CHAPTER 2

Setting the Appropriate Mix Between Active and Passive Management in the Investment Tranche of a Foreign Reserves Portfolio

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2.1 Introduction

In their evaluation of central bank practices, Morahan and Mulder (2013) find that 56 of 67 foreign reserves managers report having deviation limits around the benchmark, 86% of which are with the purpose of active management. This indicates that central banks believe that there are opportunities to earn "alpha" that can be captured through active management strategies, either with external managers or with an internal active management program. Central banks see in active management a tool by which they can react

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to potential financial market inefficiencies to enhance returns, which is often the least important objective of foreign reserves managers.[1] Furthermore, some central banks set an active management framework in order to gather market intelligence. As shown by Jeffery et al. (2016), one of the main reasons central banks conduct gathering of market intelligence is to improve the information they can use for foreign exchange reserves management operations. Particularly, they seek information related to money markets, sovereign rates, currencies, and commodities, among others.

Many institutional investors, including central banks, believe that alpha is achievable on a sustainable and scalable basis, as mentioned in Berk and van Binsbergen (2016). However, there also exists a vast literature arguing for the efficiency of financial markets and the difficulty of finding and exploiting arbitrage opportunities leading to sustainable and scalable active management returns, as shown in Fama and French (2010). Merton (2014) introduces three sources of alpha (financial services, dimensional, and traditional alpha), partly explaining the contradiction between the empirical and the theoretical research, and describes which of them are sustainable and scalable and which are not. In this chapter, his analysis is viewed through the perspective of a central bank in order to identify the availability and the sources of alpha opportunities.

If a central bank identifies its competence to assess any of the sources of alpha, then it has to determine the proper amount it will invest in these strategies. The approach taken in this chapter for traditional alpha is contrary to the usual mean-variance approximation which is regularly used in the definition of strategic asset allocation. The suggested approach follows the Kelly criterion, which maximizes terminal wealth through a maximization of the portfolio's geometric mean return.

The intuition behind using the Kelly criterion for setting the appropriate mix between active risk and benchmark risk relies on the positive features of the methodology, as risk of ruin is eliminated and the final wealth of the seemingly sustainable and scalable alpha is maximized. Given that it is almost certain that the wealth generated with this approach is higher than the wealth generated with a risk-adjusted return approach in a long-time horizon, the Kelly criterion approach is suitable for a tranche invested for a long-term horizon and whose main objective is to maximize returns. For central banks, this is the case for an investment tranche, where excess foreign reserves are invested once all the main liquidity and safety goals have already been accomplished (as shown in García Pulgarín et al. 2015).

It was Daniel Bernoulli in the eighteenth century who first used a logarithmic utility function to solve the St. Petersburg paradox.[2] Later, Kelly (1956) reviewed its properties to define an optimal fraction that a gambler should bet when she or he has noisy private information and is betting for a substantial amount of time. Among the properties that Kelly discovered were that under this technique the gambler never risks ruin, and that the terminal wealth is very likely to be the highest among all strategies. The strategy may have high volatility, and betting more (less) than the optimal fraction increases (decreases) the growth of capital. Subsequently, as mentioned in Thorp (2006), both Claude Shannon and Edward Thorp used the Kelly fraction to obtain the series of blackjack bets that maximizes the expected value of the logarithm of wealth for a gambler with a probability of success higher than one half. Afterwards, they used the Kelly fraction in order to find the appropriate percentages invested in different market stocks.

Furthermore, Thorp (2006) links the fundamental problem of a gambler and an investor. For him, the former seeks positive expectation in betting opportunities and the latter tries to find investments with excess risk-adjusted expected rates of return. Both assess the probabilities of accessing the favorable opportunities and decide how much capital to bet in those strategies. The analogy can also be made with a portfolio manager seeking to set the amount of capital to be invested in a traditional alpha strategy.

This chapter is structured in five sections. The first one is this introduction. The second section overviews Merton's definitions of the sources of alpha and analyzes whether they are available to central bank foreign reserves managers. Afterwards, the third section describes and discusses the Kelly criterion. The subsequent section shows a simulation that compares the Kelly criterion methodology to a traditional risk-return perspective to set the optimal mix between active and passive management, as suggested by Violi (2010) following the Treynor and Black model. Finally, the fifth section gives some concluding remarks.

2.2 Sources of Alpha

Merton (2014) defines the super-efficient maximum Sharpe ratio portfolio of risky assets as the combination of the passive benchmark market portfolio, which holds an efficient diversification, and the active management strategies that can be incorporated in the portfolio, given the alpha resulting from the failure of the standard Capital Asset Pricing Model

(CAPM) to fit the data. The active components encompass bottom-up strategies, top-down strategies, and efficient market timing. Given this structure, Merton (2014) considers the possibility of higher Sharpe ratios over the passive benchmark as a consequence of the failure of CAPM.

He defines three distinct sources of alpha, two which he outlines as sustainable and scalable, and one that is not. The sustainable and scalable options are the financial services alpha and the dimensional alpha. The former is the result of market frictions arising from regulations and the interaction between financial intermediaries and the market. The latter is a result of risk premiums available from dimensions of risk different from market beta, considering the fact that the CAPM fails as not all investors hold the same portion of risky assets and the market portfolio is not mean-variance efficient. The neither-sustainable-nor-scalable source of outperformance is the traditional alpha earned by asset managers who are faster, smarter, or with better models or inputs.

The financial services alpha is a result of market participants that can take advantage of the setbacks and constraints of other more regulated and controlled market participants. The impediments and restrictions include (1) leverage inefficiencies or borrowing constraints; (2) short-sale restrictions; (3) institutional rigidities from regulation restrictions or requirements; and (4) taxes and accounting rules. A class of investors with the ability to take advantage of this type of alpha are hedge funds with lighter regulations and that can identify rigidities that are binding. Other institutions can also take advantage of this type of alpha, particularly if they have (1) a strong credit standing, (2) a long investment horizon, (3) flexible liquidity needs, (4) a large pool of assets, or (5) significant reputational capital. Such financial intermediaries can follow trading strategies that ease the impact of market frictions that affect other institutions, thereby earning outsized returns. However, earning this alpha requires first identifying securities that are impacted by the market rigidities discussed above.

A central bank has very limited access to financial services alpha since it is not a financial intermediary and its usual risk constraints prevent it from investing in institutions that gain from light regulations. Although central banks in developed countries may have long investment horizons, larger pool of assets, and flexible liquidity needs, they may still be curtailed in accessing financial services alpha to safeguard their reputational capital and abide by their risk aversion standards. In the case of most central banks in emerging and frontier countries, the risk aversion constraints demand high amounts of liquidity that are usually invested under a short-time horizon. Nonetheless, some central banks could have access to this type of alpha if they took advantage of their large pool of assets, although this is more often perceived as a disadvantage as they invest most of the times in very liquid markets. Another source of this type of alpha for central banks can be through asset substitution, where liquid on-the-run US treasury bonds are replicated with less liquid off-the-run US treasuries or agency bonds, to take advantage of liquidity premiums.

Dimensional alpha[3] exists as a result of uncertainty about the future investment opportunity set, uncertainty about liquidity, uncertainty about inflation and consumption goods in the future, and the hedging roles for securities in addition to diversification. Merton (2014) indicates that the existence of this type of alpha is consistent with an efficient financial market, since this type of alpha is earned from exposure to risks that investors are willing to pay to avoid. Thus, institutions can earn this alpha if their valuation of exposure to the additional dimensions of risk (other than the market risk factor) differs from the market price of such risks. Typically, institutions that can do this are hedge funds, long-term investment funds, and private equity firms.

According to Merton (2014), the following conditions should be met for identifying a dimension of risk with a premium: (1) there is a priori reasoning supported by economic theory; (2) it is persistent through time; (3) it is pervasive across different geopolitical borders; (4) it is monotonously increasing in the exposure of the security to the risk factor; (5) the exposure to the risk factor is not sensitive to precise parameter estimates; and (6) the exposure can be scalable in a cost-effective way. Some examples of recognizable dimensions different from the market that are scalable are the size of the company, the ratio of book to market value, the ratio of profits to market value, and liquidity (see Fama and French 1996; Pastor and Stambaugh 2003).

Limitations on the asset space of foreign reserves of central banks place a constraint on central banks' ability to gain dimensional alpha. According to Morahan and Mulder (2013), from a sample of 64 central banks, only two report investing in real estate investment trusts (REITs), both of them advanced countries, while only nine report investing in equities. Most central banks invest exclusively in traditional foreign reserves asset classes (government bonds, credit-related fixed-income securities, and gold). Nonetheless, there are a few empirical dimensions of risks with additional risk premiums, which a central bank can take advantage of, particularly if the central bank has enough foreign reserves to set an investment tranche, with a longer time horizon and with the objective of maximizing returns. One of the dimensions that can be considered under this scenario is liquidity.

Finally, the last source of alpha, the traditional alpha, is the only one described by Merton (2014) as neither sustainable nor scalable. Some conditions that allow for the existence of this alpha are market participants with access to non-public information or the ability to time the market. Like many academic studies, Merton (2014) stresses the unavailability of this type of alpha. Fama and French (2010) indicate that active investment is a zero-sum game; therefore, if some active investors have positive alpha before costs, it is at the expense of other active investors. They also point out that most active management returns do not compensate for the fees charged by such managers. French (2008) elaborates on the negative net returns of active management, and estimates that the typical investor would increase her or his average annual return by 67 basis points from 1980 to 2006 if she or he switched to a passive market portfolio. Furthermore, Bernile et al. (2014) present an argument for the lack of sustainability of the traditional alpha by showing that institutions on average are not skilled and their superior intra-quarter performance reflects only possible opportunistic access to short-term local information. Given this evidence, Foster and Warren (2013) explain the puzzling prevalence of active management as reflecting investors' beliefs in their ability to dynamically manage their allocations to external managers based on their investment performance. They provide evidence that investors believe that they have an above-average ability to select good managers, and they also believe in their ability to pursue an efficient dynamic strategy to replace bad-performing asset managers. They also show that some retail investors are impaired by behavioral biases, and use available information rather poorly.

It is important to point out, however, that there exists a contrarian strand of opinion about the ability of active management to generate traditional alpha. Andonov et al. (2012) note that institutional investors add value through active management, although some alpha may be attributable to momentum. Berk and van Binsbergen (2016) find sustainability of traditional alpha for as long as ten years into the future; additionally, investors seem to be able to identify and reward these skillful asset managers, given that better-performing funds collect higher aggregate fees. Likewise, in the fixed-income space, Aglietta et al. (2012) show that active management accounts for a substantial portion of performance, when aggregated with two other sources of return (market return and return from the asset allocation policy).

Therefore, there is no consensus on whether traditional alpha is achievable in a sustainable and scalable basis. The large number of central banks with an active management program seems to indicate belief in their ability to find highly skilled asset managers. We believe that the lack of academic consensus on the benefits of active management may suggest that central banks may find it more profitable to pursue sustainable and scalable sources of outperformance.

2.3 Adding the Sources of Alpha to the Market Portfolio

Whether a central bank has access to financial services or dimensional alpha, or supports the premise of traditional alpha, selecting the risk allocation of these strategies should not be a subjective matter.

Financial services alpha should be added to the maximum allowed by the portfolio constraints, as this type of alpha is a result of market regulations and intrinsic advantages that should be maximized by any investor.

The easiest way to add dimensional alpha to the mix of the super-efficient maximum Sharpe ratio portfolio of risky assets is through a mean-variance framework that allows the inclusion of new beta sources. A central bank with a long investment horizon that has the ability to access dimensional alpha linked to liquidity strategies can follow Lo et al. (2003), and optimize over the mean-variance-liquidity frontier to account for the liquidity factor. They construct liquidity indices of each asset from five dimensions of liquidity, viz., trading volume, logarithm of trading volume, turnover, percentage bid-ask spread, and Loeb price impact function. A linear form of the aggregated liquidity metric—that depends on the portfolio weights—is then additively introduced into the mean-variance objective function.

Lastly, one possible approach to add traditional alpha is by setting an optimal fraction of allocation to alpha-generating strategies by maximizing the expected value of the logarithm wealth, as done with the Kelly fraction by gamblers and investors.[4] Contrary to the usual maximization of risk-adjusted returns, measured by the Sharpe ratio, the Kelly criterion relies on the maximization of the terminal wealth. More concretely, the criterion maximizes the portfolio's geometric mean return. Generally, this optimized portfolio is not the same one that maximizes the risk-adjusted returns. Although the Kelly criterion may result in the maximum exponential growth rate of wealth, the solution is not the most efficient in terms of minimizing short-term risk. Given this caveat, when is it relevant to use this metric to select the appropriate mix between active and benchmark strategies?

The logic behind implementing the Kelly criterion for setting the appropriate mix between active risk and benchmark risk relies on the fact that the investment tranche is managed with the return-maximization perspective. The manager of this tranche is unaffected by short-term risks and seeks to maximize long-term returns. Such a manager seeks active investment strategies under the assumption that she or he has additional information that increases the odds of a positive alpha, following the constraint of avoiding financial ruin (the size of the investment tranche reducing to zero).[5]

The optimal Kelly fraction, which avoids ruin, can be estimated as follows. Assuming an investor (gambler) with N investments (bets) to place at each time invests (gambles) a fixed portion k of available capital. If there are n successful investments and $N - n$ losses, then the capital is:

$$V_{N,n} = \left(1 + kR_w\right)^n \left(1 + kR_L\right)^{N-n} V_0 \tag{2.1}$$

where R_w is the reward when the investment is successful and R_L is the loss when the investment is unsuccessful. The growth rate is given by:

$$G = \frac{1}{N}\log\left({V_{N,n}}\big/{V_0} \right) = p\log\left(1 + kR_w\right) + q\log\left(1 - kR_L\right) \tag{2.2}$$

where p stands for the probability of a successful outcome and q for the probability of an unsuccessful one. When this log wealth is maximized, the resulting optimal Kelly fraction is:

$$k^* = \frac{pR_w - qR_L}{R_w R_L} \tag{2.3}$$

Under these conditions, as shown by Thorp (2006), the log wealth is maximized with a unique number k^*. Values lower than that level result in a positive expected growth coefficient, where the expected final wealth will be higher than the initial wealth. However, values above the optimal Kelly fraction start showing a decrease in the expected growth coefficient, even at one point making the coefficient negative (see Fig. 2.1).

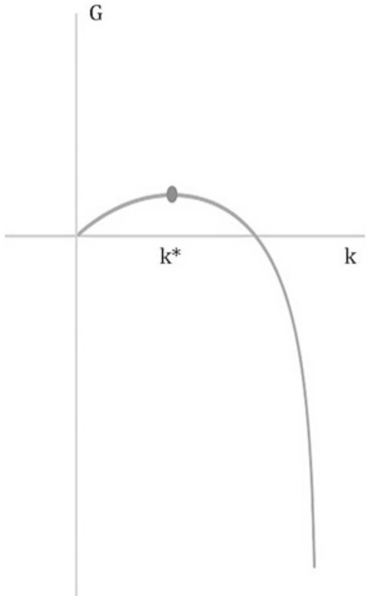

Fig. 2.1 Expected growth coefficient versus the Kelly fraction

The previous solution assumes a very simple scenario, where the investments behave as a flip of a biased coin with uneven payments. It follows a discrete probability distribution. However, the solution can be generalized to continuous outcomes and non-linear payoffs by estimating the numerical solution of:

$$V_N = V_0 \prod_{n=1}^{N} \left(1 + kR_n\right) \tag{2.4}$$

For selecting the appropriate mix between active and passive management with a single asset manager or when taking into account the whole amount of the active management program, Eq. 2.4 is solved assuming a stochastic distribution. Once the problem is expanded to more investment sources or bets, more optimal Kelly fractions are estimated. The growth rate for a discrete problem with two bets with uneven payments is given by:

$$\begin{aligned} G = {} & p_1 p_2 \log\left(1 + k_1 R_{w1} + k_2 R_{w2}\right) + p_1 q_2 \log\left(1 + k_1 R_{w1} - k_2 R_{L2}\right) \\ & + q_1 p_2 \log\left(1 - k_1 R_{L1} + k_2 R_{w2}\right) + q_1 q_2 \log\left(1 - k_1 R_{L1} - k_2 R_{L2}\right) \end{aligned} \tag{2.5}$$

When the problem of selecting the appropriate mix between active and passive management is extended to a set of asset managers, the problem is expanded to various optimal fractions. The following section describes a simulation that models different types of asset managers and compares the Kelly criterion results with the ones obtained with the Treynor and Black (1973) model.

The solution of the Kelly criterion is simple and intuitive. Moreover, in terms of leverage, the Kelly fraction depends on the product kR. Additionally, the risk of ruin is null and terminal wealth is maximized, properties that align with the objectives of an investment tranche. Furthermore, short-term volatility is not a pertinent issue when the problem is limited to defining only one fraction, the percentage allocated to the overall active management program. As no diversification benefits are considered, the difference with a Sharpe ratio-based model should not be substantial. An additional and possibly more important caveat is that the stability of profitability depends on knowing the correct parameters, which, in the context of this chapter, are the expected return distributions of asset managers.

2.4 Simulation

This section evaluates three distinct methodologies for setting the appropriate mix between active asset managers and a passive portfolio in the investment tranche of a foreign reserves portfolio. The passive portfolio is assumed to be composed by US Treasuries with a maturity between one and three years. The three methodologies to be considered are (1) Kelly criterion, maximization of the portfolio's geometric mean return; (2) the Treynor-Black model, mean-variance optimization; and (3) the alternative C, the option that assigns an arbitrary constant value of 90% to the strategy to the active asset managers. Alternative C is included in order to examine the outcomes when a significant portion is assigned to an active management strategy, without taking any leverage, constant values around 90% are expected to deliver similar results.

Violi (2010) describes the Treynor and Black (1973) model as a solution that allows an investor to set the mix of active and passive portfolio by maximizing the active Sharpe ratio. He treats the active and passive portions as two separate assets to then set a security selection framework. Hence, the problem is set with a quadratic utility function that considers the first two moments of the excess return distributions.

The simulation first considers three different asset managers, with the same expected alpha, but with distinct return distributions. The three are tested independently with the methodologies mentioned above to find the proper amount to be invested when they are mixed with the passive portfolio. In other words, we find the optimal allocation to the active portfolio separately for each of the asset managers following the three mentioned methodologies. Then, the Kelly criterion framework is tested for a portfolio that includes the three asset managers in the same portfolio. Thus, the weights are assigned considering the interaction between the three managers.

In order to set the distributions of the excess returns of the asset managers, this chapter follows Berk and van Binsbergen (2016). They use a sample of 5974 funds, gathered from the Center for Research in Security Prices survivorship bias-free database. The distribution of active returns has a positive mean value added, the percentage with less than zero is 57% and the distribution is positively skewed. In this chapter, this type of asset manager is represented with a gamma function, as shown in Fig. 2.2, identified as asset manager 1. Asset manager 2 is assumed to have the same expected value as asset manager 1, but its distribution is given by a t-student

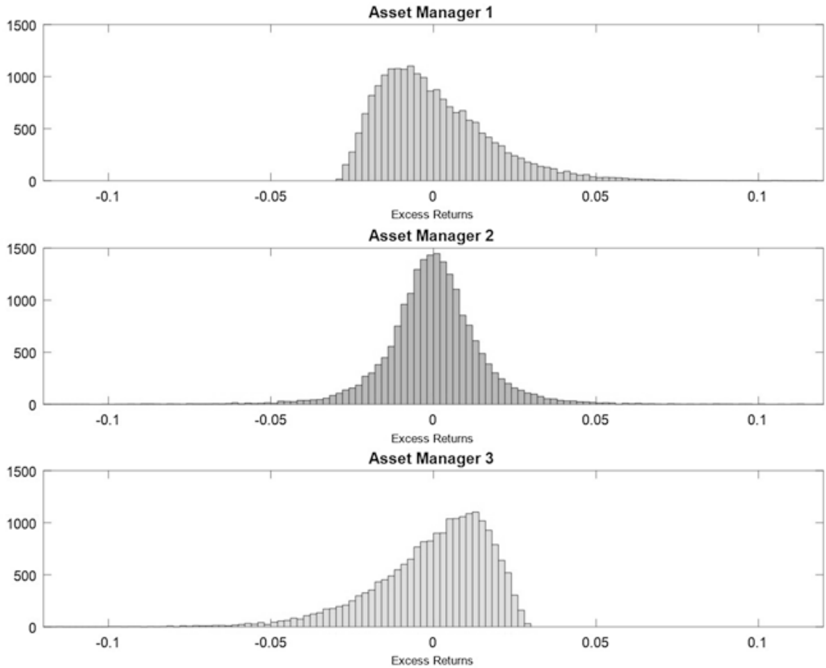

Fig. 2.2 Asset managers' excess returns distributions. The units of the Y-axis are number of funds

distribution. Finally, asset manager 3 is assumed to have the inverse marginal density function of asset manager 1, and therefore, it is negatively skewed, but the expected value is the same as the other distributions.

Table 2.1 shows the optimal fractions estimated independently under the three different methodologies for the various managers. The return distributions do not affect the amount allocated in the mean-variance model, as the methodology analyzes only the first two moments of the distributions (mean and variance). The amounts allocated with the Kelly criterion are large, but are somewhat limited by the risk of loss included in the distributions of the excess returns of the asset managers.

Figure 2.3 depicts the distributions of the terminal portfolio value when selecting the Kelly criterion as the methodology to set the mix between active and passive management. In a short-[6] and long-time horizon,[7] it can be seen that the methodology eliminates the probability of ruin. Nonetheless, the volatility and the probability of loss are high.

Table 2.2 summarizes the statistical analysis of the results of the three methodologies for the three asset managers—estimated separately. The Treynor and Black (1973) model shows a lower standard deviation; this is expected as the variance is one of the considerations within this framework. In the short-term horizon, the average cumulative excess returns are maximized with alternative C, which invests more in the asset managers compared to the other two options.

However, this option shows the highest volatility, the highest probability of loss and has a probability of ruin higher than zero for all the asset managers. The option that uses the Kelly criterion gives the highest average cumulative excess returns in a long-term horizon. This option and also the Treynor-Black optimization show a probability of ruin equal to zero and their probability of loss is very close.

As mentioned in the previous section, the methodology of the Kelly criterion can be expanded to include more than one asset manager. Figure 2.4 depicts the allocation of the portfolio once the three asset managers are

Table 2.1 Amount allocated to active asset managers

	Kelly criterion	*Treynor-Black*	*Alternative C*
Asset Manager 1	42.52%	33.24%	90%
Asset Manager 2	50.03%	32.50%	90%
Asset Manager 3	57.53%	33.13%	90%

Source: Author's calculations

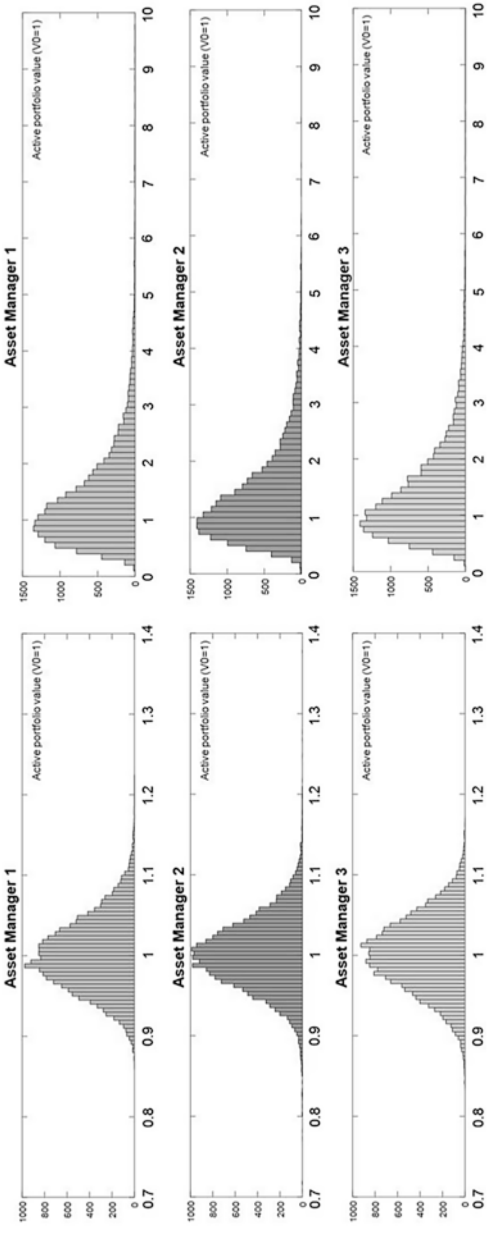

Fig. 2.3 Terminal portfolio value applying the Kelly criterion: **a**) Short-time horizon (graphs at the left), **b**) Long-time horizon (graphs at the right)

Table 2.2 Results for allocation for the overall active management program

	Standard deviation (excess returns)	*Average cumulative excess returns (long term)*	*Average cumulative excess returns (short term)*	*Probability of ruin*	*Probability of loss*
Kelly criterion					
Asset Manager 1	0.74%	53.02%	0.17%	0.00%	41.16%
Asset Manager 2	0.86%	63.69%	0.19%	0.00%	40.94%
Asset Manager 3	1.00%	76.21%	0.17%	0.00%	40.90%
Treynor-Black					
Asset Manager 1	0.58%	39.43%	0.15%	0.00%	38.21%
Asset Manager 2	0.56%	37.54%	0.14%	0.00%	38.70%
Asset Manager 3	0.57%	38.70%	0.18%	0.00%	38.59%
Alternative C					
Asset Manager 1	1.56%	45.27%	0.19%	0.65%	57.77%
Asset Manager 2	1.55%	47.69%	0.22%	0.79%	58.40%
Asset Manager 3	1.56%	43.18%	0.31%	0.98%	58.50%

Source: Author's calculations

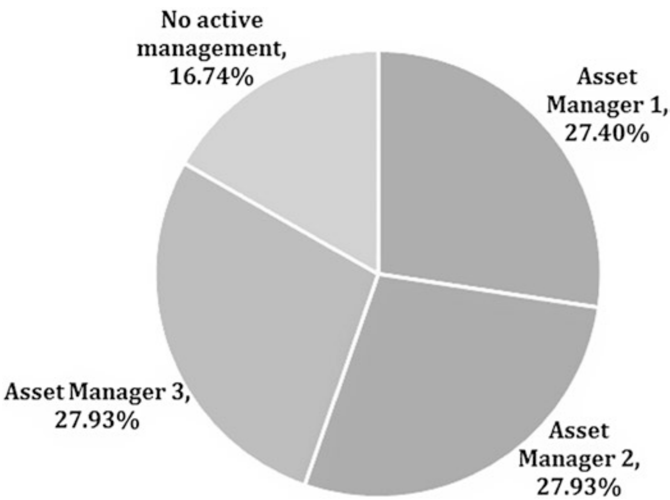

Fig. 2.4 Allocation of the asset managers within the same portfolio

Table 2.3 Results for allocation of the asset managers within the same portfolio

Standard deviation (excess returns)	0.83%
Average cumulative excess returns (long term)	126.52%
Average cumulative excess returns (short term)	0.37%
Probability of ruin	0.00%
Probability of loss	27.91%

Source: Author's calculations

considered for the same portfolio. In this case, the portion with no active management is reduced to 17%, while the rest is distributed almost equally among the three asset managers.

Table 2.3 shows the summary of the statistical analysis of the previous portfolio. The average cumulative excess returns increase both in the short-term and long-term horizons when compared with the options that considered every asset manager individually. The probability of loss decreases as in this case the negative outcomes of some active asset managers can be compensated with positive outcomes of the other active asset managers. The probability of ruin remains zero. However, the standard deviation increases compared to the options when the asset managers were considered individually.

2.5 Conclusion

This chapter reviews the three sources of alpha (dimensional, financial services, and traditional alpha) that are available for different types of investors, according to Merton (2014). The ability to access to each particular alpha relies on each investor's intrinsic characteristics; such is the case of central banks, which should consider their reputational capital and their risk aversion in order to gain exposure to them. The literature review shows contradictory conclusions as to whether a sustainable and scalable traditional alpha is feasible. Thus, to take advantage of traditional alpha strategies, a thorough analysis should be performed.

If a central bank believes that the traditional alpha is achievable, this chapter suggests setting the appropriate mix between active and passive management in the investment tranche of a foreign reserves portfolio with the Kelly criterion. The latter, considering that the behavior of an active investor resembles that of a gambler, who assumes an intrinsic advantage that gives higher probabilities of success and occasional uneven payments with higher rewards for successful outcomes and lower potential losses for unsuccessful events. Additionally, the characteristics of the Kelly criterion match those of an investment tranche of foreign reserves; more emphasis is on long-term returns than on short-term volatility.

Nonetheless, if short-term volatility is a crucial concern, the Kelly criterion can at least be considered to set an appropriate range in which the portion assigned to the active management program will fluctuate. As lower values of the Kelly fraction will still provide a positive expected growth coefficient, higher values might result in a positive probability of ruin, as shown in the empirical simulation done in this chapter. MacLean et al. (2010) show that security can be traded for lower growth by using a negative power utility function of applying a fractional Kelly strategy. Additional, it is important to note that the Kelly criterion can be extended to an active management program with various asset managers or sources of alpha.

Besides these benefits, it is important to highlight several shortcomings of the Kelly criterion. This strategy maximizes exclusively the expected logarithmic utility and ignores other possible utility functions. Furthermore, stability of the results relies on a priori knowledge of the excess return distributions of the asset managers. Moreover, despite the long-run growth properties of the strategy, it can be subject to low return outcomes and high impacts of short-term volatility.

Notes

1. The investment objectives of the foreign reserves of central banks are safety, liquidity, and return. Some central banks consider either safety or liquidity the first priority. Return is often given less importance than the other two objectives.
2. As explained by Hayden and Platt (2009), in the St. Petersburg paradox, the house offers to flip a coin until it comes up heads. The house pays $1 if heads appears on the first trial, otherwise the payoff doubles each time tails appears. The game stops, as well as the compounding, when the coin results in the first heads and the payment is given. By definition, the St. Petersburg gamble has an infinite expected value. However, most people share the intuition that no more than a few dollars should be offer to play.
3. It is feasible to link the dimensional alpha with Lo's (2012) Adaptive Markets Hypothesis (AMH). Lo suggests that the following assumptions of the relationship of risk and return are not likely under the current market conditions: (1) there is a linear relationship; (2) the relationship is constant

through time; (3) the relationship can be estimated with robust parameters; (4) all investors have rational expectations; (5) returns are stationary; and (6) markets are efficient. He recognizes that human behavior is not guided only by logical reasoning, and therefore, AMH seeks to explain how behavior is affected by the changing market conditions. One of the implications of AMH is that market efficiency is a function of the degree to which market participants have adapted to the market environment. Thus, the alpha converge to the beta as the degree of adaptability increases; investors that take advantage of this transition are investing in dimensional alpha.
4. The use of the Kelly criterion can be expanded to the other two sources of alpha; however, the scope of this chapter is to the scenario when the central banks believe to have additional information or timing abilities than the average market investor.
5. Another crucial point of the discussion is also the ability of the central bank to set an investment tranche; a rigorous analysis of the main liquidity needs should be done before going forward and setting this tranche.
6. The short time horizon is exemplified with a one-year horizon.
7. The long time horizon is considerably large, in order to represent the benefits of the Kelly criterion.

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