Chapter 6
Mesoscopic Models

Mesoscopic traffic flow models were developed to fill the gap between the family of microscopic models that describe the behavior of individual vehicles and the family of macroscopic models that describe traffic as a continuum flow. Traditional mesoscopic models describe vehicle flow in aggregate terms such as in probability distributions. However, behavioral rules are defined for individual vehicles. The family includes headway distribution models, cluster models, gas-kinetic models and macroscopic models derived from them. Most recently, hybrid mesoscopic models have appeared as a new branch on the tree: they combine microscopic and macroscopic models.

After reading this chapter, the reader will understand the basics of the traditional mesoscopic models: headway distribution models, cluster models and gas-kinetic models. Furthermore, they will understand the basics of hybrid modelling, including interface modelling and the moving coordinate system applied to hybrid models, and are able to argue about its advantages.

6.1 Headway Distribution Models and Cluster Models

Headway distribution models calculate traffic flows using time headways. The time headways are identically distributed independent random variables. The models are part of the mesoscopic family because they describe the distribution of headways of individual vehicles, while they do not explicitly trace the individual vehicles. These models are particularly well-suited to describe stochasticity (Li and Chen 2017). Examples are Buckley's semi-Poisson model (1968) and the generalized queueing model (Branston 1976).

Cluster models describe traffic flow as a flow of clusters of vehicles. Each cluster consists of multiple vehicles and within each cluster flow properties such as velocity and headway are assumed to be homogeneous. Clusters can emerge, for example,

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as an accumulation of vehicles behind a slow vehicles when overtaking possibilities are limited. Clusters can grow or decay. The most popular and famous cluster model is the one by Mahnke and Kühne (2007).

Since both headway distribution models and cluster models do not seem popular nowadays, we do not go into more detail here.

6.2 Gas-Kinetic Models

Gas-kinetic models were developed in analogy to models describing the motion of large numbers of small particles (atoms or molecules) in a gas. When applied to traffic flow, these models describe the dynamics of velocity distribution functions of vehicles. Prigogine and Andrews (1960), Prigogine (1961) first introduce gas-kinetic models describing traffic flow by the following partial differential equation:

$$\frac{\partial \tilde{\rho}}{\partial t} + v\frac{\partial \tilde{\rho}}{\partial x} = \left(\frac{\partial \tilde{\rho}}{\partial t}\right)_{\text{acceleration}} + \left(\frac{\partial \tilde{\rho}}{\partial t}\right)_{\text{interaction}} \tag{6.1}$$

with $\tilde{\rho}$ the reduced phase-space density which can be interpreted as follows. At time t, the expected number of vehicles between location x and $x + \mathrm{d}x$ that drive with a velocity between v and $v + \mathrm{d}v$ is the integral of the reduced phase-space density over this two-dimensional area:

$$\begin{aligned}&\text{expected \# of veh's in } [x, x + \mathrm{d}x) \text{ with velocity in } [v, v + \mathrm{d}v)\\ &= \int_{x}^{x+\mathrm{d}x} \int_{v}^{v+\mathrm{d}v} \tilde{\rho}(x, v, t)\, \mathrm{d}x\, \mathrm{d}v \;\approx\; \tilde{\rho}(x, v, t)\, \mathrm{d}x\, \mathrm{d}v\end{aligned} \tag{6.2}$$

where the approximation holds in the limit for an infinitesimal area with $\mathrm{d}x \to 0$ and $\mathrm{d}v \to 0$. Or, in other words, the reduced phase-space density is the expected number of vehicles in a small interval around location x, that travel with speed close to v at time t. The left-hand side of (6.1) consists of a time derivative and an advection term describing the propagation of the phase-space density with the vehicle velocity. At the right-hand side there is an acceleration term describing the acceleration towards the equilibrium velocity. The other term at the right-hand side is an interaction term, or collision term, describing the interaction between nearby vehicles.

6.2.1 Generic Gas-Kinetic Model

Paveri-Fontana (1975) improves this gas-kinetic model by relaxing the assumption that the behavior of nearby vehicles is uncorrelated, which results in an adapted interaction term. In the mid 1990s a revival of gas-kinetic models started with the development of multi-lane, multi-class and generic models (Helbing 1997; Hoogendoorn and Bovy 2001). The generic model includes a distinction between lanes, user classes, state-of-driving (free flow or platooning), flow direction, destination, desired speed, angle of movement and acceleration time). As such, the authors claim that it can also be used to model other types of particle flow, including (multi-dimensional) pedestrian flows.

We limit ourselves to the generic model as proposed by Tampère et al. (2003). It includes the reduced phase-space density as in (6.2) but more variables are considered. $\mathbf{S} = (s_1, s_2, \ldots, s_n)$ is the state vector and could include, not only velocity and position, but also any of the other variables mentioned before such as lane, user class, desired velocity, etc. The generic dynamical equation for the reduced phase-space density is:

$$\frac{\partial \tilde{\rho}}{\partial t} + \nabla \mathbf{S} \cdot \left(\tilde{\rho} \frac{\mathrm{d}S}{\mathrm{d}t} \right) = \left(\frac{\mathrm{d}\tilde{\rho}}{\mathrm{d}t} \right)_{\text{events}} \tag{6.3}$$

with the nabla operator on the state vector:

$$\nabla \mathbf{S} = \left(\frac{\partial}{\partial s_1}, \frac{\partial}{\partial s_2}, \ldots, \frac{\partial}{\partial s_n} \right) \tag{6.4}$$

The reduced phase-space density $\rho(t, S) \cdot \mathrm{d}\mathbf{S}$ can now be interpreted as the expected number of vehicles in a state 'close to' S.

6.2.2 Continuum Gas-Kinetic Models

Gas-kinetic models are usually not applied in simulations as such because they are computationally expensive. Instead, a continuum traffic flow model is derived and simulations are based on this continuum model.

The method of moments uses integration of the gas-kinetic traffic flow model (6.1) to find a continuum model. The reformulation in a continuum model, reduces some of the accuracy of detail in the model. However, its main advantage is the relative ease with which numerical simulations can be built, once the model is reformulated as a continuum model. Methods for higher order models introduced in the previous chapter can be applied to continuum models derived from gas-kinetic models.

A detailed discussion of continuum gas kinetic models or of the method of moments is out of the scope of this chapter, but we refer the interested reader to Hoogendoorn (1999) for a derivation of a continuum traffic flow model from a gas-kinetic model. Furthermore, Tampère et al. (2003, 2005) propose a continuum gas-kinetic model that explicitly includes a simple car-following model.

6.3 Hybrid Models

Hybrid models combine modelling approaches from different branches into a new model. Most hybrid models combine a car-following model with a continuum model, and are also referred to as multi-scale models. They typically apply a microscopic model to get detail and accuracy in areas and at times where that is required, e.g. the centre of an urban area. In the surrounding areas (e.g. on a free way ring road around the urban area) less detailed results are obtained with a macroscopic model, requiring much less computation time and memory. This way, simulations take advantage of the qualities of both the microscopic model (detailed results) and the macroscopic model (fast results).

6.3.1 Lagrangian Methods for Mesoscopic Models

The modelling within the micro- or macro-regions in space-time domain is done with the models discussed before and (almost) any model could be applied. The challenge lies in the modelling of the interfaces between the regions, see Fig. 6.1. To be effective, hybrid models must have a coupling on the interface between where/when traffic flow is modelled microscopically and where/when it is done macroscopically. To simplify the coupling, the Lagrangian formulation of the macroscopic model is often used. As we already saw in the previous chapter (Sect. 5.4.2) the discretized Lagrangian model is closely related to a car-following model. This makes the coupling of a discretized Lagrangian macroscopic model with a car-following model relatively easy. The continuous formulation of the macroscopic model or a discretised version of the Eulerian macroscopic model can also be applied (Leclercq 2007).

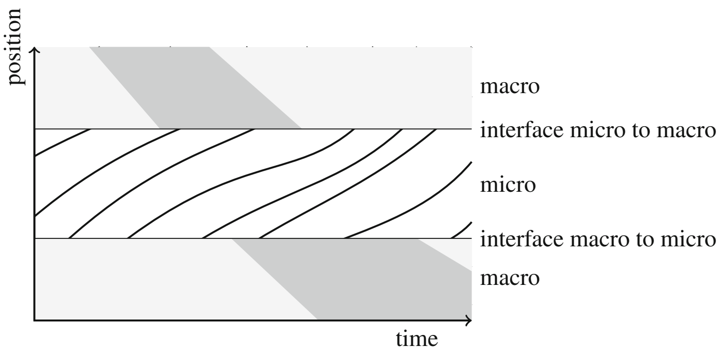

Fig. 6.1 Example of hybrid modelling: trajectories in the microscopic region and densities in the macroscopic regions

Examples of hybrid models are those combining Newell's earlier safe-distance model (Sect. 3.1.1) with the LWR model (Bourrel and Lesort 2003) and the one applying the Simplified Newell car-following model (3.4) to develop a hybrid model that couples this microscopic model with the macroscopic LWR model (Leclercq 2007). In this section, we focus on a rather generic approach as it is proposed by Moutari and Rascle (2007).

6.3.2 *Interface Modelling*

We discuss the coupling at the interface from a numerical perspective: considering how the discretized models are coupled. Therefore, at the macro-to-micro interface, groups of Δn vehicles are disaggregated into individual vehicles, while at the micro-to-macro interface, individual vehicles are aggregated into groups of Δn vehicles. For simplicity, we assume that Δn is integer, even though the method could be adjusted to also work with vehicle groups that contain any non-integer number of vehicles.

The main idea now is:

at the micro to macro interface vehicles leave the minimal microscopic region as individual vehicles. When, at time step k, Δn vehicles have left the region, they are aggregated in a vehicle group at the same location as the last vehicle in that group. See Fig. 6.2a.

at the macro to micro interface vehicles approach the minimal microscopic region as aggregated groups, but they are not allowed to enter as such. Therefore, once the front of the groups has entered at time step k, the group is disaggregated into individual vehicles, uniformly spaced over the road length that was previously taken by the group. See Fig. 6.2a.

We note that this implies that microscopic trajectories are created before the macro to micro interface and they are continued until after the micro to macro interface.

Depending on the applied microscopic and macroscopic models, the aggregated groups and the disaggregated individual vehicles inherit properties from the individual vehicles and vehicle groups, respectively. If the generic higher order macroscopic model is applied, the invariant I is inherited. The variable I can simply be averaged at the micro to macro interface: $I_j^k = \frac{1}{\Delta n} \sum_{m=1} N I_m^k$. At the macro to micro interface, the value of I_i^k for the individual vehicles equals I_j^k of the vehicle group: $I_i^k = I_j^k$.

Finally, the time step size has to satisfy the CFL condition for Lagrangian simulation (5.12). For a microscopic model with $\max \left|\frac{\mathrm{d}V}{\mathrm{d}s}\right| = v_{\max}$ and $\Delta n = 1$, the CFL condition reduces to:

$$\nu := \Delta t v_{\max} \leq 1 \tag{6.5}$$

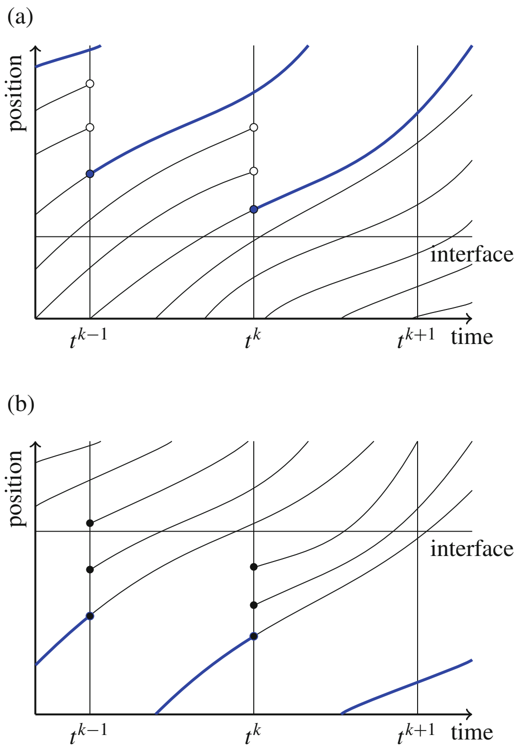

Fig. 6.2 Examples of interfaces in a hybrid model. Microscopic trajectories are indicated with thin black lines, macroscopic Lagrangian trajectories are indicated with thick blue lines. Open circles indicate the end of a trajectory, dots the start of a trajectory. The vehicle discretization in the Lagrangian model is set to $\Delta n = 3$. (**a**) In this example, every third microscopic trajectory is converted into a macroscopic Lagrangian trajectory. Whenever a new macroscopic trajectory is created, all downstream microscopic trajectories are terminated. (**b**) In this example, the macroscopic Lagrangian trajectories are converted into three microscopic trajectories. This is done when the front of the group reaches the interface, i.e. when the most upstream microscopic trajectory has left the macroscopic region. Two microscopic trajectories are created between this microscopic trajectory and the Lagrangian trajectory. Furthermore, the Lagrangian trajectory is converted into a microscopic trajectory

In most cases, this is a much stronger requirement than for the macroscopic model. This leads to a low CFL number for the macroscopic part of the simulation, which leads to added numerical diffusion and smaller times steps (and thus longer computations) than strictly necessary. Therefore, it is possible to take a larger time step only in the macroscopic region. For more details, we refer the interested reader to Moutari and Rascle (2007).

6.3.3 *Moving Interfaces*

In the examples, we have only shown interfaces fixed in space. However, the interface may also move, or one may be interested in more (or less) detail during a specific time interval. For example, it could be useful to apply a macroscopic model nighttime traffic, while moving to microscopic modelling in the urban areas of a network during peak hour. For more details about how to apply hybrid models in such cases, we refer to Joueiai et al. (2015).

Problem Set

Gas Kinetic Models

Consider the continuum gas-kinetic model as proposed by Treiber et al. (1999). (Refer to the original article for a detailed description.)

6.1 (Advanced) Reformulate the model in the Lagrangian coordinate system: i.e. reformulate the model such that it fits into the framework in Sect. 4.4.3 and define invariant I and source function g.

6.2 (Advanced) Adapt the code for a higher order model to simulate this model. Reflect on the results and compare them with previous simulations.[1]

Hybrid Models

6.3 (Advanced) Combine the code for microscopic and macroscopic modelling to build a hybrid simulation. Reflect on the results and compare them with previous simulations.[1]

Further Reading

Hoogendoorn SP, Bovy PHL (2001) Generic gas-kinetic traffic systems modeling with applications to vehicular traffic flow. Transp Res B Methodol 35(4):317–336

Joueiai M, Leclercq L, van Lint JWC, Hoogendoorn SP (2015) A multi-scale traffic flow model based on the mesoscopic LWR model. Transp Res Rec J Transp Res Board 2491:98–106

[1]Gas-kinetic and hybrid modelling are advanced topics. Numerical methods are not yet well-developed. The interested reader is encouraged to try to do some simulations, but it is not expected that this can be done easily.

Li L, Chen X (2017) Vehicle headway modeling and its inferences in macroscopic/microscopic traffic flow theory: a survey. Transp Res C Emerg Technol 76:170–188

Moutari S, Rascle M (2007) A hybrid Lagrangian model based on the Aw-Rascle traffic flow model. SIAM J Appl Math 68:413–436