator to understand and work with aspects of the professional identity of prospective teachers.

In this article, it was identified what has become focal points in the negotiations of meanings that occurred while prospective teachers analysed a multimedia case—featuring a mathematics teacher's practice—and that supported prospective mathematics teachers in the construction of their professional identity.

15.3 The Multimedia Case

The multimedia case used in this study illustrates the inquiry-based teaching practice of a 6th grade[2] teacher in a Brazilian public school, that unfolds a mathematical task entitled "The necklaces", which aims to develop algebraic thinking (Fig. 15.1), naming to find the rule of a sequence.

The multimedia case was constructed by Gepefopem,[3] in partnership with Hélia Oliveira, a professor at the University of Lisbon (UL) as part of the "UEL/UL Cooperation Network in the development and use of multimedia resources in mathematics teacher education", funded by CNPq[4] and the Araucaria Foundation and approved by the Ethics committee in Research of UEL.

The case exposition has a narrative structure, as it contains a sequential analysis of the lesson and its preparation, the implementation of this lesson and reflections produced by the teacher after the lesson. The motives behind the teacher's choices, as well as their doubts and difficulties, contribute to the prospective teachers regarding the case as realistic and authentic.

In the "before class" section, the selected task, the lesson plan prepared by the teacher, and the explanation of its intentions with respect to each phase of the class

[2]Brazilian 6th grade students are 11 years old.

[3]http://www.uel.br/grupo-estudo/gepefopem/index.html.

[4]Conselho Nacional de Desenvolvimento Científico e Tecnológico—CNPq (National Council for Scientific and Technological Development).

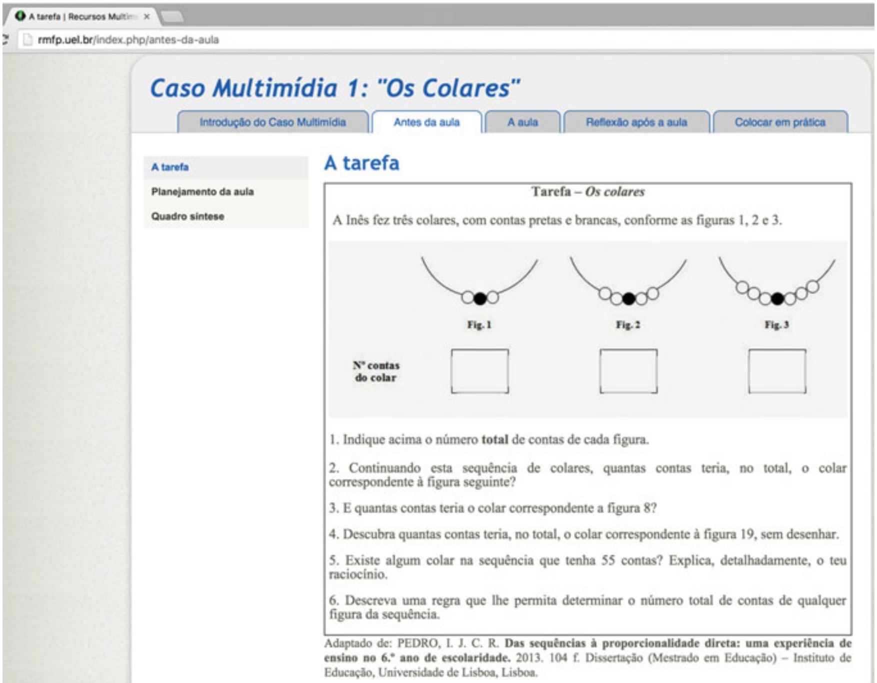

Fig. 15.1 Task "the necklaces"[5] (task translation: Inês made three necklaces, with black and white beads, as shown in Figs. 1, 2 and 3. (1) Find out the total amount of beads in each figure. (2) Without drawing, consider the pattern of this sequence of necklaces, how many beads would the necklace corresponding to Fig. 4 have? (3) How many beads would the necklace corresponding to Fig. 8 have? (4) How many beads would the necklace corresponding to Fig. 19 have? (5) Is there any necklace in this sequence that has 55 beads? (6) Describe the rule that determines the number of beads in any sequence figure

(proposition and task presentation, task performance by students, discussion of students' solutions, and systematization of learning) are presented. The featured lesson takes place during the initial part of the topic algebra. The teacher hoped that the students, through the solution and discussion of the task, could develop algebraic thinking.

In the "The classroom" section, video segments are presented for each phase of the lesson, accompanied by a transcription of teacher's and students' interventions. Questions are asked to help prospective teachers examine particular aspects of the teacher's actions which seek to promote the students learning and a suitable classroom management. In this section, the students' solutions for the task are available for analysis by prospective teachers.

In the "reflection after class" section, the prospective teachers are asked to focus on the teacher's reflections and to fill a *framework*.[5] In this *framework* the teacher's intentional actions are described for each stage of the lesson. It is also requested that the prospective teachers prepare a retrospective analysis of teacher's experience working with the multimedia case, in particular, identifying present or omitted aspects in this *framework*.

Finally, in the "in practice" section the prospective teachers is invited to elaborate a task and develop a class in the inquiry based teaching, in some context (in the school or with a group of prospective teachers).

On the left side of the multimedia case (see Fig. 15.1) are presented small text excerpts, produced by the authors of the case, termed "Synthesizing", enabling prospective teachers to systematize the main ideas the authors wish to convey with regard to the teacher's role during each phase of the lesson. Additionally, the website on which the case has been posted contains suggestions of readings on inquiry-based teaching.

15.4 Contextual Background

The work on the multimedia case formed one of the annual modules in the "Practice and Methodology of Teaching of Mathematics II" course destined to basic and secondary mathematics teachers. This course is carried out during the 4th year of an undergraduate program that prepares mathematics teachers. The multimedia case, available *online*, was analysed by twelve prospective mathematics teachers in the classroom in eight 3-h sessions, over the eight-week course. The prospective teachers worked in pairs, sharing a computer. Each group was encouraged to read and interpret the material as autonomously as possible, and they were free to seek teacher's support on details regarding the case content, the constituent material or the questions raised.

The multimedia cases offer prospective teachers an opportunity to learn about innovative practices, such as inquiry based teaching. This practice is uncommon in Brazil. Besides, it allows to develop the capacity of describing such practices, knowing the principles and strategies of classroom management and organization. In this direction, the multimedia case engages prospective teachers in the discussion about different ways of dealing with the diversity and singularity inherent in the process of teaching and learning having as starting point actual classroom situations. This knowledge about the pedagogical practice of the teacher is fundamental to the constitution of the prospective teacher's professional identity.

Each exploration session of the multimedia case generally began with a brief reference, by the teacher, on the written work of different groups produced in the previous session. Those references were intended to provide feedback for

[5] Cyrino and Teixeira (2016).

prospective teachers on how their work corresponded to the established objectives of the course. These commentaries were followed by a collective discussion on the ideas presented. Assuming a perspective of learning through negotiating meaning (Lave & Wenger, 1991), the prospective teachers had the opportunity to discuss what they had written as a product of their work at each session, first in small groups and then in the larger group. The interactions which occurred during the process of meaning negotiations were audio-recorded and then transcribed.

15.5 Methodology

This study is part of a broader Design Research project, which involves a methodological approach in which the research and development are mutually dependent (Cobb, Zhao, & Dean, 2009).

The investigation reported in this article focuses on understanding how the analysis of a multimedia case featuring one mathematics teacher's practice supported prospective mathematics teachers in the construction of their professional identity.

Concerning data collection methods, we opted to focus on the learning that the prospective teachers demonstrated through the interactions occurred during the process of meaning negotiation and written reflections that have been requested from them regarding the exploration of the multimedia resource.

The class consisted of twelve prospective mathematics teachers with little teaching experience in the classroom. Field experiences at schools occur in the 3rd and 4th year of the preservice mathematics teachers education program. These experiences had promoted the relationship between prospective teachers and teachers at the primary and secondary school level (6–17 years old) in public schools; the prospective teachers followed the work of the teachers and, at times, took on the role of teacher educator. In Brazil, the preservice mathematics teacher education occurs in undergraduate courses that enables teachers to exercise teaching from 6st to 9th grade of basic education (students aged 11–14 years), 1st to 3th grade of secondary education (students aged 15–17 years), and in youth and adult education programs.

For the data analysis, four major dimensions have been defined—the beliefs, self-image, professional knowledge and vulnerability, and sense of agency. It was considered these dimensions as constituents of the professional identity of teachers (Cyrino, 2016), which has become the focal point in the negotiation of meanings (Wenger, 1998) which took place in the examination of the multimedia case. It is presented illustrative examples of these dimensions, using excerpts from the prospective teachers' written reflections and from the collective discussion in analysis of the multimedia case.

15.6 Results

The results were organized into four sections according to the dimensions mentioned. Throughout, it was sought to characterize the contributions of the examination of the multimedia case featuring one mathematics teacher's practice for the development of prospective teachers' professional identity and for the (re)signification of their future professional practice.

15.6.1 Beliefs

Through the analysis of the multimedia case, the prospective teachers had the opportunity to clarify beliefs, sharing their repertoires. These **shared repertoires** included reification and participation aspects, namely: routines, mathematical and pedagogical concepts, classroom stories and participation in student teaching, joint discussions and impressions of the teaching and learning processes, which supported the group discussions.

For example, when prospective teachers explored the phase of the class of the "task performance by students", they shared insights about the teaching and learning of Mathematics.

> Not answering the questions directly causes students to learn and seek with the help of colleagues, in order to solve what was proposed. If the teacher answers all the students' questions, she may disrupt the students' comprehension process. In addition, the student will always wait for the teacher's response. (Written Production – 5th meeting – Dirce[6])

The shared repertoire becomes more coherent not as an activity, symbol, or specific artefact, but as part of a group practice in the process of the negotiation of meanings and therefore of learning, demonstrating the engagement of the prospective teachers in search of a joint enterprise and learning with each other.

15.6.2 Self-image

By **discussing and reflecting on their written productions** (students' solutions, answers to multimedia questions and narratives produced with reflections on the exploration of the multimedia case), the prospective teachers negotiated meanings, shared different entries and procedures, and justified their declarations. As a result, they began to value the solutions presented by the students and to understand the possibilities and limitations of the students and teachers involved in the classroom.

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[6]The names given to the prospective teachers are fictitious.

This way, prospective teachers were able to indicate whether they would or not make the same choices the teacher made and to offer suggestions for the teaching practice.

> The videos of the "task performance by students" phase are very interesting, as they teach the students by presenting their doubts and the referral given by the teacher, causing me to reflect a lot about what I would do if I were in her place. (Written Production - 8th meeting – Sibele)

Such demonstrations of recognition and solidarity legitimized the teacher's practice and allowed the emergence of challenges between them, that is, made it possible to offer provocation to reconsidering their future practices in the classroom. This process allowed the emergence of the collective recognition of difficulties and, therefore, the need for new learning (self-image). For example, one student wrote:

> The exchange of ideas with colleagues […] seems to open my mind even more, making me feel safer. I feel I still have a lot to learn. (Written Production - 6th meeting - Maya)

According to the prospective teachers, the examination of the multimedia case was an important moment mainly because it allowed the exploration of the complexities of a teaching practice that they have never experienced before, neither as a student nor in the graduation process. In this way, this was a teaching practice that they had known only from a theoretical point of view. The **future perspective**, manifested during the discussions was a temporary element of self-image, revealed the expectations of prospective teachers regarding their future at work ("how do I see myself as a teacher in the next coming years and how do I feel when I think about it?"). This component also refers explicitly to the dynamic character of self-image. It is not a fixed, static identity, but the result of an interactive process of meaning-making. Temporality permeates self-image: actions in the present are influenced by significant past experiences and by expectations about the future.

> When analysing the lessons, I was able to perceive what I should or should not do in the classroom. For example, I realized that I must pay attention to everything the students say, that it is not interesting to say directly what they should do to solve the task. I must let the students think and interpret the problem, so that they can make their decisions. (Written Production - 8th meeting - Fernanda)

It was possible to observe that beyond the satisfaction of being able to share a real classroom experience and questions relating to teaching practice (experiencing similar problems and dilemmas), this moment favoured the interaction and the harmonization of this practice with their field experience at schools, leaving them more confident to face their future professional practice (self-confidence).

> By watching the videos, we realized what took place in our field experience at schools last year. It was possible to compare. We noted aspects in which we can improve or change for this year's internship, from the preparation of lesson plans to the way we will systematize each content, considering how to choose the task, the selection criteria, and the sequencing of students' resolutions for discussion. (Written Production - 8th meeting - Edna)

Some prospective teachers stated that **(re)considering students teaching** and their future professional practice made them feel more secure (self-confident).

15.6.3 Professional Knowledge

The reflection on the teacher's practices and actions and students' productions were aspects around which the process of meaning negotiation was organized. These negotiations resulted in reifications by means of participation of prospective teachers, which consequently led to learning regarding the knowledge required for their future profession (Ball, 1990; Ball & Bass, 2002; Ball, Thames, & Phelps, 2008; Shulman, 1986; Silverman & Thompson, 2008). As an example, the following learnings have been reified:

(a) mathematical knowledge associated with algebraic thinking, such as: recognition of a pattern, Sequences (Arithmetic Progression and Geometric Progression), Function, Generalization, Rule, Variable, among others;
(b) pedagogical content knowledge, considering the teacher's need: to maintain the task's cognitive demand level; to organize questions for presenting and proposing the task; to confront different resolutions and records; to request justifications for the presented resolutions; to evaluate and explore strategies and arguments presented by students; to think about strategies to minimize students' errors and to ensure their understanding; to systematize content, concepts, and mathematical ideas from the student's resolutions; to know the curriculum and the students' cognitive development to evaluate when a certain content can be worked on; to think and propose referrals that foster communication and mathematical argumentation, among others. Some of these learnings are illustrated in the following discussion:

Trainer: In this case, does the teacher systematize any content?
Luiza: She systematizes the idea of rules, within the context of mathematics.
[…]
Maya: If the teacher was in a first year (of high school) she could have systematized a function.
Trainer: […] From these resolutions?
Maya: Yes.
Trainer But has she established any idea of a variable here? What is the meaning of the letter [she is referring to meaning of the mathematical symbols] she worked on?
Everton: She only draws attention to the fact that the letter can be replaced by 1, 2, 100… which corresponds to the figure number. I think it looks more like a letter as a generalized number than a variable.
[…]
Paulo: But when she says that the rule allows one to find the number of calculations for any figure, is it not a relation?
[…]

Everton: Yes, I think that the functional thinking is implicit, but for her to systematize the concept of function, it would take another letter to represent the amount of calculation to each figure. Then I think she would have the idea of a dependent variable.

Paulo: I see…

Everton: But, it would be possible to consider this other variable. She could have placed a *t* and said that letter is a representation for the total number of the figure…

Maya: She could go back to the table [which she constructed to record the values of *t* and *n*] as well and show that it is in the second column that the values of *t* are, and that these values will depend on the value of *n*. The (inter)dependence ratio between them. She would just need more time…

Trainer: Yes, she could hardly finish the systematization. […] But, do you think 6th year students can handle that?

Maya: I think this idea of one letter depending on the other might even be ok, but not what a function is… It would be ok for students from the 1st year (high school).

(Collective Discussion - 6th Meeting)

As the multimedia case "The necklaces" was drawn up based on a lesson developed from the *inquiry based teaching* (Elbaz–Luwisch, 2002; Oliveira & Cyrino, 2013), uncommon in Brazil, the prospective teachers regarded the teaching practice of the teacher **as an "ambitious" pedagogical practice** (Oliveira & Cyrino, 2013).

Trainer: What are the main characteristics of a task proposed in a class from the perspective of inquiry based teaching?

Maya: First, it must agree with the purpose of the lesson.

Cecília: It has to be interesting to the students.

[…]

Trainer: Somehow, the task has to instigate them, it has to awaken the will to solve it. What else?

Luiza: It ought to have several resolution possibilities. Like the "The necklaces" task.

Trainer: […] Thinking about the dynamics of the class and also about student learning, why is it important to have different resolutions?

Cecília Because of the discussion of student 'solutions, if you do not have different resolutions there will be no discussion. When they have different resolutions, each one shares their own, each has the opportunity to know the resolution of the others, thus enlarging the repertoire of how to solve the question. The student only has an idea of how to solve it, and then begins to see the others' resolutions, noting there is an easier strategy.

Trainer: As we discussed last week, this is essential for student learning, discussing different ways of solving the task, and analyzing which strategy is faster. […] What are the other characteristics of the task?

Cecília The level of complexity. […] The task has to be appropriate for the class… It cannot be easy or so difficult. In the "The Necklace" assignment, 6th graders had to think hard to solve it, and some spent a lot of time on it, but they did it.

Dirce: Yes, if this assignment were intended for smaller students, for example, students in the 2nd year of Elementary School, I do not think they would have succeeded.

Roberta: You have to take into account what they know.

(Collective discussion - 2nd Meeting)

During the discussion of the characteristics of this class, the prospective teachers had the opportunity to compare it with other perspectives of education, such as problem solving (Cai, 2003) and investigation of mathematics (Ponte, Brocardo, & Oliveira, 2003).

> It was possible to know a new perspective of teaching, a different [pedagogical practice] to take to the classroom. Something new to get out of the traditional model, in which the teacher explains on the board. At first, I had difficulty in differentiating this perspective of teaching from the methodology of problem solving. (Written Production - 3rd meeting - Amanda)

With the exploration of the multimedia case, the future teachers were able to **establish connections between observations and empirical interpretations, on the one side, different teaching perspectives studied during the program, on the other side**. When interacting with each other, prospective teachers confronted their knowledge in view of their pedagogical practices and they felt the need to resort to theoretical background to trigger new meaning negotiations. By learning new forms of reasoning (from colleagues or texts) the prospective teachers had the opportunity to interpret, challenge, respect, and validate the information mobilized in the group. This enhanced the establishment of respectful relationships between the prospective teachers which allowed them to expose their questions and concerns.

15.6.4 Vulnerability and Sense of Agency

While discussing their questions and concerns the prospective teachers experienced **vulnerability** and were challenged to call into question their certainties and convictions. This helped them develop a **sense of agency**, i.e., they had the opportunity to conciliate what must be done and what needs to be mobilized in relation to themselves (knowledge, beliefs, feelings, emotions), taking into consideration context conditions (access, resources, and support). These experiences helped them to question and get involved in meaning negotiations, i.e., they helped prospective teachers to guide their own learning process through full participation and processes of reification of their future professional identity.

When analysing the "discussion of students' solutions", one of the phases of the class, in the "The classroom" section, the prospective teachers explained some of the challenges that are posed to the teachers during this discussion.

Everton: I think the ultimate challenge here is to make the students understand the connections between different resolutions, without showing what the connections are.

Maya: Only by questioning.

Dirce: It is difficult because it is something that neither we (future teachers) nor the students are accustomed to do. Students are waiting for the teachers' resolution. So, the challenge is to make the students participate, and if they don't, we'll have no discussion. […] We must make sure that the discussion does not simply become a presentation of resolutions.

Cláudia: I think that's the greatest challenge: to make the discussion indeed become a discussion.

Trainer: And how do we do it?

Luiza: You have to relate the resolutions, seeing which one is more feasible, drawing attention to important ideas… the teacher has to ask questions, provoke the students, force them to speak… But this is all a challenge…

(Collective discussion, 7th Meeting)

Situations of vulnerability helped prospective teachers to understand that there are inherent conditions of the professional practice originated from "the relational nature of the profession which are related to the ethical character of the relation" (Oliveira & Cyrino, 2011, p. 113), once the situations may affect the efficacy of the teacher's actions and decisions, which will always be subjected to criticism and judgments. However, it is possible to balance the teachers' sense of agency and minimize the negative feelings generated during the process.

15.7 Conclusions

In the use of the multimedia case, what was discussed was not always defined by the mathematics educators. The prospective teachers had the freedom to choose, according to their previous knowledge and intentions, specific excerpts for a more detailed analysis. This was made possible by the fact that this case was on an online platform, which allowed them free access, accelerating or reviewing episodes.

The results show that by examination of the multimedia case, the prospective teachers had the opportunity to: share their repertoires, discuss their written productions, (re)consider their field experience at schools, report and discuss an "ambitious" pedagogical practice, establish connections between observations and empirical interpretations and a broader theoretical background, recognize vulnerabilities, and seek a balance in their sense of agency. These were the focal points that helped us understand how the analysis of a multimedia case featuring one mathematics teacher's practice supported prospective mathematics teachers in the construction of their professional identity.

The teacher that each student will become depends not simply on the mathematical and didactic knowledge that is developed during education, because learning implies a personal transformation. Professional identity is constructed from the individual's biography; from their various formative experiences, acting in the context of the professional practice to which they are submitted during the pre-service education process; and from the realization of field experiences. Professional identity is, therefore, influenced by several mediation systems (Lasky, 2005). The contact with the students and the professional practice of other teachers challenges prospective teachers in their outlook, and gives rise to the establishment of relations between theory and practice.

The examination of the multimedia case led to a closer relationship between the university and the school, to the extent that the prospective teachers adopted an investigative attitude regarding the teacher's pedagogical practice and the students' actions and productions. This investigative attitude triggered a process of (re)signification of their future professional practice, when they: worked collectively, prepared/organized possible materials and tasks to be developed in the classroom, shared experiences, studied and discussed mathematical concepts, worked individually or in small groups solving the task, answered questions and developed narratives in order to actively participate in their training processes, exposed their mistakes with no constraints and built a sense of agency.

There are several challenges to be faced when it is seek to develop the professional identity of prospective teachers during the education process. Among them, it is can name: the varied backgrounds of knowledge and learning of mathematics experiences from prospective teachers; the level of commitment that each one assumes in their prospective profession; the sensitivity of the teacher educator to know how to challenge the prospective teachers, in different ways, according to their different professional experiences.

Factors such as respect, trust, challenge enterprises, negotiation, dynamics and actions, valorisation of singularities, and the teachers' professional practices, cultivated by prospective teachers with the support of mathematics educators highlighted that fact that the process of examination of the multimedia case was productive and fundamental regarding the constitution of the professional identity of these prospective teachers.

It is understood that teacher education proposals that value the experiences, repertoires, and knowledge as well as enabling the assumption of learning, through negotiation of meaning, permeated by these factors, are essential to the process of mathematics teachers' education.

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Chapter 16
Exploring Pre-service Teachers' Self-perceptions of Mathematical Knowledge for Teaching

Gregory Hine

Abstract This research explored the self-perceptions of pre-service secondary mathematics teachers at one Australian university. Specifically, the researcher investigated the extent to which these teachers perceived their readiness to commence a full-time, secondary mathematics teaching position. The research relied principally on the use of a qualitative research instrument administered before and after participants undertook their major teaching practicum. Participant responses indicated varying degrees of readiness to teach secondary mathematics. An analysis of responses suggests three key findings: Pre-service teachers require further training in mathematical content; training in mathematics pedagogy; and the teaching practicum confirmed initial participant perceptions of teaching readiness.

Keywords Secondary mathematics teachers · Pre-service teacher identity Self-perceptions

16.1 Introduction

The preparation of secondary mathematics teachers for the profession is of critical importance. Scholars have argued that to teach mathematics effectively, teachers must possess considerable mathematical content knowledge (Norton, 2010; Schoenfeld & Kilpatrick, 2008). Such knowledge is concerned with the depth, breadth and connectedness of mathematical concepts and theory (Ma, 1999). At the same time, various commentators contend that pre-service teachers require training in mathematical pedagogical knowledge, which comprises a knowledge of approaches in articulating mathematical content meaningfully to students (Ball, Hill, & Bass, 2005; Harris & Jensz, 2006). The practicum experience (or teaching

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_16

internship) is also considered invaluable to teacher education programs, as this is where pre-service teachers learn first-hand the activity of teaching (Cox et al., 2013; Putnam & Borko, 2000). While there is universal agreement on the importance of the training pre-service mathematics teachers receive (Cox et al., 2013), there is considerably less agreement on what constitutes best practice for such training (Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2009; Cavanagh, 2009; Osana, Lacroix, Tucker, & Desrosiers, 2006). Building upon the work of Shulman (1999), Ball, Thames, and Phelps (2008) proposed that effective mathematics teachers require a particular type of knowledge—mathematical knowledge for teaching—which, according to a theoretical framework, encompasses both content knowledge and pedagogical knowledge. The proposed framework forms an integral component of this paper, which explores the self-perceptions of pre-service, secondary mathematics teachers completing a Graduate Diploma of Secondary Education. These self-perceptions contribute to the paucity of published literature in this genre, both in giving a 'voice' to pre-service mathematics teachers and in particular drawing attention to their professional learning needs.

16.2 Research Aims and Significance

There are two specific aims of this research project. The first is to investigate the self-perceptions of pre-service teachers enrolled in a Graduate Diploma of Secondary Education program as they prepare to teach secondary mathematics for the first time. The second aim is to explore how these pre-service teachers understand and perceive their 'readiness' to undertake such a task, based on their recent tertiary training. The significance of this research lies in the belief that the Graduate Diploma of Secondary Education course adequately prepares students for the teaching profession, and that research into this area can strengthen future efforts in preparing pre-service teachers. Collected data analyzed according to an analytical framework offered by Miles and Huberman (1994) (see Sect. 16.4.4) and in alignment with the three themes forming the theoretical framework.

16.3 Theoretical Framework

Three themes form the theoretical framework for this research, namely: Mathematical Content Knowledge (MCK), Mathematical Pedagogical Knowledge (MPK), and the domains of Mathematical Knowledge for Teaching (MKT). These themes are now articulated.

16.3.1 Mathematical Content Knowledge

There is strong agreement among researchers, scholars and policymakers alike that knowledge of mathematical content is central to its teaching (Norton, 2010). Ma (1999) asserted that teachers require a *Profound Understanding of Fundamental Mathematics*, which she described as an in-depth understanding of mathematics characterized by breadth, depth, connectedness, and thoroughness. Furthermore, Schoenfeld and Kilpatrick (2008) contended that proficient mathematics teachers have a broad and deep knowledge of the mathematics taught at school level, as well as knowing multiple methods of representation and how ideas develop from conceptual understanding. The importance of teachers' content knowledge has been recognized by the United States Department of Education (2008, p. 36): "Teachers must know in detail the mathematical content they are responsible for teaching and its connectedness to other important mathematics, both prior and beyond the level they are assigned to teach". And in Australia, Masters (2009, p. 4) noted similarly in his report on the 2008 Queensland NAPLAN performance (Ministerial Council on Education, Employment, Training and Youth (MCEETYA) that:

> Highly effective teachers have a deep understanding of the subjects they teach. These teachers have studied the content they teach in considerably greater depth than the level at which they currently teach, and they have high levels of confidence in the subjects they teach. Their deep content knowledge allows them to focus on teaching underlying methods, concepts, principles and big ideas in a subject, rather on factual and procedural knowledge alone.

In 2014, the Teacher Education Ministerial Advisory Group (TEMAG) agreed unanimously that the Australian Professional Standards for Teachers (Professional Standards) were not being effectively addressed by teacher education providers. In an Australian context, teacher education providers are universities who offer nationally accredited programs in teacher education. Consequently, the TEMAG (2014) recommended providers be required to select carefully applicants (undergraduate & postgraduate students) who possess academic skills at a requisite level. Additionally, pre-service teachers must collect evidence to demonstrate skills and capabilities for both graduation and employment, and in particular, a thorough knowledge of content they will go on to teach.

Despite the strong claims advancing the importance of content knowledge for effective teaching, ongoing debate questions the most appropriate models of teacher education including the prominence of content knowledge and how it might be best developed in teacher education programs (Cavanagh, 2009; Osana et al., 2006). For instance, Norton (2010, p. 66) underscored how there has been little research conducted on the "level of mathematics understanding that graduates typically bring to teacher preparation and the effect of teacher education courses upon that knowledge base". Investigating this issue, Miller and Davidson (2006) found that the link between teachers' background knowledge and their students' achievement is at best only mildly positive. These authors consequently suggested that prospective teachers require coursework that focuses on the foundations of a discipline rather than on studying them to greater depths (Miller & Davidson, 2006). This finding supported the earlier work of Monk (1994) who concluded generally that there appears to be no association between the number of advanced mathematics courses teachers take and how well their students achieve in mathematics. From another perspective, Ball (1990) found that prospective secondary mathematics teachers had only a cursory understanding of the concepts underlying elementary mathematics. In her research comparing Chinese and American mathematics teachers, Ma (1999) discovered that Chinese teachers—while having received less formalized instruction in mathematics than their American counterparts—had a more profound knowledge of basic mathematics and worked harder at developing effective ways to teach.

The United States and Australia, among other countries, have acted on the need to prepare mathematics teachers with sufficient content knowledge. In the United States, the Conference Board of the Mathematical Sciences (CBMS) (2001) recommended that pre-service secondary mathematics teachers complete "a 6-hour capstone course connecting their college mathematics with high school mathematics" (p. 8). With regard to the CBMS survey, a capstone course was defined as a course taken at the conclusion of a program of study for such teachers that places a primary focus on at least one of the following: (1) bridges between upper-level mathematics courses, (2) connections to high school mathematics, (3) additional exposure to mathematics content in which students may be deficient, and/or (4) experiences communicating with and about mathematics (Loe & Rezak, 2006). In Australia, as universities continue to prepare secondary mathematics teachers through either a 4-year (Bachelor of Education), 2-year (Master of Teaching) or 1-year (Graduate Diploma of Education) programs there is room to consider mandating a competency-based licensure examination focused on demonstrating a requisite level of content knowledge. While literature suggests that higher scores on licensure tests translates into increased student achievement (Sawchuk, 2011), and with all Australian graduate teachers now required to pass a competency test (LANTITE—Literacy and Numeracy Test for Initial Teacher Education Students), such an examination may well become a future certification requirement for secondary mathematics teachers. In accordance with the findings of Bonner, Ruiz, and Travis (2013), for an examination to be a useful measure of mathematics teacher competency the content would need to be aligned closely with the mathematics taught both at university and secondary level.

16.3.2 Mathematical Pedagogical Knowledge

The relationship between teachers' mathematical content knowledge and their ability to teach has been well researched and there is clear and growing evidence on the positive relationship between them (Ball et al., 2005; Harris & Jensz, 2006; Ma, 1999; Norton, 2010; Shulman, 1987, 1999). Multiple commentators have asserted that teachers require a development of pedagogical content knowledge (PCK), which has been described as an intersection of subject knowledge and pedagogical knowledge (Chick, 2012; Shulman, 1987). Shulman (1986, p. 9) defined PCK as comprising:

> The most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations - in a word, the most useful ways of representing and formulating the subject that make it comprehensible to others… Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons.

In other words, pedagogical content knowledge can be understood as knowing a variety of ways to present mathematical content and to assist students in deepening their understanding of mathematics (Ma, 1999; Shulman, 1987). The profound knowledge of mathematics and methods of representing it to students has more recently been described as mathematical knowledge for teaching (MKT) (Delaney, Ball, Hill, Schilling, & Zopf, 2008; Silvernam & Thomson, 2008). Said another way, Delaney, Ball, Hill, Schilling, and Zopf (2008) contended that in addition to the content (i.e. the 'what' of mathematics) teachers also need to know 'how' to teach mathematics. Following research into MKT, some scholars aver that implications for translating mathematical content into effective pedagogical practice are paramount in raising the profile of mathematics (Butterfield & Chinnappan, 2010). Other scholars hypothesize that MKT provides to date the promising answer to address the longstanding question of what kind of content knowledge is needed to teach mathematics well (Morris, Hiebert, & Spitzer, 2009).

Various commentators highlight the importance of combining theory and practice within teacher-education programs (Emerick, Hirsch, & Berry, 2003; Miller & Davidson, 2006; TEMAG, 2014). For example, the TEMAG (2014, p. xiii) outlined that pre-service teachers must develop a "solid understanding of teaching practices that are proven to make a difference to student learning". Furthermore, Emerick, Hirsch, and Berry (2003) argued that high quality teachers must possess both appropriate content knowledge and an ability to communicate this knowledge effectively to students. Miller and Davidson (2006, p. 58) articulated this claim, asserting that "teacher dispositions like collegiality, self-reflection, collaborative and interactive skills, and the ability to adjust personal and professional practice based on reflection are important characteristics of good teachers." Pre-service teachers often begin teacher education programs with strongly held beliefs about teaching and learning (Cavanagh & Garvey, 2012). Their own school experiences exert a powerful influence on their conceptions about the curriculum and how best to teach it, and invariably want to teach as they were taught (Scherrf & Singer, 2012). This is a critical issue in secondary mathematics education as most pre-service teachers have themselves learned mathematics in a traditional manner (Ebby, 2000). Consequently, pre-service teachers elect for a teacher-centered approach instead of opting for unfamiliar pedagogical methods (e.g. collaborative learning). This issue is exacerbated because rather than challenging pre-service teachers' prior understandings, some teacher education courses and practicum experiences have been found to reinforce them (Zeichner, 2010).

16.3.3 Domains of Mathematical Knowledge for Teaching

Building upon previous work, Shulman proposed that teaching knowledge is not a simple, uni-dimensional variable. Rather, and at the very least, teacher knowledge ought to include: content knowledge, PCK, general content knowledge, curriculum, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends, purposes and values (Shulman, 1999). From this proposal, Ball et al. (2008) analyzed extensively the work of mathematics teachers and hypothesized a conceptual framework for MKT. This framework comprises two overarching domains, Subject Matter Knowledge (SMK) and PCK, each of which consist of three subdomains. SMK is made up of the subdomains: Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Horizon Content Knowledge (HCK). PCK comprises the subdomains Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). An adapted version of the conceptual framework hypothesized by Ball et al. (2008) is offered below in Table 16.1.

For the purposes of this research, each of the six domains of mathematical knowledge for teaching is described and contextualized with an example in Table 16.2.

16.4 Methodology

16.4.1 Methods

This study was interpretive in nature, and used qualitative research methods to collect and analyze data about how pre-service, secondary teachers perceived their readiness to teach mathematics. Drawing upon the theoretical perspective of symbolic interactionism (Crotty, 1998), the researcher placed himself in the setting

Table 16.1 Domains of mathematical knowledge for teaching

Subject matter knowledge	Pedagogical content knowledge
Common content knowledge (CCK)	Knowledge of content and students (KCS)
Specialized content knowledge (SCK)	Knowledge of content and teaching (KCT)
Horizon content knowledge (HCK)	Knowledge of content and curriculum (KCC)

Adapted from Ball et al. (2008), p. 403

Table 16.2 Domains of mathematical knowledge for teaching defined

Domain	Definition	Example
Common content knowledge	The mathematical knowledge and skill used in settings other than teaching	Knowing the algorithm to multiply together two numbers
Specialized content knowledge	The mathematical knowledge and skill unique to teaching	Knowing the algorithm to multiply together two numbers connects to place value and the distributive property
Horizon content knowledge	An awareness of how mathematical topics are related over the span of mathematics included in the curriculum	Knowing how the algorithm to multiply together two numbers is related to multiplying together two polynomials
Knowledge of content and students	Knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing	Knowing that when multiplying together two numbers students may make the error of appropriately 'shifting' the terms to be added
Knowledge of content and teaching	Combines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require a mathematical knowledge of the design of instruction	Knowing what teaching strategies to employ so that students, when multiplying two numbers, learn how and why to appropriately 'shift' the terms to be added
Knowledge of content and curriculum	Represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to these programs, and the set of contradictions for the use of particular curriculum or program materials in particular circumstances	Knowing what instructional materials are available for teaching and learning multiplication of two numbers, what approach these materials take, and how effective they are

Adapted from Ball et al. (2008), pp. 389–407

of those being studied, and to consider situations from the perspective of 'the actor'. Methodologically, symbolic interactionism requires researchers to take, to the best of their ability, the standpoint of the research participants (Crotty, 1998). For this investigation, the researcher developed and used two online, qualitative surveys to collect data from participants. Participants were asked to respond to a ten-item survey prior to commencing a twelve-week teaching practicum experience. Immediately following the teaching practicum experience, the participants were asked to respond once more to the same survey. In this manner, the researcher was able to determine the extent to which any of the participants' self-perceptions of readiness had changed following their 10-week experience in the classroom. The survey items are included within this section.

16.4.2 Research Context

This research was conducted on site at the University of Notre Dame Australia (Fremantle campus). At this university, pre-service teachers completing a teaching qualification with a major (8 tertiary mathematics content units needed) or a minor (4 tertiary mathematics content units needed) in secondary mathematics education must undertake the unit EDSM04/EDSS04: Secondary Teaching Method (Mathematics). Students who undertake this unit typically undertake either a Bachelor of Education (Secondary) degree, a Master of Teaching (Secondary) degree, or a Graduate Diploma of Secondary Education. This 20-credit point unit is the only secondary mathematics pedagogy unit offered at the university, and it runs over seven weeks for a total of 21 hours contact time. During class time, pre-service teachers engage with secondary mathematical pedagogy (both for lower school and upper school students), examine key curriculum and educational policy documents, and investigate best practice approaches regarding planning, assessment, and instructional resources. The unit meets the requirements of the Australian Qualifications Framework (AQF) for secondary teachers, is nationally accredited for initial teacher education programs, and addresses a variety of Australian Institute for Teaching and School Leadership (AITSL) standards (AITSL, 2015/2011).

Over the course of this units, students are required to complete two assessment items: A Forward Planning Document (FPD) and a Reflective Practicum Workbook (RPW). When finished, the FPD comprises the plans for twelve sequential, well-detailed lessons of a chosen theme or unit of work in mathematics. In addition, one lesson from the FPD must be drafted up in considerable detail using a Lesson Plan template. Pre-service teachers complete the RPW by recording observations and reflecting upon pedagogical experiences during an *in situ* two-week Classroom Immersion period. After having recorded these observations and experiences, pre-service teachers respond to a series of reflective questions concerning mathematical content, mathematical pedagogy, classroom management strategies, and the use of instructional resources (including technology).

16.4.3 Research Participants

From the entire student population enrolled in a tertiary unit for secondary mathematics pedagogy, only those enrolled in the Graduate Diploma of Secondary Education were invited to participate in the research. Specifically, of the 20 students enrolled in this unit, 15 were purposively sampled. Of those 15 students, 10 elected to participate in the pre-practicum survey and the post-practicum survey. From the 10 participants, 6 were male (mean age = 33) and four were female (mean age = 38). Five participants had completed an undergraduate degree with a major in mathematics (at least 8 tertiary units), and five had completed an undergraduate degree with a minor in mathematics (4 tertiary units). Within the Graduate Diploma of Secondary Education, pre-service teachers with a major teaching area are trained to teach secondary students from Years 7 to 12 (typically aged 13–18 years); those with a minor teaching area are trained to teach secondary students from Years 7 to 10 (13–16 years).

16.4.4 Survey Items

Ten items comprised the pre-practicum and post-practicum surveys of this research. Survey items 1–4 were for participants to indicate specific background information regarding their age, gender, and prior tertiary studies. Survey items 5–10 directly assisted the researcher in pursuing the specific aims of the research. The research participants had been furnished with the terms *mathematical content knowledge* and *mathematical pedagogical knowledge* in the unit EDSM04/EDSS04. These items required participants to adopt a critically reflective stance towards their perceived readiness (before and after the practicum) in teaching secondary mathematics.

1. What is your gender?
2. What is your major teaching area (i.e. for Years 7–12)?
3. What is your minor teaching area (i.e. for Years 7–10)?
4. Which category below includes your age? 20–29 30–39 40–49 50–59
5. Describe your readiness to teach secondary mathematics students in terms of the mathematical content knowledge and skills you currently possess.
6. In what area(s) of mathematical content knowledge do you feel you require further training?
7. Describe your readiness to teach secondary mathematics students in terms of the mathematical pedagogical knowledge and skills you currently possess.
8. In what area(s) of mathematical pedagogical knowledge do you feel you require further training?
9. As a pre-service, secondary mathematics educator, are there any other areas you would like to receive professional training and development in?
10. Overall, describe your readiness to teach mathematics to secondary students.

16.4.5 Data Analysis Process

The researcher analyzed qualitative data collected from the ten pre-practicum and post-practicum surveys (items 5–10) according to a framework offered by Miles and Huberman (1994) which comprises the three components: data reduction, data display, and drawing and verifying conclusions. Within each of these components the researcher executed the following operations: coding, memoing, and developing propositions. According to Miles and Huberman (1994, p. 56), codes are "tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study". Codes developed by the researcher were attached to gathered data via pre-practicum and post-practicum surveys, and were selected from those data based on their meaning. In particular the codes were developed according to the domains of MKT (Ball et al., 2008), delineated in Tables 16.1 and 16.2. Memoing was then used to synthesize coded data so that they formed a recognizable cluster of information anchored in one general concept, e.g. Common Content Knowledge (CCK). Additionally, demoing helped to capture the ongoing, salient thoughts of the researcher as the coding process proceeded. Finally, the researcher generated propositions about connected sets of statements, reflected on the findings, and drew conclusions about the study.

16.5 Findings

The key findings of this research are presented briefly here, and according to the responses from survey items 5–10. Overall, participant responses concerning their self-perceived degree of readiness were geared towards MCK and MPK. The key findings—both in tabulated and discursive form—have been presented in alignment with the six domains of MKT, which are outlined in Sect. 16.3.

16.5.1 Mathematical Content Knowledge—Readiness

All of the participants (10 of 10) indicated that they were prepared to teach secondary school mathematics (to varying degrees) before the 12-week practicum experience commenced. Furthermore, all participants stated that they possessed CCK and SCK, but no participants expressed having HCK. For instance, one pre-service teacher drew attention to her CCK and SCK:

> I feel I am ready to teach Years 7 to 10, but [I] need to brush up on the content taught in Years 11 and 12 and in particular Specialist Mathematics. Often I feel I already know the content, but a refresher is needed so that the MCK is at my fingertips rather than needing to be recalled.

Another pre-service teacher stated "I feel as I am ready to teach secondary maths in terms of the MCK and skills, however [I] would feel more confident in the initial years of teaching in Years 7, 8, and 9". A third participant echoed these words, but drew attention to various perceived 'gaps' in his CCK and SCK:

> I am only now studying the highest level of mathematics that is taught in secondary schools. There are many gaps in my content knowledge, especially in topics that were covered when I was in Years 10-11 and not a very serious student, and also in tips which are not continuously emphasised throughout the school curriculum (such as project networks and some topics in statistics). However I was able to re-learn much of this in detail so that I could teach it during prac[ticum].

Four participants mentioned that there were particular 'gaps' in their MCK, most of which would restrict them to only teaching Lower School (Years 7–10) classes and not Specialist Mathematics courses.

Immediately following the 12-week practicum experience, all of the participants (10 of 10) restated their preparedness to teach secondary school mathematics. In a similar vein to the comments made before practicum, all ten participants stated they possessed both CCK and SCK; this time three participants indicated they had HCK. To illustrate, one pre-service teacher stated how she had CCK, SCK and HCK: "I have a good understanding of teaching mathematics content at all levels including setting assignments, tests and providing feedback to students and parents". Another pre-service teacher shared how he felt "confident that I have all the necessary content knowledge for Lower School and Upper School, or [I] can quickly develop it where necessary". Three participants highlighted how they still felt there were particular 'gaps' in their MCK. One female participant commented "I have appropriate content knowledge for lower secondary and basic classes in upper secondary. More work is needed to prepare for the likelihood of teaching advanced classes in upper secondary". This comment was echoed by a male participant who intimated "[My] content knowledge for Years 7–10 is good but still needs much work. I still need to review some areas and then figure out how to teach them, but generally I am pretty happy that my teaching skills can cater for this". The reported self-perceptions of pre-service teachers' MCK are shown in Table 16.3.

16.5.2 *Mathematical Content Knowledge—Further Training Needed*

Prior to the practicum experience, all participants (10 of 10) identified areas of their MCK that required further training. Of these three participants described how they felt they needed further training to consolidate their CCK, SCK and HCK. For instance, one pre-service teacher stated:

> It would be good if there were one or two units earlier in my course which cover high school mathematics in such a way that I could fill in any gaps in my knowledge about the maths that I now need to teach. I can learn this [content] as I teach but I would feel more prepared if I had some more training beforehand.

Table 16.3 Mathematical content knowledge—perceived readiness

Pre-practicum	Relative frequency	Post-practicum	Relative frequency
I feel prepared	10 of 10	I feel prepared	10 of 10
I have CCK	10 of 10	I have CCK	10 of 10
I have SCK	10 of 10	I have SCK	10 of 10
I have HCK	0 of 10	I have HCK	3 of 10

Another pre-service teacher averred “It would be useful to undertake a refresher maths course as part of P[rofessional] Development] and maybe engage with other maths teachers to look at how they teach part of the curriculum”. Six participants specified mathematical topics they believed they required further training into consolidate their SCK. These topics included: calculus, probability, matrices, proofs, and networks.

After the practicum experience, a total of six participants maintained that there were areas of their MCK that required further training. This time, participant testimony did not indicate a need for further training in CCK, and three participants stated how they felt that they required no further training in MCK. Instead, proffered responses suggested a need for participants to develop their SCK and HCK. For example, one pre-service teacher described how he required HCK: “It would be nice to see the structure of the upper school courses, and how the content of lower school courses fits in with these”. Again, participants expressed particular mathematical topics which they required further SCK and HCK, namely: geometry, differential and integral calculus, trigonometry, probability, quadratics and matrices. Participant responses indicating a need for further training in MCK are summarized in Table 16.4.

16.5.3 Mathematical Pedagogical Knowledge—Readiness

All participants (10 of 10) claimed that they were prepared to teach secondary mathematics with regard to pedagogical knowledge. Out of these participants, a variety expressed that they currently possessed any combination of KCS (5 of 10), KCT (6 of 10), or KCC (3 of 10). One pre-service teacher shared her self-perception of readiness in KCS and KCT: “I feel I am ready to teach secondary maths in terms of the mathematical pedagogical knowledge and skills; however, [I] would feel more confident in the initial few years of teaching Years 7, 8 and 9”. Another pre-service teacher described her readiness in terms of KCS and KCT:

> I feel I have all the skills (from a pedagogical perspective) because the pedagogical knowledge was dealt with so comprehensively. I particularly valued the shift to teaching mathematics in the context of real-life examples (e.g. exploratory) and using student-centered [lessons] rather than teacher [centered lessons].

Table 16.4 Mathematical content knowledge—further training needed

Pre-practicum	Relative frequency	Post-practicum	Relative frequency
I need CCK	3 of 10	I need CCK	0 of 10
I need SCK	8 of 10	I need SCK	4 of 10
I need HCK	10 of 10	I need HCK	5 of 10
I need none	0 of 10	I need none	3 of 10

Most of the participants (6 of 10) described how they felt their postgraduate training in general pedagogy (including the unit EDSM04/EDSS04) had prepared them to teach secondary mathematics. To illustrate, one pre-service teacher commented how he had developed his KSC and KCC:

> I know a lot of pedagogical theories that I will take into account when planning lessons and teaching, but in practice I can only integrate a few of them into my lessons. I do feel well prepared in terms of general pedagogical knowledge, and I'm looking forward to developing my pedagogical knowledge that is specific to teaching mathematics.

Other commonly proffered responses indicated that participants felt confident in their KCT (6 of 10). At the same time these participants predicted that their KCS and KCC would develop quickly during the practicum experience.

Following the practicum experience, all participants (10 of 10) reaffirmed they possessed sufficient pedagogical knowledge to teach secondary mathematics. Most participants stated that they felt confident in their KCS or KCT (or a combination of these domains), while only two expressed feeling confident with the KCC domain. For example, one pre-service teacher drew attention to his KCS and KCT: "My skills are relatively strong, [and I] need more repetitiveness so that they become habits. I have picked up many different things to engage with the students and motivate their learning". These comments were echoed by a female pre-service teacher, who commented "I am very ready to successfully apply what I have already learned, and my pedagogical skills are constantly growing and evolving. I will need to find a range of ways to develop my skills and to learn more about how to better teach mathematics". A third pre-service teacher emphasized how she had developed a strong sense of KCT: "I feel that I need to improve my teaching strategies. I think I can get the content knowledge across but I think I need to develop more ways to make the lessons more interactive and enjoyable". A summary of participant self-perceptions of readiness in MPK is presented in Table 16.5.

16.5.4 Mathematical Pedagogical Knowledge—Further Training Needed

Before the practicum experience, a total of seven participants mentioned that they required further training in one or more kinds of MPK. One pre-service teacher indicated she required further training to develop her KCS, KCT and KCC:

Table 16.5 Mathematical pedagogical knowledge—readiness

Pre-practicum	Relative frequency	Post-practicum	Relative frequency
I feel prepared	10 of 10	I feel prepared	10 of 10
I have KCS	5 of 10	I have KCS	7 of 10
I have KCT	6 of 10	I have KCT	7 of 10
I have KCC	3 of 10	I have KCC	2 of 10

> I have learned a lot about Bloom's Taxonomy, constructivism and other broad areas but very little about specific ways of teaching maths. Being creative and observing other teachers' own techniques are both important but I would feel more well-prepared to teach mathematics if I could learn more about specific strategies that have been found to be effective most of the time/when used properly. This sort of information helps me to better evaluate my own ideas and the teaching strategies that I observe.

Drawing attention to a perceived need to further develop his KCT and KCC, a pre-service teacher remarked "I think I require more training on how to formulate a more interesting lesson. I think if we were provided with more examples of interactive lessons across a variety of mathematical areas it would be easier to develop our own variations of interactive lessons". While other participants claimed some training was needed in any combination of the KCS, KCT and KCC domains, three participants expressed they did not require any further training.

After the practicum experience, most of the participants (7 of 10) indicated that they required further training in one or more forms of MPK. For instance, one pre-service teacher stressed how he required further training in the KCS and KCT domains, especially with regards to "how to break down the simple stuff. I am finding when teaching Year 7/8 I assume too much. Many do not know their [multiplication] times tables and so simplifying fractions becomes difficult. Techniques for scaffolding these gaps would be great". In addition, pre-service teachers offered a variety of statements concerning the MCS and MCT domains, including a need to "create a learning environment in which every student is engaged", "learn a few different teaching style ideas, but nothing too major", and "watch other teachers teach maths and sharing notes with them". Three participants reported that they did not require any further training concerning MPK. A summary of participants' self-perceptions regarding additional MPK is tabulated in Table 16.6.

16.5.5 Further Professional Development

All ten participants identified at least one area of professional development that they needed further training in before commencing the practicum experience. Common responses included the use of technology (3), increased MCK (3), the use of resources (2), and training in drafting assessment items (2). Immediately following the practicum experience, all participants were again able to identify at least one

Table 16.6 Mathematical pedagogical knowledge—further training needed

Pre-practicum	Relative frequency	Post-practicum	Relative frequency
I need KCS	4 of 10	I need KCS	5 of 10
I need KCT	5 of 10	I need KCT	5 of 10
I need KCC	7 of 10	I need KCC	2 of 10
I need none	3 of 10	I need none	3 of 10

Table 16.7 Further professional development

Pre-practicum	Relative frequency	Post-practicum	Relative frequency
Graphics calculators	3 of 10	Graphics calculators	5 of 10
MCK	3 of 10	MCK	4 of 10
Resources	2 of 10	Classroom management	2 of 10
Assessments	2 of 10	Special needs education	2 of 10

area of professional development that they needed further training in before commencing a full-time teaching position. These responses included the use of technology (5), increased MCK (4), improved classroom management (2), and training in special needs education (2). A summary of these responses is offered in Table 16.7.

16.5.6 Overall Readiness to Teach Mathematics

All participants (10 of 10) reported that they felt ready to teach secondary mathematics prior to the practicum experience. From the proffered testimony, and despite their avowed readiness to teach, six participants stated they required further training in the SCK and HCK domains—two of these also averred they needed to work on their KCS and KCT. For example, one pre-service teacher stated "I would say I am competent in teaching mathematics to Years 7–10 but I think I need a lot of work on teaching strategies". Two participants expressed they felt ready and did not need any further training. Following the practicum, all participants (10 of 10) re-affirmed their readiness to teach secondary mathematics. In a similar manner to pre-practicum responses, participants indicated a need to upskill in the domains of SCK (3 of 10), HCK (4 of 10), KCT (3 of 10), and KCC (3 of 10). In particular, three pre-service teachers reported feeling ready to teach Lower School classes effectively, but both their MCK and MPK in Upper School courses required attention. Four participants shared feeling ready to teach all year levels and did not need any further training. Participant responses regarding an overall readiness to teach mathematics are summarized in Table 16.8.

Table 16.8 Overall readiness to teach mathematics

Pre-practicum	Relative frequency	Post-practicum	Relative frequency
I feel prepared	10 of 10	I feel prepared	10 of 10
I need SCK	6 of 10	I need SCK	3 of 10
I need HCK	6 of 10	I need HCK	4 of 10
I need KCS	2 of 10	I need KCT	3 of 10
I need KCT	2 of 10	I need KCC	3 of 10

16.6 Discussion

The ten surveyed participants were able to articulate their self-perceptions of 'readiness' before and after the 12-week teaching practicum. Following the conceptual framework which underpinned the study itself (Ball et al., 2008), an analysis of these self-perceptions revealed three key findings with regards to how ready participants felt to undertake a full-time teaching position, and what areas of professional development in which they required further training. These key findings included (i) a clearly expressed need for pre-service teachers to develop their MCK, (ii) a clearly expressed need for pre-service teachers to develop their MPK, and (iii) the practicum was an instructive experience for pre-service teachers' regarding their perceived readiness to teach secondary mathematics. The findings will now be discussed.

Despite all ten participants acknowledging feeling 'ready' to teach secondary mathematics before and after the practicum experience, the majority asserted a need to develop their MCK further. For instance, when asked about the MCK needed before practicum commenced all participants were able to identify any combination of CCK, SCK or HCK. Following the practicum, seven participants could still identify specific areas of MCK which they required further training. The HCK domain of MCK was acknowledged as that which participants required the most professional development, (10 before practicum; 5 after practicum). This finding supports claims made in current literature that MCK is central to its teaching (Norton, 2010), and that proficient mathematics teachers require a broad and deep knowledge of the mathematics taught at school level (Ma, 1999; Masters, 2009; Shoenfeld & Kilpatrick, 2008). In light of the six domains of MKT (Ball et al., 2008), the tendency for pre-service teachers to self-perceive CCK and SCK as areas of need suggests they recognize the importance of focusing on the foundations of mathematics instead of studying this discipline to considerable depth (Miller & Davidson, 2006). To amplify, participants tended to outline their MCK first in terms of operationalizing the content with rules, algorithms and properties (i.e. CCK) before highlighting a need to learn how various ideas are connected topically (i.e. SCK). The HCK domain was cited as being most in need of further development; participants tended to identify a lack of Upper School content (including Specialist Mathematics) and a lack of teaching experience as barriers to possessing this knowledge. Whilst a lack of teaching experience could be an expected response for those undertaking a Graduate Diploma of Education (Secondary), a need to consolidate MCK taught within secondary schools is surprising—especially given the undergraduate background of candidates.

In an identical manner to the MCK responses, all participants insisted they were ready to teach secondary mathematics in terms of their MPK. At the same time as many as seven (before practicum) and five (post-practicum) participants highlighted domains of MPK which required professional development. In particular, seven participants asserted they possessed KCS and KCT before practicum; following the practicum, five acknowledged needing further training in these same domains. And while there was an expressed need for KCC before practicum (7 of 10), this number decreased significantly to two post-practicum. Of those who stated further training in MPK was needed, emphasis was placed on having repeated opportunities to teach mathematical concepts and to try different teaching approaches. To a lesser degree, participants identified that their future professional development could focus on techniques to better engage Lower School students and students at educational risk. Collective testimony aligns with research conducted into MPK, and supports the previous work of Hine (2015) who investigated the MCK of aspirant middle school teachers. First, such testimony suggests the importance of both MPK in mathematics education (Delaney et al., 2008, Silvernam & Thomson, 2008), and of having opportunities to practice MPK approaches during teacher education (Emerick et al., 2003; Miller & Davidson, 2006). Second, and while no participants drew attention to how they themselves were taught mathematics at secondary school, several participants expressed they wished to acquire KCC so they could deeply influence student learning (Emerick et al., 2003; TEMAG, 2014).

The practicum experience confirmed most of the participants' pre-practicum self-perceptions of their readiness to teach secondary mathematics. For example, there appeared to be little variation in the pre-practicum and post-practicum responses offered (see Tables 16.3, 16.4, 16.5 and 16.6). While this research did not seek to examine the practicum experience in any way, participant testimony suggested that the 12-week, school-based experience was invaluable preparation for their careers as secondary mathematics teachers. Analyzed testimony revealed that the practicum experience was an opportunity for pre-service teachers to learn from a more experienced teacher (i.e. a mentor teacher) as they modelled MCK and MPK in secondary mathematics classrooms. Equally, the practicum acted as the first teaching experience for the participants as they discerned their own teaching style, identified their preferred method for creating a mathematical learning environment, engaged with mathematical content, and experimented with various pedagogical approaches. In addition, participants were able to identify and confirm various domains of MCK and MPK required for their own professional development. As such the participants' experiences have underscored the importance of combining theory and practice within teacher-education programs (Emerick et al., 2003; Miller & Davidson, 2006; TEMAG, 2014).

16.7 Conclusion

This research project investigated the self-perceptions of pre-service teachers enrolled in a Graduate Diploma of Secondary Education program as they prepared to teach secondary mathematics for the first time. Concurrently, the researcher explored how these pre-service teachers understood and perceived their readiness to undertake such a task, based on their recent tertiary training. The shared self-perceptions were analyzed according to a framework developed from the six domains of mathematical knowledge for teaching (Ball et al., 2008). Despite all participants (10 of 10) asserting feeling ready to teach mathematical content, a majority stressed that they required additional training in mathematical content knowledge, particularly in Upper School content. These self-perceptions remained unchanged before and after participants engaged in their practicum teaching experience. Similarly, all participants stated they felt pedagogically ready to teach mathematics; however, a significant number articulated a need to further develop their mathematical pedagogical knowledge. In light of these findings, it is therefore incumbent on pre-service mathematics teachers and mathematics teacher educators alike to remain cognizant that assertions of 'readiness' to teach secondary mathematics are complex, multi-faceted constructs. An analysis of participant testimony illustrates that the multi-faceted nature of these assertions comprises a general need for pre-service teachers to develop their professional skills and knowledge further in mathematical content, pedagogy, and curriculum.

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Chapter 17
Exploring the Initial Convictions and Mindset of Prospective Mathematics Teachers Towards Modelling

Rina Durandt and Gerrie J. Jacobs

Abstract Modelling, as a problem solving strategy forms part of the South African mathematics secondary school curriculum since 2011. In many instances, mathematics teachers are incapacitated to teach modelling effectively, although literature also reveals the opposite. This study reports on the convictions (beliefs) and mindset (attitude) of prospective mathematics teachers, based on their initial engagement with a modelling task. Significant attitudinal differences emerged between the two genders, and between participants with diverse mathematics competency levels. Participants enjoyed and valued the modelling task, but seemed to lack confidence and motivation in pursuing it in future. In striving to develop the modelling and mathematical proficiency of prospective teachers, a well-planned programme that gradually nurtures participants' modelling mindsets and confidence is recommended.

Keywords Mathematics teacher education · Prospective secondary mathematics teachers · Prospective mathematics teachers' mindset towards modelling · Mathematics teachers' competencies and modelling
Mathematics teachers' gender and modelling

17.1 Background Context and Purpose

To teach mathematics effectively a particular 'knowledge-in-action' to unpack and expand mathematical ideas is usually required. A framework developed from Shulman's (1986) categories by Ball, Thames, and Phelps (2008) outlines expected knowledge components of effective mathematics teachers. The framework firstly

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_17

incorporates subject matter knowledge, which contains how topics in the subject are connected. Pedagogy content knowledge, which includes an understanding of how students grasp mathematical content, what skillful mathematics teaching means and what the mathematics curriculum entails, comprises an interconnected second component. A profound understanding of both components is essential to effectively teach mathematics as research reports from Baumert et al. (2010, p. 138) underlined,

> One of the major findings of qualitative studies on mathematics instruction is that the repertoire of teaching strategies and the pool of alternative mathematical representations and explanations available to teachers in the classroom are largely dependent of the breadth and depth of their conceptual understanding of the subject.

Teachers are expected to present content and to inspire students to discover mathematical relations, but they also have to understand and anticipate students' reasoning and potential misconceptions (Turnuklu & Yesildere, 2007). Learning is constructed from the classroom environment and student activities, and teachers' beliefs, knowledge, judgements and thoughts therefore influence decision making in the classroom. Teacher education programmes have a huge role to play in shaping prospective teachers' convictions (beliefs) and mindset (attitudes) in an appropriate manner and in providing a functional foundation for their mathematical pedagogical content knowledge.

Olanoff, Lo and Tobias (2014) argued for a stronger focus on regular and non-standard approaches to problem solving in prospective teacher education programmes. Authentic problem solving (in relation to mathematical modelling) is progressively used to great effect in enhancing students' mathematical competencies and mathematics teachers' pedagogy and subject content knowledge (Buchholtz & Mesrogli, 2013). From a South African perspective, Adler and Davis (2006, p. 272) reason that the ways in which teachers interact with mathematics, when teaching, have significant implications for mathematics teacher education and raises the question "whether the mathematical education of teachers can and does provide opportunities to learn these ways of knowing and using mathematics"? Their research underscores two critical elements required for teaching mathematics (in contrast with the elements that mathematicians require), namely "unpacking" (an interpretation of mathematical results and processes) and "decompression" (an understanding and clarification as the student engages with specific mathematical thinking and reasoning). Mathematical modelling, the process of generating mathematical representations in attempting to solve real-life problems (Blum et al., 2002), generates authentic learning experiences and is considered as pedagogical strategy in teacher education to develop these critical elements. The educational need for adopting mathematical modelling is either to teach modelling competencies or mathematical content (Stillman, Galbraith, Brown, & Edwards, 2007). Soon and Cheng (2013) argued that teachers may not be able to appreciate the benefits and importance of developing their students' mathematical modelling competencies if they themselves were not adequately exposed to such tasks and activities. The ideal is for prospective teachers to eventually 'model' (use) modelling in their classrooms.

Former research by Ng (2013), conducted with pre-service and in-service teachers in Singapore, revealed challenges in fostering a positive climate towards modelling. The authors also underlined that work is needed to help change the mindset of in-service teachers to adopt modelling as a pedagogical strategy. The purpose of this study is to explore the convictions (beliefs) and mindset (attitude) of a group of prospective mathematics teachers at a South African public university, based on their first mathematical modelling experience. This investigation forms part of a broader design-based research (DBR) project, which strives to develop a set of guidelines, aimed at the effective integration of modelling into the formal education programme of prospective secondary school mathematics teachers at a South African university. The intention of this investigation[1] is to find relatively acceptable answers to two research questions that will inform, to some extent, the next iteration in the broader DBR project. The research questions are: (1) what are the initial convictions and mindset of prospective mathematics teachers towards modelling and (2) are there differences between the convictions and mindset of the two gender groups, and between participants' performance in mathematics?

17.2 Theoretical Perspectives

17.2.1 Theoretical Elements Concerning to Teachers' Convictions and Mindset

The authors are of the opinion that the education of prospective mathematics teachers, especially in the current South African school context, has a vital influence on their initial practices, convictions (beliefs), mindset (attitudes) and early effectiveness as secondary school teachers (Adler & Davis, 2006). Aligned with the abovementioned assumption, the *first* element of the theoretical framework that underlies this inquiry is the "Learning to Teach Secondary Mathematics" (LTSM) framework (Peressini, Borko, Romagnano, Knuth, & Willis, 2004). This framework suggests that the particular competencies (knowledge and skills) a prospective teacher acquires are primarily influenced by the specific context (or teaching situation) in which it happens. Furthermore, teachers' knowledge, convictions and mindset interact with teaching-learning situations. This suggests that mathematics teacher education is "usefully understood as a process of increasing participation in the practice of teaching, and through this participation, a process of becoming knowledgeable in and about teaching" (Adler, 2000, p. 37). In this enquiry, we tried to ensure the claims of the LTSM framework by actively involving prospective

[1]This DBR study is conducted over three phases and the investigation forms part of phase 2, iteration 1. Each phase has a unique focus and the analysis contribute towards testing, improving and understanding the design.

teachers in the modelling task and exposing them to mathematical content knowledge—adopting mathematical modelling as a pedagogical strategy in teaching mathematics.

The *second* element of the theoretical framework that underlies this research is the view of Schackow (2005) in respect of mathematics teachers' convictions and mindset. Schackow (2005) describes convictions (beliefs) as the individual ways in which teachers grasp their role(s). Teachers' convictions on how mathematical themes should be taught are deeply rooted, usually related to their own experiences as mathematics learners (especially during their formal education) and are difficult to change. These convictions are primarily rational in nature, and they play an important role in the development of their (and their students') mindsets. Dweck (2006) and Boaler (2016) differentiated between two different mindsets. On the one hand, a 'growth' mindset, cultivate a belief in the minds of prospective teachers that they can improve their abilities through continuous practice and effort. On the other hand, with a 'fixed' mindset prospective teachers belief that their abilities are unchangeable and predetermined. Research reports confirm people with a growth mindset believe in the development of intelligence and outperform those with a fixed mindset (Boaler, 2016) as they focus on "learning, believed in effort, and were resilient in the face of setbacks" (Dweck, 2010, pp. 26–27). On the contrary, those with a fixed mind-set are too concerned about setting the 'correct' impression and "became discouraged or defensive in the face of setbacks because they believed that setback reflected limitations in their intelligence" (Dweck, 2010, pp. 26–27). In this inquiry, we attempted to ensure the viewpoint of Schackow (2005) by exposing prospective teachers to mathematical modelling during their formal education and cultivating a belief in their minds in the hope that they will improve their abilities in the future through continuous practice and effort.

17.2.2 Mathematical Modelling and Teaching

This inquiry relates to the discussion about what mathematical modelling brings to teaching and, in particular, about prospective mathematics teachers' convictions and mindset towards model-eliciting tasks. In this direction, we will draw on the following notions developed by the literature on mathematical modelling: levels of modelling tasks (Tan & Ang, 2012); design guidelines for a mathematical modelling learning experience (Tan & Ang, 2012); evaluation criteria for mathematical models (Meyer, 2012), originally described by Meyer in 1984; and the modelling cycle according to curriculum planners Balakrishnan, Yen, and Goh (2010). Next, we present these theoretical notions.

Tan and Ang (2012) discuss three different levels of mathematical modelling tasks. The focus of Level 1-tasks is on students obtaining mathematical modelling skills that may perhaps be used in forthcoming tasks. Traditional problem solving (or the typical textbook problem) fits the description of such a task as these problems are carefully defined and require specific mathematical procedures.

The emphasis with Level 2-tasks is guiding students to apply known models to new situations, ensuring to some extent, meaningful engagement in the modelling process. Such tasks usually have a slight vagueness. A Level 3-task is the most authentic open-ended type. At this level, students should not only review task constraints, but also search for possible approaches to solve the problem—they should build new models. According to Ng (2013), a decent modelling task should consist of a real-world context, open-endedness, unstructuredness and complexity to provide a platform to experience the complete modelling cycle. In this inquiry prospective mathematics teachers were exposed to a Level 3-task, *World Cup Rugby 2015* (view Sect. 17.3.2).

Tan and Ang (2012) suggested a framework, to guide the design of a mathematical modelling learning experience. Such a framework involves addressing the following five questions:

- WHICH level of learning experience?
- WHAT is the skill or competency?
- WHERE is the mathematics?
- HOW to solve the problem or model?
- WHY is this experience a success?

The first two questions lead to the formulation of clear learning goals for specific modelling tasks. The third question asks from the designer to be conscious of the mathematics underlining the activity and the required mathematical competency needed by students to complete the task. The forth question determines if the task is a good fit for the learning goals and guide teachers in their facilitation of students' learning. The last question prompts teachers to monitor the modelling process and highlights that "self-monitoring can increase teachers' sense of competence and control and in turn, their motivation to carry out such modelling tasks" (Tan & Ang, 2012, p. 715). These five questions informed the planning of the model-eliciting activity developed in this inquiry.

Meyer (2012) highlighted the relevance of carrying out an evaluation of the *mathematical model* at the end of the modelling cycle as it makes a vital difference in simulating and improving real life decision-making. In this direction, in the report "Guidelines for Assessment and Instruction in Mathematical Modeling Education" (GAIMME), it is explained, "the value of a model is determined by its ability to provide reasonable solutions to a given problem" (COMAP-SHIAM, 2016, p. 197) and it is advisable to examine the model. Meyer (2012) discussed six evaluation criteria for mathematical models. These include the accuracy and precision of the answer, the realism (based on correct assumptions) and robustness of the model, its applicability to other situations and the usefulness of its conclusions. In this inquiry, the authors used the criteria discussed by Meyer (2012) to examine the mathematical models presented by prospective teachers (view Sect. 17.4.2).

A variety of mathematical modelling cycles exist in the literature (Doerr, Ärlebäck, & Misfeldt, 2017). In this inquire we will adopt the modelling cycle proposed by Balakrishnan et al. (2010). Such cycle involves: "*mathematisation*", representing the proses to present the real-world problem mathematically using a model; "*working with mathematics*", representing decision-making using appropriate mathematics to solve the problem; "*interpretation*", representing the sense-making of the solution in terms of its relevance and appropriateness to the real-world situation; and "*reflection*", examining the assumptions made and subsequent limitations of the suggested solution. The four sequential phases included in this cycle represented the activities prospective teachers engaged with during the modelling process carried out during this inquire.

Research reports indicate students often experience difficulties moving between these phases and underline that it is important to develop students' competencies in each of them (Stillman et al., 2007). For example, Buchholtz (2017, p. 49) explains how an "out-of-school" activity (in a European setting) such as a mathematical city walk can be used to develop competences in mathematising. In this inquiry, we investigated how prospective teachers moved through the modelling cycle (view Sect. 17.4.2). This analysis fulfilled a vital role in informing the next phase of the broader DBR study.

17.2.3 Connecting Students' Attitude, Mindset, Performance and Teacher Intervention

This section is based on two key assumptions; (1) teaching strategies can send a growth-mindset message to students, and (2) prospective teachers' attitudes toward modelling are influenced by their experiences of the lecturer's approach and teaching throughout their formal pre-service education.

Attitudes form a central part of a person's identity. The affective domain of learning typically features three dimensions: *emotions*, *attitudes* and *beliefs* (Papageorgiou, 2009). Attitudes are seen by Philipp (2007, p. 259) as

> Manners of acting, feeling, or thinking that show one's disposition or opinion. Attitudes change more slowly than emotions, but they change more quickly than beliefs. Attitudes, like emotions, may involve positive or negative feelings, and they are felt with less intensity than emotions. Attitudes are more cognitive than emotions but less cognitive than beliefs.

Attitude and mindset are closely related, and in some case viewed as synonyms. According to Fang, Kang, and Liu (2004, p. 298) mindset is "something that occurs in a person's head" that can influence a person's attitudes and actions. Many prospective teachers generally have either a positive or a negative attitude towards mathematics. Ma and Wilkins (2002) put the vital role of teacher attitudes into perspective, by stating that students who believe that teachers have high expectations of them tend to have a more positive attitude towards mathematics. Blum et al. (2002, p. 161) wrote, "beliefs, attitudes and emotions play important roles in the development of critical and creative senses in mathematics", and these are essential to achieve modelling aims. The quality of mathematics teaching and the nature of teacher attitudes have a pertinent influence on students' attitudes towards mathematics and eventually also on their achievement. Dweck (2006) cautioned against the messages teachers communicate to students—by often praising their ability and intelligence teachers can harm their motivation and performance as they become more afraid to make mistakes. On the contrary, teachers rather have to show interest in students' development and growth. It seems that positive teacher attitudes towards mathematics likewise stimulate favourable attitudes in students, but so are negative attitudes in students generally caused by inappropriate teaching practices and undesirable teacher attitudes (Henderson & Rodrigues, 2008; Yara 2009).

Anhalt and Cortez (2016) claim it is vital that prospective teachers develop a good understanding regarding mathematical modelling throughout their formal education. They examined the development of 11 prospective teachers' (from an American public university) understanding of the topic through the implementation of a modelling course in a teacher preparation programme. Their study revealed that although most participants initially had misconceived ideas about mathematical modelling, they developed the correct understanding of the modelling process throughout the course. At the end the prospective teachers were able to translate the modelling cycle into practice in the context of a Level 3-task and understood the strong relation between modelling activities and promoting mathematical practices. Kuntze, Siller, and Vogl (2013) investigated the self-perceptions of modelling-specific pedagogy content knowledge of Austrian in-service and pre-service teachers and found it were not positive. They recommended intensified professional development support for both teacher groups. The research findings from Anhalt and Cortez (2016), Kuntze et al. (2013) and others reveal the experiences of prospective teachers regarding modelling during their formal education indeed shape their convictions and mindset.

17.3 Research Design and Method

17.3.1 Research Paradigm and Design, Sampling and Participants

The study's research paradigm relates to an attempt to measure prospective mathematics teachers' modelling competencies as well as their mindset in respect of a model-eliciting task. The study was thus conducted from a *pragmatic* worldview (Creswell, 2013) as the researchers are concerned with finding a solution to the problem. According to Creswell (2013) pragmatic knowledge claims are focused on understanding the problem (refer to Sect. 17.1) and therefore pragmatists apply a variation of approaches to inform their understanding. This worldview forms the philosophical underpinning for the mixed methods design of this investigation, and for the broader project. Such a research design (Teddlie & Tashakkori, 2009) allowed the researchers to *explore* the mindset of prospective mathematics teachers towards modelling and the differences between gender and achievement groups, to *explain* their thinking and planning strategies and to *describe* some valid principles to incorporate in teacher education programmes. For this study, a pre-designed worksheet/document and an open-ended questionnaire were administered to collect the qualitative data and a survey to collect the quantitative data.

The 49 participants were secondary school (Grade 10–12) prospective mathematics teachers at the University of Johannesburg in 2015 with very little school experience. They were studying full-time and the main elements of their demographics are *male* (63%), *black* (almost 80%), *indigenous language* speaking (also almost 80%), *22 years or younger* (57%), no *prior exposure* to model-eliciting challenges (100%) and having scored *70% or more* in Grade 12 for mathematics (63%).[2] Participants were exposed to a mathematical modelling activity during the last week of the first semester, which is during the last week of May 2015. Proportional stratified sampling was used to randomly assigning participants to groups of four or five, in such a way that each group had at least a high(er), a moderate and a low(er) achiever. The group selection was motivated by various reasons; to form comparable groups due to the complexity of the modelling task, the collaborative nature of problem-solving activities in mathematics and in particular modeling tasks, the lack of experience of participants in modelling tasks, and to create an inviting climate for dialogue and reflection activities which are vital in the development of modelling competencies (Stillman et al., 2007).

17.3.2 The Modelling Task/Activity

The activity was based on both the design guidelines of Tan and Ang (2012) and a pilot study conducted by the researchers in the previous year (Durandt & Jacobs, 2017) involving another mathematical modelling activity and group of students. The session lasted for almost two hours during a scheduled time slot. A 20 min presentation (by one of the researchers fostering a positive attitude towards modelling) served as introduction on the purpose and nature of the research explaining ethical measures, what modelling entails, its pedagogical value, and the phases of a typical modelling cycle. The model-eliciting activity, labelled *World Cup Rugby 2015* (adapted from a traditional textbook problem) relates to participants' mathematics curriculum (applications of integration). It is an unstructured, open-ended, complex and incomplete 'real world' modelling task [Level 3-task according to the

[2]In South Africa, different Grade 12 qualifications are available, e.g. the National Curriculum Statement (known as the Curriculum and Assessment Policy (CAPS) since 2011), mostly offered by public schools and the Grade 12 examination of the Independent Examinations Board (the IEB) mostly offered by private/independent schools and colleges. The two curricula differ in respect of scope and depth, though not substantially, while the exit level outcomes of both in respect of the subject Mathematics are compatible and thus accepted by both public and private South African universities.

World Cup Rugby 2015 Task

The 2015 Rugby World Cup Tournament (RWT) will be hosted by England from 18 September to 31 October 2015 and the final will be played at the well-known Twickenham Stadium.

The official rugby ball is made of the following features: three-ply backing material for good shape retention, standard grip, grippy rubber surface, hand stitched, and synthetic latex bladder for excellent air retention. The design features the official Tournament social media hashtag, and ball comes inflated for safeguarding a good smell and shape (approximate weight 460g & pressure 10PSI). Tournament rules stipulate each of the twenty teams should supply a sufficient number of the official rugby balls for team practices and preparation. Tournament officials are responsible to supply the match ball only. The South African Rugby Union (SARU) would like to receive a recommendation on the maximum number of official rugby balls that can be transported to England for the RWT 2015 on a South African Airways (SAA) flight. Provide the SARU with a plan on how they can perform relevant calculations. You need to explain the method you used as the SARU would like to apply this method to other areas.

Data collected from the SARU stipulates the official match ball is oval or egg shaped and made from four different panels. For a size 5 rugby ball (relevant for full sized rugby), the length should be approximately 30cm, the length circumference approximately 77cm and the width circumference roughly 62cm. In flight safety regulations by SAA stipulates the weight per new ball (460g) and optimal ball pressure of approximately 10PSI can be allowed (standard ball size). Luggage capacity is restricted to a maximum of 1 cubic meter per flight.

Fig. 17.1 The 'World Cup Rugby 2015' modelling task

classification by Tan and Ang (2012)]. The activity, displayed in Fig. 17.1, contained information regarding specific features of the official rugby ball, tournament rules and regulations, in-flight safety regulations, and specific requirements from the South African Rugby Union (SARU).

Participants were expected to pave the way through the modelling cycle of Balakrishnan et al. (2010) (*mathematisation*, *working with mathematics*, *interpretation*, and *reflection*) with the purpose of making recommendations to SARU on the maximum number of official inflated rugby balls that can be transported via a crate to England for the World Rugby Tournament in September 2015.

Their recommendations must also allow SARU to apply the design in other situations. The ten prospective teacher groups were required to report on the strategies and methods they employed in order to come up with possible solutions and also to critique their suggested solutions. The researchers carefully monitored the experiment and group interactions and each group recorded their strategies, processes and suggested solutions on a predesigned worksheet.

17.3.3 Quantitative Design, Data Collection and Analysis Procedures

A questionnaire was used to collect information from the participants the day after their initial exposure. It contained demographical items, an item on their Grade 12 performance in mathematics, as well as the items of a recognized instrument, used for gaining student teacher attitudes towards mathematics as subject. The original Attitudes towards Mathematics Inventory (ATMI) (Schackow, 2005), was adjusted towards mathematical modelling, keeping its items and dimensions intact. The new Attitudes towards Mathematical Modelling Inventory (ATMMI) still consists of four dimensions, namely *value* (whether mathematical modelling knowledge and skills are worthwhile and necessary, including 10 items), *enjoyment* (whether problem-solving and model-eliciting activities are enjoyable, including 10 items), *self-confidence* (expectations about mastering mathematical modelling, including 15 items) and *motivation* (the desire to learn more about mathematical modelling and to teach it, including 5 items). Each of the 40 items used a Likert-type response scale, ranging from 1 (*Strongly disagree*) to 5 (*Strongly agree*).

Sweeting (2011, pp. 53–54) classifies teacher attitudes towards mathematics as a subject on five levels, which is labeled as "strongly negative, negative, neutral, positive and strongly positive". Using this classification, positive scores on the *enjoyment* and the *value* dimensions (maximum 50) would be 41 or more. Positive scores on the *self-confidence* dimension (maximum 75) would be 61 or more and on the *motivation* dimension (maximum 25) 21 or more. A positive *ATMMI total* (incorporating all four dimensions—maximum 200) would thus be 161 or above.

Analyses of the data, including normality testing, reliability measures and testing for attitudinal differences between groups of participants, were performed via the Statistical Package for the Social Sciences (SPSS, version 23). Cronbach's alpha coefficients were calculated in respect of the four dimensions, as well as participants' total ATMMI scores. Table 17.1 portrays the coefficients (all of them > than .8), implying that all items in each dimension indeed measure the same construct, confirming the questionnaire's reliability (internal consistency).

Table 17.1 Reliability (internal consistency) measures of the attitudes towards mathematical modelling inventory (ATMMI)

ATMMI dimension	Cronbach's alpha coefficient
Enjoyment (including 10 items)	.891
Value (including 10 items)	.857
Self-confidence (including 15 items)	.925
Motivation (including 5 items)	.872
ATMMI total (including 40 items)	.893

17.3.4 Qualitative Design, Data Collection and Analysis Procedures

One predesigned worksheet, containing the different phases of the modelling cycle (Balakrishnan et al., 2010), was completed by each group during the modelling session and submitted at the end of the session. These working documents reflected the work from each group in relation to the task proposed. Figure 17.2 demonstrates how the participants in Group 1 represented the real-world problem mathematically. All participants were enrolled in a course in the applications of calculus and could therefore relate to the method of calculating volumes of solids of revolution. However, participants were not necessarily familiar with calculating the volume of an ellipse (consistent with the shape of a rugby ball). Group 1 selected a function familiar to them to construct a solid of revolution. They identified the relevant information from the task and used the familiar root function $f(x) = \sqrt{x}$ to rotate about the x-axis. This solid of revolution then represented half of the rugby ball. They continued setting up an integral, working between the limits $x = 0$ and $x = 15$, to calculate the volume in cubic centimeters. If the result is multiplied by two they could find an accurate estimation of the volume of a match size rugby ball. In addition, they calculated the radius of the ellipse using the given information on the circumference of the ball. This method allowed Group 1 to mathematise the modelling task and proceeded with phase two of the modelling cycle. Not all groups followed the same approach, although most groups tried to use well-known mathematical content from their current mathematics course or previously studied. These working documents, containing the mathematical work of groups, their proposed mathematical model and solution to the real-life problem, were analyzed according to the modelling cycle described by Balakrishnan et al. (2010), and model evaluation criteria from Meyer (2012).

The questionnaire, including four open-ended questions, was administered the following day. These questions requested individual participant's feedback on four aspects, namely: (1) their perceptions of their group's biggest challenges during the modelling activity, (2) their views on the open-ended nature of the modelling task, (3) their post-activity mindset towards mathematical modelling, and (4) concrete suggestions on how the university can support a prospective teacher to become an

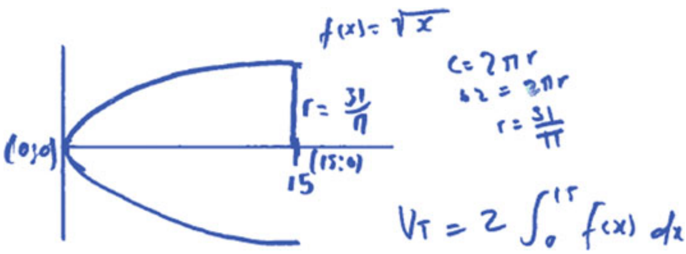

Fig. 17.2 Representation of the first modelling phase mathematisation by Group 1

even more effective teacher of mathematical modelling. For analysis the "codes-to-theory" model for qualitative inquiry by Saldaña (2016) was used. This model refers to the systematic arrangement of data, first cycle coding, re-reading and second cycle coding. In short, the data (collected from the open-ended questionnaire) were arranged in categories and sub-categories that lead towards more general abstract constructs such as themes or concepts. In particular, process coding was exploited for this analysis. According to Saldaña (2016), process coding is particular useful if participants interact with the purpose of reaching a specific goal. This method of coding uses "-ing" words such as "reading", to illustrate action.

Furthermore, the researchers purposefully addressed the strategies to maintain trustworthiness of findings from qualitative data, as originally recommended by Lincoln and Guba (1985). These strategies include the transferability of data (by providing necessary detail on the context of the fieldwork), dependability (by encouraging other researchers to repeat the study), confirmability (by applying triangulation of data collection and analysis methods) and credibility of data (by providing a thick description of the methodology).

17.4 Findings and Discussion

17.4.1 Quantitative Findings

The researchers expected the majority of the participants to portray relatively positive attitudes towards this model-eliciting activity (considering they are all prospective mathematics teachers).

Table 17.2 reveals six in ten participants *enjoyed* the activity, while just more than half regarded modelling competencies as *valuable*. Just three in ten were *confident* that they could master modelling, and felt *motivated* to learn more about it in future. In an overarching sense, almost half of the participants exhibited positive

Table 17.2 Distribution and descriptive statistics of ATMMI scores

ATMMI dimensions	Scoring intervals	N	%
ENJOYMENT [N = 48] [M = 42.10; SD = 6.33]	40 or lower	19	39.6
	41–50	29	60.1
VALUE [N = 38] [M = 39.63; SD = 6.73]	40 or lower	18	47.4
	41–50	20	52.6
SELF-CONFIDENCE [N = 44] [M = 51.32; SD = 11.79]	60 or lower	27	70.5
	61–75	13	29.5
MOTIVATION [N = 49] [M = 17.61; SD = 4.82]	20 or lower	34	69.3
	21–25	15	30.7
TOTAL ATMMI SCORE [N = 36] [M = 152.25; SD = 24.27]	160 or lower	19	52.8
	161 or higher	17	47.2

attitudes towards the modelling activity, but a single modelling experience is not sufficient to develop an attitude towards a complex pedagogical perspective such as modelling.

The Mann-Whitney U test, as non-parametric statistical technique was used to detect differences between the medians of the responses of two genders and two performance groups (based on their achievement in mathematics in Grade 12). This test is considered appropriate, because participants' responses are not normally distributed, are measurable on an ordinal scale, are comparable in size and independent (responses from one subgroup do not affect the responses of another) (Milencović, 2011).

Table 17.3 present the *ranks* and Table 17.4 the *test statistics* of students' overarching attitudes towards this modelling experience, with gender and mathematics achievement in Grade 12 as grouping variables.

The findings firstly indicate that prospective female mathematics teachers in this study (Mdn = 135) have a significantly lower (at the 95% confidence level) overarching attitude towards this mathematical modelling activity than their male counterparts (Mdn = 168.5, U = 82.50, p = .020). The findings secondly indicate that prospective mathematics teachers in this study, who have scored 69% or less for mathematics in their Grade 12 year (Mdn = 141.0) have a significantly lower (at the 95% confidence level) overarching attitude towards this mathematical modelling activity than their counterparts, who have scored 70% or more (Mdn = 168.5,

Table 17.3 Ranks relating to total ATMMI scores

Demographic attributes	Groups	N	Mean rank	Sum of ranks
Gender [N = 36]	Female	14	13.39	187.50
	Male	22	21.75	478.50
Achievement in mathematics in Grade 12 [N = 36]	Less than 70%	14	14.14	198.00
	70% or more	22	21.27	468.00

Table 17.4 Test statistics[a] in respect of total ATMMI scores

	Gender[b]	Achievement in mathematics in Grade 12[c]
M Mann-Whitney U	82.500	93.000
W Wilcoxon W	187.500	198.000
Z Z	−2.322	−1.981
A Asymp. Sig. (2-tailed)	.020[d]	.048[d]
E Exact Sig. (1-tailed)	.019[d]	.049[d]

[a]Grouping variables: gender and achievement in mathematics in Grade 12
[b]Female participants are compared to male participants
[c]Participants, who scored 69% or less in mathematics in Gr 12 are compared to participants, who scored 70% or more for mathematics in Gr 12
[d]Significant at the 95% level of confidence

$U = 93.0$, $p = .038$). Cohen's effect sizes (r = .390 and .393 respectively) are in the medium to high interval (Milencović, 2011, p. 77), which imply that the findings have both moderate (to high) practical significance.

17.4.2 Qualitative Findings

Qualitative findings from data collected by the open-ended questionnaire indicate three main categories based on Saldaña's (2016) model for analysis (refer to Sect. 17.3.4). These findings reflect participants' initial convictions and mindset after a single modelling session. It is important to highlight that such convictions could likely change after more and more in-depth exposure to modelling activities. The following categories emerged from the analysis: (1) *challenging nature of tasks*, (2) *mindset towards modelling* (with sub-categories *positive mindset* on the one side and *negative mindset* on the other) and (3) *supporting teacher education*.

The first category, *challenging nature of tasks*, describes participants' general responses on the modelling activity. Participants (44 from 49) mostly experienced the task as overwhelming and difficult to understand, especially at the beginning of the session. For example, one participant commented "We could not understand the problem thus we could not even attempt to solve the problem".

The second category, *mindset towards modelling*, provides insight on the positive (24 from 47) or negative (14 from 47) participants' mindset after the modelling activity. Some participants (9 from 47) experienced mixed emotions, ranging from scary and frustrating to exciting and effective. For example a participant said, "It is exciting but not easy, I have mixed emotions".

The third category, *supporting teacher education*, represents participants' opinions on the type of support they require to become more effective modelling teachers and the necessary structure to achieve such a goal. A number of possibilities have been listed including more frequent exposure to modelling tasks, a course/module on modelling, linking mathematical modelling with relevant methodology courses and some others.

Each worksheet document (from 10 groups) was examined according to the elements of the modelling cycle (Balakrishnan et al., 2010) and certain modelling and mathematical competencies required (Stillman et al., 2007) by groups to proceed through the cycle. These qualitative findings are useful to answer our first research question. Table 17.5 represents the findings by indicating the number of groups showing evidence in their documents of the specific assessment criteria.

To support and crosscheck the findings from the list (as displayed in Table 17.5), the different models (presented by each group) were further evaluated according to the six evaluation criteria presented by Meyer (2012). An assessment of the models revealed only 4 of 10 groups presented an accurate answer, although 8 groups presented a realistic and precise model and only 3 of 10 group models were relatively robust. Sixty percent of the models could be applied in other situations, for example if the size of rugby balls change.

Table 17.5 Groups' modelling and mathematical competencies

Modelling and mathematical competencies required to complete the modelling cycle	Number of groups
Identifying from the available information what is relevant and what is irrelevant	9
Making simplified and relevant assumptions to enable mathematics to be applied	10
Recognising relevant variables	10
Presenting the real-world problem mathematically (mathematisation)	10
Selecting appropriate mathematical formulae/s	10
Using acquired mathematical knowledge to solve the problem	10
Demonstrating the need to acquire new mathematical knowledge	5
Choosing appropriate methods of checking and testing the model	1
Indicating the selection of technology or other resources to confirm calculations and/or to investigate other possibilities	3
Linking and critically checking mathematical results with the real-world situation	6
Considering implications of decisions and results	7

The qualitative findings indicate almost all 10 groups could shift effortlessly through the first two phases of the modelling cycle. Nine groups could differentiate between relevant and irrelevant information and all groups could represent the real-life problem mathematically and then use appropriate acquired mathematical knowledge to solve the problem. The findings also indicate that almost half of the groups had trouble in the third and fourth phases of the modelling cycle. Four groups could not make sense of the mathematical solution in terms of relevance and appropriateness to the real-world situation and three groups did not consider implications of decisions and results. In addition, five groups demonstrated the need to acquire new mathematical knowledge and only one group used alternative methods to check results.

Finally, qualitative data findings 'tell the story' of the exposure of participants to a challenging mathematical modelling task. In particular, participants' testimony communicated how the modelling cycle created opportunities for them to experience shortcomings, and to view inaccuracies as valuable opportunities in which to invest. These are vital messages to stimulate a growth-mindset (Dweck, 2006) and would be considered in the next DBR phase of the broader project.

17.5 Conclusion

Since 2011, mathematical modelling has been integrated into the mathematics curriculum of South Africa's secondary (Grade 10–12) public schools. The solid relationship between a positive mindset towards and achievement in mathematics has been adequately recognised (Dowker, Ashcraft, & Krinzinger, 2012; Durandt & Jacobs, 2017; Schukajlow, Kolter, & Blum, 2015; Sweeting, 2011). In this context, the goal of this study was to explore the convictions and mindset of a group of prospective mathematics teachers at a South African university, based on their initial exposure to a model-eliciting task. An investigation of participants' mindsets (attitudes) towards the modelling task and convictions (beliefs) about mathematical modelling afterwards, has shown that they enjoyed and valued the activity. A majority of qualitative responses indicated participants experienced the activity as challenging, although about half of the participants made positive remarks about the experience and they mostly felt inspired to further develop their modelling competencies. The participants generally revealed a confidence deficiency with regards to approaching and handling the modelling task. Subsequent analyses revealed that females, as well as prospective teachers who are less mathematically competent, largely exhibited a fixed mindset towards mathematical modelling.

The theoretical lens through which this study was viewed, the Learning to Teach Secondary Mathematics framework (Peressini et al., 2004), *firstly* asserts that how a prospective mathematics teacher acquires a particular set of knowledge and skills and the specific teaching context in which it happens fundamentally influence what they eventually learn. It *secondly* stipulates that mathematics teachers' knowledge, especially their convictions and mindset, are shaped through increased participation in the practice of teaching itself.

17.6 Suggestions

The researchers realised right from the start that a once-off modelling experience would not be insufficient to prepare prospective teachers for this complex mathematical topic, and that one modelling task cannot shape their confidence and beliefs adequately. Prospective mathematics teachers should develop professionally during their formal education with regards to their own mathematical modelling content and pedagogical knowledge. Such professional development should ideally be based on a well-planned set of guidelines that take cognisance of and gradually cultivate a growth mindset in prospective teachers.

Acknowledgements This work is based on the research partially supported financially by the National Research Foundation (NRF) of South Africa, Unique Grant No. 106978.

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Chapter 18
Conclusion and Looking Ahead

Marilyn E. Strutchens, Rongjin Huang, Despina Potari and Leticia Losano

Overall this collection of articles is very forward thinking and focused on ensuring that prospective secondary mathematics teachers develop positive dispositions toward teaching and learning mathematics, develop pedagogical and content knowledge that provides PSMTs with the background knowledge to handle most situations, and experience teaching and learning in their course work and field experiences that lead them toward becoming effective teachers of mathematics.

Much like many of the articles reviewed by Strutchens (2017) in the Topical Survey: The Mathematics Education of Prospective Secondary Teachers Around the World (Strutchens et al., 2017), most of the articles focused on field experiences are at a small scale and much of the work has not been replicated in other places. However, Peterson and Leatham in this monograph replicated their work with some extended goals, and the Martin and Strutchens chapter shares how researchers across universities are replicating the previous work of Peterson and Leatham related to paired placements within their Networked Improvement Community. Their work is in alignment with Strutchens et al.'s (2017) suggestion that, "a

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M. E. Strutchens et al. (eds.), *Educating Prospective Secondary Mathematics Teachers*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-91059-8_18

collaboration of researchers across multiple programs may be able to create a sufficient sample size to undertake larger-scale investigations" (p. 46). Also featured in this monograph is an article Mohr-Schroeder, Jackson, Cavalcanti, and Delaney) which examined how a robotics course in an educator preparation program that required a field experience in an informal learning environment impacted its participants. This chapter highlights an innovative practice which benefits both prospective teachers and their students. Heinrich's and Kilic's chapters focus on the the importance of prospective teachers having extended time in the field during practicum placements to work with students in order to hone their lesson planning, noticing, and decision making skills. Overall, this set of papers reflects the myriad of experiences that prospective teachers need to have during practicums and student teaching in order to fully develop the craft of teaching.

The technology based papers presented in the monograph illuminate the impact that technology can have on teacher preparation programs and teacher candidates. These articles pay particular attention to PSMT's development of technological pedagogical content knowledge (TPACK) as a set of knowledge as well as an orientation that reviews technology as a critical tool for identifying mathematical relationships (Huang & Zbiek, 2017). In addition to exploring the PSMT's perspectives of the use of technology in Moreno and Llinares' chapter, Akcay and Boston provided an account about how teaching a methods course focusing on intentional selection and use of tasks with high levels of cognitive demand could help develop PSMT's capacities in designing technology-based lesson plans with high-level cognitive demand tasks and maintaining a high-level of implementation and anticipation of student responses. Moreover, the chapter by Zbeik describes a holistic and integrated framework for designing programs and teaching courses that prepare PSMTs to effectively use technology during mathematics teaching. However, designing and implementing programs for preparing PMSTs to become "proficient with tools and technology designed to support mathematical reasoning and sense making, both in doing mathematics themselves and in supporting student learning of mathematics" (AMTE, 2017, p.11) on a large scale remain challenging.

The papers included in the professional identity section present different opportunities where PSMTs can reflect and discuss their beliefs, attitudes, and convictions about becoming a mathematics teacher. In this way, they analyze varied settings in which pre-service education can contribute to the development of PSMTs' professional identities. In line with the articles reviewed by Losano and Cyrino (2017), Hine's and Cyrino's papers highlight the importance of different field experiences as key settings for the development of PSMTs' professional identities while Durant and Jacobs' work is focused on the potentiality of content courses for developing PSMTs' beliefs and attitudes about pedagogical strategies. Cyrino's work shows that through the analysis of a multimedia case featuring one mathematics teacher practice PSMTs developed an investigative attitude towards pedagogical practice and (re)signified their identities and their future professional practice. Hine's work underlines the complexity and multi-faceted character of

PSMTs self-perceptions. In this way, despite all Hine's research participants asserted they felt ready to teach, a significant number articulated a need to further develop their mathematical content knowledge and their mathematical pedagogical knowledge. Durant and Jacobs' results underline that PSMTs beliefs and attitudes about mathematical modelling as a pedagogical strategy are shaped through increased participation in the practice of teaching. Therefore, pre-service education should offer progressive and coherent opportunities that gradually cultivate a growth mindset in PSMTs regarding the pedagogical value mathematical modelling and, simultaneously, develop their confidence with regards to approaching and handling modelling tasks. The three articles show the complexity, variety, and richness of research on PSMTs professional identities and dispositions. Particularly, notions such as identity, beliefs, self-perceptions, and attitudes provide rich insights into why PSMTs make certain decisions or assume particular stands (inside and outside the classroom) and into how mathematics teacher educators may assist PSMTs in developing their autonomy and agency.

Understanding and teaching mathematical modeling, developing assessment criteria and grading student work, reasoning for proportional relations and facilitating student mathematical argumentation were topics discussed in the teacher knowledge chapters. The reported studies emphasize the process of how teacher knowledge can be developed, an approach that seem central in the current research on teacher knowledge (see Potari & Ponte, 2017). In particular, the four papers in the teacher education section, show that the development of prospective teachers' knowledge can be facilitated through a number of teacher education practices such as: engaging prospective teachers in identifying and interpreting critical incidents; in solving mathematical tasks, transforming them into classroom tasks and identifying their main mathematical ideas; marking exam's responses, interpreting students' understanding and defining learning goals.

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