Chapter 1
Introduction to Traffic Flow Modelling

Population growth and economic growth come with an increase in traffic demand and—more often than not—increased levels of congestion and accompanying delays, pollution and decrease in safety. There are several strategies to reduce congestion, keep cities liveable, clean and safe and limit travel time increase. Examples are encouraging people to travel using modes of transport that put less strain on the transportation network, to encourage people to travel at different times or on different routes, to apply traffic management to use roads in a more efficient way or to expand the road network. For all these measures, it is important to know how traffic flow will actually look: where and when will there be congestion, what are the bottlenecks and where is the road capacity already sufficient? Traffic flow models support this assessment by describing and predicting traffic on roads. For example, they model the number of vehicles on the road and their speeds. Using the models, travel times and congestion can be predicted.

To describe and predict traffic appropriately, real world observations are used to build theories, models and discretized models of traffic flow. By doing simulations based on these models the performance of roads or traffic networks can be assessed. In turn, this information is used for traffic management or the (re)design of roads and road networks.

This process is called the traffic flow modelling cycle, which is shown in Fig. 1.1 and is discussed in more detail in the next section. The rest of this chapter discusses some of its elements in more detail and the scope of this book is detailed in the last section.

The reader of this chapter will understand the importance and context of traffic flow modelling, they will have an understanding of the different types of traffic flow data and some important phenomena that can be identified in them. Furthermore, the reader will be able to work with the key variables of traffic flow modelling, which forms in important ingredient of the models that are discussed in the following chapters. Finally, the reader becomes familiar with classifications of traffic flow models and the genealogy of traffic flow modelling, including the four families.

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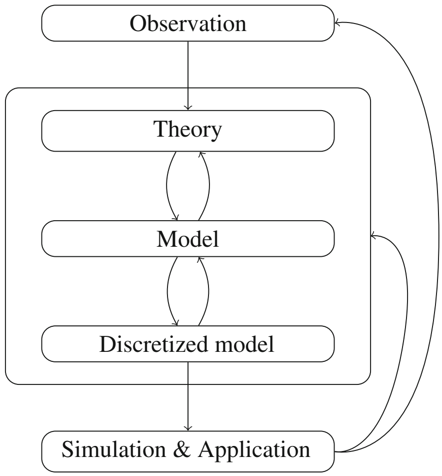


Fig. 1.1 Traffic flow modelling cycle. The genealogical tree focusses on the models. This book discusses the models more in depth, also including theories behind models and discretization methods

1.1 Traffic Flow Modelling Cycle

Traffic flow modelling is a largely inductive process: traffic observations are used to build a theory about the behaviour of individual drivers and vehicles or about traffic flow in general. Subsequently, that theory is used to build a model, discretize it and apply it in simulations. A simple example is the observations by Greenshields (1934) of vehicles passing his camera in the 1930s, see Fig. 1.2. Plotting the distance between the vehicles (spacing) and their change in position in consecutive photographs leads to a theory that spacing and speed are related. Subsequently, this leads to a model with a linear relationship between spacing and speed.

In more general terms, the development and application of traffic flow models is schematized in Fig. 1.1. As a first step, data is collected using, for example, loop detectors, cameras or GPS devices that many vehicles have on-board, such as navigation systems or mobile phones. Alternatively, data is collected using lab experiments for example with a driving simulator. These observations are analyzed and phenomena that characterize traffic flow are recognized.

In the second step, observations are used to build a theoretical framework. The theoretical framework consists of (mainly qualitative) statements and (behavioural) assumptions. For example, it is assumed that drivers perceive short space headways as more dangerous at high velocities than at low velocities. This is assumed to be the reason why at low velocities shorter headways are maintained. Another assumption is that drivers only react to their leaders and not to their followers.

In the third step, the theoretical framework is used to build a traffic flow model. The model consists of a set of equations, sometimes supplemented with a set of (behavioral) rules. For example, the theory about short headways at low velocities and long headways at high velocities is quantified in a fundamental diagram. It expresses the average vehicle velocity as a function of the average headway. Alternatively, a car-following model is developed that describes how a following

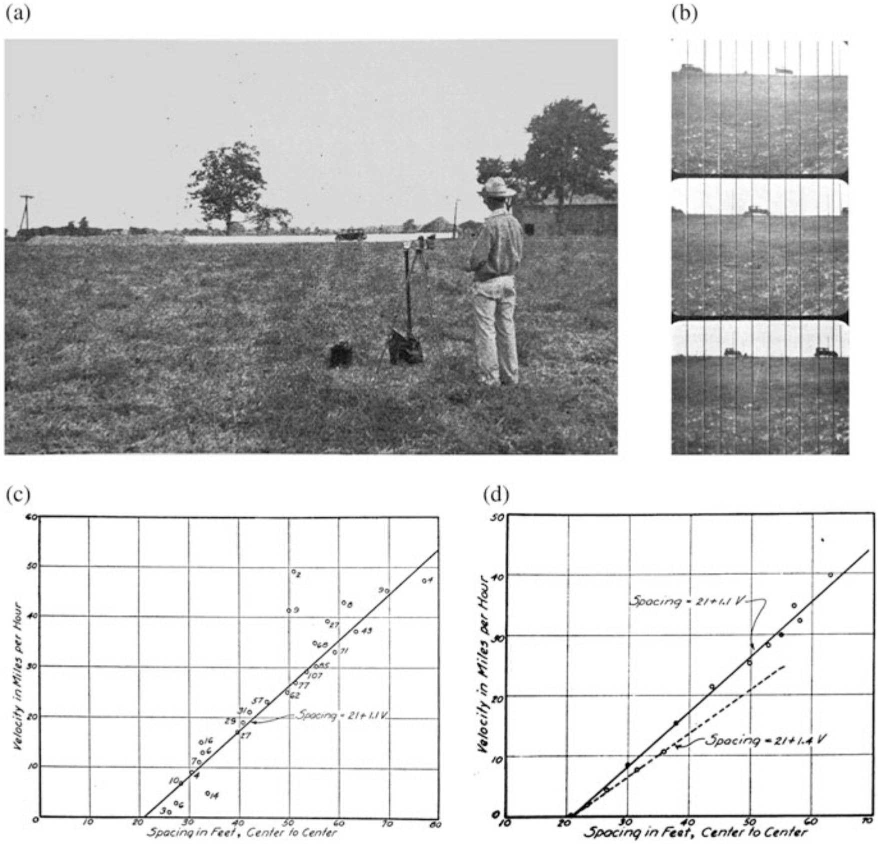


Fig. 1.2 Greenshields making field observations and turning this into a simple traffic flow model. Pictures reproduced with permission from Greenshields (1934). (**a**) Making field observations. (**b**) Resulting photographs. (**c**) Plotting data: speed against spacing. (**d**) Determining a linear relationship between spacing and speed

vehicle reacts to its leader(s), at which distance the leading vehicle is followed, and how the distance depends on the speeds of both leading and following vehicle.

The models can not be used directly in applications using computer simulation. Therefore, discretization is applied in the fourth step. In most simulation tools, time is divided into discrete time steps. Furthermore, depending on the model, also space or other continuous variables are discretized. Numerical methods are applied to approximate the new traffic state each time step. This results in a discrete traffic flow model.

Finally, the discrete traffic flow model is implemented in a computer program, resulting in a simulation tool. By applying this tool, and combining it with input such as data from traffic sensors, traffic state estimation and predictions can be made. Simulation results are compared to observations to calibrate the parameters and to validate the simulation tool.

Traffic flow models have many applications, for different purposes. They include:

- State estimation & short term predictions to inform travellers
- State estimation & short term predictions for traffic management
- Decision support for (semi-)autonomous vehicles
- Long term assessment of development plans, e.g. the (re)design of a transportation network
- Assessment of the impact of traffic on safety and emissions
- Design of evacuation plans

Naturally, different applications, call for different type of models. For example, when the goal is to inform commuters about the expected traffic situation if they'd decide to go home within 30 min, or to decide about activating traffic management measures in the next few minutes, it is most important to have almost instantaneous access to state estimation or prediction. In the more extreme case of decision support for autonomous vehicles, there is an even more urgent need of immediate information about, for example, driving into congestion. On the other end of the spectrum, when redesigning a network, it may be less important to get results quickly. However, in this application, many different scenarios may need to be calculated, for example using different socio-economic scenarios as input. Furthermore, besides computational speed, accuracy also plays a role. For example, when making long term plans, it may not be very interesting to know exactly what time of the day congestion will occur, but when informing travellers about the travel time if they would leave now, it is very important to know exact time and location of congestion.

Because of the difference in applications, different types of models have been developed, each of them more suited for certain applications than for others. Those interested in more detailed discussions of applications are referred to e.g. Treiber and Kesting (2013); Elefteriadou (2013).

1.2 Observations and Phenomena

While this book focusses on the central part of the traffic flow modelling cycle, this section introduces some of the other elements, in order to place the content in context and to support the reader in applying the materials covered in this book.

1.2.1 Observing Traffic

Traffic can be observed in many different ways. Most data comes from 'real world' observations where there is no intervention by researchers. Usually, the main purpose for collecting this data is not research or traffic model development but traffic management and information. Data is collected to get insights about actual traffic and then manage traffic accordingly or to inform road users. Typically, data collection is done using loop detectors which count the number of passing vehicles, sometimes also including information on their speed or the vehicle length. Other ways to collect data for traffic management and information include the use of camera's and systems to collect trajectory data from GPS or other in-car devices.

Observations are also collected using lab experiments where drivers are instructed to drive on a certain closed off road (network) or with driving simulators. Lab experiments have proven useful for qualitative model development, but are only limited applicable for quantitative observations, for example because safety perception in a driving simulator can be different from when driving on a real road. Driving simulators have seen a rapid development over the last years and the interested reader is referred to Auberlet et al. (2014) for more details. An other interesting data source are camera recordings from helicopters, which are gathered with the special purpose of research.

Summarizing, traffic can be observed from three perspectives, which are illustrated in Fig. 1.3:

- Local (fixed position): a loop detector, camera or other sensor that observes traffic passing at a certain point along the road.
- Instantaneous (fixed in time): a camera or other sensor that captures the traffic on a longer road stretch at a certain time (e.g. a picture taken from a helicopter)
- Trajectory (moving with vehicle): an in-car device or other sensor that collects data about the position of the vehicle over a certain time period.

A fourth perspective combines the first and second: observing traffic over a limited space and time period. For example, this type of observations combines a series of local observations over a few minutes to an hour and over a few hundred meter to a few kilometer. These observations can be obtained using for example camera's placed on a high building or bridge. The perspectives are compared in Table 1.1. How exactly to derive the variables introduced in the table is subject of Sect. 1.3.

Finally, different types of observations are combined to get a more complete image of traffic flow. In many applications, such as traffic state estimation and traffic

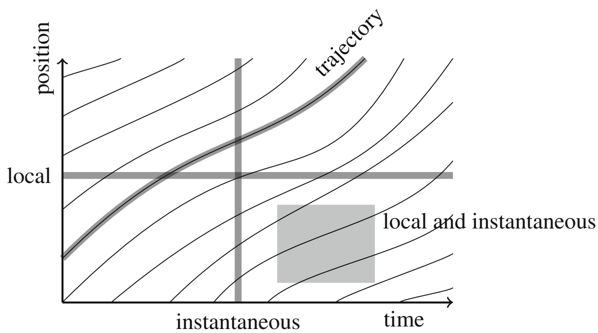


Fig. 1.3 Three ways to observe traffic

Table 1.1 Comparison of different types of traffic data

Type	Principle data	Derived variables
Local	Vehicle counts	Time headways, flows
Instantaneous	Vehicle positions	Space headways, densities
Trajectory	Location, time	Trajectories
Local and instantaneous	Counts at series of positions	Headways, flows, densities

state prediction, data from different types of sensors is fused to give a consistent image of the (current or future) traffic state. The applied techniques, including Kalman filtering, are beyond the scope of this book, the interested reader is referred to van Lint and Djukic (2012); Sun and Work (2017).

1.2.2 Phenomena in Traffic

Special patterns can be observed in traffic flow data, they are usually referred to as phenomena. Traffic flow models are designed to reproduce or predict these patterns. The simplest of those—which we would usually not call 'phenomena'—are low speeds at high densities (i.e. short headways) and long headways (i.e. low densities) at high speeds. Related to this is the observation that flow (or throughput) is highest at an intermediate density level, also known as critical density. If densities are well below the critical value, there are so few vehicles, that the flow (density $\times$ speed) is low, if densities are well above the critical value, the vehicle speed is so low that again the flow is low. Other patterns—or traffic flow phenomena—that can be observed include hysteresis and stop-and-go-waves.

1.2.2.1 Hysteresis and Capacity Drop

In general, hysteresis can be defined as follows. Consider a system where some variable (e.g. speed) changes when an other variable (e.g. space headway) changes. The system shows hysteresis when the change of dependent variable (speed in the example) lags behind the change in the other variable. Observations often show that accelerating takes longer than decelerating: when headways become larger, as is the case when leaving a queue, it takes longer for vehicles to adapt their speed to these new headways then when they enter a queue and slow down. An other example is the capacity drop observed at bottlenecks. Just before congestion sets in, the flow (capacity) through the bottleneck is high. Vehicles don't slow down in the bottleneck. However, slightly later, when the bottleneck has become active and a jam has developed upstream of the bottleneck, the flow (capacity) through the bottleneck is lower, see for example Fig. 1.4.

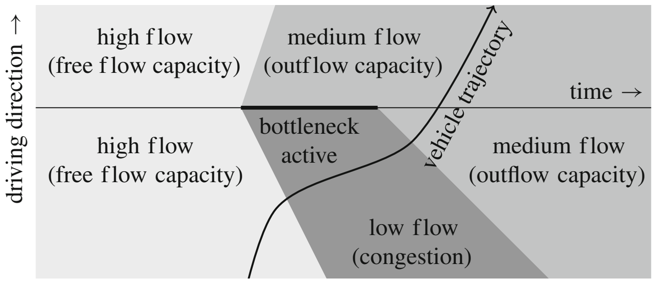


Fig. 1.4 Example of capacity drop: initially (on the left), the bottleneck is inactive and flow is high, at the 'free flow capacity'. When the bottleneck is activated (e.g. through a random event, a slight disturbance), congestion starts developing upstream of the bottleneck and—most importantly—the outflow out of the bottleneck drops to 'outflow capacity'

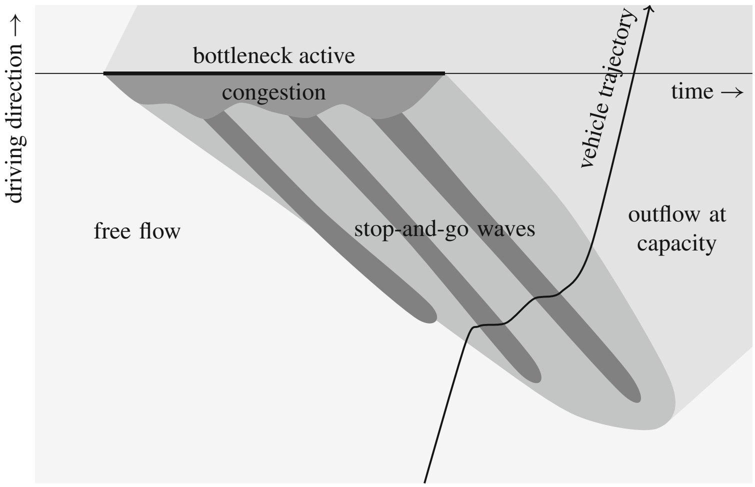


Fig. 1.5 Example of stop-and-go waves: upstream of a bottleneck is congestion. Within the congested area, stop-and-go waves are created: the waves travel upstream, vehicles encounter alternating relatively light congestion (high speed, low density) and heavy congestion (low speed, high density)

1.2.2.2 Stop-and-Go Waves

Stop-and-go-waves (also known as wide moving jams) are sometimes observed by drivers in congestion: alternatingly a driver has to slow down and can speed up again, see Fig. 1.5. This typical pattern can be very persistent, but poses a challenge to modelling. Again, delayed reactions (hysteresis) are proposed as a possible cause for their existence.

1.3 Traffic Flow Models

Traffic flow models have been developed and used since the beginning of the twentieth century. Traffic flow models are part of a long history of mathematical modelling of physical and other systems. Scientists and engineers use mathematical models as simplified representations of real-world systems. They are applied to explain and predict weather or chemical reactions, behaviour of materials or humans, fluid or traffic flow, etc. In this section we present a short overview of the traffic flow modelling efforts up to date.

Since the introduction of the first traffic flow model in the 1930s the number of models has increased. We only focus on the ones that are still most relevant in practical and scientific applications, but we can still identify about 50 different models, many of which have been developed over the last two decades.

To gain some basic insight into traffic flow modelling and the principle variables involved, we shortly discuss the main concepts of agent based and continuum traffic flow modelling. They are discussed in much more detail in the following chapters. Furthermore, we introduce other classifications of models and make a link between traffic flow models and models of other complex systems.

1.3.1 Agent-Based Models and Their Variables

Microscopic (or agent-based) traffic flow models are often considered the most intuitive, as they describe the behaviour of individual vehicles and trace their trajectories through space. They describe the longitudinal and lateral behaviour of individual vehicles, often based on assumptions regarding human factors and driving behaviour. Only longitudinal behaviour is discussed here. Vehicles are numbered to indicate their order: n is the vehicle under consideration, $n-1$ its leader, $n+1$ its follower, etc., see Fig. 1.6. The behaviour of each individual vehicle is typically modelled in terms of the position of the front of the vehicle x_n, speed $v_n = \mathrm{d}x_n/\mathrm{d}t$ and acceleration $a_n = \mathrm{d}v_n/\mathrm{d}t = \mathrm{d}x_n^2/\mathrm{d}t^2$. The speed typically depends on a few of the following factors:

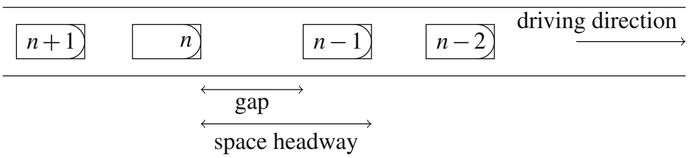


Fig. 1.6 Vehicle numbering in microscopic traffic flow models (and macroscopic models in Lagrangian formulation)

current speed of the vehicle under consideration
current speed of the leading vehicle or possibly multiple leading vehicles
space headway distance of the vehicle to its leader
individual properties of driver and vehicle e.g. desired speed, reaction time, maximum acceleration, braking power

In this book, 'space headway' (or spacing) is defined as in Fig. 1.6: the distance between the front of the leader and the front of the vehicle under consideration. Furthermore, the 'gap' is the distance *between* the two vehicles. It should be noted that some authors, exclude the vehicle length from the space headway and define it as the distance between the vehicles, or they include the vehicle length of the vehicle under consideration instead of the vehicle length of the leader.

1.3.2 Continuum Models and Edie's Definitions

Most traffic flow models are based on the assumption that there is some relation between the distance between vehicles and their speed: if headways are short, drivers tend to lower their speed. This relation can be described, or modelled through, the fundamental diagram.

Originally Greenshields studied the relation between the variables spacing and velocity. However, the fundamental diagram can also be expressed in other variables such as density (average number of vehicles per unit length of road) and flow (average number of vehicles per time unit), see Fig. 1.7. These variables were first defined rigorously by Edie (1965). Figure 1.8 illustrates the definitions. Flow is defined as the flow in an area A with length dx and duration dt which is determined by the number of vehicles $N(A)$ that travel through the area and the distance y_n they

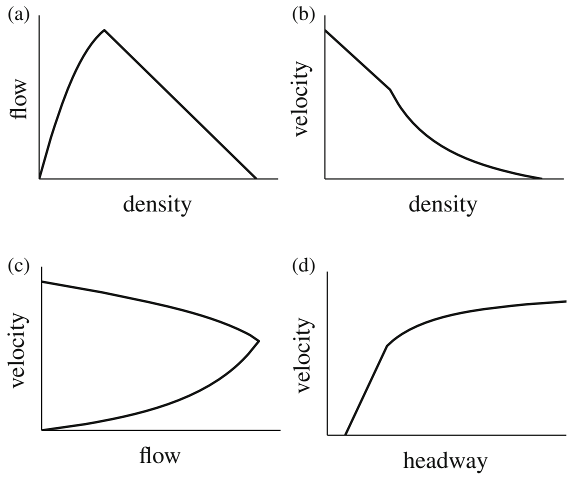


Fig. 1.7 Fundamental diagrams in different planes. (**a**) Density-flow plane. (**b**) Density-velocity plane. (**c**) Flow-velocity plane. (**d**) Spacing-velocity plane

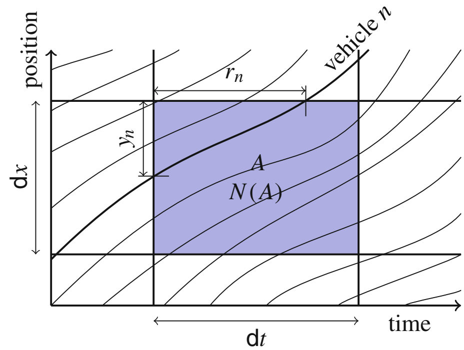


Fig. 1.8 Time-space region with some vehicle trajectories to illustrate Edie's definitions of flow and density. $N(A)$ is the number of vehicles that travel through the area A

travel through the area:

$$q_{\text{area}} = \frac{\sum_{n=1}^{N} y_n}{\mathrm{d}x\,\mathrm{d}t} \tag{1.1}$$

Similarly, density is defined as the density in an area using the time r_n vehicle n is present in the area:

$$\rho_{\text{area}} = \frac{\sum_{n=1}^{N} r_n}{\mathrm{d}x\,\mathrm{d}t} \tag{1.2}$$

Finally, this leads to the intuitive definition of velocity in an area by dividing the total distance traveled by the total time spent:

$$v_{\text{area}} = \frac{q_{\text{area}}}{\rho_{\text{area}}} = \frac{\sum_{n=1}^{N} y_n}{\sum_{n=1}^{N} r_n} \tag{1.3}$$

These are workable definitions to extract flow, density and velocity from observations of a large area A with many vehicles N. They can even be applied to long road sections observed over a short period of time or, vice versa, short road sections observed over a long period of time. However, for other applications such as macroscopic traffic flow models, flows, densities and velocities at points in (x, t) are considered. Therefore, we have to assume that traffic is a continuum flow. In Sect. 1.3.4 we argue why this is a reasonable assumption. The assumption implies that N becomes continuous (instead of discrete). Furthermore, N is assumed to be continuously differentiable in x and t. Edie's definitions are then not applicable directly. However, by decreasing the area A such that it becomes a point, the definitions of flow, density and velocity become meaningful at points.

The local and instantaneous flow, density and velocity are found using the procedure described by Leutzbach (1988). To find the local and instantaneous flow (the flow at a point in (x, t)) we decrease the length of the road section: $\mathrm{d}x \to 0$.

This yields the local flow through a cross section x. Afterwards, we decrease the time $\mathrm{d}t \to 0$ and find:

Definition 1.1 ((Local and Instantaneous) Flow)

$$q(x,t) = \lim_{\mathrm{d}t\to 0}\lim_{\mathrm{d}x\to 0}\frac{\sum_{n=1}^{N} y_n}{\mathrm{d}x\,\mathrm{d}t} = \lim_{\mathrm{d}t\to 0}\underbrace{\frac{N(x,[t,t+\mathrm{d}t])}{\mathrm{d}t}}_{=q_{\text{local}}(x)} \tag{1.4}$$

To find the local and instantaneous density we decrease the time: $\mathrm{d}t \to 0$. This yields the instantaneous density through a cross section t. Afterwards, we decrease the length $\mathrm{d}x \to 0$ and find:

Definition 1.2 ((Local and Instantaneous) Density)

$$\rho(x,t) = \lim_{\mathrm{d}x\to 0}\lim_{\mathrm{d}t\to 0}\frac{\sum_{n=1}^{N} r_n}{\mathrm{d}x\,\mathrm{d}t} = \lim_{\mathrm{d}x\to 0}\underbrace{\frac{N([x,x+\mathrm{d}x],t)}{\mathrm{d}x}}_{=\rho_{\text{instant}}(x)} \tag{1.5}$$

Finally, similar to Edie's definition of velocity (1.3), we define the local and instantaneous velocity:

Definition 1.3 ((Local and Instantaneous) Vehicle Velocity)

$$v(x,t) = \frac{q(x,t)}{\rho(x,t)} \tag{1.6}$$

In addition, we define local and instantaneous spacing:

Definition 1.4 ((Local and Instantaneous) Vehicle Spacing)

$$s(x,t) = \frac{1}{\rho(x,t)} \tag{1.7}$$

The above definitions of flow, density, speed, and spacing are used throughout this book. They form an essential ingredient for mesoscopic and macroscopic models.

1.3.3 Classifications of Models

To get better insight in traffic flow models and their usefulness to certain applications, they can be classified into groups with similar properties. Many classifications of traffic flow models have been proposed, including classifications based on the type of variables and equations that are used to describe the processes:

Scale of independent variables continuous, discrete, semi-discrete (independent variables are usually time and space, or sometimes vehicle count);
Stochasticity deterministic or stochastic variables and processes;
Type of model equations (partial) differential equations, discrete or static models;
Operationalization analytical or simulation, where simulation often involves discretization of the model equations;
Number of variables and parameters to distinguish between 'efficient' models with few variables and parameters but still able to reproduce and predict traffic realistically and 'inefficient' models.

Other classifications are based on how detailed traffic and driver behavior is described and potential applications:

Level of detail of representation sub-microscopic, microscopic (agent-based), mesoscopic, macroscopic (continuum), network-wide or hybrid models combining different levels of detail;
Level of detail of underlying behavioral rules individual, collective
Scale and type of application networks, links, intersections, urban roads, freeways;
Number of phases described by the model mainly to distinguish between models that show states such as free flow, congestion, stop-and-go traffic differently.
Phenomena that can be explained or reproduced by the model

Throughout the book, some categories listed above are used for sub-classification of models. Furthermore, the concepts have been used as criteria to assess traffic flow models, with properties that are often considered to be desirable such as:

1. The model only has few parameters.
2. Parameters are (easily) observable and have realistic values.
3. Relevant phenomena are reproduced and predicted by the model.
4. The model allows fast computations for state estimation or prediction.

To establish the genealogy of traffic flow models, we mainly use the classifications based on the type of model equations (static vs dynamic, with discrete or continuous flow) and the level of detail, as discussed further below. The interested reader is referred to e.g. Hoogendoorn and Bovy (2001b); Lesort et al. (2003); van Wageningen-Kessels et al. (2011); Bellomo and Dogbe (2011); van Wageningen-Kessels et al. (2015) for a more detailed discussion of classifications and reviews of traffic flow models.

1.3.4 Traffic Flow, Fluid Flow and Other Complex Systems

Traffic flow models are often related to and derived in analogy with models for fluid flow. For example, the seminal paper on macroscopic traffic flow modelling (Lighthill and Whitham 1955b), was published as 'On Kinematic Waves Part 2' together with Part 1 discussing flood movement in rivers (Lighthill and Whitham 1955a). In turn, traffic flow models have recently inspired researchers to model pedestrian crowds and animal swarms in a similar way, cf. Bellomo and Brezzi (2008), other articles in that special issue of Mathematical Models and Methods in Applied Sciences on Traffic, Crowd, and Swarms, and Bellomo and Dogbe (2011). Furthermore, similarities between vehicular traffic, pedestrian and granular flow have been recognized and conferences on Traffic and Granular Flow are organized bi-annually, the most recent ones being held in Delft (The Netherlands) in 2015 and in Washington D.C. in 2017 (Knoop and Daamen 2016; Transportation Engineering Group at the George Washington University 2017). Finally, Helbing (2008) relates traffic flow to systems that might even seem more diverse such as those related to collective decision making, risk management, supply systems and management strategies.

1.4 Approach and Scope of This Book

In this book, we discuss the traffic flow models and some of its underlying theories and numerical methods for computer simulation. The main modelling approaches are introduced and positioned in the genealogical tree of traffic flow models, which shows the historical development of traffic flow modelling, see page 15. For most models, this book also introduces the reader to useful approaches for computer simulations.

1.4.1 The Genealogical Model Tree

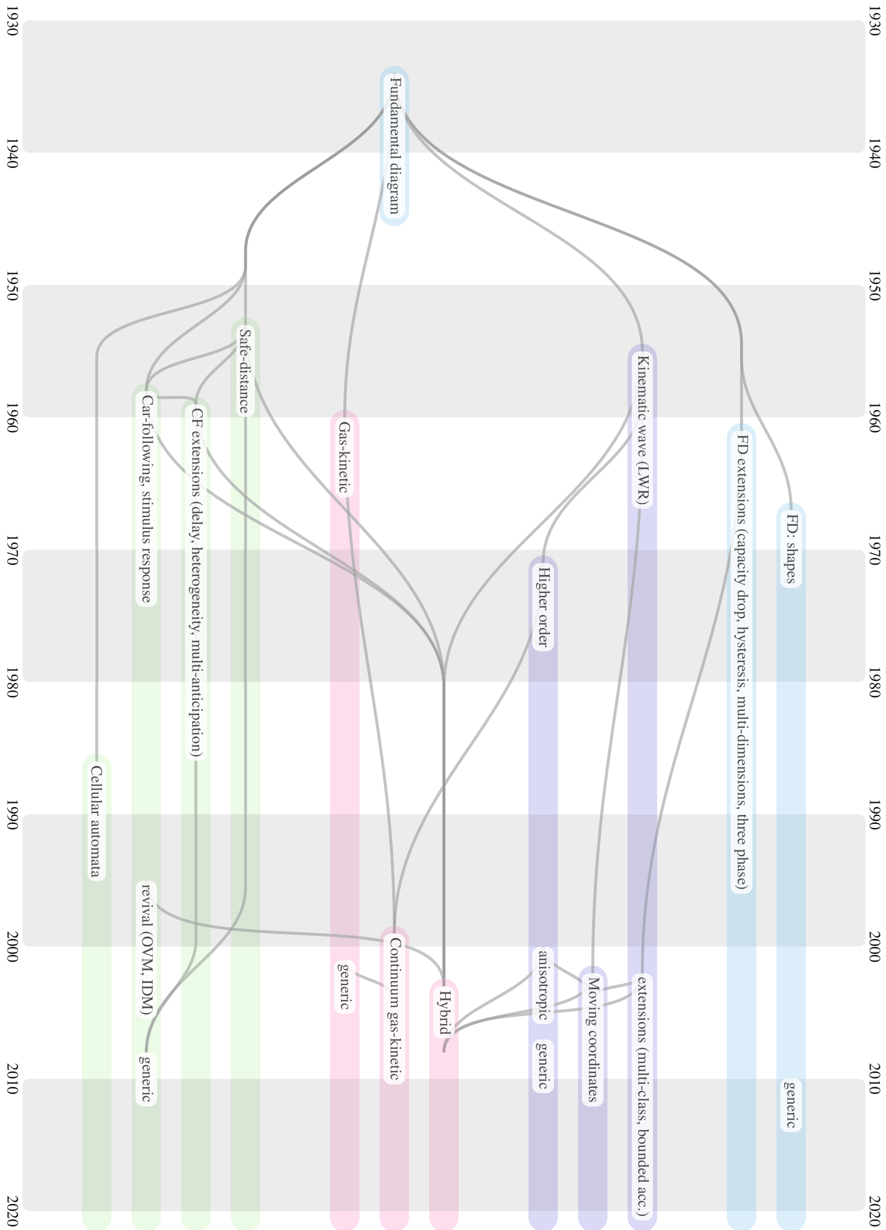
The main part of this book follows the historical lines of the development of traffic flow models since they were first studied in the 1930s. This approach shows better how traffic flow models have developed and how different types of models are related to each other. To show the historical development of traffic flow models we introduce a genealogical tree of traffic flow models, see page 15.

The historical development of traffic flow models shows the emergence of four families. All models in the tree have one common ancestor: the fundamental relation (or fundamental diagram). The other three families consist of micro-, meso- and macroscopic models. After the introduction of the fundamental diagram in the 1930s, microscopic and macroscopic models were introduced simultaneously in the 1950s. Mesoscopic models are about a decade younger. Particularly over the last two decades, the fundamental diagram and all three types of models have been developed further and many offshoots can be recognized.

The fundamental diagram family is the only one with static models. The fundamental diagram, which constitutes a family of its own, does not describe any changes in traffic state, but only links how 'busy' the road is (usually in terms of number of vehicles per kilometer) to how fast the vehicles are driving. In other words: the fundamental diagram relates the vehicle headway (front-to-front following distance) to vehicle speed, in a static way. How headways and speeds change is described by micro-, meso- and macroscopic models. These models are dynamic: they describe how traffic states change over time, for example when congestion is created and how it dissolves. Those three types of models are categorised further according to their level of aggregation of the variables. Microscopic models describe vehicles as individual agents, each with their own headway and speed, they distinguish and trace the behaviour of each individual vehicle. Macroscopic models aggregate the vehicles into a continuum flow approximation, with variables averaged over multiple vehicles, e.g. average speed of vehicles on a certain section, or average number of vehicles passing that section per time unit. Mesoscopic models have an aggregation level in between those of microscopic and macroscopic models or combine both approaches. Classical mesoscopic models describe vehicle behaviour in aggregate terms such as in probability distributions, while behaviour rules are defined as individual vehicles. Hybrid models are a much younger branch of mesoscopic models. They model traffic at different scales: adapting the scale according to the needs for accuracy and computational speed in that area.

1.4.2 Numerical Methods for Computer Simulation

To apply a traffic flow model, it is often included in a computer simulation. Therefore, the dynamic model is discretised in time. The traffic state is not computed for every moment in time, but instead only at discrete instances, with time steps of usually 0.5–2 s. In microscopic models, the traffic state consists of the position of the vehicles, usually in combination with their speed and possibly other variables. In macroscopic models, the traffic state consists of variables like density and speed. Mesoscopic models use combinations of those variables or even other variables. When doing a time step, the current state (at time t) is used to approximate the state at the next time step (at time $t + \Delta t$). This way, when the initial state is known, subsequent time stepping can predict future traffic states.


1930
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Fundamental diagram
Cellular automata
Car-following, stimulus response
revival (OVM, IDM)
generic
CF extensions (delay, heterogeneity, multi-anticipation)
Safe-distance
Gas-kinetic
generic
Continuum gas-kinetic
Hybrid
Higher order
anisotropic
generic
Moving coordinates
Kinematic wave (LWR)
extensions (multi-class, bounded acc.)
FD extensions (capacity drop, hysteresis, multi-dimensions, three phase)
FD: shapes
generic

How exactly the discretisation in time and—where applicable in space or an other independent variable—is approached and how the new state is computed for each time step, is discussed for some of the most widely used models.

1.4.3 Other Aspects of Traffic Flow Modelling

We focus on models that include a clear set of rules describing the behaviour of drivers, vehicles and/or traffic flows. These models are usually relatively easy to understand and are shaped by a set of mathematical equations, that can often be solved analytically for (very) simple problems. However, the human behaviour in traffic is often much more complex than can be described by these models. Therefore, artificial intelligence models have been developed (Aghabayk et al. 2015). They describe traffic flow using for example fuzzy logic or neural networks. Artificial intelligence models are not discussed in detail in this book.

Furthermore, this book focusses on models and simulation methods for longitudinal traffic flow on homogenous roads. We aim to provide the reader with knowledge and tools to be able to build their own model and simulation of a heterogeneous road with no entries, exits or intersection. Aspects of traffic modelling that are not discussed in detail in this book are:

- models for lateral behavior (lane changing) that can be included in microscopic models
- node models to include intersections, merges and diverges in traffic flow models
- demand and origin destination modelling: we assume that in the applications the number of vehicles that want to travel over a certain road is given.
- calibration and validation of models and simulation tools: we focus on the qualitative aspects of the models and refer the reader to other publications for details on how to properly estimate parameter values.
- details of applications: there is a broad variety of applications of traffic flow model and we suggest the reader—where applicable—to refer to other publications specifically dealing with their application.
- network flows: there is a steady growth in the literature about how to model flows in networks, without modelling each road individually, but instead using network (or macroscopic) fundamental diagrams. We consider this scale too course for the scope of this book.
- psychology and decision making of the driver, technology of the vehicle and their interaction: again, these topics are gaining more and more research interest but we consider this scale to detailed for the scope of this book.

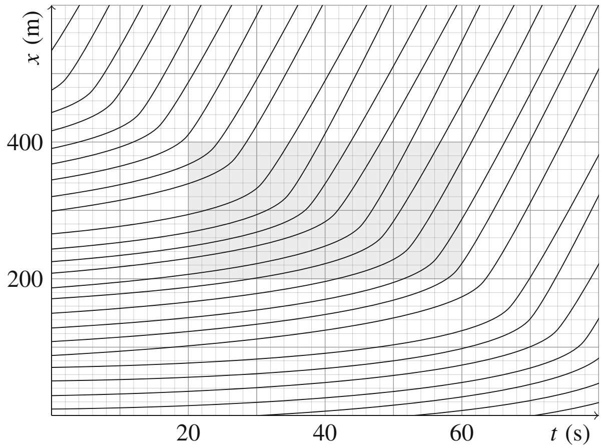

Fig. 1.9 Example of observed trajectories

Problem Set

Calculating Traffic Variables

Consider the trajectories in Fig. 1.9.

1.1 Use Edie's definitions to calculate the average density, speed and flow in the grey area from $t = 20$ s to $t = 60$ s and from $x = 200$ m to $x = 400$ m.

1.2 Use Edie's definitions to calculate the instantaneous density at the boundaries of the grey area, i.e. at $t = 20$ s and at $t = 60$ s.

1.3 Use Edie's definitions to calculate the local flow at the boundaries of the grey area, i.e. at $x = 200$ m and at $x = 400$ m.

Wardrop User Equilibrium and Braess Paradox

This problem set is based on the ideas on user equilibrium introduced by Wardrop (1952) and the Braess paradox (Braess 1968). Consider a simple road network consisting of 4 roads, as in Fig. 1.10a. The travel times on the links from A to C and from B to D are always 15 min. The travel times on the other links (from A to B and from C to D) are longer when the number of vehicles on those links (q_{AB} and q_{CD}, respectively) are higher. To be precise, the travel time in minutes is $T_{AB} = q_{AB}/10$ and $T_{CD} = q_{CD}/10$, respectively. 100 vehicles and their drivers want to travel from A to B. Finally, assume a user equilibrium (Wardrop's first principle): there is no driver for which the travel time would decrease if they would chose a different route.

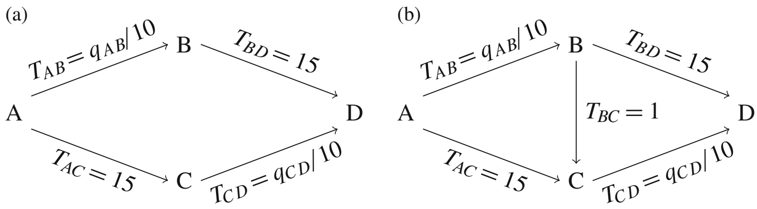

Fig. 1.10 Networks illustrating Wardrop's principle and the Braess paradox. (**a**) Initial network (**b**) Network with extra link added

1.4 (Advanced) In a user equilibrium, how many of the 100 drivers travel via B and how many travel via C? What is their travel time?

Now consider a fifth road being build, as in Fig. 1.10b. It connects B with C, only taking one minute to travel.

1.5 (Advanced) In a user equilibrium, how many of the 100 drivers travel stay on the original routes via B and C, respectively (i.e. not making use of the new route) and how many take the new route and travel $A \to B \to C \to D$? What are their travel times?

The 'traffic flow models', relating the travel time with the number of vehicles on the road, or even using a constant travel time, are very simplistic. Using more advanced and realistic models—as introduced in the next chapters—will also give more realistic travel times and it will be harder to find an example of the Braess paradox.

Identifying Phenomena in Data

There are many online tools to view current or previous traffic states, such as Google Maps (maps.google.com) or the collection of traffic states at http//traffic-flow-dynamics.org/traffic-states.

1.6 (Advanced) Go to any of these websites and identify instances of congestion, free flow, capacity drop and stop-and-go-waves.

Further Reading

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