Chapter 3
Microscopic Models

The earliest family in the model tree incorporating dynamics are microscopic models. They are based on the assumption that drivers adjust their behaviour to that of the leading vehicle. Microscopic modelling has shown to be a fruitful line of thought, which is illustrated by the large part of the model tree taken up by this family (see the model tree on page 15). Microscopic models describe the longitudinal (car-following) and lateral (lane-changing) behaviour of individual vehicles. We focus on longitudinal behaviour.

Most microscopic models are car-following models: they describe the movement of each vehicle based on the behavior (movements) of the vehicle(s) in front of it. This chapters also discusses the most recent branch of microscopic models, namely cellular-automata and numerical methods for microscopic models.

After reading this chapter, the reader will understand the basics of the most popular microscopic models, including their main features. They understand how extensions of microscopic models to include heterogeneity, multi-anticipation and time delay will improve them and being able to adapt a simple model in these directions. They can reflect on the desired properties of such models, including stability, and are able to assess simple models. Finally, the reader learns about the application of numerical methods applied to microscopic models, understand the impact of the choice of numerical method on stability and accuracy and they will be able to apply simple methods themselves.

3.1 Safe-Distance Models

In microscopic models, vehicles are numbered to indicate their order: n is the vehicle under consideration, $n-1$ its leader, $n+1$ its follower, etc., see Fig. 1.6. The state of each individual vehicle n is modelled in terms of the position of the front of the vehicle x_n at time t. Different types of models have different ways to describe or

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predict the movement of the vehicles, and thus how their position changes over time. Most models reflect human factors and behaviour, and are built using assumptions on how humans react and drive (Saifuzzaman and Zheng 2014).

The earliest car-following models include a car-following rule based on safe following distance. Pipes (1953) proposes to express the position of the leader as a function of the position of its follower:

$$x_{n-1} = x_n + T v_n + s_{n,\text{jam}} \tag{3.1}$$

with $s_{n,\text{jam}} = l_n + d$ the minimum rear-to-rear headway, i.e. the distance between the rear of the leader $n-1$ and the vehicle under consideration n in a jam. This is also illustrated in Fig. 3.1. $T v_n$ is interpreted as the 'legal distance' *between* vehicle $n-1$ and n: the extra distance that—together with the minimum rear-to-rear headway $s_{n,\text{jam}}$—makes up the actual rear-to-rear headway.

Assuming $s_{n,\text{jam}} = s_{\text{jam}}$ equal for all vehicles n, the model can be reformulated expressing the speed v_n as a function of the position x_n and the leaders' position x_{n-1}, or of the space headway (spacing) $s_n = x_{n-1} - x_n$:

$$v_n = \frac{x_{n-1} - x_n - s_{\text{jam}}}{T} = \frac{s_n - s_{\text{jam}}}{T} \tag{3.2}$$

which then leads to the following spacing-speed and density-speed relation:

$$V^*(s) = \frac{1}{T}(s - s_{\text{jam}}) \quad \text{or} \quad V(\rho) = \frac{1}{T\rho_{\text{jam}}}\left(\frac{\rho_{\text{jam}}}{\rho} - 1\right) \tag{3.3}$$

The 'fundamental diagrams' are shown in Fig. 3.2. Note that for low densities, the fundamental diagram is not realistic as the speed goes to infinity.

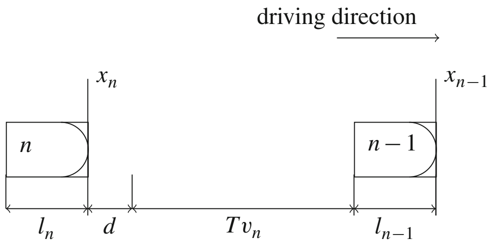

Fig. 3.1 Parameters of Pipes' safe-distance model

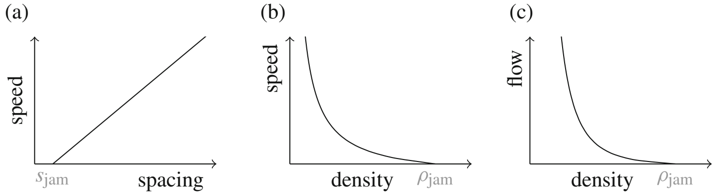

Fig. 3.2 The fundamental diagram of Pipes' safe-distance model. (**a**) Spacing-speed. (**b**) Density-speed. (**c**) Density-flow

3.1.1 Safe-Distance Models with Delay

Safety is closely related with delay: car-following behaviour is only safe when the delayed response to a change is taken into account. The first models including delay were introduced around 1960 (Kometani and Sasaki 1961; Newell 1961). Kometani and Sasaki's model is derived from basic Newtonian equations of motion. It is assumed that a drivers act such that they can avoid a collision even if their leader would act 'unpredictable'. Effectively, jam spacing s_{jam} in Pipes's model is replaced with a velocity-dependent term. Furthermore, their formulation includes a time delay τ. A positive τ represents that it takes some time between a change in the behaviour of a vehicle and the actual reaction of its follower to this change. The time delay τ includes the reaction time of the driver, but it can also depend on their perception and the time it takes between noticing that action is needed/desired and actual braking or accelerating due to limitations of the vehicle.

Several decades after the introduction of Newell's 1961 model, it was simplified (Newell 2002):

$$x_n(t + \tau_n) = x_{n-1}(t) - s_{\text{jam},n} \tag{3.4}$$

In this model, a vehicle follows the trajectory of its leader, translated by delay time τ_n and jam spacing $s_{\text{jam},n}$. Delay time τ_n and jam spacing $s_{\text{jam},n}$ may differ for each vehicle and driver. This model was later extended to include differences between 'timid' and 'aggressive' drivers (Laval and Leclercq 2010). Timid drivers would keep a longer distance when the leader decelerates into congestion, while aggressive drivers tend to keep shorter following distances, see Fig. 3.3. The formulation clearly shows that delay leads to hysteresis: the current behaviour of a vehicle/current traffic state depends on previous behaviours/states. With the correct parameter settings, the timid/aggressive car-following models gives simulation results showing stop-and-go waves.

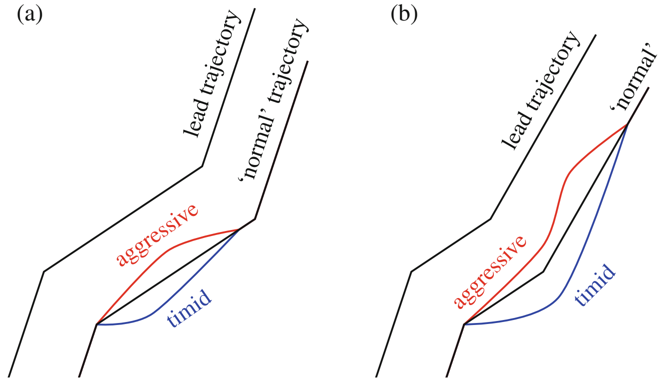

Fig. 3.3 Trajectories in the timid/aggressive model. Timid drivers decelerate and accelerate fast and therefore keep longer headways throughout a congestion wave. Aggressive drivers keep shorter headways. (**a**) Long wave. (**b**) Short wave

3.1.2 High Speeds Versus Safety

Gipps (1981) refines safe-distance car-following models with delay by assuming that 'the driver travels as fast as safety and the limitations of the vehicle permit':

$$v_n(t+\tau) = \min\left\{ \overbrace{v_n(t) + 2.5 a_{\max}\tau \left(1 - \frac{v_n(t)}{v_{\max}}\right)\sqrt{0.025 - \frac{v_n(t)}{v_{\max}}}}^{\text{limited by self}}, \right.$$
$$\left. \underbrace{a_{\min}\tau + \sqrt{a_{\min}^2\tau^2 - a_{\min}\left(2\big(s_n(t) - s_{\text{jam}}\big) - v_n(t)\tau - \frac{v_{n-1}(t)^2}{a_{\min}}\right)}}_{\text{limited by safe distance}} \right\} \tag{3.5}$$

with $a_{\max}$ maximum acceleration, $a_{\min}$ maximum deceleration (minimum acceleration), $v_{\max}$ the desired (maximum) velocity and s_{jam} jam spacing. Jam spacing is the front-to-front distance between two vehicles at standstill. Effectively, this approach introduces two regimes: one in which the vehicle itself limits its velocity (the upper part in Eq. (3.5)), and one in which the safe distance to the leader limits velocity (the lower part in the equation).

An other approach to model 'safe' behaviour is to include the collision probability in a model for acceleration (Hamdar et al. 2008). The authors propose to model the probability of a collision and adjust the acceleration (or deceleration) accordingly. The main idea is as follows: Driver behaviour is influenced by

1. the predicted velocity distribution of the leading vehicle
2. the speed of the leading vehicle relative to the own speed
3. the gap between the rear bumper of the leading vehicle and the own front bumper.

These three factors determine the probability of a collision, for any given acceleration. However, accelerating also gives a certain utility: e.g. driving at a speed close to the maximum speed. A combined utility function is proposed to weigh the collision probability against the utility of accelerating. Finally, the utility is maximized and the vehicle is modelled to accelerate or decelerate with the corresponding optimal acceleration/deceleration.

3.2 Stimulus-Response Models

The second branch of car-following models consists of stimulus-response models. It is assumed that drivers accelerate (or decelerate) as a reaction to three stimuli:

1. own current velocity $v_n = \frac{\mathrm{d}x_n}{\mathrm{d}t}$
2. spacing with respect to leader $s_n = x_{n-1} - x_n$
3. relative velocity with respect to leader (receding rate) $\dot{s}_n = \frac{\mathrm{d}s_n}{\mathrm{d}t} = v_{n-1} - v_n$

In the late 1950s and early 1960s there was a rapid development of stimulus-response models and the efforts consolidated in the now famous GHR-model, named after Gazis et al. (1961):

$$\underbrace{a_n(t)}_{\text{response}} = \underbrace{\gamma \frac{\left(v_{n-1}(t)\right)^{c_1}}{\left(s_n(t-\tau)\right)^{c_2}}}_{\text{sensitivity}} \underbrace{\dot{s}_n(t-\tau)}_{\text{stimulus}} \tag{3.6}$$

$\gamma \frac{(v_{n-1}(t))^{c_1}}{(s_n(t-\tau))^{c_2}}$ is considered as the sensitivity of vehicle/driver n. γ is the sensitivity parameter and c_1 and c_2 are parameters that are used to fit the model to data. The receding rate $\dot{s}_n(t-\tau)$ is observed at delay time τ ago and is considered as the stimulus, the acceleration $a_n(t)$ as the response, hence the name 'stimulus-response' model.

Since those early developments, a lot of work has been done in calibrating and validating this and other similar models. However, in 1999, Brackstone and McDonald concluded that stimulus-response models are being used less frequently, mainly because of contradictory findings on parameter values. Nevertheless, since the mid 1990s many new models have been developed and stimulus-response models are popular again.

3.2.1 More Recent Stimulus-Response Models: OVM and IDM

Bando et al. (1995) introduce the optimal velocity model assuming that drivers accelerate (or decelerate) to their optimal velocity, which is a function of the headway:

$$a_n(t) = \gamma \left(V\big(s_n(t)\big) - v_n(t)\right) \tag{3.7a}$$

$$V(s) = c_1\big(\tanh[c_2(s - c_3)] + c_4\big) \tag{3.7b}$$

with γ the sensitivity parameter and c_1, c_2, c_3 and c_4 parameters of the optimal velocity function $V(s)$. As discussed in more detail in Sect. 2.2.2, the optimal velocity function can be interpreted as the fundamental diagram of the model: it represents the spacing-speed relationship at equilibrium.

In the intelligent driver model by Treiber et al. (2000) the acceleration function includes two important components:

1. acceleration towards the maximum speed $v_{\max}$
2. acceleration/deceleration to obtain the space gap that is desired at the current speed and current change in gap (e.g. deceleration/lower acceleration when approaching the leader).

The acceleration function is described by:

$$a = a_{\max}\left(1 - \left(\frac{v}{v_{\max}}\right)^{\delta} - \left(\frac{S(v,\dot{s})}{s}\right)^{2}\right) \tag{3.8}$$

with $a_{\max}$ the maximum acceleration, v_{free} the free flow velocity (desired maximum speed) and δ the acceleration exponent. $S(v, \dot{s})$ denotes the space gap function, describing the desired spacing as a function of the speed and the change in spacing (i.e. difference in speed with leader):

$$S(v,\dot{s}) = s_{\text{jam}} + Tv - \frac{v\dot{s}}{2\sqrt{a_{\max}a_{\min}}} \tag{3.9}$$

with $a_{\min}$ the maximum deceleration (minimum acceleration), s_{jam} the jam spacing and T the minimum time headway.

The fundamental diagram (2.8) can be derived using the acceleration equation (3.8) and the space gap function (3.9). At equilibrium both the acceleration a and the change in spacing $\dot{s}$ are zero. Substituting this into (3.8) and subsequently substituting (3.9) gives:

$$a = a_{\max}\left(1 - \left(\frac{v}{v_{\max}}\right)^{\delta} - \left(\frac{s_{\text{jam}} + Tv}{s}\right)^{2}\right) = 0 \tag{3.10}$$

Table 3.1 Typical parameter values of the IDM

Maximum speed $v_{\max}$	30 m/s
Jam spacing s_{jam}	7 m
Reaction time T	1.5 s
Maximum acceleration $a_{\max}$	1 m/s^2
Maximum deceleration $a_{\min}$	1.5 m/s^2
FD shape parameter δ	1

Rewriting gives the fundamental diagram (2.8):

$$S(v) = \left(s_{\text{jam}} + Tv\right)\left[1 - \left(\frac{v}{v_{\max}}\right)^{\delta}\right]^{-1/2} \tag{3.11}$$

with s_{jam} the jam spacing and T the minimum time headway, v_{free} the free flow velocity (desired maximum speed) and δ the acceleration exponent, see Fig. 2.10.

Typical values for parameters in the IDM are given in Table 3.1. They are used in the simulations to produce the figures in this chapter. Readers are encouraged to play around with the parameter settings in the Problem Set at the end of this chapter.

3.2.2 Simulation Results with a Stimulus Response Model

The results of a simple simulation using IDM are shown in Fig. 3.4. It shows 15 vehicles on a 800 m long ring road. They are waiting in a queue until the first one starts driving at $t = 0$. Because they drive on a ring road, the first vehicle catches up with the last one and after about 1 min, the spacing and speeds are almost equal for all vehicles. The parameters of the model are given in Table 3.1. Furthermore, for the numerical method, an explicit Euler scheme with time step size $\Delta t = 0.5$ s were used. The relevance of the numerical method is discussed in Sect. 3.6.

3.2.3 Generic Model and Stability

Already in the earliest days of stimulus-response models Chandler et al. (1958) introduced a generic formulation:

$$a(t) = f\left(v(t), s(t), \dot{s}(t)\right) \tag{3.12}$$

In this formulation, it clear to see that the acceleration a is a response to the stimuli speed v, spacing s and change in spacing $\dot{s}$. It is interesting to note that, after reformulation, most safe-distance models also fit in this framework.

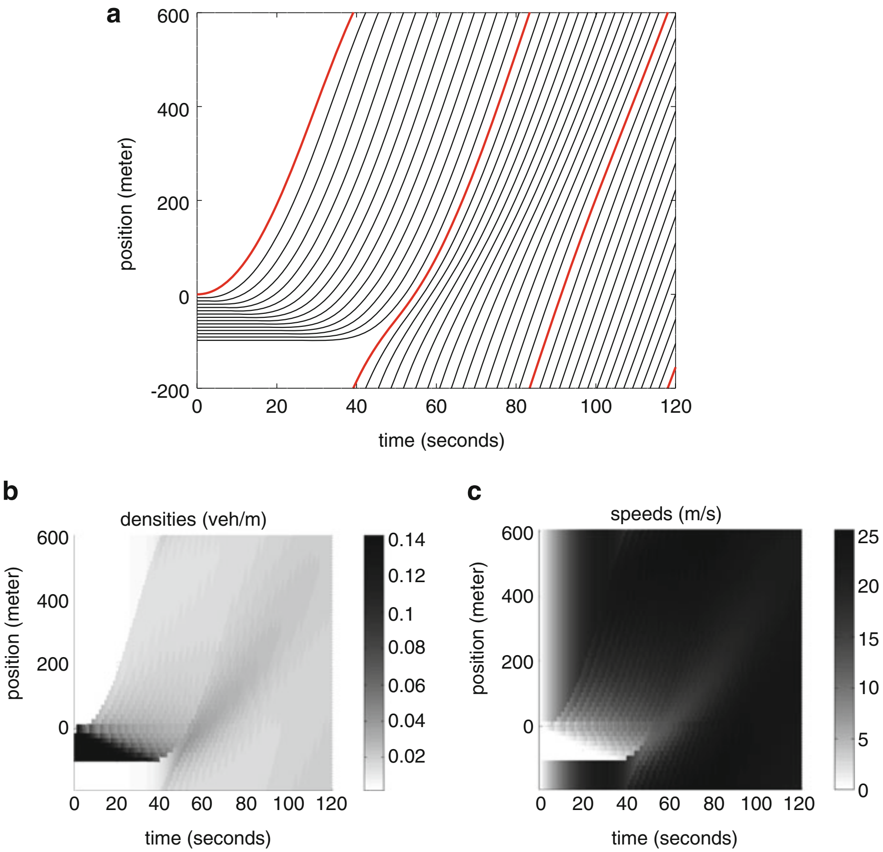

Fig. 3.4 Simulation results with IDM on a ring road, with an initial queue that dissolves, as described in Sect. 3.2.2. The same results are presented in different ways. (**a**) Trajectories: thick red trajectory is that of the first vehicle that starts driving at $t = 0$. The other trajectories (black) are of vehicles that were waiting in a queue behind the first one. (**b**) Densities. (**c**) Speeds

3.2.3.1 Requirements for Car-Following Models

More recently, Wilson (2008), Wilson and Ward (2011) use the generic formulation (3.12) to qualitatively asses stimulus-response models. They perform stability analyses and put forward constraints on the function f and its parameters. Firstly, for any car-following model, it should be possible to derive an equilibrium fundamental relation from the steady state solution of f:

$$\forall s > 0, \ \exists v = V(s) > 0 \text{ such that } f(v, s, 0) = 0 \tag{3.13}$$

Secondly, driving behaviour should be 'rational':

1. If velocity increases, the vehicle accelerates less (or decelerates more): $\mathrm{d}f/\mathrm{d}v < 0$.
2. If headway increases, the vehicle accelerates more (or decelerates less): $\mathrm{d}f/\mathrm{d}s \geq 0$.
3. If relative velocity increases, the vehicle accelerates more (or decelerates less): $\mathrm{d}f/\mathrm{d}\dot{s} \geq 0$.

If these conditions are satisfied, the model satisfies certain desirable stability conditions, as discussed in more detail below.

3.2.3.2 Stability of Car-Following Models

When looking at the dynamics of a system, it is important to know whether it is stable. In general, a system is stable if, when it is brought out of its current state, it will go back to that state (see Fig. 3.5a). In a traffic flow context, this means that when there is a perturbation (e.g. one driver suddenly brakes), traffic is stable when all vehicles will return to their initial speed. There are three types of questions to ask about the stability:

1. Does the fluctuation grow over time, will it stay within certain limits, or will it 'die out'?
2. Does the fluctuation stay at the same position or does it move, does it also affect other vehicles?
3. If the fluctuation moves, does it move with the vehicle that initially experienced the perturbation or does it move with a different speed, maybe even in the other direction?

These kind of questions are addressed by defining two types of stability that are of special interest in traffic flow: local stability and string stability. Local stability is about a perturbation growing both in amplitude and number of affected vehicles and becoming permanent, see Fig. 3.5b. If the flow is locally unstable, it will not return to its original, unperturbed state, and this type of instability is undesired. Wilson and Ward (2011) show that if the rational driving behaviour conditions are satisfied, then the car-following model is local stable. It is important to note, however, that, for a given model, the rational driving conditions may be satisfied for certain parameter values, but not for others.

The second type of stability is string stability , which is about a perturbation that might grow but each vehicle themselves will eventually return to the original unperturbed state Fig. 3.5c. This type of stability is linked to stop-and-go waves. The idea is that if a model is string unstable, then the fluctuation will grow and after it has passed tens or hundreds of cars, nonlinear effects will take over and trigger stop-and-go waves. The key difference with local stability is that the frame of reference is permitted to move: the fluctuation will remain, but it is allowed to move upstream, leading to recovery from the perturbation by vehicles that were affected

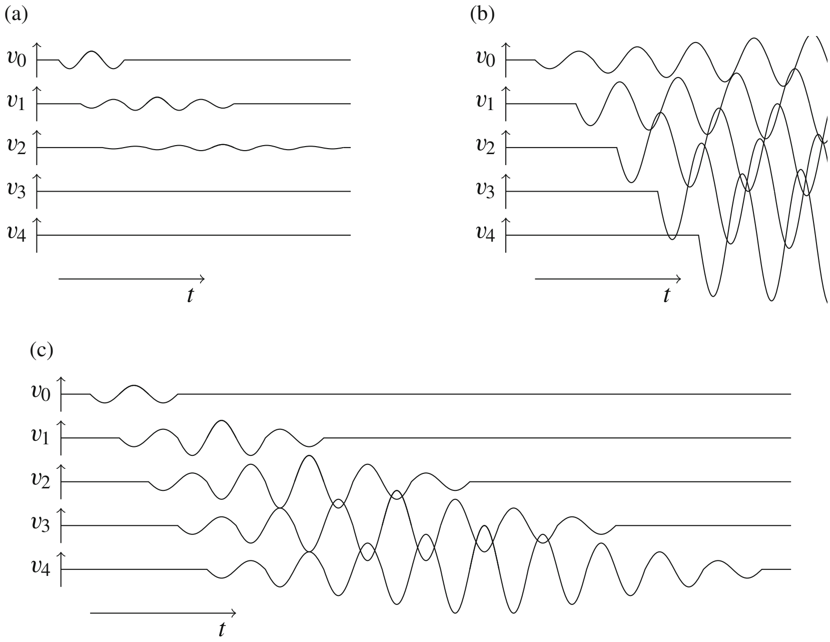

Fig. 3.5 Stability in car-following. The top line indicates the development of the speed (v_0) of the leading vehicle, which has a small perturbation. The other lines indicate the development of the speed of the following vehicles. (**a**) Stability. (**b**) Local instability (**c**) String instability

before. Wilson and Ward (2011) show that it largely depends on its parameter values whether the model will exhibit string instability. The problem set at the end of this chapter will explore this further and challenge the reader to do some stability analysis themselves.

Except for the type of stability, also its propagation direction is important. Will any propagation move downstream, in the driving direction of the vehicles, at the same speed, faster or slower? Or will it stay at the same location or move upstream, and at which speed? To model stop-and-go waves realistically, the string instability should propagate upstream, at the same speed as an observed stop-and-go wave, which is more or less equal to the slope of the congestion branch of the fundamental diagram. This could be checked by simulation, but a more rigorous approach is, again, based on analysis of the acceleration function f in (3.12). However, the analysis is beyond the scope of this book. The interested reader is referred to Wilson and Ward (2011), Ward and Wilson (2011) for more details.

3.3 Action Point Models

The third, and last, branch of car-following models consists of action point models, first introduced by Wiedemann (1974). However, a decade earlier, Michaels (1965) discussed the underlying concept that drivers would only react if they perceive that they approach a vehicle. Therefore, the approach rate or the headway must reach some perception threshold before a driver reacts. The main advantage of action point models is that they incorporate, in contrast to other car-following models, that:

1. at large headways driving behaviour is not influenced by that of other vehicles, and
2. at small headways driving behaviour is only influenced by that of other vehicles if changes in relative velocity and headway are large enough to be perceived.

If driving behaviour is influenced by that of others, any of the previously introduced safe-distance or stimulus-response models can be used to describe the influence quantitatively.

3.4 Cellular-Automata Models

Cellular-automata models are usually categorized as microscopic models, even though they are a different, and much younger, branch of the model tree. Just as in car-following models, the movement of individual vehicles is modelled. The main difference with car-following models is that space, and sometimes time, is discretized. Moreover, the velocity is discretized. Therefore, they are in general computationally more efficient.

In a cellular-automata model, the road is partitioned into cells of usually 7.5 m long. In a cell either a vehicle might be present or not. The model consists of a set of rules, that determine when the vehicle will move to the next (downstream) cell. The rules may be stochastic or deterministic. The model by Nagel and Schreckenberg (1992) is regarded as the prototype cellular-automata model. In this model, each time step each vehicle is advanced a few (or zero) cells according to the following algorithm:

1. If velocity is below maximum velocity, then accelerate: $\tilde{v} \to \min(\tilde{v} + 1, v_{\max})$.
2. If headway is too small, then decelerate: $\tilde{v} \to \min(\tilde{v}, \tilde{s}_{\text{jam}} - 1)$.
3. Decelerate at random: $\tilde{v} \to \max(\tilde{v} - 1, 0)$ with probability π.
4. Move: $\tilde{x} \to \tilde{x} + \tilde{v}$.

In the algorithm, $\tilde{v}$ denotes the normalized vehicle velocity in number of cells per time step, $\tilde{v}_{\max}$ denotes the normalized maximum vehicle velocity and $\tilde{s}_{\text{jam}}$ the normalized jam spacing in number of cells. $\tilde{x}$ is the cell number and π is the deceleration probability.

More recent developments combine cellular-automata models with the optimal velocity car-following model (Helbing and Schreckenberg 1999) or three phase theory (Kerner et al. 2002). Some of the most popular cellular-automata models are compared by Knospe et al. (2004).

3.5 Extensions

Microscopic models can relatively easily be adopted to other (assumed) behaviours of vehicles and drivers. Extra variables or parameters are added to reflect differences between types of vehicles, to reflect look-ahead behaviour, to reflect reaction-time delays or to include lane changes and lateral behaviour.

3.5.1 Heterogeneity

Most car-following models described and analysed in literature assume homogeneous vehicle-driver units, that is: vehicles and drivers all behave identically. However, since each vehicle is modelled and simulated individually, it is relatively straightforward to take into account heterogeneity. In that case model parameters such as desired (maximum) velocity, sensitivity and reaction time may vary over vehicles and drivers. In fact, most simulation tools based on car-following models are multi-class, that is: they do take into account heterogeneity.

3.5.2 Multi-Anticipation

Simple car-following models only take into account reaction to the immediate leader. However, in multi-anticipation models more than one leading vehicle influences the behaviour of a driver. For example, the generic model (3.12) with multi-anticipation would then be:

$$a_n(t) = f\left(v_n(t), s_n(t), \dot{s}_n(t), s_{n-1}(t), \dot{s}_{n-1}(t), \ldots, s_{n-N}(t), \dot{s}_{n-N}(t),\right) \tag{3.14}$$

The index n is used to denote the vehicle under consideration, $n-1$ is its leader, etc. N is the number of leaders that may influence the behaviour.

3.5.3 Time Delay

Time delay is introduced to reflect that drivers do not instantaneously react to any changes, but instead take a while to change their behavior (Bando et al. 1998; Treiber et al. 2006; Yu et al. 2014). For example, the generic model (3.12) with delay would then be:

$$a_n(t) = f\big(v(t-\tau), s(t-\tau), \dot{s}(t-\tau)\big) \tag{3.15}$$

with $\tau \geq 0$ the delay time. This is a commonly used extension of the models. However, including delay has a big impact on the mathematical properties of the model, and especially on the stability. However, this is beyond the scope of this book. The interested reader is referred to Wilson and Ward (2011), Ward and Wilson (2011) for more details about the theory. They are also encouraged to explore the issues using simulations, described in the Problem Set at the end of this chapter.

3.5.4 Lateral Movements

Until now, we have only discussed longitudinal (car-following) behaviour. However, an agent based traffic flow simulation for multi-lane roads is not complete without a model for lateral movements (lane changes). There are many models for lane changing, most of them making a distinction between:

mandatory lane change occurs when a driver moves to a different lane because of their route choice, e.g. from on ramp to main road to enter the freeway, from main road to off ramp to leave the freeway, from one lane to the next because the first lane will end or is blocked.

discretionary lane change occurs when a driver seeks a speed advantage, this often includes overtaking.

Other types of lane change in models can include random lane change (without apparent reason) or forced merging (when a driver creates a gap to enter a congested lane). Most lane change models consist of the following three steps:

1. Decide about necessity of lane change
2. Choose target lane
3. Decide whether to accept gap

Gap acceptance models include choices about whether the gap (distance between new leader and new follower) is big enough, but also about whether the necessary deceleration or acceleration are acceptable. A more detailed review of lane change models can be found in Rahman et al. (2013), Treiber and Kesting (2013).

3.6 Numerical Methods for Car-Following Models

The previously introduced car-following models are continuous in time and space. Therefore, for any case or simulation that is not very simple or even trivial, numerical integration needs to be used. Time is sliced into discrete time steps and the positions, and possibly states like velocity, acceleration and lane of all vehicles are computed.

One of the most simple and widely applied schemes is the Euler method:

$$x_n^{k+1} = x_n^k + \Delta t v_n^k \tag{3.16a}$$

$$v_n^{k+1} = v_n^k + \Delta t a_n^k \tag{3.16b}$$

Where k indicates the previous time step and $k+1$ the new time step: x_n^k, v_n^k and a_n^k are the position, speed and acceleration, respectively, of the n-th vehicle at time $t = t_0 + k\Delta t$. a_n^k is calculated according to the car-following model, e.g.

$$a_n^k = f\left(v_n^k, x_{n-1}^k - x_n^k, v_{n-1}^k - v_n^k\right) \tag{3.17}$$

with function f as the generic model as in (3.12). Furthermore, $v_{n-1} - v_n$ approximates the change in spacing because $\dot{s} = \frac{\mathrm{d}s}{\mathrm{d}t} \approx \frac{\mathrm{d}}{\mathrm{d}t}(x_{n-1} - x_n) = v_{n-1} - v_n$. This method is, for example, applied in the traffic simulators SUMO (Krajzewicz et al. 2012) and AIMSUN (Casas et al. 2010).

3.6.1 *Advanced Numerical Methods*

Other, more advance methods have been proposed and compared by Treiber and Kanagaraj (2015). The most promising of them is the ballistic update scheme which uses the same Euler update for the velocity (3.16b). The position is updated using the average of the velocity at the old time and that of the new time:

$$\begin{aligned} x_n^{k+1} &= x_n^k + \Delta t \frac{v_n^k + v_n^{k+1}}{2} \\ &= x_n^k + \Delta t v_n^k + \frac{(\Delta t)^2}{2} a_n^k \end{aligned} \tag{3.18}$$

The ballistic method outperforms the Euler method in terms of accuracy and computational complexity. Furthermore, they show that other methods (such as Runge Kutta) that are often used for ordinary differential equations resulting from other types of systems, perform worse because of the lack of ‘smoothness’ in traffic flow. Traffic is not ‘smooth’ in the sense that oftentimes there are sudden changes in traffic state such as a decrease in speed and increase in density at the upstream front of congestion. In fact, the non-smoothness can even in the most simple numerical methods lead to negative velocities. Therefore, in simulations, heuristics are applied to ensure that vehicles stop behind their leader without driving backward.

3.6.2 Numerical Methods and Delay

Many car-following models include a delay term, such as in (3.15). Therefore, the time step Δt needs to be taken such that the delay τ is a multiple thereof: $\tau = r\Delta t$, with r integer. When including delay, the acceleration function f is not considered at current time t, but at the earlier time $t - \tau = t - r\Delta t$, which leads to the following formula for the acceleration:

$$a_n^k = F\left(v_n^{k-r}, x_{n-1}^{k-r} - x_n^{k-r}, v_{n-1}^{k-r} - v_n^{k-r}\right) \tag{3.19}$$

The difference between time delay τ and time step size Δt is important. Even though they have the same dimension (time) and can be of similar size (≈0.1–1.5 s), they have a different interpretation and changing them should affect simulation results in a different way. τ is a model parameter—reflecting a time delay that is present in real car-following behaviour. τ should be small (or zero) when one wants to model that drivers (and vehicles) react quickly (or instantaneously) to a change in their leaders' behaviour, it should be large when the reaction takes longer. Δt is a parameter of the numerical method: indicating the size of the time steps in the simulation. A very small time step (approaching zero) gives a very accurate approximation of the time-continuous model. A somewhat larger time step size will give less accurate approximation but in many cases also shorter computation times. This difference is explored further in the Problem Set.

Problem Set

Microscopic Model and Fundamental Diagram

Consider Gipps' Safe-Distance model with delay as in Sect. 3.1.2. We modify the equation slightly to allow speed to become zero:

$$v_n(t+\tau) = \min\left\{ v_n(t) + 2.5 a_{\max}\tau\left(1 - \frac{v_n(t)}{v_{\max}}\right), \right.$$
$$\left. a_{\min}\tau + \sqrt{a_{\min}^2\tau^2 - a_{\min}\left(2\big(s_n(t) - s_{\text{jam}}\big) - v_n(t)\tau - \frac{v_{n-1}(t)^2}{a_{\min}}\right)} \right\} \tag{3.20}$$

This adaptation may lead to unrealistic acceleration/deceleration behaviour at very low speeds, but that does not interfere with our current goal of deriving and understanding the fundamental diagram.

3.1 Consider only the safe distance branch of (3.20) (that is the lower part of the equation). Derive the spacing-speed fundamental diagram and draw it.

The other ('free') branch introduces more realistic behaviour at high spacings.

3.2 What is the speed at very high spacings, according to (3.20)? Note: consider the free branch (the upper part).

3.3 Add the free flow branch to the previously drawn fundamental diagram (Problem 3.1).

Simulations

Simulations can give better insights into models. Sample code can be found on the website (http://extras.springer.com) of this book.

3.4 Run a simulation to reproduce the results in Fig. 3.4.

3.5 Adapt the provided code in one or more of these directions:

- make the initial queue longer
- change parameter values of the model parameters (maximum speed, jam spacing, reaction time, maximum and minimum acceleration shape parameter δ)
- replace the IDM with the OVM
- include delay
- change the time step size
- replace the Euler explicit time stepping method with the ballistic update scheme.

Compare the results of different setups and reflect on your insights: is this more (or less) realistic? Do you see other phenomena? Compare your results with those in literature.

Stability in IDM

As a base case, we consider the Intelligent Driver Models as introduced in Sect. 3.2.1, with the parameters as in Table 3.1. Again, we refer to the sample code available on the website.

3.6 Assess the platoon stability of the model by determining the sign of the derivatives of the acceleration function.

3.7 (Advanced) Asses the string stability of the model using simulations:

- Simulate vehicles driving over a ring road, using the default parameters.
- Vary the initial state of the vehicles to find when the model is string stable and when it is string unstable.
- When the results show instability, does the instability propagate upstream and/or downstream? Is this realistic?

Stability in Other Models

3.8 (Advanced) Use the adaptations in Problem 3.5 to explore (string) stability of:

- IDM with different parameter values
- OVM
- IDM or OVM with delay

and the interplay between the time step size or the numerical method and stability.

Simulations of Other Models

Problem 3.1 (Advanced) Adapt the simulation of IDM for a simple cellular automata model (see Sect. 3.4). Use the same queue test case and compare the results with those of other simulations that you have done.

Further Reading

Aghabayk K, Sarvi M, Young W (2015) A state-of-the-art review of car-following models with particular considerations of heavy vehicles. Transp Rev 35(1):82–105

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Orosz G, Wilson RE, Stépán G (2010) Traffic jams: dynamics and control. Philos Trans R Soc A Math Phys Eng Sci 368:4455–4479

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