



# Micro-Meso Mechanics Based Modeling of Damage Evolution in Cross Ply Laminates Composites

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**Abstract.** This paper develops a numerical model to study the crack density evolution and delamination initiated from the transverse crack tips for micro-racked ply in cross-ply laminates subjected to tensile loading. A representative unit cell using an energy based model is performed to study the growth of a new transverse crack and delamination process between two existing cracks in the 90° layer of a cross-ply laminate. However, an improved damage mesomodel for laminates allowing the calculation an energy standpoint, to the damaged laminate micromodel. A shear lag model was adopted to evaluate the stress distribution. The solution presented is obtained by using finite element analysis which implements micro-meso progressive failure analysis, while the effect of the stacking sequences has been done by varying the thickness of the 90° plies.

**Keywords:** Damage · Crack · Density · Delamination · Laminate

## 1 Introduction

The composite materials are one of the most widely used engineering materials in mechanical structures due to their high strength and relatively light weight. Such materials, despite the misunderstanding of certain aspects of their behavior. To exploit this design freedom, proper predictive tools are needed. However, the development of reliable numerical models is a challenging task, because the failure process is complex, involving different processes such as transverse ply cracking, fiber/matrix debonding, delamination, fiber breaking, and fiber pull out (see e.g. Green et al. 2007; Hallett and Wisnom 2006; Pierron et al. 2007). Often the damage evolution occurs over an important part of the loading, and a significant interaction is observed between the different mechanisms, leading to final fracture.

When a cross-ply laminate is loaded with the tensile axis aligned parallel to the 0° ply direction, it is generally believed that transverse matrix crack first appears in the weaker 90° plies. These small cracks quickly grow to extend laterally to span the entire 90° layer and then penetrate into neighbouring 0° plies. These cracks are more or less uniformly spaced. With applied loading, their number increases and then reaches a

saturation level. This level has been termed as Characteristic Damage State (CDS). Transverse cracks induce local stress concentrations at crack tips and involve inter-laminar delamination between  $0^\circ$  and  $90^\circ$  layers. Delamination develops from the crack tips and releases the local stress concentrations.

In order to predict the laminate failure which involves all these damage mechanisms evolution until final fracture, numerical tools are needed. However, this work intend to the development of such a predictive methodology to study the damage evolution in composite laminates. The existing approaches are based either on refined concepts on the microscale (which thus, provide precise information on specific mechanisms, but whose formalisms do not convene for structural analysis). The micromodel model is directly linked to the microscale and is able to take into account several mechanisms observed at microscopic level (the level of microcracking is quantified by a cracks density and delaminated process). However, The Damage Mesomodel based for Laminates developed since twenty years is one of those computational approaches. This model provides global measures of the degradation and no information on the microstate of degradation. The intact ply is modelled as a continuum, a relatively straightforward way to represent failure is with a continuum damage model, in which the stiffness of the material is reduced gradually after a certain failure criterion has been violated.

The present study develops both precise micromechanical models and mesocontinues models to evaluate the damage mechanisms and their evolution. The micromechanics is performed on a representative unit cell using an energy based model to study growth of a new transverse crack and delamination process between two existing cracks in the  $90^\circ$  layer of a cross-ply laminate. The objective is to build a continuum damage mechanics model which is quasi equivalent, from an energy standpoint, to the damaged laminate micromodel. Consequently, the potential energy stored in any part of a complete structure must be the same on the microscale and on the mesoscale.

A numerical application has been carried to cross-ply laminate of type  $[0_n/90_m]_s$  under tensile loading. In addition, the solution algorithm using finite element analysis which implements progressive failure analysis is presented. Effect of stacking sequence is focused particularly on the  $90^\circ$  ply thickness which mostly affects the transverse cracks initiation, propagation, cracks density and delamination process.

## 2 Computational Damage Model

### 2.1 Mesocontinuum Damage Model

In the approach of “mesomodel damage of laminates” (see Ladeveze 1986; Ladeveze et al. 2000; Allix and Ladeveze 1992; Ladeveze and Lubineau 2000), one assumes that the behaviour of any composite laminate for any loading and any stacking sequence can be modelled using ply failure criterion. The ply takes into account diffuse damage, transverse microcracking, and fibre breakage. The level of each mesoconstituent is quantified by damage indicators, whose evolution is governed by damage forces through an assumed damage law. An important point of the model is that the state of damage is assumed to remain constant throughout the thickness of a single layer (of course, it can

vary from one layer of the laminate to the next one). In other words, the main hypothesis of this computational model is that the damage evolution law is intrinsic to a ply and does not depend on the stacking sequence in which the ply is placed.

One of the most widely used approaches of continuum damage mechanics models based on energy potentials. The plies are taken to be homogeneous and orthotropic and the damage to be constant throughout the ply thickness. This mesoscale model is based on thermodynamic expressions, where the damage can be described by three internal variables  $d_{11}$ ,  $d_{22}$  and  $d_{12}$  corresponding to the loss of stiffness in the fiber, transverse and shear directions, respectively:

$$\begin{aligned} E_{11} &= E_{11}^0(1 - d_{11}) \\ E_{22} &= E_{22}^0(1 - d_{22}) \\ E_{12} &= E_{12}^0(1 - d_{12}) \end{aligned} \quad (1)$$

where  $E_{11}^0$ ,  $E_{22}^0$  and  $E_{12}^0$  are the initial stiffness of the material.

Assuming the existence of plane stresses and small perturbations, the associated elastic strain energy of a ply at a given stage of damage is:

$$E_D = \frac{1}{2} \left[ \frac{\sigma_{11}^2}{E_{11}^0(1 - d_{11})} - \frac{2\nu_{12}^0\sigma_{11}\sigma_{22}}{E_{11}^0} + \frac{\langle\sigma_{22}\rangle_+^2}{E_{22}^0(1 - d_{22})} + \frac{\langle\sigma_{22}\rangle_-^2}{E_{22}^0} + \frac{\sigma_{12}^2}{2G_{12}^0(1 - d_{12})} \right] \quad (2)$$

The transverse tension energy and compression energy are split in order to describe the unilateral feature resulting from the opening and closing of the micro-cracks. In the case of a transverse compression load, micro-cracks close up and thus the transverse direction behaviour remains undamaged. From this potential, thermodynamic forces associated are defined in each unidirectional ply:

$$\begin{aligned} Yd_{11} &= \frac{\partial E_D}{\partial d_{11}} = \frac{\sigma_{11}^2}{2E_{11}^0(1 - d_{11})^2}, \\ Yd_{22} &= \frac{\partial E_D}{\partial d_{22}} = \frac{\langle\sigma_{22}\rangle_+^2}{2E_{22}^0(1 - d_{22})^2}, \\ Yd_{12} &= \frac{\partial E_D}{\partial d_{12}} = \frac{\sigma_{12}^2}{2E_{12}^0(1 - d_{12})^2} \end{aligned} \quad (3)$$

In order to account for the coupling between the transverse and the shear during the development of the damage, an equivalent associated force  $Y$  was defined in each time during the history of the loading in ply:

$$Y_{eq}(t) = Yd_{12} + bYd_{22} \quad (4)$$

Generally, the coupling factor  $b$  is determined from experiments.

The development of the internal variables depends on these equivalent thermodynamic forces. When tensile is applied, the  $d_{11}$  develops sharply because of the brittle behaviour of the fibres in the unidirectional plies:

$$d_{11} = 0 \quad \text{if} \quad Yd_{11} < Yd_{11}^c \quad \text{else} \quad d_{11} = 1 \quad (5)$$

$Yd_{11}^c$  is the critical parameter, which defines the ultimate force corresponding to the tensile failure.

The evolutionary laws governing the internal variables  $d_{12}$  and  $d_{22}$  in each unidirectional ply, which depends on the associated equivalent forces (Eq. 4), still remain to be defined. In shear and transverse direction, a first approximation of the damage development, which corresponds to a linear law with respect to the square root of the equivalent associated force  $Y$ , shows a good fit with the experimental data:

$$d_{12} = \frac{\langle \sqrt{Y} - \sqrt{Y_0} \rangle}{\sqrt{Y_c} - \sqrt{Y_0}} \quad (6)$$

$$d_{22} = cd_{12} \quad \text{if} \quad Yd_{22} < Yd_{22}^c \quad \text{else} \quad d_{22} = 1 \quad (7)$$

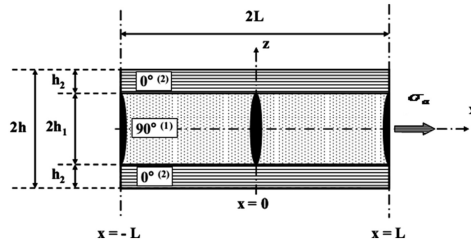
where the constant parameters  $Y_0$  and  $Y_c$  correspond to the threshold and the critical values of the development of  $d_{12}$  (which varies from 0 to 1) and the coefficients  $c$  stands for the ratio between the transverse damage and the shear damage, generally it equals 1.  $Yd_{22}^c$  is the critical parameter, which defines the ultimate force corresponding to transverse failure.

## 2.2 Micromechanics Damage Model

Up to now, there has been numerous theoretical and experimental works on the micromechanics of laminates (see Dvorak and Laws 1987; Hashin 1985a, b; Nairn and Hu 1994; Berthelot 2003); the micromechanics approach provides a relatively good understanding of damage mechanisms. A series of researchers attempted to quantify the stiffness reduction in a transversely cracked laminate. Aveston et al. (1971) proposed a so called ‘‘shear lag model’’ to modelling the stress transfer between fibers and matrix. This model is based on one dimensional analysis and was employed by Garrett and Bailey (1977) and others (Highsmith and Reifsnider 1982; Smith and Ogin 1999) to estimate the effective stiffness in a cracked laminate. Hashin (1985a, b) derived a variational solution to the stress field in a cross ply laminate under the assumption of uniformly distributed transverse cracks in  $90^\circ$  layer. His approach was a two dimensional analysis and adopted and modified by many authors to obtain the closed form solution of the normal and shear stresses as well as the axial stresses in the cracked  $90^\circ$  layers. More recently, the extended shear-lag analysis and variational approach studied the longitudinal displacements in the  $0^\circ$  and  $90^\circ$  layers, and considers the delamination initiation and growth from the tips of the transverse cracks (Nairn and Hu 1992; Takeda and Ogihara 1994; Berthelot and Le Corre, 2000).

**2.2.1 Stress Analysis**

Considering a baseline cross-ply laminates  $[0_n/90_m]_s$  with the midplane  $90^\circ$  plies, the loading state considered here is only the simple axial tensile load ( $\sigma_a$ ), as shown in Fig. 1.



**Fig. 1.** Cross ply laminate under uniaxial tensile load.

A shear lag model is employed to estimate the average stress of the  $0^\circ$  and  $90^\circ$  plies. The average stress of the  $0^\circ$  and  $90^\circ$  plies is supposed constant along their thickness:

$$\sigma_{22}(x) = \sigma_a \frac{E_{22}^0}{E_{xx}^0} \left[ 1 - \frac{ch\left(\beta \frac{x}{h_1}\right)}{ch\left(\beta \frac{L}{h_1}\right)} \right] \tag{8}$$

$$\sigma_{11}(x) = \frac{1}{h_2} [h\sigma_a - h_1\sigma_{22}(x)] \tag{9}$$

With:

$$E_{xx}^0 = \frac{t_{90}E_{22}^0 + t_0E_{11}^0}{t_{90} + t_0} \tag{10}$$

where  $\beta$  is the shear-lag parameter (Laws and Dvorak 1988):

$$\beta = \frac{(\alpha + 1)G_{23}(h_2E_{11}^0 + h_1E_{22}^0)}{E_{11}^0E_{22}^0h_2h_1^2} \tag{11}$$

$h_1, h_2$  is the thickness of the  $90^\circ$  and  $0^\circ$  plies.  $\alpha$  is the assumed shape index of the crack open displacement. For the cross-ply laminates  $[0_n/90_m]_s$ , it varies from 0 to 2 in different shear-lag models and reflects the assumed displacement function of the cracked  $90^\circ$  plies. It can be used as a global parameter to characterize the matrix crack effect.

The equilibrium of longitudinal applied force in an element of the  $90^\circ$  layer leads to the relation (Berthelot and Le Corre 2000):

$$\frac{d\sigma_{22}(x)}{dx} = -\frac{1}{h_1} \tau(x) \tag{12}$$

where  $\tau(x)$  is the shear stress at the interface between  $0^\circ$  and  $90^\circ$  layers.

### 2.2.2 Computational Micromodel

For cross-ply laminates,  $90^\circ$  plies are susceptible to the transverse cracks and they result delamination at the interfaces of the transversely cracked  $90^\circ$  plies and the adjacent  $0^\circ$  plies. The micromodel is characterized by periodic micro patterns described by cracking density ( $d$ ), at least locally, which is consistent with most practical situations. The level of microcracking is quantified by a cracking rate  $a$  defined by  $a = h_1/L$ . The local delamination is described at each transverse crack tip by a local delamination ratio  $r$  defined by  $r = l_d/h_1$  ( $r \in [0; 0.4]$ ) (Fig. 2).

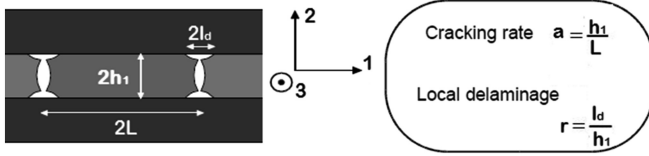


Fig. 2. Parameters of the micromodel

The evolution of these microvariables is governed by energy release rates. Considering the elementary cell of length  $2L$  of cracked laminate limited by two pre-existing cracks,  $x = L$  and  $x = -L$  (Fig. 1). Expression of the strain energy release rate based on Nairn model (Nairn 1989; Nairn and Mendels 2001) which associated to nucleation of the new transverse crack between the existing cracks is a function of applied stress  $\sigma_a$  and of cracking density:

$$G_{\max}(\sigma_a, d) = G_{\max}(\sigma_a) \times f_d(d) \quad (13)$$

With:

$$G_{\max}(\sigma_a) = \frac{1}{\beta} \frac{E_{22}^0}{E_{xx}^0} \frac{1}{E_{11}^0} \left(1 + \frac{h_1}{h_2}\right) (\sigma_a)^2 \quad (14)$$

$$f_d(d) = 2 \tanh\left(\frac{\beta}{4d}\right) - \tanh\left(\frac{\beta}{2d}\right) \quad (15)$$

We suppose that the strain energy release rate reached the critical value, us a following forme:

$$G_{\max}(\sigma_a, d) = G_c \quad (16)$$

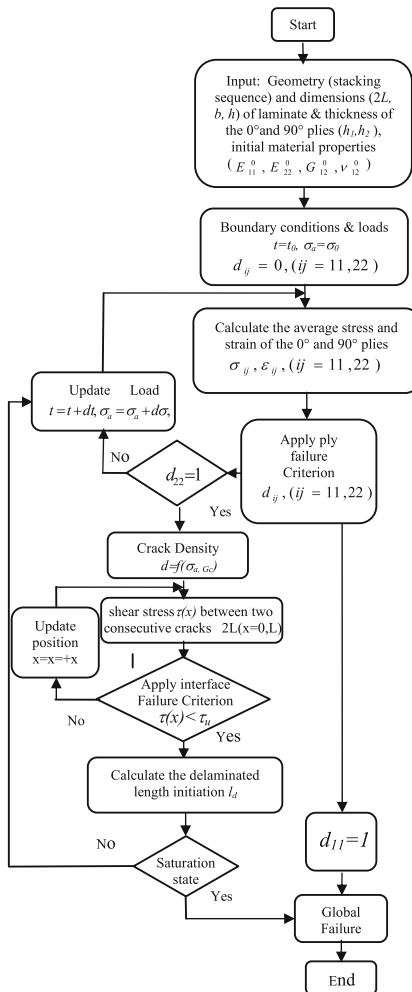
Replacing the Eqs. (13)–(15) in (16), we can find the final form for estimation the cracks density:

$$2 \tanh\left(\frac{\beta}{4d}\right) - \tanh\left(\frac{\beta}{2d}\right) = \frac{G_c}{\left[\frac{1}{\beta} \frac{E_{22}^0}{E_{xx}^0} \frac{1}{E_{11}^0} \left(1 + \frac{h_1}{h_2}\right)\right] (\sigma_a)^2} \quad (17)$$

### 3 Application and Result

A numerical application has been carried out on the cross-ply laminates constituted from continuous carbon fibres and epoxy resin of type IM7/977-2 with long carbon fibres of intermediate reference module IM7 and with epoxy matrix of reference 977-2. The different stacking sequence  $[0_n/90_m]$  is the quasi-isotropic laminate (dimensions  $140 \text{ mm} \times 12.7 \text{ mm} \times 1.1 \text{ mm}$ ). The stacking sequence effect can be studied by varying the structural stiffness ratio of the two plies ( $0^\circ$  and  $90^\circ$ ). This can be done by varying the thickness of the  $90^\circ$  plies. In the present application, three configurations are analyzed:  $[0_3/90_3]_s$ ,  $[0_2/90_4]_s$  and  $[0/90_5]_s$ .

A computational flow chart for the progressive failure analysis of the cross ply laminates used in this application is presented in Fig. 3.



**Fig. 3.** Flow chart of progressive failure analysis used for modelling the damage in cross-ply laminates.

All these numerical steps can summarised by six points: (1) Finite element modelling is used by including the geometry (stacking sequence) and specimen dimensions ( $l, b, h$ ), thickness of the  $0^\circ$  and  $90^\circ$  plies ( $h_1, h_2$ ) and initial material properties ( $E_1^0, E_2^0, E_{12}^0, \nu_{12}^0$ ) (2) For each time ( $t$ ) and load ( $F$ ) step, the finite element analysis is performed and the on-axis stresses/strains at each element are obtained. (3) The stresses/strains at each element are compared with the material allowable values and are used to determine whether some elements have failed according to the failure criteria or not. (4) In mesoscopic scale if  $d_{22} = 1$ , the first transverse crack is created, which provide to calculate the density cracks. (5) For a given elementary cell between two consecutive cracks, the delaminated length was estimated if the shear stress at the interface with the position  $x$  reached the ultimate shear stress  $\tau_u$ . The delamination process was continued until saturation state of microcracks, else the applied load is increased and the analysis continues. (6) finally, In macroscopic scale, the global failure is happen by fibre breakage in the  $0^\circ$  plies ( $d_{11} = 1$ ).

### 3.1 Effect of Stacking Sequence on the First Ply Failure

Materials are initially degraded by micro-cracks which are associated mainly with matrix micro-cracking (diffuse damage). This zone is characterized by non linear evolution (Fig. 4). Further loading will cause micro-cracks to coalescence and form a macro-crack leading to brittle fracture which is assumed to propagate into the entire thickness of the off-axis and will extend until it is stopped by fibres crossing the crack paths. Initiation and propagation of the first transverse crack is significant if a  $90^\circ$  layer thickness becomes more than the thickness of the  $0^\circ$  plies.

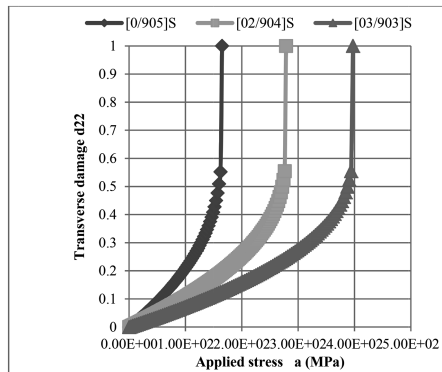


Fig. 4. Effect of stacking sequence on the first ply failure

### 3.2 Effect of Stacking Sequence on Crack Density Evolution

Figure 5 shows the crack density as a function of applied load. Continued loading will generate new transverse macro-cracks until a saturated level of matrix macro-cracks is



reached an asymptotic state which called the characteristic damage state (CDS). Hence more stress needs to be applied to form a new crack. The crack density (cracks/length) increases if a 90° layer thickness becomes less than the thickness of the 0° plies (Fig. 5).

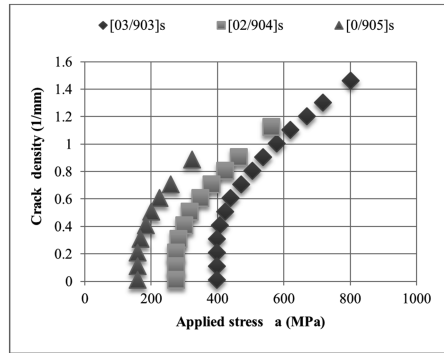


Fig. 5. Effect of stacking sequence on the crack density

### 3.3 Effect of Stacking Sequence on the Delamination Process

Figure 6 shows the delamination initiated length from the transverse crack tips versus cracks density. It is observed that When the thickness ratio  $h_1 = h_2$  is equal to 1 (case of the  $[0_3/90_3]_s$  laminate), the delamination process initiates only for high crack densities. when the tensile load applied to laminate is increased, delamination process do not progress more and laminate fractures rapidly because the higher crack density in the 90° plies causes local stress redistribution in the laminate which leads to additional loads on 0° plies. For  $[0_2/90_4]_s$  and  $[0/90_5]_s$  sequences, the delamination process initiates for low crack densities, as a result, the delamination length is considered before a new transverse crack was created by applying the load to laminate. So, Fig. 7 and 8 reports an example of the stress variations for 2 cases of transverse cracking density for  $[0/90_5]_s$  laminate.

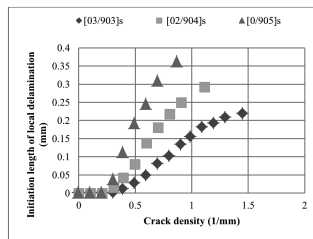
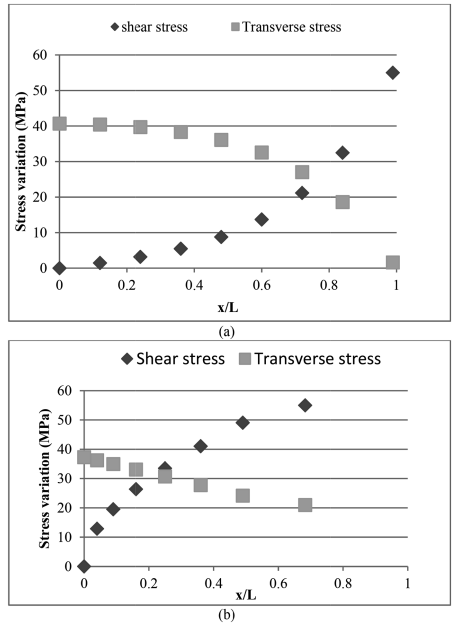
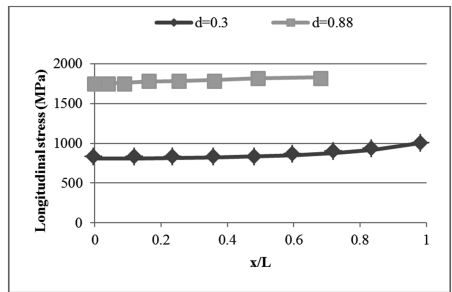


Fig. 6. Effect of stacking sequence on the delamination process



**Fig. 7.** Variation of shear and transverse stress in  $[0/90_5]_s$  laminate: (a)  $d = 0.3$ , (b)  $d = 0.88$



**Fig. 8.** Variation of longitudinal stress in  $[0/90_5]_s$  laminate for two cases:  $d = 0.3$  and  $d = 0.88$

### 4 Conclusions

To study the damage mechanisms in composite laminate, it is absolutely essential to use a multiscale computational strategy. The mesomodel provides global measures of the degradation and no information on the microstate of degradation. However, the micromodel is able to take into account several mechanisms observed at microscopic level, such that the nucleation and growth of a transverse crack and delamination process from the transverse crack tips.

The modelling results showed the effects of different stacking sequences with thickness variation of the 90° layers. It is observed that there is a significant effect of 90° plies thickness on the stress initiation and propagation of transverses cracking. As the thickness of 90° plies decreases and becomes less than the thickness of the 0° plies, the stress to micro-cracks initiation and propagation decreases and the level of characteristic damage state (CDS) increases. However, when the 90° layers thickness are less than 0° layers thickness, the cracks density increases and affects the entire microcracking process and leads to instantaneous fracture events as the cracks in the 90° plies act to raise the stress in adjacent 0° plies to cause early fibre fracture. It is also found that the number of broken fibres did not increase because of the development of local delamination at the s0°/90° interface in the vicinity of the crack. At loads close to failure in the quasi-static tensile test, the same local delamination appeared. So, when the 90° layers thickness are more than 0° layers thickness, the micro-cracks are suppressed entirely and the laminate fails before propagation of micro-cracking, while the final failure is mainly occurred by delamination process.

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