Fingers-on Geometry: The Emergence of Symmetry in a Primary School Classroom with Multi-touch Dynamic Geometry

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Abstract In this chapter, we describe a research project with first grade children using a multi-touch dynamic geometry sketch. We approach our analysis through the lens of inclusive materialism (de Freitas & Sinclair, 2014), which considers the intra-actions involved in the child-device-geometry assemblages and thus to the way in which new mathematical ideas emerge in this assemblage. Drawing on the design experimentation methodology (de Freitas, 2016), we analyse the assemblage in order to study how concepts such as symmetry arise. We therefore seek to investigate the way digital technology can become a device for producing new concepts. We focus particularly on how the multi-touch environment, in which geometry objects can be continuously dragged with fingers, occasions new gestures and body motions that provide the basis for emerging geometrical ideas.

Introduction

In this chapter, we experiment with the concept of symmetry in a grade one classroom where students interact with a dynamic geometry environment (DGE) on multi-touch tablets. In mathematics, a concept, in general, is seen as a robust, cohesive idea that represents all of its manifestations. For example, if the concept of symmetry is understood 'fully', a person should be able to apply, answer questions, and understand symmetry in all its instantiations. We believe this reductive approach to learning mathematical concepts relies too heavily on knowing as 'stored' mental knowledge. This perspective ignores the relevance that both tools and activities have on what it means to know a mathematical concept. Some

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researchers have challenged the representationalist view of knowledge, arguing for a more performative practice of *knowing*, which happens in time, in context and through action (e.g., Pickering, 1995). That is, rather than interpreting experimental results in an abstract way, inductively drawing out a “schema” that a given child has constructed, they refuse the dichotomising of action and thought, focussing instead on the multiple ways in which bodies and materials engage in knowing. We challenge the idea that symmetry is a concept that is slowly developed and acquired, then schematised in a stable way in the brain. Instead we use the theoretical framework of inclusive materialism (de Freitas & Sinclair, 2014) which draws attention to the material nature of mathematical concepts, and thus to the concept as indeterminate, mobile and imbricated with the activities of children with tools. Following this set of assumptions, we also utilize a method of diffractive analysis (Barad, 2007, 2010, 2012; de Freitas, 2016), which will be described in more detail in a subsequent section of the chapter.

The Importance of Symmetry in the Curriculum

Symmetry is an important mathematical topic that supports spatial reasoning and patterning. The topic appears in different forms in various curricula typically starting at a reasonably young age. In our jurisdiction, symmetry appears in the curriculum in grade 4 with line symmetry followed by two-dimensional shape transformations in grade 5, and then again, later in grade 9 with line and rotational symmetry. These stand-alone topics in grades 4, 5 and 9 are not the only time symmetry is addressed. Although symmetry is formalised in each of these grades, the word ‘symmetry’ is referenced in other mathematical areas such as problem solving and also in working with two- and three-dimensional objects. Other topics that draw on symmetry include analog clocks, direction, graphs, and working with parallel and perpendicular lines.

In primary school, children usually encounter symmetry as a property of shapes. They maybe asked, for example, whether a given shape (a butterfly, a square, etc.) is symmetric. Folding is frequently used as a means to determine whether a given shape is symmetry. In this study, in which we used a dynamic geometry environment, we used a more transformational approach involving motion, in which symmetry is the result of a reflection. The act of reflecting is an isometric transformation that ‘reflects’ a pre-image from one side of a line of symmetry to the other. In such a transformation, the image and the pre-image are equidistant to the line of reflection; and, the line joining the image and pre-image is perpendicular to the line of symmetry. These properties remain invariant as the pre-image is dragged on the screen.

Prior research has shown that children have a great deal of knowledge about symmetry long before they learn about it formally in geometry classes. For example, children spontaneously construct symmetrical figures during informal play at the pre-school age (Seo & Ginsburg, 2004). However, the importance of this implicit understanding has been under-utilized in mathematics education as well as under-represented in the research literature. Even when studied, the research has focused on “the development of children’s ability to tell symmetrical figures apart, not to understand the relation between them” (Bryant, 2008, p. 34). For example, Bornstein and Stiles-Davis (1984) link the developmental progression of 4–6-year-olds with types of line symmetry. Namely, they found that 4-year-olds discriminated only vertical line symmetry, 5-year-olds, vertical and horizontal line symmetry, and 6-year-olds, vertical, horizontal and oblique symmetry. However, their study focussed exclusively on the visual identification of symmetry, rather than on the relationship between the various elements involved, such as the line of symmetry, the relationship of equidistance between the line of symmetry and both the pre-image and the image.

Based on their research of mathematics learning in the early years, Clements and Sarama (2004) propose that children should work with symmetry in the pre-K through to grade-2 years. They offer a developmental trajectory in which children begin at the pre-K level to create shapes that have line symmetry, then work in kindergarten and grade one to identify symmetry in 2-D objects. In grade two, children identify the lines of symmetry of various shapes. This trajectory also focusses more on identification than on properties of symmetry and relations between the pre-image and the image in reflectional symmetry, which, following Duval (2005), we see as significant parts of geometric thinking that can be engaged even at the early years.

In their study involving children in grades 2/3, Ng and Sinclair (2015) found that the use of a dynamic geometry environment developed dynamic and embodied ways of thinking about symmetry after engaging in teacher-guided explorations of a pre-constructed sketch called the “Symmetry Machine”. In this sketch, which is also used in the present research, symmetry is preserved as different components of a diagram are moved, including the line of symmetry. While that research was conducted in a whole classroom setting using an interactive whiteboard, the present study also included as an addition the use of a classroom set of iPads, so that each student had the opportunity to directly manipulate the sketch.

As we will develop in the next section, we take symmetry to be a concept that cannot be separated from the tool, nor the user with which it is instantiated. In this study, when we speak of the concept of symmetry, we do not abstract it from the movement of fingers, eyes, bodies of students, nor the iPads, sketches and classroom dynamics. This choice is based on the idea of intra-actions (Barad, 2007) and the notion of assemblage.[1]

[1]*Assemblage* is a notion introduced by Gilles Deleuze and Félix Guattari, and later used both by Bruno Latour and Karen Barad. The article on Deleuze in the Stanford Library of Philosophy glosses it as follows: “‘assemblages’, that is to say, an emergent unity joining together heterogeneous bodies in a ‘consistency’” (http://plato.stanford.edu/entries/deleuze/).

Theoretical Framework

In this chapter, we integrate post-humanist and new materialist perspectives into both how we see and analyse mathematics teaching and learning. These perspectives, essentially, attempt to de-centre the human as the primary—or, indeed, only—agent in the learning process and to find ways of accounting for how matter matters in that process. These perspectives are rooted in broader philosophical developments associated with 'the ontological turn' that has emerged from feminist studies (see Barad, 2007; Braidotti, 2013; Haraway, 2008). Within the context of mathematics education, these perspectives have been adapted and refined to the context of educational research, in the form of *inclusive materialism* (de Freitas & Sinclair, 2014), which looks closely at the material specificities of mathematical experiences. This approach positions itself "within a tradition in which abstract thought and materiality are assumed to be entwined" (p. 3).

de Freitas and Sinclair draw primarily on the work of Barad and her concept of intra-action. Barad (2007) contrasts intra-action with interactions, where the latter assumes the coming together of entities that have pre-defined properties and characteristics. In intra-action, entities can be seen to be emerging from activity, that is, activity occurs first, and that activity creates and integrates the 'bounded' entities such as the iPad, the child and the mathematics. Combined with the post-humanist view, there is a shift away from individuating the student as an independent and well-defined body and how she is acquiring knowledge. Instead, the fixed boundaries of that body are disrupted in order to attend to the evolution of a tool, child and mathematics assemblage. Inclusive materialism, consequently, takes mathematical concepts to be material and emergent from particular intra-actions. Because the concept is material, and because—as Barad argues based on her analysis of experiments in physics, such as the two-slit experiment,[2] matter is indeterminate, concepts as well partake of the indeterminacy of matter. This challenges the traditional view of individuals abstracting conceptual knowledge from engaging with material objects. Rather than focussing on epistemological concerns, those related to what is learned by the student, inclusive materialism attends to ontological concerns, that is, what is the emergent material assemblage that gives rise to meanings.

In an inclusive materialist framework, mathematical concepts arise out of intra-actions between student and material and activity. In particular, we expand traditional approaches that see symmetry as a distinct idea or concept. We challenge the common pedagogical approach whereby different tools and different tasks will move students closer to the bigger picture of symmetry. Symmetry is not a *form* that

[2]Specifically, two slit experiments appeared to show that light was a particle or a wave, depending on the experimental apparatus that scientists used. Instead of seeing particles and waves as ontologically antithetical, as in classical (non-quantum) models of physics, Barad suggests that light, and therefore matter more generally, is *ontologically indeterminate*. It takes on a specific ontological form—it becomes determinate—through intra-actions with the measurement apparati.

is somehow buried *fait accompli* in matter and waiting to be conjured or evoked. We see symmetry as an emergence of different meanings from an entanglement of tool, task and student. Because of this view of symmetry, as a mathematical concept, the questions we can ask about teaching and learning are different. We avoid questions that assume strict boundaries between students, mathematics and tools, and that position mathematical concepts as fixed ideas waiting to be abstracted from experience. Our question is more ontological in nature since we will be concerned with what happens to an assemblage over time, how it changes, ruptures or renews.

Methodological Framework

In this chapter we explore a diffractive methodology that draws on Barad's agential realism (2007, 2010, 2012). A diffractive analysis involves the reading of data through multiple theoretical insights in such a way to gain unpredictable and productive emergences; for example, Barad read Neils Bohr, the physicist, through Jacques Derrida, the philosopher. A diffractive analysis is thus less concerned with reflecting objectively a particular event and instead seeks to offer, in the words of Haraway (1992), "a mapping of interference" (p. 300). In the context of educational research, Lenz Taguchi (2012) uses the method of diffractive analysis to interpret data gathered from discussions she had with a boy who had made a bark boat. Her goal is to "make visible new kinds of material-discursive realities that can have transformative and political consequences" (p. 265).

What distinguishes Barad's diffractive analysis from that of Lenz Taguchi is that Barad's approach includes an experimental device, or an apparatus. Indeed, in her case, the apparatus is a machine used in physics laboratories that interferes with the environment (in the example she provides, light) and produces a new phenomenon (patterns on a screen). Experiments using this apparatus enable Barad to explore new ontologies, such as: What is light? What is matter? She sees the experimental interventions that she studies—with theoretical physics—as delving into the indeterminacy of matter, while also being 'the condition' of determinate meaning.

de Freitas (2016) has suggested that as educational researchers, we too, could conduct experiments that involve a *diffractive apparatus*. Such an experiment would be designed to explore new ontologies and to better understand the relations between matter and meaning that emerge in a particular classroom situation, for example. Imagine, as will be the case in this chapter, that the apparatus not only includes a particular educational digital tool, but also students' bodies and movements. A diffractive apparatus experiment would differ from methods based on theories of tool use in mathematics education research because of the way in which the apparatus is not simply taken as a mediator of learning (as in the theory of semiotic mediation elaborated by Bartolini Bussi and Mariotti (2008)), or a tool that students use in order to learn particular concepts (as in the theory of instrumental genesis (Artigue, 2002)). Instead, the tool is part of a diffractive apparatus that produce effects that help us see how meanings about the concept of symmetry and how it is entangled with the physical. This may sound surprising, given that symmetry is not exactly a new concept and that it is usually considered to be characterised by logical determination. And yes, what a diffractive analysis might show, in an experiment, is that the indeterminate nature of matter, which is a fundamental assumption of Barad's, entails indeterminacy about symmetry as well.

We thus use de Freitas's (2016) mobilisation of Barad's diffractive model in her elaboration of how an experimental device 'interferes' with the environment. Barad is exploring the indeterminacy of matter and de Freitas elaborates that the indeterminacy results partly from these devices that are part of experiments and consequent data collection. This diffractive methodology offers ways of exploring new ontologies and insight into the relationship between matter and meaning. In this chapter, our diffractive apparatus included DGE sketches that were designed in *Web Sketchpad* and used both in a whole classroom setting with a projector, and in pairs, with iPads. Given our theoretical framing, we assume that a concept is never a singular representation, nor is it an essence or form. Rather, in our diffractive apparatus, symmetry is indeterminate and will take on a specific ontological form in intra-action with the dynamic geometry apparatus (and other parts of the material surroundings). As such, our question becomes, what determinations of symmetry arise from our experimental setting? We invite the reader to consider the following as a thought experiment that tries to imagine what it would mean to adopt the theoretical perspectives we have outlined. We recognise that given the novelty of the methodological approach, there is bound to be some tension between our traditional focus on individual children and their actions with tools and on concepts, and our new attempt to focus on intra-actions. Nevertheless, we contend that a consideration of symmetry from this perspective will open opportunity for alternative, yet productive insights.

Research Setting

At the start of 2016, from January to April, as part of a research project, we visited a grade one classroom in a public French-Immersion elementary public school in a North American west coast school. (Since this is the first year that the children are learning French, they often speak in English during class, and the teacher also sometimes addresses them in English.) We went every week and spent just over one hour working very closely with the regular teacher in organizing activities, discussing curriculum directions, and taking turns to teach. Nathalie (the second author) taught the class almost every time we visited. Sean also participated in teaching but was more often working with individual students during group work. Typically we went in the mornings before lunch. The classroom had a carpeted area in the front of the room where students often gathered as a group, there were also five tables set up around the room, to the sides and back of the room, where students could sit and work. Six students could sit at a table. Every session began in a whole classroom interaction, with the students at the carpet and an overhead projector connected to an iPad on which designed sketches were shown. This would be followed by pairwise explorations on the iPads, at the tables.

All classroom activities were videotaped, in the whole class gatherings, when students were sitting on the floor. The camera captured what was on the projected screen, all the children and the teacher. When students worked in pairs on the iPad around the room the video was focussed on one pair, and sometimes two, if the students were close enough to each other.

The Apparatus and Sketch Design

The apparatus we look at in this chapter is a web-based variation of *The Geometer's Sketchpad* (Jackiw, 1991, 2001) that is used on the iPad. Different sketches with various functionalities were designed (available at www.sfu.ca/geometry4yl/). The sketches are open and exploratory in that there are no instructions explicitly given. In this chapter we work with the discrete Symmetry Machine sketch. In this sketch, there is a vertical line in the middle of the screen, which is the line of symmetry. On either side of the line are six coloured squares, two blue, two red, and two purple (Fig. 1a). When a coloured square is touched on the screen and moved, its image square moves so as to preserve the reflectional symmetry of the diagram as a whole. These squares move discretely on a square grid background. Dragging any square on one side of the line of symmetry will also move the corresponding square on the other side of the line of symmetry (see Fig. 1a, b). The discrete motion, as well as the use of the grid, was intended to help the children attend to the distance between a square and the line of symmetry. The line itself can be moved, right or left, which will move six of the squares in order to maintain symmetry. The line has a red point on it and when that point is dragged, the line can be rotated around so as to create diagonal or horizontal lines of symmetry (Fig. 1c).

Outline of the Lesson (53 min)

The lesson we report on in this chapter was our first lesson using the Symmetry Machine. At the beginning of the lesson, the Symmetry Machine was projected on the front screen, the students were seated together on the floor in front of the screen and Nathalie was towards the back of the room with the iPad. Nathalie engaged the students in some questions relating to the sketches. The later part of the lesson had students working in pairs on the iPad at individual tables. Students were given set Symmetry Machine diagrams on paper and asked whether they could re-create the diagrams using the Symmetry Machine on the iPads. Not all of the diagrams were symmetric.

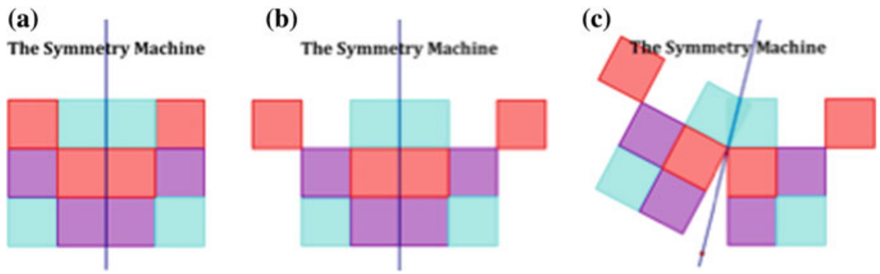

Fig. 1 **a** The discrete symmetry machine; **b** after dragging one block away from the line; **c** after rotating the line of symmetry using the point visible near the bottom of the line of symmetry

Becoming Symmetry

As per our stated methodology, our analysis of the video data follows the concept of symmetry as it becomes manifested through the experimental device of the iPad and the movement of the children. Below we present snapshots of that evolution. There are three main ones in the first ten minutes, while in the whole classroom configuration. Recall that this was the first time that the students were being formally introduced to the word 'symmetry' and were engaging in mathematical activity focussed on creating and manipulating symmetric shapes. Indeed, the first lesson was designed in order to introduce the students to symmetry by investigating its behaviour on the iPad, and not through a description or static example of it.

Symmetry as Twoness, Movement and Holes

The first visible effects of the diffractive analysis are that 'symmetry' was seen as something that involves twoness.[3] In the first lesson, the students were gathered together as a group on the floor while Nathalie projected the image of the Symmetry Machine on a screen at the front of the classroom. There were six coloured blocks on either side of the vertical line. At first, many finger puppets were made on the

[3]Readers may find that their own conceptions of symmetry also involve some kind twoness as well. It is also implicit in more traditional ways of working with symmetry with young children where one evokes folding (so one side matches the other side, there being two sides) or mirroring (where what's in the mirror is the same as what is being mirrored, thereby also involving two things). We argue that in these situations, the emphasis is on sameness rather than on twoness. This is because attention is usually focussed on one side of the symmetry line, rather than on symmetry as a transformation of one shape to another. Since our goal is to study the determinacies of symmetry in this experimental setting, we examine the emergence of any and all such determinacies, whether they seem familiar, or not.

projected screen, but as soon as Nathalie moved the top, right-most square (which was red) to the right (which of course moved the corresponding red square on the other side of the line) the sound of student gasping was heard (see Fig. 2a).

4:20 Nathalie: What happened when I moved the red square?

4:21 Jonathan: Deux (*Two*) (two fingers up in the form of a peace sign (see Fig. 2b, left). Deux (*Two*).

4:22 Nathalie: Deux. Qu'est-ce que tu veux dire Jonathan? (*Two. What do you mean Jonathan?*)

4:27 Jonathan: Deux carrés bougent (*Two squares are moving*).

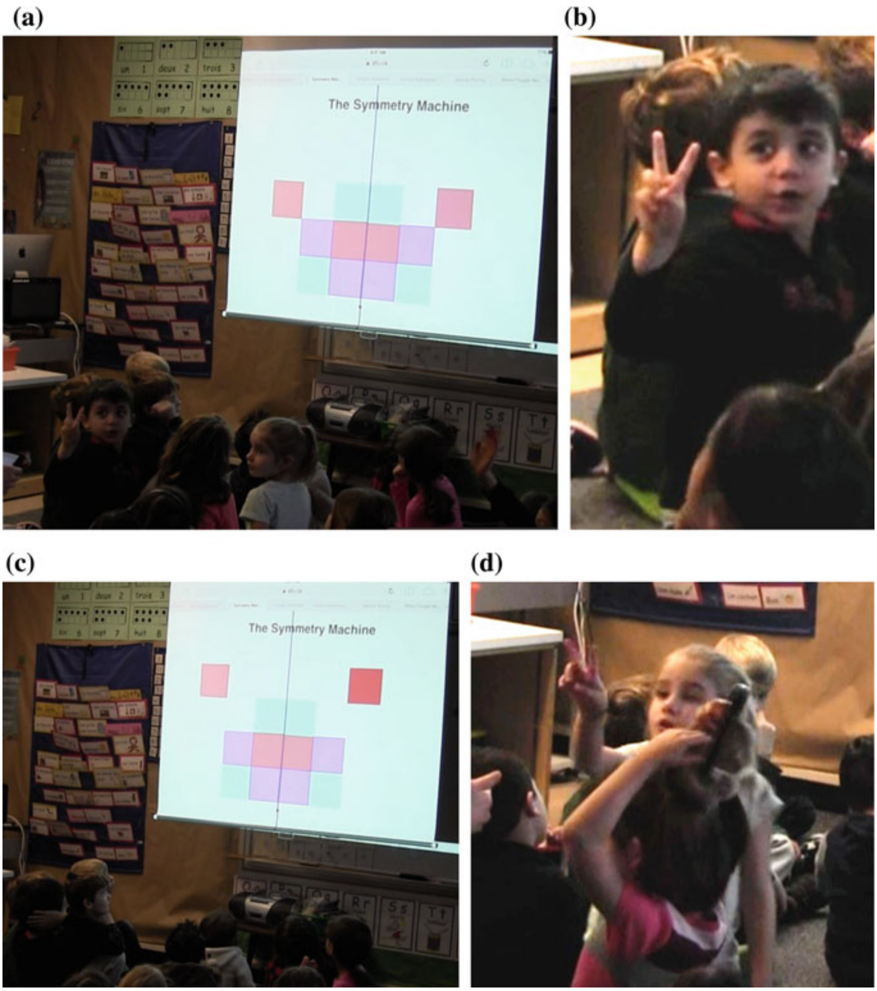

Fig. 2 **a** Students gasp; **b** deux: two fingers up; **c** two red squares up; **d** peace: two fingers extended

4:34 Nathalie: Weston?

4:37 Weston: Deux carrés blancs. (*Two white squares.*)

4:40 Nathalie: Deux carrés blancs. Ah oui, il y a deux carrés blancs maintenant! (*Two white squares. Oh, yes, there are two white squares now!*)

Nathalie then moved the same red square up (see Fig. 2c) and several voices said "wow!" She asked what happened when the red square was moved towards the top. One voice said "deux carrés bancs", then several voices said "trois" (*three*) and then several other voices said "quatre" (*four*). When the same red square was moved towards the right, several "wow" exclamations were heard again as well as several numbers, including "four", "five" and "six".

Over the course of this period of time (1 min 22 s), two ideas emerged in relation to symmetry. The first is the notion of twoness, which occurs both in language and in gesture. Recall that the students were not looking at Nathalie's finger on the iPad, so they would just be able to see the squares move on the overhead screen. And that motion happened in pairs. Had they seen a finger move one square, as might be the case on an interactive whiteboard, they might have focussed less on the square being moved and more on what was happening to the image square. The idea of twoness emerges several minutes later, when the children begin working in pairs on the iPads (see Fig. 6a, where Ava is explaining how to move the squares). The students seemed to want to fill in the white spaces on the top row and Ava turns around, kneeling, and put her right arm up with two fingers extended (Fig. 2d), as in the peace gesture.

While the first comment "deux carrés bougent" addresses the motion of the squares, without saying anything specific about the way the motion happens (such as, moving away from the line), the next verbal comments focus less on what's moving than on what gets left behind. The two white squares are the empty squares that appear once the red square has moved. The movement of the red square leaves a kind of hole, which is like the negative space of the sketch. This hole is also characterized by its parity, first in the two white squares, then in the four white squares and finally in the six white squares. Interestingly, there are many more than two or even six white squares in the sketch, so the parity seems to focus specifically on the space created by the moving squares. It is worth remarking that had the squares not moved, holes could not have formed, so the iPad as an apparatus enables the discrete motion of squares to intervene in the concept of symmetry in a novel way.

Symmetry as Bringing Together

In the initial activity, when the students were seated on the carpet as a group and Nathalie was moving the squares, she asked what would happen when she moved one of the purple squares (right-most square in the second row of Fig. 2a) upward. There was an approximate two-minute length of time during which the students

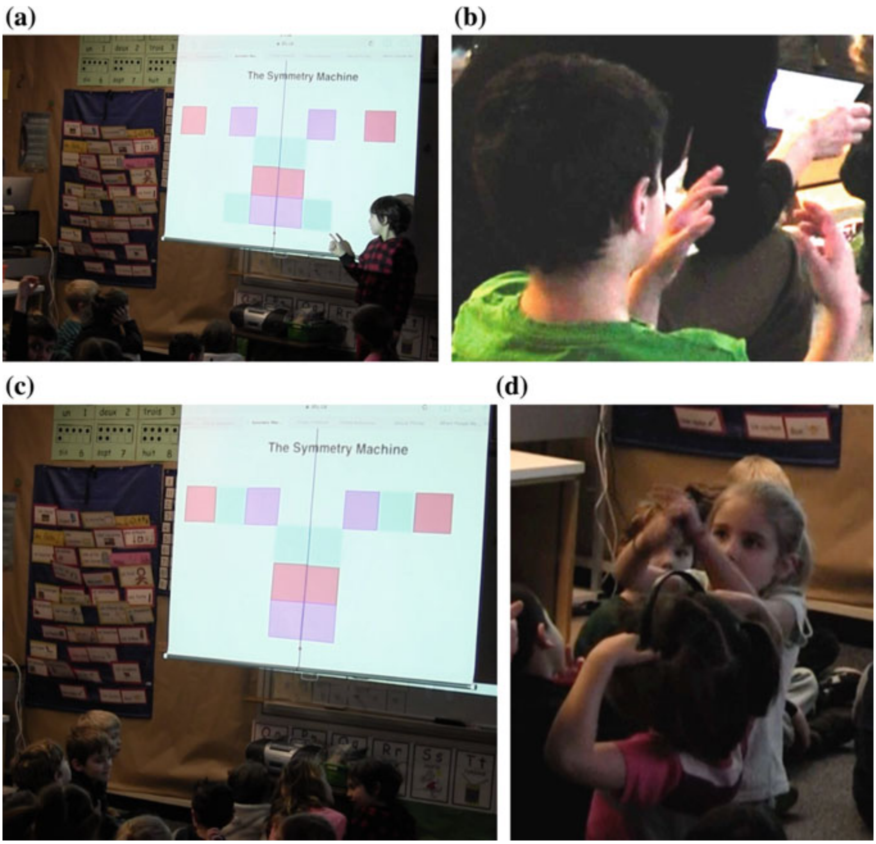

Fig. 3 **a** What will happen; **b** thumb and finger gesture; **c** top row; **d** Ava's gesture

struggled to predict what would happen. Although there was a lot of mumbling, no one said anything that could be heard in the videotape recording. Nathalie moved the purple square (as shown in Fig. 3a) and then asked the students to predict what would happen if she moved the blue square (right-most square in the third row of Fig. 2a). One voice said, "it will make a white square". Michael's response was inaudible, but it was accompanied by a two-handed gesture in which his palms face each other and the thumb and fingers on each hand are a mirror image of each other (see Fig. 3b). After making this gesture, he began clapping his hands (and knees). He clapped once with his hands, once on his knees, four times on his hands, once on his knees, once on his hands, once on his knees, three times hands, one knees, he brings his hands together and rubs them, once on his knees and finally he brought his hands together and rubbed them.

Michael was sitting on a chair at the back of the room, while everyone else is on the floor, so the other students could not see what he was doing. The initial gesture, and then the more dynamic gesture of clapping (but without sound), both express the sense of twoness seen before, as well as the motion in the two hands coming together. Symmetry has moved from the projector screen to his own two hands. This shift of symmetry from the screen to Michael's own space, is a movement of symmetry, visual to physical, discrete to continuous, technology to body. The students then directed Nathalie to move the purple and blue squares up. Then, despite not being able to see Michael's gestures, Jessica made a clapping gesture as she explained that she wanted Nathalie to "put them both together", in order to fill in the white spaces on the top row of Fig. 3c. Several other children were asking for the same thing, but also mentioning the blue, purple and red squares. Ava made the same gesture shown in Fig. 3d and then brought her other arm up and moved her two hands together, as in a clapping gesture. She was speaking as she made the gestures, but her voice could not be heard above the other voices.

Ava's sequence of gestures combines the ideas of twoness and of bringing together (which includes the motion). But it also echoes the gestures of both Jessica and Michael, even though it is not at all evident that she had seen them. The bringing together does not just describe the way in which the coloured squares on either side of the line move, but in both Jessica and Ava's interventions, it also describes the filling of the white spaces. Furthermore, the point of contact of the two hands clapping can be seen as actualising the line of symmetry—that is, bringing forth an object (the line of symmetry) that was not previously present. Indeed, when the two squares touch, like when the hands touch, they do so right on the line of symmetry. Although that line is visible on the sketch, it has not been referred to yet.

Symmetry as Making Recognizable Shapes

After a few squares had been moved on the screen, including the red one (twice) and the purple one (up) (see Fig. 4a), Nathalie again asked what had happened. Several children shouted out numbers, then Jonathan turned around and said "I" (identifying it as a recognizable letter). Jonathan continued by saying, "it's cutting the I" and lifted his hand and moved it down vertically. When asked what would happen if the purple square was moved up, another boy said, "it looks like a creeper". After moving more squares, as in the configuration shown in Fig. 4a, Nathalie moved the blue square on the bottom row to the right and someone said, "it's a T". Several children then began to ask Nathalie to move the blue square up so that it would reach the top row, eventually obtaining the diagram shown in Fig. 3c. At that point, several students said "whoa" and also shouted out "un T" (a T). Once there, as reported in the previous section, the students wanted Nathalie to move the squares so as to fill in the top row. Once the three squares had been moved into position, several students said, "that's a T" and one student said "awesome". Over the next few minutes, the children came up one by one to move other squares. Each time they did so, the other students commented on what "it looks like". For example, when the configuration in Fig. 4c was made, one child said, "it looks like a Chinese temple".

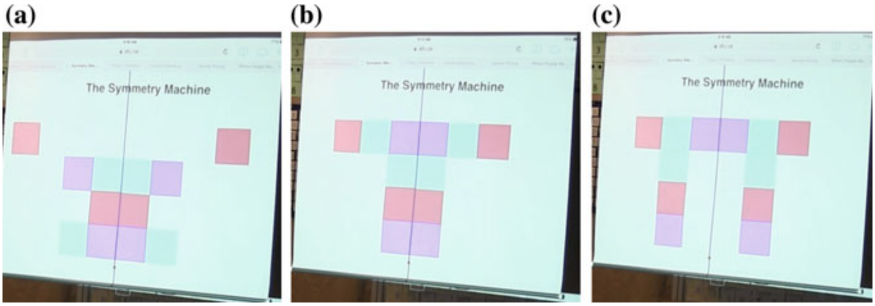

Fig. 4 **a** Two reds and a purple moved; **b** a 'T'; **c** Chinese temple

From a focus on the local, that is, on individual squares and how and where they move, there's now a shift to seeing the collection of 12 squares as a whole, to a more global perception. The global perception of symmetry is typically the first one that students encounter, when they are asked to consider the symmetric nature of a heart, for example. In this case, it is only when the squares are moved into a certain configuration, that the children begin to talk about one whole shape, referring to it as a letter of the alphabet and a Chinese temple. This idea of the Symmetry Machine producing letters initiated the recognition of a T-like shape and the subsequent movement of the square to produce Fig. 4b. Thereafter, the talk was focussed on what the configuration looked like rather than on the number of white squares or the twoness.

Over the next ten minutes, the children worked on the task of trying to create a diagram that has been taped to the whiteboard using the Symmetry Machine. In turn, they explained where the squares should move. They described moving squares on the left as well as squares on the right. Throughout, they focussed on the overall, global configuration. They got several of the squares into place, but some children began to engage in other activities, so the classroom teacher decided that it was time to move to the pairwise activities with the iPads.

Symmetry as Joint Movement

In the pairwise activity, the children were asked to reproduce a series of symmetric diagrams that were given to them on a piece of paper. Alik and Ava were given the diagram shown in Fig. 5a (bottom of the figure). Alik initially said that they could not make it and pointed to a square that is not symmetric with its corresponding square, saying "that square should be here" (pointing to the purple square to the right of the line of symmetry and then to the white square to its right). Nathalie urged them to try anyway. Ava put her finger on the purple square to the left of the line and moved it down, towards the line. Then Alik put his finger on the purple square to the right of the line and started moving it towards and away from the line

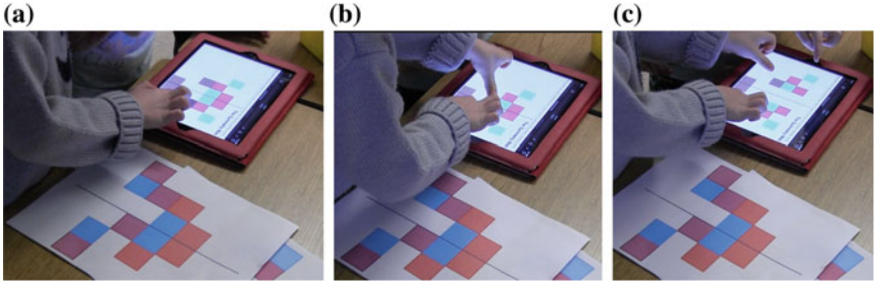

Fig. 5 **a** Paper sketch; **b** thumb and index finger; **c** line of symmetry translated

(see Fig. 5a). He then put his thumb on the square (Fig. 5b) and moved both his thumb (left hand) and his index finger (right hand). He ended up moving the line of symmetry and translated all the squares on the left of the line one unit away from the line of symmetry (see Fig. 5c). Both Ava and Alik were surprised. He then used two index fingers to move the purple squares towards each other and Ava did the same with the two blue squares. They each moved the purple square several times, towards and away from the line and concur that they could not make the diagram. Alik said to Nathalie, "when we move one, it just …" and then moves the purple square towards the line. He then pointed to the two purple squares on the piece of paper and explained that it's not symmetric because "that one (pointing to the purple square to the left of the line) is here" and "that one (pointing to the purple square to the right of the line) is on the line". Alik then went back to the Symmetry Machine and used one index finger, moving one purple square repeatedly back and forth. He then said, "there's supposed to be one there and one there" pointing to the right and to the left of the line.

Symmetry as Lining up

Nathalie then offered a new diagram (Fig. 6a) and Alik and Ava each moved one square. They then paused and Alik said, "it's not symmetric". When asked why, Alik pointed to the purple square at the top left of the piece of paper. Ava pointed to that square too, then to the other purple square on the top right of the piece of paper (Fig. 6b). Nathalie asked "you can't make it?" and Ava shook her head and said, "because those two (placing one side of her hand on the page to form a diagonal line between the two purple squares) are supposed to be" (placing her two hands to form a line perpendicular to the line of symmetry (see Fig. 6c).

This sequence gives rise to yet new symmetry concepts. While it is tempting to see in the students' reasoning about the Symmetry Machine that they are showing awareness of the equidistant property and the perpendicularity property, it is evident from their actions, and especially their gestures, that the Symmetry Machine

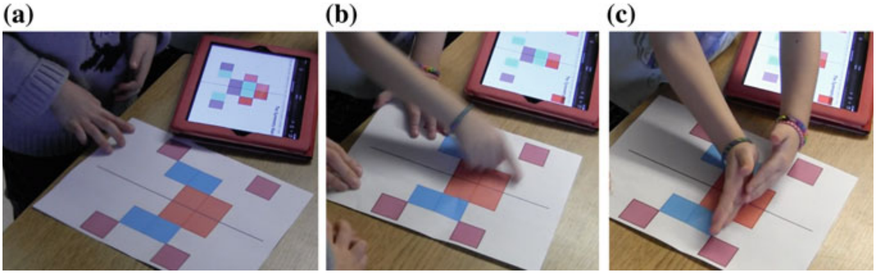

Fig. 6 **a** New diagram; **b** pointing; **c** perpendicular to line of symmetry

intervenes to give rise to new properties. In the first case, there is a shift from one-handed to two-handed dragging, then back again to one-handed dragging. In the first shift, Alik seems to want to force the squares to move in non-symmetric ways and when he sees that this is not possible (it just moves the line of symmetry), he goes back to the one-handed dragging, repeatedly making the squares come towards the line or move away. Therefore, it is less about the distance away from the line, which would emerge from a static configuration, than about the joint movement towards and away from the line. That the joint movement can be controlled by one square only emerges in the shift from the two-finger back to the one-finger dragging.

In terms of the perpendicularity, Alik and Ava barely move any of the squares from the Symmetry Machine before deciding that the diagram in Fig. 6a is not symmetric. Alik's continued pointing to the top left purple square suggests that he thinks it is out of place. The subsequent double pointing of Ava, which goes from one square to the other creates a virtual line that she then actualizes with her gesture. That line is not perpendicular to the line of symmetry (nor to the other 'lines' joining corresponding squares). With her second gesture, Ava shows what the correct line should look like, not necessarily in terms of the perpendicularity, but in reference to the pair of red squares that are already there.

Discussion and Conclusion

Reiterating our objective of this chapter, we are not addressing epistemological issues of what was learned or how learning takes place, for this infers the concept either to be a priori, independent of context and tools or to be the result of the mediation of tools (which might subsequently become expunged from knowledge). Instead we focus on the *Web Sketchpad* as part of an experimental apparatus that can highlight the indeterminacy of material engagement.

We used a diffractive apparatus because it helps us see how meanings about symmetry are not only entangled with the physical, but can also be considered intrinsically indeterminate. As noted in the highlighted episodes, symmetry takes different actualisations and meanings at different times. It is the varying material effects that allow us to follow the concept and not individual understandings. For example, when Jonathan moved his hand up and down along the line of symmetry saying “it’s cutting the I”, this gesture initially emerges from his observation of the projected sketch. His gesture is also aligning with the line of symmetry on the screen so that when he is moving his hand up and down he is expressing, and in fact, materially actualising a line of symmetry. The intra-action of his gesture and the sketch confirm each other and become an assemblage of meaning making of symmetry. While the focus on the concept of symmetry enabled us to carry out the diffractive analysis, we found it more difficult to write about the assemblage. Indeed, our writing, following conventional style, evoked individual children doing individual actions (gesturing, dragging, speaking). More methodological innovation will be required in future work in order to adequately follow the entailments of our theoretical perspectives.

Nonetheless, through using *Web Sketchpad* as part of a diffractive apparatus, new meanings of symmetry emerged. The space provided by the Symmetry Machine created new ways of instantiating symmetry. The concept of symmetry was expressed in numerous ways, as reflected in the subsection titles of the previous section: as twoness, as making holes, as bringing together, as making recognisable shapes, as joint movement, as well as lining up. In each case, students were intricately tied to the Symmetry Machine and the activity by forming gestures and body motions (e.g., like clapping) expressing symmetry in both a material and indeterminate way. In the clapping gesture of Ava, Jessica and Michael, the line of symmetry is actualised in bringing their hands together. Symmetry is seen as instantiated in movement, in alignment and in bringing together. Our analysis, which focused on the shifting nature of the concept of symmetry, enabled us to attend more carefully to the gestures that emerged (from movements of objects on the screen, as well as from child to child) over the course of the lesson. We connect these gestures to Michel Serres’ (2011) assertion that “there is nothing in knowledge which has not been first in the entire body, whose gestural metamorphoses, mobiles postures, very evolution imitate all that surrounds it” (p. 70). Serres is suggesting that the origin of knowledge is *not* understanding, which is about explanation and inference, but instead, is in the building of memory in the body, through gestures and movement. The mobility of the DGE can thus be seen as crucial to the changing ways in which the children moved and the continued new meanings for symmetry that emerged.

We are interested in challenging the a priori notion of symmetry and drawing attention, in particular, to the mobile device and how it influences and makes symmetry in different ways. We do not tie things up succinctly. Indeed, wrapping up this study with a cohesive conclusion is to contradict the very assumptions we began our study with. We merely tend to the assemblage of the diffractive apparatus and embrace the new and becoming of symmetry. By focussing on these new meanings, we did not track the ruptures and losses of meanings that resulted from the changing assemblage, such as the slipping away of the numerical value of two associated with the initial movements of the squares. In future work, more attention could be paid to this aspect of changing assemblages, to highlight the continuation of the mobility of symmetry.

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