Collaborative Engagement Through Mobile Technology in Mathematics Learning

Mina Sedaghatjou and Sheree Rodney

Abstract When a group of students come together to engage in negotiation about mathematical ideas and activities, they draw on each other's cultural experiences for a shared understanding of mathematical meanings. This chapter considers how mobile technologies, along with children's collaborative engagements, can enhance mathematical learning. We adapted previous findings regarding touchscreen-based interactions to assess and analyse how mathematical learning occurs when learners interact with mobile technologies and with their peers. We also utilized StudioCode software to analyse children's interactions with a mathematical tool in order to better understand their collaborative practices and how they reflect using touch-screen-based devices. Our conclusions emerge from children's use of an iPad application called *TouchCounts*, which aims to develop number sense. Overall, we found that the one-to-one multimodal touch, sight, and auditory feedback via a touchscreen mobile device served to assist children's collaborative engagement and helped children develop their number sense.

Keywords Engagement · iPad · Numbers · Touchscreen-based device Mathematics · Mobile technologies · Interaction · Collaborative engagement Reflection

Mobile Technology in Learning Mathematics

In recent years, it has become apparent that there is a shift in the way society perceives and uses technology to enhance learning, from the ancient Greek invention of the abacus, to slide rules, calculators and now more sophisticated and complex inventions such as digital technologies. Papert (1980) articulated the changing ways

M. Sedaghatjou (✉) · S. Rodney
Brock University, St. Catharines, Canada
e-mail: msedaghatjou@brocku.ca

S. Rodney
e-mail: sgilling@sfu.ca

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in which technology enhances learning and, for him, the invention of computers has affected "the way people think and learn," (p. 3) and occasioned debate about how these changes enhance mathematics learning. Noss and Hoyles (1996) focused on "what the computer makes possible for mathematical meaning-making" (p. 5) suggesting that interaction with computers may facilitate children's development of mathematical meanings. In keeping with Papert's idea, many researchers have sought to integrate digital technologies into mathematics learning environments (Clements, Sarama, Yelland, & Glass, 2008; Hollerbands, Laborde, & Strasser, 2008; Laborde, Kynigos, Hollebrands, & Strasser, 2006; Sinclair, Arzarello, 2010). In particular, researchers (Ainley & Ainley, 2011; Drivjers, Mariotti, Olive, & Sacristán, 2010; Hoyles & Lagrange, 2010; Sedaghatjou & Campbell, 2017; Sinclair, Chorney, & Rodney, 2016) have also suggested that the use of mobile technologies, if integrated with a suitable pedagogical structure, can make mathematics more pleasing, meaningful, practical, and engaging. That is to say, mathematics learning can be more appealing when children are provided with experiences involving the use of mobile technologies. These experiences enable children to develop strategies to better understand mathematical concepts while maintaining a sense of connection with their peers during an activity.

In this chapter, we consider ways in which touchscreen interactions with mobile technologies influenced the way a group of pre-school children engaged in mathematics learning. By mobile technologies we mean devices such as Personal Digital Assistant (PDA) Devices, iPads, cell phones, iPods, e-readers and similar handheld devices, which are increasingly being used as educational tools (Moyer-Packenham et al., 2016). We discuss how collaborative engagement—the convergence of common features and mutually constructed practices—along with mobile technologies, facilitate the development of the ways children reflect on their work and attach meanings to mathematical ideas. Furthermore, we discuss the collaborative engagement of a group of young children as they perform the task of "make 100" using *TouchCounts*.[1]

TouchCounts is a multimodal iPad application that provides children with the opportunity to create and represent numerical quantities. For example, by touching the screen in *TouchCounts*' "Operating World" with three fingers, a circle called a "herd" containing three small discs and labelled "3" appears, while *TouchCounts* says the number "Three" (Fig. 1a). When there are more than two herds on the screen, children can perform the gesture of *pinch-in* (Fig. 1b) for addition. *TouchCounts* represents larger quantities with larger circles. The *pinch-out* action (Fig. 1c) splits a herd. This gesture performs a subtraction operation.

[1] http://touchcounts.ca/.

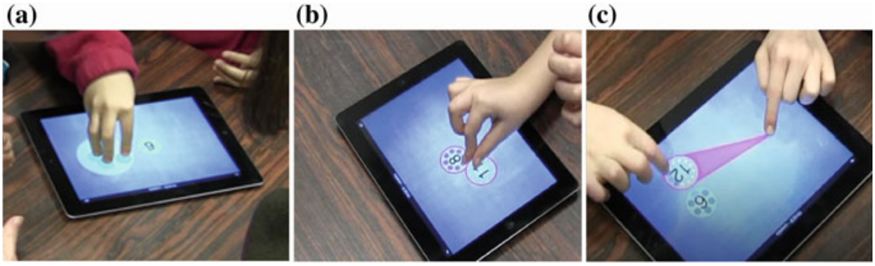

Fig. 1 **a** In *TouchCounts'* "Operating World" each touch makes a herd of discs, as *TouchCounts* displays and speaks the number; **b** represents the action of pinch-in (gestural representation for addition) and **c** pinch-out (gestural representation for subtraction)

Research Method

Participants and Data Collection

This exploratory case study is part of a larger research project that aimed to understand the various ways that young children, aged from three to six, develop numerical abilities through embodied interactions. The study explores one way that children share mathematical meaning when they work together with *TouchCounts*. Data was collected over a four-month period in a classroom of a daycare facility located in a Canada. All 27 children registered in the daycare were free to join and leave the activities as they pleased. This means that we provided the opportunity to play with *TouchCounts* to all children, even those without parental permission to participate in the study. However, only children whose parents or guardians had signed the consent form, were video recorded. During the data collection process, children were not simultaneously exposed to formal mathematical instruction.

In the section that follows, we present evidence of collaborative engagement where a group of young children converged independently in a separate corner of the classroom engaging in their own "play" with *TouchCounts*. We noticed children's collaborative activities manifested through an "investment" (Newman et al., 1992) directed toward reaching their own goal of "making 100" or "the biggest number in the galaxy" using *TouchCounts*. Newman et al. (1992) refer to this notion as engagement. Engagement is the psychological and perhaps physical investment directed towards learning a concept, or mastering a skill that mathematical work is intended to attain. Similarly, children's engagement is exhibited through the purposeful effort they direct toward the "educationally [and mathematically] purposeful activity, which contributes directly to a desired outcome" (Hu & Kuh, 2001, p. 3).

We Are so Good, Right John?

One day, while the senior researcher (Nathalie) was interviewing some of the children in a corner of the classroom, Mina (the research assistant and first author) noticed three boys—Sam (4 years and 8 months), Tom (5 years and 4 months), and John (4 years and 11 months)—gathering in a separate group playing with *TouchCounts*, and decided to record their activities and interactions. The names of children in this interaction are fictitious in order to conceal identity and maintain confidentiality. The children's stated overarching aim was to make big numbers. Prior to us working with them, John and Tom were able to count up to twenty starting from any given number, whereas Sam was able to count up to 20 only in a sequential form starting from one.

Phase 1 and 2: Making a Big Number and Celebration

Sam, Tom and John are gathered around the iPad and managed to create the number 203. Tom pinches a 'one' to the herd, *TouchCounts* says "Two hundred and four". The children notice Mina.

1. Sam looks back at her, smiles and says [in a high pitch]: "We made two hundred and four! We made one hundred, and then, we made two hundred and four", with surprise.
2. Mina says, "Excellent!" Tom adds a '1'–'204'. *TouchCounts* says: "Two hundred and five." Tom says to his teammates, "Hey look at this" [smiles]. Both John and Sam happily scream: "Oh, what the heck!" Tom and Sam pinch a '3'–'205'. *TouchCounts*: "Two hundred and eight."
3. The children run towards Nathalie and other kids, jumping up and down proudly: "Look at our number, we made the biggest circle in the galaxy!" (pointing at 208 on the iPad) (Fig. 2a). Nathalie looks at the number and says: "Wow, two hundred and eight!", with an excited tone. Children scream, jump, clap, and celebrate their group work.
   Note: **a** The children show their 'big' number, '208' to Nathalie; **b** Sam on the left is upset. He asks others to work as a team. **c** The children share fingers to reach their target number, '100'; **d** They are proud that they could make 100 collaboratively.
4. Sam resets *TouchCounts* and the children make new herds on the screen. Tom says: "We are making the biggest number in the galaxy," Sam continues: "We are making one hundred and two thousand and one!" John emphasizes: "No! we are trying to make *one hundred.*"

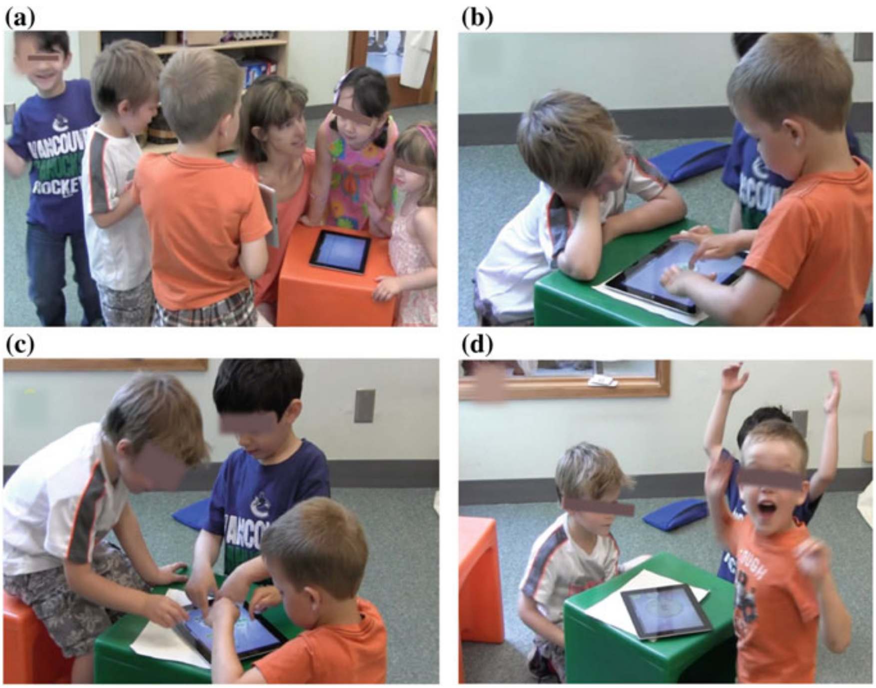

Fig. 2 Children collaborate in making 100

Phase 3: Making 100

5. Sam requests: "So, after that, a hundred and two thousand and one?" John rejects Sam's request and pinches herds intending to make a 100 all by himself. Sam wants to pinch herds, but John disregards this request and says, "Let me do it, Sam." *TouchCounts*: "Thirty two, forty two, forty three, fifty five …" This appears to disappoint Sam: "But John, let's make one hundred *as a team*" [very low pitch and disappointed voice] (Fig. 2b).
6. John accepts. Sam smiles, creates herds of numbers one after another with tensed fingers, "I'll make all these [numbers] and then you put them on John, okay?" says Sam excited and happily. Working together, John uses both hands to assemble the herds that Sam rapidly creates. Tom also helps John to gather herds. All fingers and eyes are on the iPad. The three children make numbers with shared fingers (Fig. 2c).
7. *TouchCounts* says: "Forty five." Sam keeps creating herds. *TouchCounts* does not allow children to both pinch herds, and create new herds, simultaneously. So, John asks Sam to wait until he puts all herds on the screen together. "Sam? No, let me first do the numbers, then you can do it again."

8. Sam says, “John look at the big circle we made.” *TouchCounts*: “Sixty six.” Sam says (with a surprise) “Sixty six!” John pinches 66 and 22. *TouchCounts*: “Eighty eight.” “Hey, we got to eighty eight!” Sam says cheerfully. Tom: “Hey did we make the biggest number in the galaxy?” John says, “No, we got to make one hundred.” Sam answers Tom, “No, *one hundred* and two thousand and *one*! That’s the biggest number on the galaxy”. Tom says, “No, a trillion.” Sam refine his statement, “Okay, a trillion *one hundred* two thousand *and one*.” John pinches 9 and 88.
9. It seems children realized that they are getting close to 100. John ‘holds’ the big circle of 97 to check it for few seconds. This is the first time that this action happens. Tom and Sam make some more ‘ones’, but no other numbers and not using more than one finger. John adds ‘one’ at a time to the big herd of 97 to reach 100. *TouchCounts*: “Ninety seven, ninety eight, ninety nine, one hundred.” John claps, screams and says with lots of joy, “one hundred, *we* got to one hundred, *we* got to one hundred *here*!” (Fig. 2d).
10. Reaching 100 did not occur by chance, as we observed how children made and added ‘ones’ precisely, one at a time as a team. Tom says: “We are so good, right John?” “Yes, we are!” John responds.

Collaborative Engagement

The previous episode demonstrates how some children “made 100” in their small group without instruction or supervision. They were *engaged* with the mathematical idea and their peers through fashioning an “educationally purposeful” (Hu & Kuh, 2001) activity and setting the goal of “making one hundred” [4 and 8]. Fredericks, Blumenfield and Paris (2004) suggest that *engagement* has three dimensions—behavioral, affective and cognitive—that provide insights into how student’s collaborative engagement can be identified. According to Fredericks et al. behavioral engagement involves active contribution in learning activities; affective engagement refers to children’s attitudes towards the activities; and cognitive engagement deals with the strategies used to thoughtfully involve children in understanding mathematics. These three different dimensions of engagement were evident in the observed children’s group activities. For example, behavioral engagement was involved in the children’s active contribution in “making 100”, while affective engagement was observed in the children’s attitudes towards making numbers “as a team” [2–9]. In our study children’s collaborative engagement resulted in them deriving recreational value from this activity, that is, engaging in the task for their own pleasure. In addition, the boys were proud to see what they have accomplished through team work. Using *TouchCounts*, children were making numbers collaboratively, and shared their moment of joy and accomplishment with the researchers. They also commended each other for being “so good” in creating numbers [10]. The behavioral and affective engagements occurred as a consequence of the children's cognitive and mathematical engagement and understanding.

Donato (2004) offers different perspectives on the notion of collaboration. Using Gee's (2003) idea of *affinity groups,* in which "groups are continually immersed in practice and share common features" (p. 286), he argued that collaboration involves recognition of individuals engaged in the larger activity, bonding with each other, and learning mainly in cooperation with each other with knowledge usually dispersed among the members. A second perspective of collaboration comes from Petrovsky (1985), who suggested that "collaboration implies group conventionality and disregards the individual as distinctive and imitative of the social" (p. 286). Taken together, the two definitions imply that an individual within a group should not be treated as a unique or isolated agent. This also affirms the *participatory* nature of learning (Lave & Wenger, 1991). In agreement with the ideas of Donato and Petrovsky, we found children were connected with each other, learning through combined efforts, and using the mathematical tool to negotiate and achieve a group aim. The implicit cultural and social relevance of collaboration was also evident as the children influenced the construction of meanings through communal engagement.

So far, this section has explored theories of engagement and collaboration separately. To explore collaborative engagement as an integrated concept, involving the use of a mathematical tool, we adopt Gee's definition of *collaboration* and Fredericks's et al. (2004) and Hu and Kuh (2001) perspective on *engagement*. Therefore, in this chapter, *collaborative engagement* is thought of as the practice wherein children's attitudes, active contributions, and mathematical strategies promote mathematics learning using mobile technologies. It refers to harmonized learning activities and practices, within which individuals build and practice a joint mathematical engagement, using mobile technology. In the episode, *TouchCounts* provided children a collaborative engagement environment, facilitating their mathematical negotiation, contribution and exploration. The local social interaction shaped the children's goal recognition strategies, group construction and role-assignment (e.g. Tom and Sam make numbers, and John pinches them to create new numbers).

Video Timeline: An Analytical Tool to Trace Paths of Interactions

In this section, we suggest an innovative methodology for analysing video data. This methodology helped us to assess mathematical collaborative engagement as the children interacted with *TouchCounts*. The tool we used to support our analysis is *StudioCode*: a professional video tool that captures, codes and categorizes video data for later review and analysis.

Using Interaction Theory to Determine Codes and Categories

To analyse video data, we adapted Vogel and Jung's (2013) video-coding procedure augmented by Arzarello et al. (2014) theory of touchscreen-based interactions to develop "active" versus "basic" action categories. The theory of interaction describes basic actions as the simple ways of interacting with the touch interface. Therefore, interactions that appear to be made either randomly or with no plan are marked as basic actions. Combinations of basic actions are classified as "active" actions. For example, tapping the reset button in *TouchCounts'* "Operating World" or tapping randomly are defined as basic actions; however, tapping with multiple fingers to create a given number is categorized as an active action. In this study, active actions were identified mostly as the interaction that the individual learner (or group of learners) take(s) to attain the achieved outcome or solve the given problem (See Table 1).

During the data analysis, codes and categories were identified and modified based on different modes of interactions with the touchscreen application. The first step was to review the video data on multiple occasions to generate initial codes. This provided familiarity with the content, suggested possible codes, and reduced pre-coding bias. Next, we verified whether categories and codes were informed by

Table 1 Basic and active actions when using *TouchCounts*

Basic actions		Active actions	
Tap	(a) To reset (b) Single or multiple (randomly or without a plan)	Tap	(a) Single finger (b) Multiple fingers
Hold	To know the application or by random, or without a plan	Hold	User make a contact with a herd and continues contact (a) To check (b) To slide
Slide/ Swipe	User puts finger on the screen and moves it randomly in any direction without touching a herd	Drag	(a) To organize (flick/drag): user grabs a herd and goes in a specific direction (b) **Pinch-in** (user makes twin drags and brings contact together without lifting fingers- to add herds) • With one hand or multiple hands • Two herds or multiple herds
		Pinch-out/ Spread	User performs two drags and pushes them apart without breaking contact (For subtraction —not observed in this episode) (a) With one hand (b) With two hands

the chosen theory (in this case, Arzarello et al. (2014) theory of interaction). In the final step, the researcher developed and checked the integrated system of coding and created a standardized system of codes via rating accordance. Rating accordance rates and verifies the degree to which categories and codes conform to the theory being used. This can be contextually varied depending on the scope of study, theoretical framework, and/or available coding software. The process of developing the video coding system is illustrated in Fig. 3.

After identifying active and basic codes as discussed in Table 1, the researchers coded video data and captured both the frequency and duration of actions. StudioCode allows timeline analysis and connects codes to each video-segment. The [video] timeline in StudioCode provides a chronologically organized, multi-layered, graphical representation of the codes, narratives, and comments from the video recording. It enables coding of a variety of verbal and visual cues (such as gestures, body position) to precisely capture what occurred during a specific time. We used the software to comprehensively detect different interactions among the children while they interacted with *TouchCounts*.

In StudioCode's timeline, each row represents its corresponding code on the left and demonstrates the distribution, frequency, and length of each occurrence chronologically. The resulting coded timelines are summarized and synchronized below. Phases 1 and 2 correspond to the children creating a big number and celebrating, and Phase 3 to the group work resulted in creating one hundred.

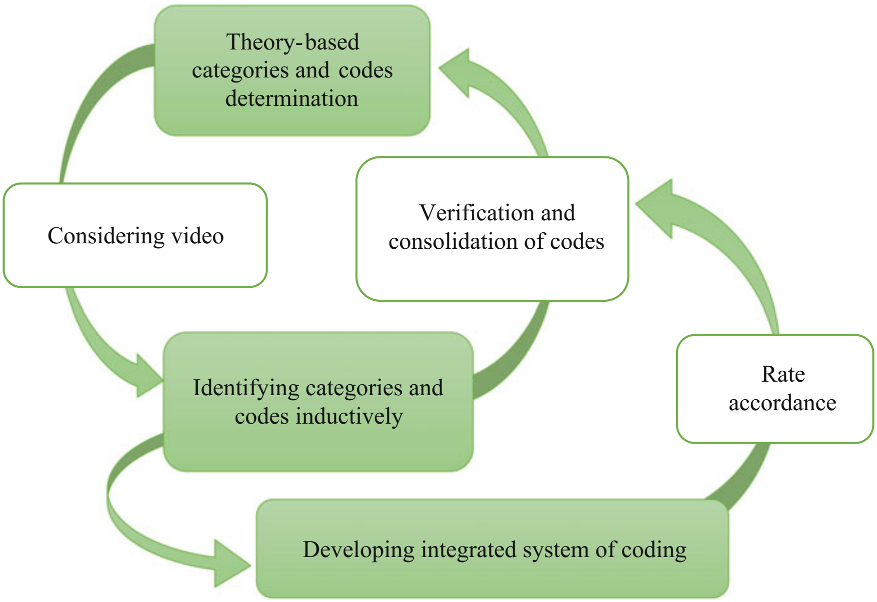

Fig. 3 Video coding process for determination of categories and codes. (Adapted from Vogel and Jung 2013, p. 3)

Figure 4 comprises the three separate timelines for Tom, Sam, and John along with performed active actions as the codes in rows that are synchronized. As seen in Fig. 4, there are common and unshared rows in each timeline that indicate performed or unperformed active actions for each child during the reported time interval. For instance, the active action of "holding to check" was observed only once in John and Tom's interaction with *TouchCounts*.

At first glance, the timelines show a periodic form of activity (working in turns) among group members. This could be either because children took turns while they worked as a team or it might demonstrate a limitation of *TouchCounts* in not allowing a user to create new herds, while another user is simultaneously gathering existing herds. We also exported the "frequency matrix" of StudioCode to Statistical Package for the Social Sciences (SPSS) software to create a scatterplot graph that displays frequency distribution of observed active actions (Fig. 5). The frequency matrix is a table where the rows indicate active actions in the video timeline and the columns indicate different forms of a given active action that are coded in the instances of each row.

Rather than attempting to demonstrate a correlation between codes, we used a scatterplot to indicate the distribution of active actions across the video timeline. We observed higher numbers of active actions between 3:50 and 4:20 (as indicated in the horizontal box at the top of Fig. 5) when children were able to successfully reach 100 with their collaborative engagement. During this period, we observed that the children progressed from creating many herds by multi-tapping, to precisely making single-tap 'ones' after getting close to 97. That is to say, as a group, Tom and Sam made 'ones' on the screen and John used those 'ones' to reach one hundred (Fig. 5). This might be interpreted as signifying the children's awareness

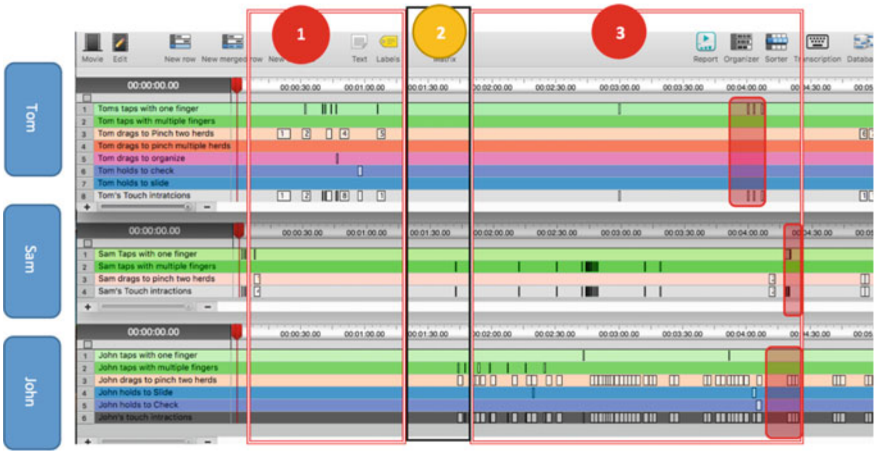

Fig. 4 Synchronized coded timeline for children's interactions: In phase one, children make a big number; in phase two they cellebrate; in phase three they make 100. The highlighted boxes in the third phase show the children's active actions after reaching 97

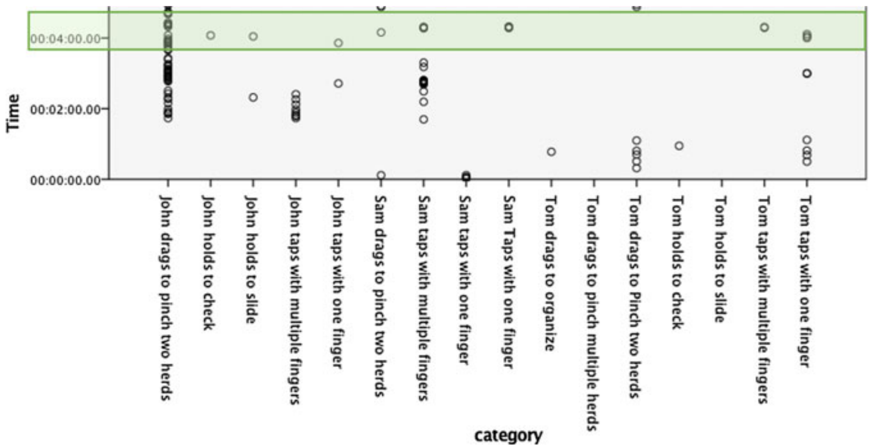

Fig. 5 Frequency distribution of active actions over time for Tom, Sam and John

of approaching one hundred. The children's shared active actions while interacting with each other and *TouchCounts* also demonstrates their collaborative engagement. The graph also shows an increase in the distribution of active actions during the time period under investigation.

In addition, more sophisticated forms of interaction were observed just before completing the task of making 100, such as 'hold to check' by John (03:58). Children also become proficient in pinching herds, working in turns, and collaborating to make 100. This precision in using the tool cannot be separated from the precision of articulating each number by gestures (Sinclair & Heyd-Metzuyanim, 2014).

The analysis above demonstrates how collaborative engagement, assisted by mobile technologies, appear to facilitate the development of mathematics learning. In agreement with Harper and Quaye (2009), we found that engagement entails more than participation. It also incorporates "feelings and sense-making as well as [bodily] activity" (p. 5), adding to Hu and Kuh's (2001) definition of engagement. The video-timeline data demonstrates the children's bodily interactions with the mathematical tool *TouchCounts* through basic and active interactions, while they were collaboratively engaged in mathematical practice to achieve their goal of "making a hundred."

Reflection

One of the key features of mobile technology, which makes it fruitful for adaptation in collaborative mathematical activities, is its potential to help children reflect on their work. In the context of using mobile technology, we refer to reflection as a catalyst for linking and revisiting learning. Rodgers (2002) introduced four criteria of reflection, inspired by Dewey's (1916) long-historical perspectives on the subject. The first criterion of reflection involves a meaning-making process; one that moves the learner through various levels of experience. The second criterion posits reflection as a systematic and rigorous way of thinking where the learner draws on past experiences that are similar or different from the new experience. The third criterion involves a mind-set that place emphasis on personal and intellectual growth of the individual and others who are a part of the community. Finally, reflection is embedded in a community of interaction with others, which implies that reflection is a necessary component of collaborative engagement and is, therefore, of paramount importance to our research.

Schön's (1983) model of reflection presents an alternative approach. In this model, two phases of reflection: reflection-in-action (learning through practice) and reflection-on-action (learning after the event) are described. Our primary focus here is on reflection-in-action, emphasizing that the reflection observed was temporally extended across the entire activity, rather than occurring at a discreet moment at the conclusion of the activity. This means that children's meaning-making processes can take place at any point throughout the activity. The reflection-in-action was evident when John was leading the group to reach one hundred as a milestone before making "the biggest number in the galaxy." We observed that using *TouchCounts* supported the children's reflection-in-action and also reflection-on-action. It facilitated conversations and collaborative practice among children themselves, and also between children and adults (Cochrane & Bateman, 2010). Through the experience of working with *TouchCounts*, children learned they might pass 100, and then 200 to reach 204 (See John's explanation in [1]). They also drew the conclusion that "the biggest number in the galaxy" has the "biggest circle in the galaxy," implying a relationship between circle size and number. This was because of their continuous reflection on the size of circles appearing on the screen at different intervals (reflection-in-action); the bigger the number-the larger the circle (limited to the size of the screen).

Using the Affordances of Mobile Technologies to Enhance Engagement

According to Lai et al. (2007), the term 'affordance' originates in the work of Gibson (1977) and means "the relationship between an object's physical properties and the characteristics of a user that enables particular interactions between user and object" (p. 328). The development of this relationship has opened up diverse trajectories for learners to construct and comprehend mathematical knowledge (Sacristan et al., 2010). Mobile technology offers the ability to simultaneously connect to and explore visual, symbolic, and numerical representations in a dynamic way (Sacristan & Noss, 2008). They allow the learner the flexibility to quickly rearrange information and re-engage with activities from new perspectives (Calder, 2005; Clements, 2000). They also provide a system of networking where interaction and collaboration within a structured system is used to share and discuss issues relating to mathematics (Sinclair, 2005).

Mobile technologies also provide cognitive benefits based on their various functionalities and the purpose for which they are used. Results from studies (Gadanidis & Geiger, 2010; Pierce & Stacey, 2010) have shown that the use of mobile technological tools supports the learning of mathematics skills such as problem-solving, reasoning, computational thinking, and justifying.

For example, although we are not arguing whether or not the proposed "biggest number in the galaxy" by children is mathematically meaningful or not, we found it interesting when children shared their knowledge of "the biggest number in the galaxy", as it surprisingly almost always had a '*one*' at the end (e.g. "a trillion one hundred two thousand and *one*"). Presumably, this is the influence of one of the *TouchCounts'* affordances, which promotes creating larger numbers by adding a 'one' to any given number. We suggest two distinct explanations for this phenomenon. First, the capability of *TouchCounts* to encourage the development of an early exploration of the set of natural numbers as an infinite[2] set by young children. Second, the opportunity for children to validate their assumptions openly as far as the actuality of the designed tool allows. This is what Stone and Minocha 2005) defined as a good user interface design in that it facilitated easy, natural, and engaging interaction, which in turn allows users to carry out their required tasks or goals in a natural and logical order. In this sense, children initially proposed a "trillion", as "the biggest number in the galaxy" and their continuous adding of the aforementioned special '*one*' is a manifestation of a fundamental number theory axiom, which proves that the set of natural numbers is an infinite set. Based on set theory, natural numbers are the counting numbers that usually represent the cardinality of a (non-zero sized) set with each natural number being "built upon" the previous number add one. $A_1 = 1$, $A_2 = 1 + 1 = 2$, $A_3 = 2 + 1 = 3$, …, $A_{n+1} = A_n + 1$. So, regardless of which number is suggested as *the biggest number*, children can envision a number that is greater, by adding a 'one'.

Conclusion and Limitation

Although smart phones, tablets and other handheld devices are increasing popular among young children (Hilda et al., 2015), research regarding the implications of mobile technology on mathematics teaching and learning is still relatively new. As

[2]Suppose x is the greatest natural number, then there is $x + 1$ that $x + 1 > x$ (proof by contradiction).

mathematics education researchers, we sought to gain better understanding of the relationship among collaborative engagement, mobile technology, and mathematics learning. We initially examined collaboration and engagement as separate entities, and then examined them collectively to operationalize the idea of a group of children working collaboratively on a purposeful activity towards a shared outcome. We used a case study to show that social interactions among children engaging with mobile technology shape goal recognition and role-assignment strategies. We also showed that mathematical meanings were developed when children worked together on a task mediated by mobile technology. They were able to "make one hundred" collectively, and demonstrated early understanding of a fundamental idea of number theory: specifically, that the set of natural numbers is an infinite set. Furthermore, the children were able to envision that an amount can get greater if 'one' or more is added.

In addition, collaborative engagement using *TouchCounts* was most notable during teamwork, goal setting, negotiations, sharing knowledge, and joint construction of practices. We also discussed the ways children discarded individual practices to collaboratively achieve goals. The diverse paths that children utilized in achieving their goal were evidence that the affordances of *TouchCounts* provided cognitive benefits for children and that they were able to use their problem-solving skills to test and justify conjectures. As an analytic tool, the video timeline showed the emergence of a remarkable amount of bodily engagement as the children completed active actions (Arzarello, Bairral, & Danè, 2014). In this case study, while we specifically examined the relationship between collaborative engagement and mathematics learning through mobile technology, we suggest that StudioCode is a powerful and sophisticated data analysis tool for other mathematics research investigating different modes of interactions, communication, and gestures in educational contexts. As a result, we believe that it would be beneficial to further explore how the use of mobile technology might also contribute to older children's collaborative engagements.

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Mina Sedaghatjou is a mathematics educator, whose primary research interest focuses in the areas of mathematics and STEM education with the use of emerging technology. Mina engages in qualitative and quantitative research approaches to explore how using technology for teaching and learning contributes to embodied cognition and conceptual understanding. She currently serves as an assistant professor at Brock University.

Sheree Rodney is a Ph.D. candidate in mathematics education in the Faculty of Education at Simon Fraser University. She is a Jamaican native with experiences at various levels of the education system (Elementary, Secondary and Tertiary) and also serves as a Mathematics Specialist and National School Inspector with the Jamaican Ministry of Education. She earned both Bachelor's and Master's degrees at the University of the West Indies Mona, in Jamaica and a Teaching Diploma at the College of Agriculture Science and Education also in Jamaica. Her research interest is the integration of digital technology in mathematics classrooms with a specific focus on the relationship between embodied curiosity and digital technology tools.