CHAPTER 9

Regime Identification for Sovereign Bond Portfolio Construction

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9.1 Introduction

Financial markets are closely linked to the business and credit cycles. They experience periods of persistent high or low volatility and go through risk-on and risk-off episodes. Certainly, return distributions vary with the state of the economy. As a consequence, the behaviour of portfolio returns can vary significantly over shifting economic and financial conditions—in other words, it can substantially change over each *regime*.

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Traditional asset allocation algorithms do not typically incorporate regime-specific information to construct optimal portfolios. In this chapter, we introduce a state-dependent investment strategy based on a set of indicators that we believe are useful in identifying economic and financial regimes. Importantly, it should be noted that the objective of this chapter is not normative. We are not proposing an alternative asset allocation approach; rather, our intention is to better compare the properties of portfolios which are, and are not, optimised taking state-conditional information into account.

To this end, we apply in this chapter a multi-step approach to portfolio construction. First, the state space is characterised by separating "regular" from "distressed" market environments, using a selected regime indicator. We then obtain distributions of asset class returns conditional on the regime indicator. Finally, we execute a dynamic asset allocation algorithm on the mean-variance space, optimising a portfolio over expected conditional return distributions.

While the existing literature on regime identification has focused, in particular, on equity markets, we illustrate this approach for an investment universe consisting of four of the most important and liquid developed government bond markets: the United States, the United Kingdom, Germany, and Japan. Furthermore, we analyse the properties of the portfolio construction method for different assumptions on currency numeraires (specifically, those often used by sovereign investors), different utility concepts and different levels of risk tolerance. Then, we compare these results with traditional asset allocation methodologies, such as simple mean-variance and Bayesian optimisation.

We show that the portfolios optimised across regimes have properties markedly different from those optimised using conventional asset allocation approaches. They imply diversified bond weightings with a lower inclination to corner solutions, and display higher mean returns at broadly comparable volatilities. Accordingly, the Sharpe ratios of the regime-optimised allocations indicate better risk-adjusted returns. Yet, as we show, they imply fatter-tailed return distributions. These findings may indicate that the regime-optimised allocations are exposed to an additional risk factor that, when priced, could give rise to an expected excess return over standard portfolios. From a theoretical perspective, this makes sense: if the optimised portfolios are adequately diversified within each financial or economic regime, resulting risk exposure must be mostly of systematic nature and thereby should carry a premium.

The rest of this chapter is organised as follows: Sect. 9.2 provides a brief literature review documenting the notion of economic regimes and the issues that arise when applying the concept to the analysis of financial markets. In Sect. 9.3 we propose three indicators for identifying regimes. Section 9.4 first demonstrates that these three measures are useful in characterising the future return distributions of our universe of developed market sovereign bonds, and then describes and applies our regime-optimal asset allocation framework. Section 9.5 concludes.

9.2 Regime Identification

The term *regime* has been used extensively in various fields: in finance, in economics, and even in politics.[1] The concept of multiple regimes received early formal treatment by Nicholas Georgescu-Roegen (1951) in a study of linear economic models. The author discussed the idea that different phases of the business cycle could be represented in a multiple regime model.

Later on, regime identification was addressed by Goldfield and Quandt (1973). They were among the first to incorporate the concept of regime switching into an econometric model. This approach was later popularised by Hamilton (1989), who explicitly modelled two states representing the aggregate business cycles: expansion and recession.

On an ex-post basis, for example, information published by the National Bureau of Economic Research (NBER)—responsible for determining official recessionary periods in the United States—could be used to identify regimes.[2] Based on this classification, return distributions of financial assets could be estimated separately for periods when the US economy is expanding or when it is contracting (Fig. 9.1). Though simple-sounding, several issues arise when applying such an approach to investment decision making. First, the expansion regime takes up most of history (e.g., about 80% of the past 26 years). Clearly, not all expansions since 1990 have been characterised by the same asset class behaviour. Second, asset classes can sometimes behave as if there is a looming recession, though macroeconomic data may not reflect so. To illustrate this point, we fit the NBER recession probability using two different probit models: one purely based on macro data and another using market indicators (Fig. 9.2). In both cases, the empirical probability of facing an economic downturn presents a spike when the NBER says the US economy is contracting.

However, the market-based model presents additional spikes in the last couple of years; the period going from October 2015 to February 2016 stands out the most. During this time, oil prices experienced one of the sharpest falls in history, sparking deflationary pressures. At the same time, investors were worried that China's economy may face a hard landing.

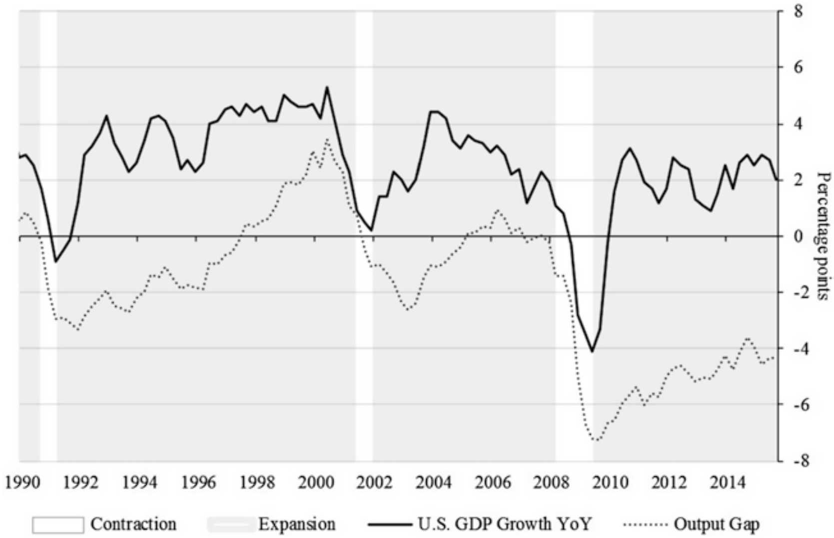

Fig. 9.1 US real GDP quarterly annualised GDP growth and output gap

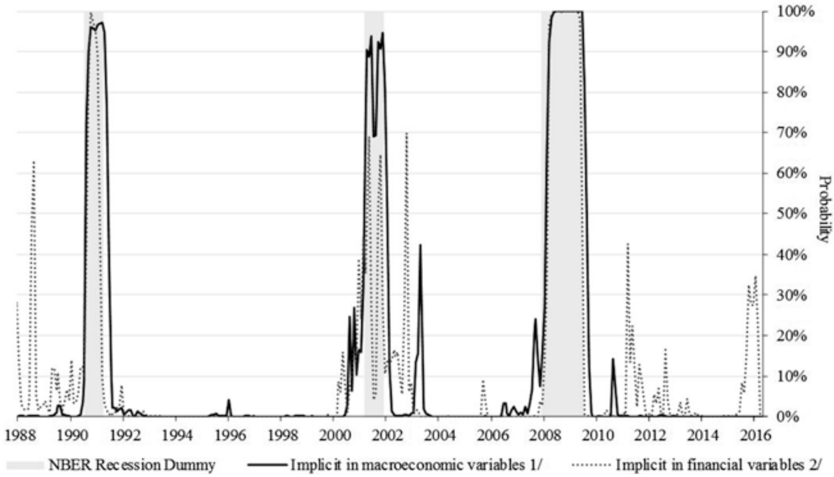

Fig. 9.2 US probability of recession implicit in selected variables

Though these worries later dissipated, market-based measures appeared to be pricing it in. Yet, the US recession probability based in macroeconomic data remained close to zero as economic fundamentals in the United States were not deteriorating. In conclusion, using the NBER classification as a regime indicator to model financial markets' behaviour, our forecasts would miss the changes in the conditional distribution of asset reeturns observed in the data.

Third, because the NBER classifies a period as either expansion or recession after it already happened (often, quite late), its data is actually of little use for real-life asset allocation purposes. A similar case can be made for most *ex-post* and macro-based dummy variables (i.e., financial crises, stages of the interest rate, or business cycles). This was pointed out by Blitz and van Vliet (2011), who propose a timelier leading indicator of the US business cycle that allows to split the state space into finer scenarios. However, we believe their approach still does not control for other important issues, such as country-risk concentration, which we discuss later on.

A fourth important point is that the frequency of the data can affect estimation results. Sometimes, in asset allocation, the periodicity of a sample is chosen in order to reflect the length of the investment horizon. To capture the phases of the business cycle, it would make sense to use a quarterly or annual frequency. However, this can introduce a small-sample bias to our estimations. By using a quarterly sample (e.g., when basing our estimations on GDP data), very few observations become available, making it harder to make adequate and trustworthy statistical inference. From the point of view of regime identification, this means that we receive the *regime signals* less frequently—an unattractive feature. The appropriate data frequency thus involves a trade-off between sample size and investment horizon.

Finally, the choice of the appropriate regime indicator is complicated by the fact that the regimes of different asset classes may not be perfectly synchronised. Even if assuming that one state variable is sufficient to summarise the regime in a particular country, relying on only one economy's data (in this case, the United States), may not be appropriate for portfolios with assets from multiple geographies. Including multiple state variables, one for each of the different regimes governing the assets in a global portfolio is difficult due to multiple reasons. Not least of which is the difficulty in estimating the joint probability distribution of the multiple state variables.

9.3 Alternative Regime Indicators

For market participants, it can be a daunting task to characterise the financial and economic environment given the wealth of data that is published every day. Not accounting for outliers in the data can easily lead to misspecification of conditional asset return distributions. For example, practitioners may be faced with investor preferences to use a well-known market indicator such as the VIX index or a corporate credit spread to define the states of the world; however, these indices are restrictive in nature as they only consider asset- and country-specific behaviour. For a multi-asset, multi-country investor, objective measures to define states of the world are much harder to find.

To achieve a regime identification process that is *rule-based, systematic, transparent,* and *less subjective*, we introduce in this section mathematical models that capture the underlying data structure. With the objectives of summarising a broad group of signals, achieving a fine enough partition of the state space and avoiding ambiguity in its interpretation, we propose the following three measures.

(a) Macro Fragility Index

The Macro Fragility Index (MFI) is defined as the variance explained by the principal components of a chosen set of macroeconomic indicators.[3] The time series plots the MFI obtained using monthly industrial production and consumer price indices for a set of developed countries (the United States, Canada, the United Kingdom, Germany, and France), beginning 1975 (Fig. 9.3). A 36-month rolling window is applied to estimate a time series of the measure.

If the total variation in this group of economic variables can be explained to a large degree by a few factors, then this is an indication of higher macro risk concentration. Additionally, this measure offers a way to summarise the economic cycle of multiple economies simultaneously.

(b) Financial Turbulence Index

The Financial Turbulence Index (FTI) is the time series of the Mahalanobis distance (i.e., the square root of the multivariate Z-score) of the return matrix of several asset classes.[4] The FTI is estimated from monthly returns of global bonds, equities, and commodities starting in 1977 (Fig. 9.4).[5] The

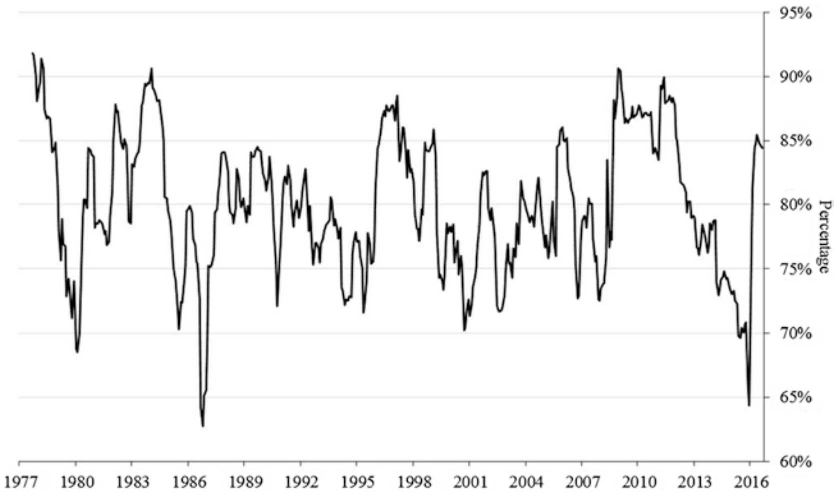

Fig. 9.3 Macroeconomic fragility index

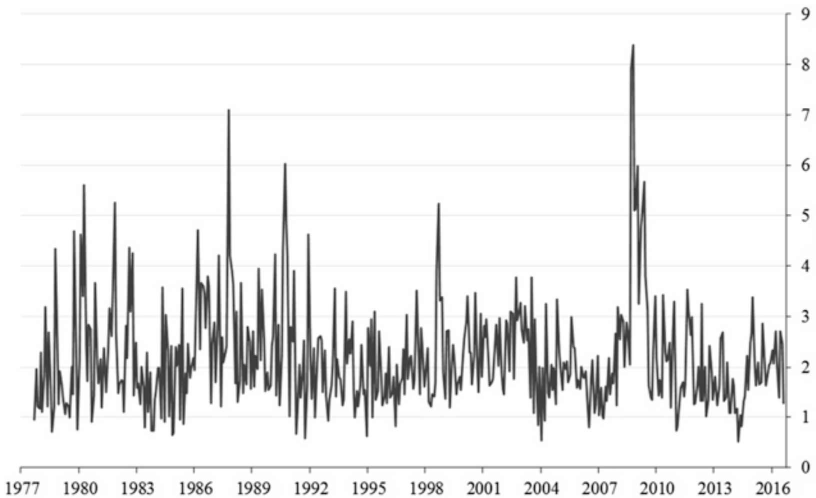

Fig. 9.4 Financial turbulence index

higher the FTI is, the more it signals an extraordinary realisation of joint returns away from the average. In this sense, the measure can prove helpful in defining regimes under which the market is more *turbulent*.

(c) Systemic Risk Index

In a similar fashion to the MFI, the Systemic Risk Index (SRI) is defined as the variance explained by the first factor of a principal component analysis over the return matrix of a selected set of asset classes (Fig. 9.5). High values of this index indicate periods in which the returns are well explained by only one factor. This means that the multi-country, multi-asset class volatility is concentrated which may indicate systemic risk. As in the case of the MFI, a three-year rolling window is used for the estimation. Opposed to the FTI it does not measure *concentration*, but the *level* of risk in the financial system.

As noted, these measures are estimated at a monthly frequency. Note, however, that these measures could be constructed at a weekly, and even daily, frequency, depending on the availability of underlying data. Higher frequency indicators may have some applications such as for early-warning indicators. However, higher frequency indicators must be used with caution, because daily data may contain greater noise.

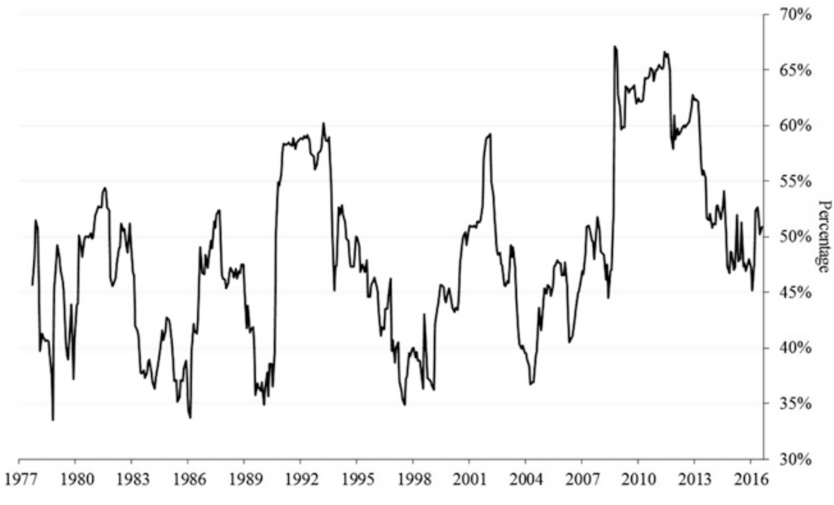

Fig. 9.5 Systemic risk index

9.4 Empirical Analysis

In this section, we assess the fitness of the proposed regime indicators for predicting future bond return distributions over several investment horizons. Subsequently, we attempt to construct regime optimised portfolios using an out-of-sample approach.

9.4.1 Predictive Power of the Regime Indicators

Now that systematic indicators have been defined—and before constructing an *ex-ante* investment strategy—the properties of these measures to characterise future returns are assessed. This analysis is performed for an investment universe comprising government bonds from four markets: the United States, the United Kingdom, Germany, and Japan, with constant durations ranging from one to ten years in one-year steps.

Using the full available data history from January 1975 to August 2016, the following predictive regression for the monthly local currency returns is performed:

$$r_t^i = \alpha + \beta I_{t-k} + \varepsilon_t.$$

Here, r_t^i is the month over month total return of the i-year government bond $i \in \{1,2,3,\ldots,10\}$ and I_{t-k} is the k-th lag of regime indicator $\in\{MFI, SRI, FTI\}$, $k \in \{1,3,6,9,12\}$. That is, the regime indicators' predictive power is tested for 1, 3, 6, 9, and 12 month-ahead returns. ε_t is the regression error.

SRI and FTI show some predictive power (Table 9.1).[6] Concretely, the SRI seems to do a decent job in explaining short-end bond returns across all economies and several months ahead. Additionally, the relationship between this indicator and total returns seems to be inverse and decreasing along term. However, the FTI can explain some returns in both Germany and the United Kingdom, especially in the medium term, and the relationship between them and financial turbulence appears positive. In contrast, the MFI does not perform well-explaining future returns for any investment horizon.

As a mean-variance algorithm will be used to construct portfolios, it is also important to explore if the MFI, SRI, and FTI can predict future volatility. To this end, predictive regressions of the following form are performed:

Table 9.1 T-statistics of β coefficient in the predictive regression using monthly local currency returns

		MFI				*SRI*				*FTI*			
	k=	1	3	6	12	1	3	6	12	1	3	6	12
US	1 year	0.04	0.20	0.78	0.44	−2.94***	−3.55***	−4.00***	−3.80***	1.26	−0.09	0.10	−0.68
	2 years	−0.25	−0.11	0.93	0.37	−1.57	−2.26**	−2.74***	−2.34**	1.49	−0.06	0.32	−0.68
	3 years	−0.43	−0.31	1.00	0.38	−0.75	−1.41	−1.89*	−1.45	1.53	−0.12	0.32	−0.61
	4 years	−0.52	−0.41	1.01	0.41	−0.28	−0.91	−1.36	−0.89	1.52	−0.22	0.21	−0.57
	5 years	−0.56	−0.46	0.99	0.43	−0.04	−0.63	−1.06	−0.53	1.49	−0.32	0.05	−0.55
	6 years	−0.58	−0.48	0.96	0.44	0.09	−0.47	−0.88	−0.28	1.47	−0.40	−0.12	−0.53
	7 years	−0.59	−0.48	0.93	0.44	0.15	−0.39	−0.78	−0.11	1.44	−0.48	−0.28	−0.52
	8 years	−0.59	−0.47	0.89	0.44	0.18	−0.35	−0.71	0.01	1.42	−0.53	−0.44	−0.51
	9 years	−0.59	−0.46	0.86	0.43	0.20	−0.32	−0.67	0.10	1.40	−0.58	−0.57	−0.50
	10 years	−0.58	−0.45	0.83	0.42	0.20	−0.31	−0.64	0.17	1.38	−0.61	−0.69	−0.49
Germany	1 years	0.82	0.46	0.51	0.68	−4.59***	−4.47***	−4.71***	−4.72***	0.64	0.30	−0.24	−0.54
	2 years	0.96	0.27	0.26	0.70	−3.32***	−3.35***	−3.42***	−3.37***	1.61	0.40	−0.68	−0.97
	3 years	1.02	0.13	0.07	0.76	−2.25**	−2.30**	−2.36**	−2.27**	1.69*	0.16	−1.03	−1.05
	4 years	1.10	0.06	−0.01	0.86	−1.57	−1.56	−1.68*	−1.54	1.57	−0.17	−1.31	−1.01
	5 years	1.19	0.03	−0.02	0.99	−1.14	−1.06	−1.27	−1.07	1.41	−0.48	−1.54	−0.97
	6 years	1.28	0.03	0.01	1.10	−0.85	−0.73	−1.01	−0.75	1.25	−0.74	−1.74*	−0.92
	7 years	1.36	0.03	0.07	1.19	−0.64	−0.50	−0.84	−0.52	1.09	−0.93	−1.89*	−0.89
	8 years	1.42	0.05	0.14	1.26	−0.49	−0.33	−0.71	−0.36	0.94	−1.08	−2.00**	−0.85
	9 years	1.47	0.06	0.20	1.31	−0.37	−0.20	−0.62	−0.23	0.79	−1.18	−2.08**	−0.82
	10 years	1.50	0.08	0.26	1.34	−0.27	−0.10	−0.55	−0.13	0.66	−1.26	−2.13**	−0.79
UK	1 year	0.54	0.52	−0.04	−0.40	−2.47**	−3.29***	−3.65***	−4.01***	1.45	0.16	0.04	−0.53
	2 years	0.75	0.46	−0.88	−0.61	−1.16	−1.64	−2.38**	−2.30**	1.27	0.48	0.41	−0.94
	3 years	0.87	0.41	−1.40	−0.46	−0.40	−0.49	−1.55	−1.30	0.81	0.51	0.31	−0.99
	4 years	0.94	0.39	−1.45	−0.24	0.02	0.14	−1.06	−0.80	0.45	0.42	0.01	−0.90
	5 years	0.99	0.38	−1.21	−0.02	0.23	0.44	−0.78	−0.53	0.17	0.28	−0.37	−0.76

	6 years	1.03	0.37	−0.89	0.17	0.34	0.57	−0.62	−0.39	−0.06	0.15	−0.79	−0.61
	7 years	1.07	0.37	−0.59	0.33	0.39	0.61	−0.53	−0.31	−0.25	0.03	−1.19	−0.46
	8 years	1.09	0.37	−0.35	0.47	0.42	0.62	−0.47	−0.27	−0.40	−0.07	−1.55	−0.33
	9 years	1.10	0.38	−0.15	0.57	0.43	0.62	−0.44	−0.24	−0.53	−0.15	−1.87*	−0.21
	10 years	1.12	0.38	0.00	0.66	0.44	0.61	−0.41	−0.22	−0.63	−0.21	−2.13**	−0.12
Japan	1 year	0.85	0.60	0.42	0.18	−3.48***	−3.60***	−3.70***	−4.57***	0.81	0.22	0.48	0.09
	2 years	0.96	0.51	0.47	−0.20	−2.33**	−2.39**	−2.56**	−3.84***	1.19	0.45	0.16	0.23
	3 years	0.88	0.34	0.50	−0.44	−1.71*	−1.75*	−1.89*	−3.36***	1.33	0.41	−0.24	0.24
	4 years	0.75	0.16	0.48	−0.54	−1.37	−1.37	−1.48	−3.00***	1.40	0.26	−0.42	0.16
	5 years	0.60	−0.02	0.42	−0.54	−1.17	−1.12	−1.20	−2.72***	1.46	0.05	−0.39	0.04
	6 years	0.45	−0.19	0.33	−0.50	−1.05	−0.94	−0.99	−2.47**	1.52	−0.16	−0.27	−0.10
	7 years	0.31	−0.34	0.23	−0.43	−0.96	−0.80	−0.81	−2.25**	1.58	−0.35	−0.15	−0.24
	8 years	0.19	−0.48	0.12	−0.35	−0.89	−0.69	−0.67	−2.06**	1.63	−0.51	−0.03	−0.37
	9 years	0.07	−0.59	0.02	−0.28	−0.84	−0.60	−0.55	−1.89*	1.66*	−0.65	0.06	−0.48
	10 years	−0.03	−0.70	−0.08	−0.22	−0.81	−0.53	−0.44	−1.73*	1.69*	−0.76	0.14	−0.57

Source: Authors' calculations using return data from Bloomberg and Bank of America/Merrill Lynch

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

$$\sigma_t^i = \alpha + \beta I_{t-k} + \varepsilon_t.$$

Here, σ_t^i is the 12-month rolling volatility of local currency returns for the i-year government bond $i \in \{1,2,3,\ldots,10\}$. The corresponding t-statistics, adjusted for overlapping sample issues using Hansen & Hodrick (1980) procedure (Table 9.2).

In line with previous results, MFI does not show predictive power for the future return, except in some Japanese Government Bond cases (results available on request). However, SRI and FTI show significant statistical power. Specifically, the SRI has a positive relationship with the volatility of some short- and medium-term maturities across all countries. The FTI performs well in a greater part of the term structures and across most investment horizons. The relationship between this indicator and future bond return volatility is positive and increasing with duration.

9.4.2 Portfolio Construction

Given some evidence of predictive power of the previously introduced regime indicators, we now proceed to establish an investment strategy that is *regime-optimal*. We define a regime-optimal portfolio as the *best* asset allocation to hold during the predicted state of the regime space.

Past examples of applications of a state-space-based approaches for constructing portfolios can be found in the literature. Clarke and de Silva (1998) suggest a method to expand the optimal frontier when considering multiple regimes. We apply the approach of Ang and Bekaert (2004), who take into consideration the effect of high volatility environments on the equity market. Blitz and van Vliet (2011) use a modified version of the NBER economic cycle indicator described above to capture the time-variation of risk and return properties in US markets. More recently, and from a sovereign investor's perspective, Cruz-Lopez and Rivadeneyra (2014) set up an approach to maximise the expected value of international reserves in the states of the world where they are most likely to be liquidated. They choose foreign exchange rates as state variables to differentiate between different regimes.

Our approach offers a more general setting: by recognising that asset portfolio investors may have different objectives, goals, and reaction functions, we define our state space by using a set of indicators that encompass a broader amount of information.

Table 9.2 T-statistics of β coefficient in the predictive regression of 12-month rolling volatility using monthly local currency returns

		MFI				*SRI*				*FTI*			
	k=	1	3	6	12	1	3	6	12	1	3	6	12
US	1 year	0.31	−0.09	−0.43	−0.81	−1.12	−1.25	−1.33	−1.49	2.23**	1.90*	1.47	0.16
	2 years	0.22	−0.14	−0.47	−0.88	−1.71*	−1.87*	−1.87*	−1.72*	2.22**	1.93*	1.46	0.14
	3 years	0.20	−0.14	−0.45	−0.91	−1.85*	−2.02**	−2.00**	−1.75*	2.38**	2.15**	1.65*	0.22
	4 years	0.24	−0.09	−0.40	−0.93	−1.75*	−1.92*	−1.93*	−1.66*	2.67***	2.56**	2.02**	0.40
	5 years	0.31	−0.01	−0.31	−0.92	−1.48	−1.63	−1.71*	−1.51	3.09***	3.12***	2.56**	0.64
	6 years	0.39	0.09	−0.20	−0.91	−1.11	−1.25	−1.39	−1.31	3.53***	3.73***	3.18***	0.92
	7 years	0.46	0.20	−0.07	−0.87	−0.73	−0.84	−1.04	−1.07	3.83***	4.16***	3.70***	1.19
	8 years	0.52	0.29	0.06	−0.82	−0.39	−0.49	−0.70	−0.82	3.89***	4.25***	3.93***	1.39
	9 years	0.57	0.37	0.19	−0.76	−0.13	−0.20	−0.41	−0.58	3.75***	4.10***	3.90***	1.52
	10 years	0.60	0.44	0.30	−0.70	0.08	0.03	−0.17	−0.35	3.53***	3.87***	3.73***	1.59
Germany	1 year	1.61	1.46	1.35	0.62	−0.88	−0.86	−0.94	−1.10	2.95***	2.53**	1.59	0.76
	2 years	0.92	0.53	0.28	−0.42	−0.54	−0.56	−0.64	−0.73	3.15***	3.06***	2.09**	0.47
	3 years	0.83	0.40	0.13	−0.57	−0.40	−0.42	−0.56	−0.54	3.22***	3.20***	1.86*	0.24
	4 years	0.70	0.28	0.05	−0.66	0.08	−0.03	−0.32	−0.36	3.55***	3.61***	1.95*	0.29
	5 years	0.45	0.05	−0.12	−0.78	0.88	0.61	0.06	−0.18	3.98***	4.18***	2.31**	0.54
	6 years	0.29	−0.06	−0.16	−0.82	1.58	1.12	0.36	−0.07	4.15***	4.52***	2.74***	0.87
	7 years	0.33	0.04	0.01	−0.72	1.92*	1.33	0.46	−0.05	3.96***	4.55***	3.10***	1.17
	8 years	0.52	0.30	0.30	−0.53	1.95*	1.34	0.44	−0.06	3.60***	4.40***	3.36***	1.40
	9 years	0.75	0.58	0.61	−0.32	1.84*	1.26	0.39	−0.08	3.25***	4.20***	3.50***	1.56
	10 years	0.96	0.83	0.86	−0.13	1.69*	1.16	0.33	−0.10	2.97***	4.01***	3.56***	1.67*
UK	1 year	0.76	0.69	0.61	0.12	−1.49	−1.48	−1.53	−1.91*	1.43	1.99**	1.70*	0.97
	2 years	0.35	0.29	0.24	0.20	−1.76*	−1.82*	−1.95*	−2.09**	2.10**	2.59***	1.86*	0.66

(*continued*)

Table 9.2 (continued)

		MFI				SRI				FTI			
	k=	1	3	6	12	1	3	6	12	1	3	6	12
	3 years	0.09	0.05	0.04	0.14	−1.76*	−1.87*	−2.08**	−2.12**	2.08**	2.54**	1.81*	0.57
	4 years	−0.01	−0.03	−0.02	0.14	−1.67*	−1.80*	−2.05**	−2.05**	1.87*	2.41**	1.79*	0.61
	5 years	0.03	0.02	0.05	0.20	−1.50	−1.63	−1.90*	−1.92*	1.63	2.31**	1.83*	0.75
	6 years	0.16	0.16	0.21	0.30	−1.23	−1.35	−1.64	−1.73*	1.41	2.24**	1.94*	0.99
	7 years	0.33	0.34	0.42	0.41	−0.94	−1.02	−1.33	−1.53	1.23	2.18**	2.06**	1.27
	8 years	0.48	0.51	0.62	0.51	−0.66	−0.71	−1.02	−1.33	1.07	2.10**	2.14**	1.55
	9 years	0.62	0.66	0.80	0.60	−0.44	−0.45	−0.75	−1.16	0.95	2.01**	2.19**	1.78*
	10 years	0.73	0.78	0.94	0.67	−0.27	−0.25	−0.53	−1.02	0.86	1.93*	2.20**	1.97**
Japan	1 year	−0.37	−0.06	0.28	0.30	−1.63	−1.84*	−2.16**	−2.78***	0.73	0.88	0.89	−0.29
	2 years	−0.27	−0.19	−0.04	−0.34	−1.65*	−2.02**	−2.68***	−3.24***	1.13	1.28	1.22	0.12
	3 years	−0.21	−0.33	−0.43	−1.05	−1.76*	−2.17**	−2.94***	−3.42***	1.34	1.47	1.36	0.41
	4 years	−0.25	−0.45	−0.66	−1.42	−1.96*	−2.35**	−3.13***	−3.62***	1.45	1.59	1.47	0.58
	5 years	−0.34	−0.59	−0.82	−1.63	−2.14**	−2.52**	−3.27***	−3.80***	1.58	1.72*	1.57	0.71
	6 years	−0.47	−0.73	−0.97	−1.77*	−2.25**	−2.62***	−3.31***	−3.87***	1.71*	1.83*	1.66*	0.82
	7 years	−0.59	−0.84	−1.07	−1.82*	−2.31**	−2.67***	−3.28***	−3.87***	1.83*	1.91*	1.72*	0.91
	8 years	−0.69	−0.93	−1.13	−1.81*	−2.33**	−2.69***	−3.23***	−3.82***	1.91*	1.95*	1.75*	0.97
	9 years	−0.77	−0.99	−1.16	−1.75*	−2.34**	−2.69***	−3.18***	−3.75***	1.96*	1.97**	1.77*	1.01
	10 years	−0.82	−1.02	−1.16	−1.69*	−2.34**	−2.68***	−3.13***	−3.67***	2.00**	1.97**	1.76*	1.03

Source: Authors' calculations using return data from Bloomberg and Bank of America/Merrill Lynch

Note: All t-statistics are Hansen-Hodrick adjusted

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

Naturally, the definition of *best* can vary depending on the investor's preferences and constraints. For example, take an investor with mean-variance preferences and a one-month regime-predicting horizon looking to maximise risk-adjusted returns. If the regime split has two states of nature $\{s_1, s_2\}$ and he foresees that the second state will prevail during the following month, the best portfolio to hold over the next 30 days could be the one which delivers the highest Sharpe ratio during said regime. The investor can subsequently re-adapt the portfolio if he foresees another regime switch. Alternatively, the investor could determine the probability of observing each of the two states during the following month and weight two state-optimal portfolios accordingly.

This definition highlights the importance of the regime identification process, which is of dynamic nature: the distribution of future bond returns is conditional on the state of the world. Naturally, a succinct definition of the state space and a methodology for forecasting such regimes are required, and are detailed as part of our investment set-up. It is also important to note at this point that, the methodology under which a portfolio is optimised is assumed to work, in principle. We take as given the portfolio optimisation process, and instead focus on pinpointing the value added to sovereign bond portfolios through regime identification.

9.4.2.1 Methodology

Regime-optimal portfolios are constructed by performing standard mean-variance (SMV) optimisation separately on risk and return estimates obtained conditionally. That is, based on each regime indicator, the historical returns of every bond prior to January 2000 are classified into two states: a high (H) and low regime (L), using the indicator's medians—a rather simple two-state split. Subsequently, for each set of returns from the high (s_H) and low (s_L) states, separate mean-variance optimisations are performed.

As a robustness check, these optimisations are executed for different currency numeraires and apply different rules by which a portfolio is selected from the efficient frontier. Next, the weights of the low and high regime portfolios are averaged either statically or dynamically. In the static approach, a constant weight of 50% is assigned to the weights of the low and high regime portfolios, respectively; in the dynamic approach, the weight is based on the expected value of the regime indicator (Table 9.3) at a given point in time; means and covariances are estimated in the sample from January 1985 to January 2000.

Table 9.3 Alternative assumptions used for portfolio construction

Portfolio selection criterions	• Minimum volatility • Maximum return/volatility ratio • Maximum Sharpe ratio • Target durations of 2, 4 and 6 years • Target volatilities of 2%, 4% and 6% • Maximum annualised loss probability of 2%, 5% and 10%
Currency numeraire assumptions	USD, EUR, GBP, JPY, and SDR

In detail, this portfolio construction process consists of the following steps:

1. Classifying the historical asset returns into low (s_L) and high (s_H) regime observations for each of the three regime indicators: MFI, SRI, and FTI. Any historical return observation is considered a high regime observation if the respective regime indicator exceeds its median during that period, and vice versa. This is the definition of our state space $\{s_L, s_H\}$.
2. Calculating low and high regime conditional means ($\mu|\ s_i$) and covariances ($\Sigma|\ s_i$) for each indicator and currency numeraire (thus in total 2 (#of regimes) × 3 (# of indicators) × 5 (# of numeraires) = 30 sets of means and covariances).
3. Calculating mean-variance efficient frontiers for each set of means and covariances: a low regime efficient frontier and a high regime efficient frontier.
4. Selecting one portfolio from the set of mean-variance efficient portfolios. We show the alternative selection criteria (Table 9.3).
5. For each regime indicator, the weights to place on the low $\left(w_L^*\right)$ and high $\left(w_H^*\right)$ regime optimal portfolios $\left\{w_L^*, w_H^*\right\}$ are determined using either a static and dynamic approach. Under the static weighting scheme, the low and high regime portfolios are weighted by w_i^* = 50% each. With the dynamic weighting, the low and high regime portfolios are weighted based on the expected value of the corresponding regime indicator. The expected value is obtained from an autoregressive process of order 1 with a projection horizon of 12 months, and represents our regime forecasting algorithm.[7]
6. Finally, as an aggregation method for both the dynamic and static approach, "combined" regime optimised portfolios are calculated as weighted averages across the regime identification criteria.

The combined regime-optimal portfolios are compared to standard mean-variance (SMV) optimal portfolios and a Bayesian approach (BAY) where the first moment of the prior distribution of expected returns is obtained by scaling expected return with the corresponding volatility. To be comparable to the regime-optimal portfolios, these portfolios are determined using in-sample data up to January 2000.

9.4.2.2 Results

The regime-optimised portfolios show a degree of diversification in-between that observed for the SMV and BAY portfolios. We calculate the portfolio weights resulting from the alternative portfolio construction techniques for the selection criterion of a target duration of four years separately for the alternative numeraire assumptions (Fig. 9.6). Clearly, the SMV portfolios on the left show higher bond concentration with corner solutions: the allocation to a single yield curve node can go as high as 50% of the portfolio. Under the Bayesian approach, the portfolios appear to be well diversified with few asset classes at zero weight and maximum asset weights not much higher than 10%. The regime-optimised portfolios appear to be more diversified than the SMV but less than the BAY portfolios. Furthermore, the results hold for the average portfolio composition across *all* selection criteria. Again, the SMV approach also shows more concentration in comparison to the Bayesian optimisation and the regime-optimised allocation.

With regard to the portfolios' risk and return profile, we present the mean returns and conditional returns-at-risk at a 95% confidence level over the out-of-sample period (Fig. 9.7). The dots represent either the SMV optimal portfolios or the Bayesian portfolios and the static or dynamic regime-optimal portfolios. The individual dots for each optimisation approach refer to the combination of different numeraires (5) with different selection criterions (12)—thus 60 dots for each approach. The regime-optimal portfolios show different features than SMV and BAY portfolios.

Compared to the SMV optimisation allocations, the regime-optimal portfolios appear to show mostly superior risk-return combinations for both the dynamic and static approaches. This is evidenced by their corresponding dots, which are mostly northeast of those produced from the SMV approach (two upper panels of Fig. 9.7). However, compared to the Bayesian approach, the regime-optimised portfolios do not show superior risk-return properties. The static approach appears to show less favourable risk-return combinations than the Bayesian approach for some of the currency numeraires.

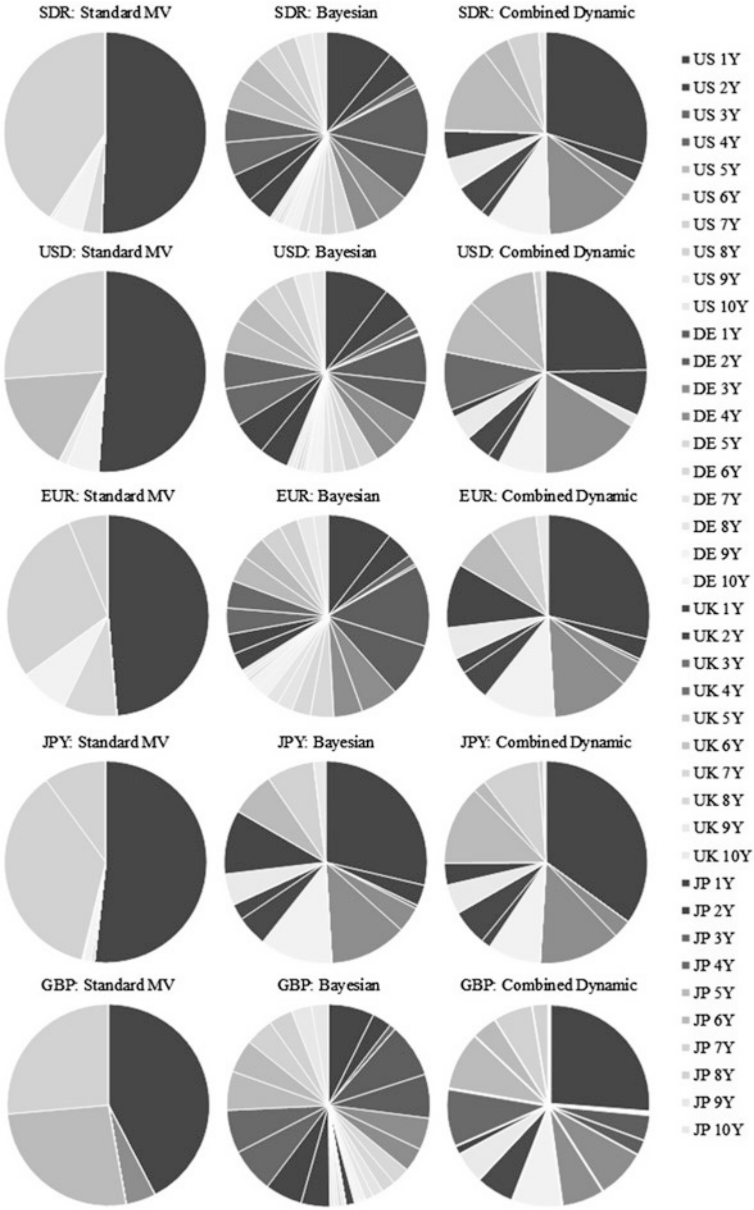

Fig. 9.6 Composition of optimised portfolios for a target duration of four years

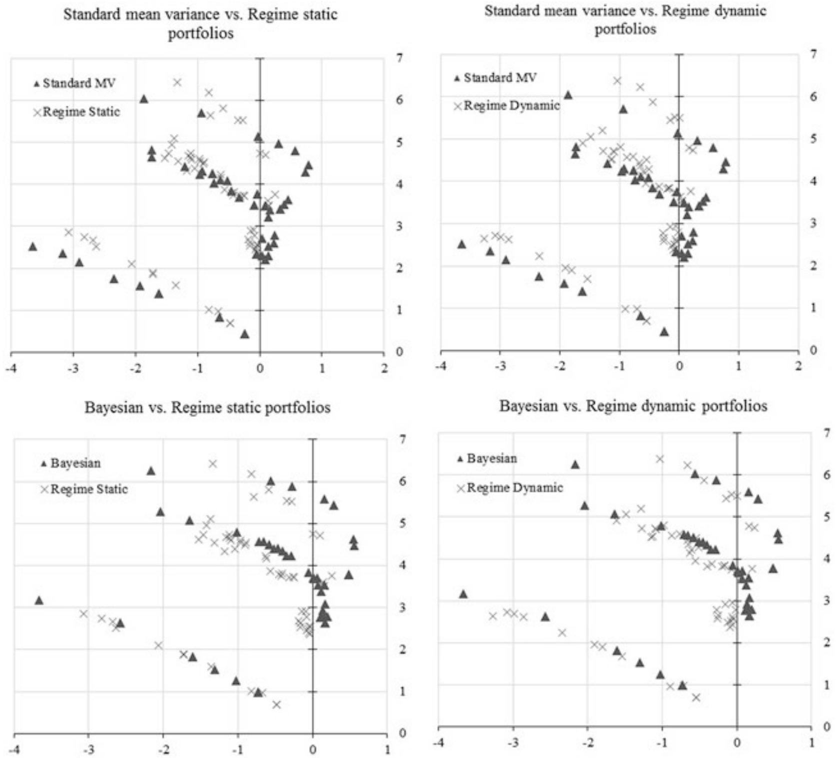

Fig. 9.7 Risk-return plots of regime portfolios versus standard mean-variance and Bayesian portfolios

Across the five numeraires, the combined regime portfolios (both static and dynamic) result in consistently higher mean returns than the full sample SMV and BAY optimisations (Table 9.4). At the same time, return volatilities of the combined static regime portfolios are broadly comparable to the SMV and BAY portfolios (slightly higher for the EUR and JPY numeraire and slightly lower for the GBP) while the combined dynamic portfolios tend to have, on average, slightly higher volatility. Also, average duration tends to be slightly higher for the combined regime portfolios—evidence of further risk taking.

Table 9.4 Absolute risk-return properties of standard, Bayesian, and regime portfolios

			MFI		*SRI*		*FTI*		*Combined*	
Numeraire/statistics	Standard MV	Bayesian	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic
SDR										
Mean return	3.3%	3.6%	4.1%	4.0%	3.6%	3.5%	3.9%	4.3%	3.7%	4.0%
Volatility	2.1%	2.1%	2.4%	2.3%	2.0%	2.0%	2.3%	2.6%	2.1%	2.3%
Sharpe ratio	0.72	0.86	0.94	0.94	0.88	0.86	0.91	0.95	0.89	0.94
Return-at-risk	0.2%	0.3%	−0.1%	0.4%	0.2%	0.1%	−0.2%	−0.5%	−0.2%	0.0%
Cond. return-at-risk	−0.1%	0.0%	−0.5%	−0.1%	−0.4%	−0.4%	−0.6%	−1.1%	−0.5%	−0.5%
Loss prob.	3.2%	2.1%	5.8%	3.2%	4.7%	4.2%	5.3%	6.8%	5.3%	5.8%
Modified duration	4.0	4.1	4.9	5.2	3.8	3.9	4.5	5.2	4.2	4.5
USD										
Mean return	3.4%	3.7%	4.1%	4.1%	3.7%	3.6%	4.0%	4.4%	3.8%	4.0%
Volatility	2.2%	2.2%	2.3%	2.3%	2.1%	2.1%	2.4%	2.7%	2.2%	2.3%
Sharpe ratio	0.70	0.86	0.96	0.97	0.88	0.85	0.90	0.95	0.88	0.95
Return-at-risk	0.1%	0.1%	−0.1%	0.4%	0.0%	0.1%	−0.4%	−0.7%	−0.3%	−0.2%
Cond. return-at-risk	−0.3%	−0.2%	−0.5%	−0.1%	−0.4%	−0.4%	−0.8%	−1.3%	−0.6%	−0.6%
Loss prob.	4.7%	3.2%	5.3%	2.6%	5.3%	4.7%	5.8%	6.8%	5.8%	5.8%
Modified duration	4.2	4.4	4.9	5.1	4.0	4.0	4.7	5.5	4.3	4.5
EUR										
Mean return	3.4%	3.7%	4.1%	4.0%	3.6%	3.6%	3.9%	4.3%	3.8%	4.0%
Volatility	2.2%	2.3%	2.6%	2.5%	2.2%	2.2%	2.4%	2.7%	2.4%	2.5%
Sharpe ratio	0.68	0.75	0.84	0.82	0.76	0.74	0.80	0.86	0.78	0.82
Return-at-risk	0.3%	0.2%	0.0%	0.2%	−0.1%	−0.1%	−0.2%	−0.3%	−0.2%	−0.1%
Cond. return-at-risk	0.0%	−0.1%	−0.5%	−0.1%	−0.5%	−0.5%	−0.6%	−1.1%	−0.5%	−0.5%
Loss prob.	2.6%	2.1%	4.7%	2.1%	5.3%	6.3%	5.3%	6.8%	5.3%	5.3%
Modified duration	3.8	3.9	4.6	4.9	3.6	3.7	4.2	4.8	3.9	4.2

JPY									
Mean return	1.4%	1.7%	2.3%	2.3%	1.7%	1.6%	2.1%	2.5%	1.9%	2.1%
Volatility	1.3%	1.5%	2.1%	2.0%	1.5%	1.4%	1.9%	2.5%	1.7%	1.9%
Sharpe ratio	0.93	1.03	0.99	1.02	0.99	1.01	0.98	0.93	0.99	1.00
Return-at-risk	−1.0%	−0.7%	−1.0%	−1.0%	−0.8%	−0.7%	−1.1%	−1.7%	−0.9%	−1.2%
Cond. return-at-risk	−1.6%	−1.4%	−1.9%	−1.8%	−1.2%	−1.2%	−1.8%	−2.1%	−1.6%	−1.7%
Loss prob.	13.7%	12.6%	16.8%	13.7%	16.8%	16.8%	19.5%	19.5%	18.9%	16.8%
Modified duration	3.5	3.7	4.7	4.9	3.3	3.2	4.3	5.0	3.9	4.1
GBP										
Mean return	4.4%	4.9%	5.1%	5.0%	4.9%	4.8%	5.1%	5.5%	4.9%	5.1%
Volatility	2.5%	2.6%	2.7%	2.6%	2.5%	2.5%	2.7%	3.0%	2.5%	2.7%
Sharpe ratio	0.60	0.76	0.82	0.81	0.80	0.75	0.81	0.86	0.78	0.83
Return-at-risk	0.9%	0.9%	0.6%	1.1%	0.5%	0.5%	0.5%	0.1%	0.4%	0.5%
Cond. return-at-risk	0.4%	0.2%	−0.2%	0.4%	−0.2%	−0.2%	−0.4%	−0.9%	−0.3%	−0.2%
Loss prob.	0.0%	1.6%	2.6%	0.5%	2.6%	3.7%	4.2%	4.7%	4.2%	2.6%
Modified duration	4.6	5.0	5.1	5.2	4.4	4.5	5.1	5.7	4.7	4.9

Source: Authors' calculations using return data from Bloomberg and Bank of America/Merrill Lynch

Notes: Summary statistics for the absolute return distribution of portfolios averaged over the individual selection criterions for the respective optimisation approaches. The figures are annualised and the return-at-risk and conditional return-at-risk are calculated for a confidence level of 95%

Indeed, tail risks appear to be markedly higher for the combined regime portfolios. That is, the combined regime portfolios show almost consistently lower returns-at-risk and higher loss probabilities. However, these differences are larger than what can be attributed to observed differences in volatilities and durations. Risk-adjusted returns—measured on the basis of the Sharpe ratio—of the combined regime portfolios are consistently higher compared to the SMV portfolios and mostly higher compared to the BAY portfolios, with exception of the JPY numeraire.

Next, we turn to an excess return perspective to analyse how regime portfolios perform relative to their corresponding SMV counterparts. We also show the evolution of the cumulative excess returns of the regime portfolios—averaged for the individual regime indicators separately—over the in-sample and out-of-sample period (Fig. 9.8). While we observe a fairly continuous increase in the cumulative return of the combined regime portfolio, the allocations based on the individual regime indicators perform quite differently over time. Both the Macro Fragility and Financial

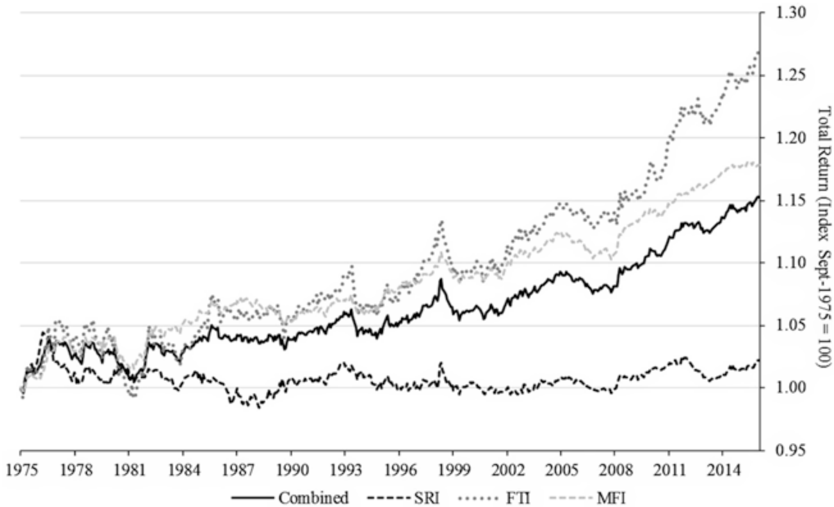

Fig. 9.8 Evolution of cumulative excess returns of regime portfolios over standard mean-variance portfolios

Turbulence-based portfolios show continuously increasing cumulative excess returns while the Systemic Risk-based portfolio implies essentially a sideways evolution of the cumulative excess return.

While taking a closer look to the summary statistics for the excess returns of the regime optimal and Bayesian optimal portfolios over SMV ones, we find that the combined regime portfolios consistently show positive excess returns for all numeraires with significance levels between 90% and 95%, and the excess returns of those dynamically rebalanced are slightly higher (Table 9.5). Nonetheless, the BAY portfolios show a consistent excess returns relative to the SMV portfolios. While the level of the excess returns is lower, they have higher statistical significance.

9.4.2.3 Stylised Facts

Regime-optimal portfolios demonstrate markedly different properties than portfolios based on SMV optimisation in an out-of-sample backtest. They imply more diversified holdings and show a lower inclination to corner solutions. In addition, the regime portfolios show higher mean returns at broadly comparable volatilities. Accordingly, their Sharpe ratios indicate better risk-adjusted returns. The excess returns of the combined regime portfolios compared are statistically significant and gradually increasing over time. At the same time, the tails of the regime portfolios are markedly fatter while return-at-risks are lower and loss probabilities are higher.

This combination of statistically significant excess returns, comparable return volatilities and fatter tail distributions may indicate that the regime portfolios constitute a factor. Arguably, positive factor returns could arise from a combination of two sources:

(a) SMV portfolios may turn out to be insufficiently diversified and risk-return inefficient in the out-of-sample period. The fact that the Bayesian portfolios show excess returns over them—at comparable volatilities and tail properties—may support this notion.
(b) Secondly, regime-optimised portfolios could be riskier than SMV portfolios, as indicated by fatter tails at comparable volatilities. The regime portfolios may therefore be compensated for the risk of regime switches in the asset return distributions. That is, the regime-based portfolios are a combination of allocations that are optimised for conditional asset return distributions in low and high fragility, turbulence,

Table 9.5 Excess returns relative to the standard mean-variance portfolio over the out-of-sample period

		MFI		*SRI*		*FTI*		*Combined*	
	Bayesian	Static	Dynamic	Static	Dynamic	Static	Dynamic	Static	Dynamic
SDR									
Mean	0.30%	0.77%	0.71%	0.32%	0.26%	0.58%	1.02%	0.44%	0.66%
T-statistic	(2.66)***	(2.70)***	(2.82)***	(1.51)	(1.37)	(2.37)**	(2.40)**	(2.01)**	(2.44)**
Standard deviation	0.46%	1.17%	1.04%	0.88%	0.78%	1.00%	1.74%	0.89%	1.11%
USD									
Mean	0.32%	0.70%	0.67%	0.31%	0.23%	0.59%	1.05%	0.39%	0.65%
T-statistic	(2.59)**	(2.47)**	(2.59)**	(1.52)	(1.29)	(2.34)**	(2.40)**	(1.85)*	(2.39)**
Standard deviation	0.50%	1.16%	1.05%	0.85%	0.74%	1.03%	1.80%	0.87%	1.12%
EUR									
Mean	0.22%	0.65%	0.54%	0.23%	0.17%	0.47%	0.88%	0.36%	0.53%
T-statistic	(2.73)***	(2.73)***	(2.64)***	(1.26)	(1.05)	(2.25)**	(2.33)**	(1.95)*	(2.32)**
Standard deviation	0.33%	0.97%	0.84%	0.73%	0.67%	0.85%	1.55%	0.76%	0.94%
JPY									
Mean	0.35%	0.87%	0.85%	0.29%	0.26%	0.75%	1.16%	0.54%	0.76%
T-statistic	(2.62)***	(3.00)***	(3.17)***	(1.42)	(1.40)	(2.45)**	(2.41)**	(2.28)**	(2.58)**
Standard deviation	0.55%	1.18%	1.10%	0.83%	0.75%	1.26%	1.97%	0.97%	1.20%
GBP									
Mean	0.41%	0.63%	0.57%	0.49%	0.39%	0.64%	1.07%	0.46%	0.67%
T-statistic	(2.69)***	(2.23)**	(2.31)**	(1.56)	(1.42)	(2.15)**	(2.17)**	(1.73)*	(2.09)**
Standard deviation	0.62%	1.17%	1.01%	1.29%	1.12%	1.22%	2.01%	1.08%	1.32%

Source: Authors' calculations using return data from Bloomberg and Bank of America/Merrill Lynch

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

and systemic risk regimes, respectively. However, the information on asset return distributions cross regimes, that is, unconditional volatilities and unconditional correlations, do not enter the portfolio construction.

In summary, the return distributions of regime-optimised portfolios differ significantly from those derived on the basis of standard techniques. The regime-optimised portfolios show superior Sharpe ratios—but contrary to our prior—they also imply fatter-tailed return distributions. With these results, regime-optimal portfolios appear to be a less obvious choice as a technique for robust optimisation. However, they may constitute an independent risk factor which could give rise to an expected excess return over standard portfolios.

9.5 Concluding Remarks

Regime identification algorithms can prove useful in many situations: for historical analysis, to better understand how financial markets have behaved under different scenarios; for forward-looking tests, because having the ability to foretell regimes could inform us about investor behaviour going forward; and, as shown in this chapter, for portfolio construction. By studying the properties and implications of regimes and regime changes, we can set up a state-dependent investment strategy.

A regime-based approach to portfolio construction has the flexibility to adapt to changing economic conditions. To perform it, indicators are required that allow to adjust in a timely fashion to changing states of the world. In this chapter, we propose three measures: the MFI, the FTI, and the SRI—all of which allow us to partition the state space into "low" and "high" risk states.

Furthermore, we show that the proposed regime indicators are useful in predicting future developed market government bond return distributions for several investment horizons. And, given some evidence of predictive power from the regime identification measures, we establish a multi-step algorithm to perform dynamic asset allocation.

This method seems to perform well when compared to SMV algorithms but faces challenges vis-à-vis a Bayesian approach. Though regime-optimal portfolios display higher Sharpe ratios, they represent higher tail-risk strategies, therefore being a less preferable choice when the investor's target is minimising the probability of loss. This is usually the case for more conservative sovereign investors. However, the higher excess returns delivered by regime-optimal portfolios appear to provide some evidence that they are a result of greater exposure to risk premia.

This approach can be used to support strategic or tactical asset allocation decisions; however, it should be adjusted for some practical issues. First, the usefulness of these (or other regime indicators) could be explored in a broader asset class universe; for example, one comprising equity, credit and even commodities markets, such as gold. Second, the dynamic optimisation methodology can be extended to allow for automatic updating of the optimal "high" and "low" regime return distributions, thereby permitting the conditional efficient frontiers to be refreshed as often as the portfolio is rebalanced. Third, a finer partition of the state space could be defined. Fourth, one could also try to calibrate the optimal rebalancing horizon—this could help minimise transaction costs and find statistical evidence of excess returns for medium and long-term investors. Finally, while our use of regime identification aimed to construct a dynamic portfolio *along* regimes—perhaps, one can try and construct a portfolio that is robust *across* different states of the world.

Notes

1. See Brida, Anyul & Punzo 2006. "A review on the notion of economic regime" for a review of the basic notions and definitions of economic regime and regime switching.
2. Refer to the website http://www.nber.org/cycles.html for the US Business Cycle Expansions and Contractions dates and durations.
3. Previous applications of principle component analysis for regime identification include Billio et al. 2010. "Measuring systemic risk in the finance and insurance sectors"; Pukthuanthong and Roll 2009. "Global market integration: An alternative measure and its application"; and Kritzman et al. 2011. "Principal components as a measure of systemic risk".
4. Mahalanobis 1927. "Analysis of race-mixture in Bengal", used several characteristics of the human skull to analyse dissimilarities between various castes and tribes in India. He later proposed a more generalised statistical measure, the Mahalanobis distance, which takes into account both the standard deviations of individual dimensions and the correlations between

dimensions (see Mahalanobis 1936. “On the generalised distance in statistics”). For applications of the measure in finance see Chow et al. 1999. “Optimal portfolios in good times and bad”; Kritzman and Li 2010. “Skulls, financial turbulence, and risk management” and references therein.

5. Specifically, we use the total return indices of U.S. Treasuries, investment grade global corporate bonds, the MSCI World U.S. and Non-U.S. equity indices and the GSCI Commodity Index.
6. For simplicity, only *t*-statistics and significance level for typical intervals are shown. The intercept and slope values are available upon request.
7. The expected value is normalised based on data from the in-sample period. The weight of the low-regime optimal portfolio corresponds to the normalised value of the expected regime indicator (x) and the weight of the high-regime portfolio corresponds to 1 minus the normalised value ($1 - x$). The dynamic regime optimal portfolio then is: $w^*_{\text{dynamic}} = x \cdot w^*_L + (1 - x) \cdot w^*_H$.

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