Chapter 13
Modeling and Co-Design of Control Tasks over Wireless Networking Protocols

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Abstract In this chapter, we provide a brief overview of the state of the art on control over wireless communication protocols and present some recent advances in the co-design of controller and communication protocol configuration (i.e., scheduling and routing) subject to stochastic packet drops.

13.1 Introduction

Wireless networked control systems (WNCS) are distributed control systems where the communication between sensors, actuators, and computational units is supported by a wireless communication network. WNCSs have a wide spectrum of applications, ranging from smart grids to remote surgery, passing through industrial automation, environment monitoring, intelligent transportation, and unmanned aerial vehicles, to name few.

The use of WNCS in industrial automation results in flexible architectures and generally reduces installation, debugging, diagnostic, and maintenance costs with respect to wired networks (see e.g., [3, 33] and references therein). However modeling, analysis, and co-design of WNCS are challenging open research problems since they require to take into account the joint dynamics of physical systems, communication protocols, and network infrastructures. Recently, a huge effort has been made in scientific research on WNCSs, see e.g., [5, 8, 10, 19, 24, 28, 31, 40, 43, 68, 74, 78, 81, 84] and references therein for a general overview.

The challenges in analysis and co-design of WNCSs are best explained by considering wireless industrial control protocols. In this chapter, we focus on a networking protocol specifically developed for wireless industrial automation, i.e., WirelessHART, [35–37]. Indeed WirelessHART is not a niche technology, as many high-impact technological companies, such as Siemens, ABB, Emerson, sent to the

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market devices and industrial automation solutions based on the WirelessHART protocol. Due to its novelty, plenty of research activity on WirelessHART is still in progress in order to analyze the real capabilities of the standard and the application limits. In particular, the WirelessHART specification creates the opportunity for designers to implement ad hoc algorithms to configure the network configuration: we fill this gap by providing novel algorithms for co-designing controller and network configuration from a control performance point of view, and in particular, we investigate the design of redundancy when routing actuation data to a LTI system connected to the controller via a wireless network. We show with an example that the optimal co-design of controller gain and routing can strongly improve the control performance [21].

Pursuing the objective described above, we first consider the modeling, stability analysis, and controller design problems in a purely nondeterministic setting, when the actuation signal is subject to switching propagation delays due to dynamic routing [50]. We show how to model these systems as pure switching linear systems and provide an algorithm for robust stability analysis. We show that the stability analysis problem is NP-hard in general and provide an algorithm that computes in a finite number of steps the look-ahead knowledge of the routing policy necessary to achieve controllability and stabilizability.

We then consider, in a stochastic setting, the case when actuation packets can be delivered from the controller to the actuator via multiple paths, each associated with a delay and with time-varying packet loss probability. The *packet dropouts* have been modeled in the WNCS literature either as stochastic or nondeterministic phenomena [43]. The proposed nondeterministic models specify packet losses in terms of time averages or in terms of worst-case bounds on the number of consecutive dropouts (see e.g., [40]). For what concerns stochastic models, a vast amount of research assumes memoryless packet drops, so that dropouts are realizations of a Bernoulli process [31, 68, 74]. Other works consider more general correlated (bursty) packet losses and use a transition probability matrix of a finite-state (time-homogeneous) Markov chain (see e.g., the finite-state Markov modeling of Rayleigh, Rician, and Nakagami fading channels in [72] and references therein) to describe the stochastic process that rules packet dropouts (see [30, 74]). In these works, WNCS with missing packets are modeled as time-homogeneous Markov jump linear systems, which are an important family of stochastic hybrid systems that we use to model packet losses. In particular, it has been shown (e.g., in [21, 30, 74, 77]) that discrete-time Markov jump linear systems (MJLS, [17]) represent a promising mathematical model to jointly take into account the dynamics of a physical plant and nonidealities of wireless communication such as packet losses. A MJLS is, basically, a switching linear system where the switching signal is a Markov chain. The transition probability matrix of the Markov chain can be used to model the stochastic process that rules packet losses due to wireless communication. However, in most real cases, such probabilities cannot be computed exactly and are time-varying. We can take into account this aspect by assuming that the Markov chain of a MJLS is time-inhomogeneous, i.e., a Markov chain having its transition probability matrix varying over time, with variations that are arbitrary within a polytopic set of stochastic matrices.

Given such mathematical model, the first problem we address is providing necessary and sufficient conditions for the stochastic notion of mean square stability (MSS). Some recent works addressed the above problem: in [2], a sufficient condition for stochastic stability in terms of linear matrix inequality feasibility problem is provided, while in [16] a sufficient condition for MSS of system with interval transition probability matrix, which in turn can be represented as a convex polytope [38], is presented in relation to spectral radius; in general, only sufficient stability conditions have been derived for MJLS with time-inhomogeneous Markov chains having transition probability matrix arbitrarily varying within a polytopic set of stochastic matrices. We derive necessary and sufficient conditions [60] for MSS of discrete-time MJLS with time-inhomogeneous Markov chains. Having solved the stability problem, we extend the framework of MJLSs replacing the time-inhomogeneous Markov chain with a time-inhomogeneous Markov Decision Process and provide the optimal solution of the finite time horizon LQR problem considering the issue of joint minimization of costs of continuous and discrete control inputs for the worst possible disturbance in transition probabilities [61].

In addition to the investigations described above, we also addressed the problem of stabilizing a WNCS in presence of long- term link failures and malicious attacks. More precisely, we addressed the co-design problem of controller and communication protocol, and in particular routing and scheduling, when the physical plant is a MIMO LTI system and the communication nodes are subject to failures and/or malicious attacks. We first characterize by means of necessary and sufficient conditions the set of network configurations that invalidate controllability and observability of the plant. Then, we investigate the problem of detecting and isolating communication nodes affected by failures and/or malicious attacks and provide necessary and sufficient conditions for the solvability of this problem. This latter line of research is not illustrated in this chapter for space limitations, and we refer the interested reader to the papers [24–26].

This chapter is organized as follows. In Sect. 13.2, we provide a high-level description of the WirelessHART communication protocol. In Sect. 13.3, we first define our mathematical framework (i.e., time-inhomogeneous MJSLSs), which takes into account accurate models of packet dropouts; then we show with a motivating example that co-designing the controller and the routing strategy can lead to a strong improvement of the control performance. In Sect. 13.4, we summarize and discuss our technical results on co-design of controller and network configuration. In Sect. 13.5 we draw conclusions and directions for future work.

13.2 The WirelessHART Protocol

In this section, we introduce WirelessHART, one of the most relevant protocols currently used in industrial environments, emphasizing the features that will be analyzed and addressed in our mathematical models and co-design algorithms for WNCS. WirelessHART, [35–37] is one of the first wireless communication standards specifically designed for process automation applications. The standard has been finalized in 2007, at the beginning of 2010 it has been ratified as an IEC standard, and is based upon the physical layer of IEEE 802.15.4. When WirelessHART was developed, many requirements deemed critical for the industrial environment were not defined in IEEE 802.15.4, thus further specifications have been added in the data link layer (DLL). This new MAC protocol combines frequency hopping with a TDMA scheme utilizing a centralized a priori slot allocation mechanism. Indeed, it is commonly thought that TDMA-based protocols offer good opportunities for energy-efficient operation of sensor nodes, as they allow them to enter sleep mode when they are not involved in any communications. Such allocation of the shared channel regularizes the dynamical behavior introduced by multi-hop transmission: indeed, TDMA schemes avoid collisions and thus induce a periodic time-varying behavior, where delays and transmission times are well predictable, which can be nicely analyzed by considering sophisticated mathematical models like Markov Jump Linear Systems or Markov Decision Processes. In the results, illustrated in this chapter, we concentrate on modeling the joint dynamics of a closed-loop dynamical system and the WirelessHART data link and network layers.

About **data link layer**, the timing hierarchy of WirelessHART can be split in three timescale layers, as depicted in Fig. 13.1. The lowest layer consists of individual time slots: within each time slot, one data packet and the corresponding immediate acknowledgment packet are exchanged. A time slot in WirelessHART has a fixed length of 10 ms., and two types of time slots are available: *dedicated time slots* (slot is allocated to one specific sender–receiver pair) and *shared time slots* (more than one device may try to transmit a message). Within a dedicated time slot, transmission

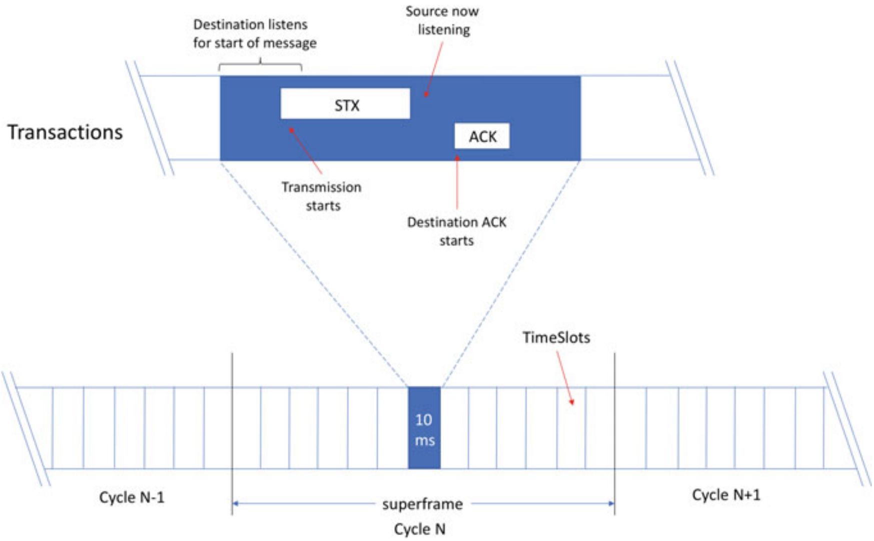

Fig. 13.1 WirelessHART timing hierarchy

of the source message starts at a specified time after the beginning of a slot. This short time delay allows the source and destination to set their frequency channel and allows the receiver to begin listening on the specified channel. Since there is a tolerance on clocks, the receiver must start to listen before the ideal transmission start time and continue listening after that ideal time. Once the transmission is complete, the destination device indicates by transmitting an ACK whether it received the source device data link packet successfully or not, indicating in this latter case a specific class of detected errors. At the second layer, a contiguous group of time slots of fixed length forms a superframe. At the third layer, a contiguous group of superframes forms a network cycle. Within each cycle, each field device obtains at least one time slot for data transmission, but certain devices may have more time slots than others because they provide data with more important requirements or have additional forwarding duties.

About **network layer**, in advanced applications point-to-point communications are unreliable also because it is very difficult to place devices to maintain line of sight all the time. The best architecture solution for wireless communication is a mesh topology network, which can provide multipath redundancy. Nodes, in fact, can multicast the same information to one or more of its neighbors with the goal of mitigating the risk of unplanned outages and ensure continuity of operation by instantly responding to and reducing the effects of a point of failure anywhere along the critical data path. In order to guarantee timely and reliable data delivery, routing topology and transmission schedule are centrally computed by a network manager device (which has global knowledge of the network state) and then disseminated to all devices in the network. WirelessHART allows two strategies to route packets: graph routing and source routing.

In *source routing*, a single fixed route of devices is decided by the source node and written in the header of the packet. Then, each device in the route forwards the packet to the next specified device until the destination is reached. There are no alternate routes in this mode, so if any device fails on a route, the whole route fails. Source routing is mostly used for network diagnostics.

In *graph routing*, the network manager defines a set of *routing graphs*, consisting of a set of acyclic directed graphs each connecting a source node to a destination node through some relay nodes on the network, and communicates them to each device. When a source node needs to send a packet, it writes a routing graph ID in the header of the packet to be sent. As the packet arrives at each node, the node forwards (or consumes, if it is the destination node) the packet according to the corresponding routing graph. Each relay node can be configured with multiple neighbors to create redundancy in the packet's forwarding. Thanks to such redundancy, graph routing is mostly used for sensing and actuation data communication.

As discussed above, the WirelessHART standard specifies the communication stack as well as the interfaces and tasks for the devices comprising a WirelessHART network. However it does not specify how these tasks should be accomplished, which provides interesting opportunities to develop improved and optimized solutions. An example is the exploitation of routing redundancy by jointly configuring the scheduling at the data link layer and the routing graph at the network layer. WirelessHART does not specify the algorithms and performance metrics to be used for scheduling and routing, and a designer must implement the best policies according to the specific application: we will provide in the next sections methods for co-designing controller and network from the control performance point of view.

13.3 Mathematical Framework and Motivating Example

In this section, we first introduce our reference mathematical framework and then provide an example showing that the joint design of network communication policies and control has a relevant impact on the control performance of a closed-loop system.

13.3.1 Time-Inhomogeneous Discrete-Time Markov Jump (switched) Linear Systems

Linear systems subject to abrupt parameter changes due, for instance, to environmental disturbances, component failures, changes in subsystems interconnections, changes in the operation point for a nonlinear plant, etc., can be modeled by a set of discrete-time linear systems with modal transition given by a discrete-time finite-state Markov chain. This family of systems is known as discrete-time Markov(ian) jump linear systems, often abbreviated as MJLSs.

The transition probabilities of a Markov chain are frequently time-varying and unavailable to the modeler, and a large body of research has been devoted to deal with these uncertainties and also to the identification of the Markov chain using available observations (see [13] and references therein for an introduction to the topic of estimation of such transition probabilities, which always introduces estimation errors). In order to account for uncertainties and time-variance inherent to real-world scenarios, the time-inhomogeneous polytopic model of transition probabilities is very general and widely used in the literature.

In this section, we present a rigorous mathematical model of MJLSs with polytopic uncertainties on transition probabilities and also the model of their natural extention, i.e., Markov jump switched linear systems (MJSLSs).

A discrete-time Markov jump linear systems can be defined as follows:

$$\begin{cases} \mathrm{x}_{k+1} = A_{\theta_k}\mathrm{x}_k + B_{\theta_k}\mathrm{u}_k + H_{\theta_k}\mathrm{v}_k, \\ \mathrm{y}_k = F_{\theta_k}\mathrm{x}_k + G_{\theta_k}\mathrm{w}_k, \\ \mathrm{z}_k = C_{\theta_k}\mathrm{x}_k + D_{\theta_k}\mathrm{u}_k, \\ \mathrm{x}_0 = \boldsymbol{x}_0,\ \theta_0 = \vartheta_0,\ \mathrm{p}_0 = \boldsymbol{p}_0, \end{cases} \tag{13.1}$$

where $k \in \mathbb{T}$ is a discrete-time instant, $\mathbb{T}$ is a discrete-time set, $\mathbb{T} = \mathbb{Z}_0$, with $\mathbb{Z}_0$ indicating the set of all nonnegative integers and $\mathbb{Z}$ the set of integers. Then, x_k is a vector of n_x either real or complex *state* variables of the Markov jump linear system, where $n_\mathrm{x} \in \mathbb{Z}_+$ with $\mathbb{Z}_+$ the set of positive integers, and $\mathrm{x}_k \in \mathbb{F}^{n_\mathrm{x}}$ with $\mathbb{F}^{n_\mathrm{x}}$ an n_x-dimensional linear space with entries in $\mathbb{F}$. Note that $\mathbb{F}$ indicates the set of either real numbers $\mathbb{R}$ or complex numbers $\mathbb{C}$: in general, when studying MJLSs, it is a standard practice to work with complex fields [17], but one can consider complex operators acting on $\mathbb{C}^{m,n}$ as real (block) matrices acting on $\mathbb{R}^{2m,2n}$ [49].

For what concerns other system variables in the aforementioned state-space representation of an MJLS, u_k stands for a vector of n_u *control input* variables, $\mathrm{u}_k \in \mathbb{F}^{n_\mathrm{u}}$; then, $\mathrm{v}_k \in \mathbb{F}^{n_\mathrm{v}}$ and $\mathrm{w}_k \in \mathbb{F}^{n_\mathrm{w}}$ are vectors of exogenous input variables, known as *process noise* and *observation noise*, respectively; $\mathrm{y}_k \in \mathbb{F}^{n_\mathrm{y}}$ represents a vector of *measured state* variables available to the controller; $\mathrm{z}_k \in \mathbb{F}^{n_\mathrm{z}}$ denotes a vector of measured *system output* variables. Clearly, $n_\mathrm{u}, n_\mathrm{v}, n_\mathrm{w}, n_\mathrm{y}, n_\mathrm{z} \in \mathbb{Z}_+$.

In the following, we write $\mathbb{F}^{m,n}$ to denote a set of matrices with m rows, n columns, and entries in $\mathbb{F}$. Consequently, the elements of the system matrices A_{θ_k}, B_{θ_k}, etc., are also defined on a field of either real or complex numbers $\mathbb{F}$. The subscript θ_k indicates that system matrices active in a time instant k are determined by the value of the *jump* variable θ_k, which is a random variable having the set $\mathbb{M} \triangleq \{i \in \mathbb{Z}_+ : i \leq N\}$ as its *state space*, where $N \in \mathbb{Z}_+$ is the cardinality of the set, formally $|\mathbb{M}| = N$. The set $\mathbb{M}$ is generally referred to as the (index) set of *operational modes* of the Markov jump linear system. We denote by θ_k as the identity function of the set of operational modes, i.e., $\theta_k : \mathbb{M} \to \mathbb{M}$, and $\forall i \in \mathbb{M}$, we have that $\theta_k(i) = i$.

For every operational mode, there is a correspondent system matrix, and the collection of the system matrices of each type is generally represented by a sequence of N matrices, which are not necessarily all distinct. Specifically, $\boldsymbol{A} \triangleq (A_i)_{i=1}^N \in {}_\mathrm{N}\mathbb{F}^{n_\mathrm{x},n_\mathrm{x}}$ is a sequence of the so-called *state matrices*, each of which is associated to an operational mode of the (switching) system. Noticeably, ${}_\mathrm{N}\mathbb{F}^{m,n}$ indicates a *linear space* made up of all *N-sequences of* $m \times n$ *matrices* with entries in $\mathbb{F}$. Similarly, $\boldsymbol{B} \triangleq (B_i)_{i=1}^N \in {}_\mathrm{N}\mathbb{F}^{n_\mathrm{x},n_\mathrm{u}}$ is an N-sequence of *input matrices*; $\boldsymbol{C} \triangleq (C_i)_{i=1}^N \in {}_\mathrm{N}\mathbb{F}^{n_\mathrm{z},n_\mathrm{x}}$ is a sequence of *output matrices*; $\boldsymbol{D} \triangleq (D_i)_{i=1}^N \in {}_\mathrm{N}\mathbb{F}^{n_\mathrm{z},n_\mathrm{u}}$ is a sequence of *direct transition* (also known as feed-forward or feedthrough) *matrices*; $\boldsymbol{F} \triangleq (F_i)_{i=1}^N \in {}_\mathrm{N}\mathbb{F}^{n_\mathrm{y},n_\mathrm{x}}$ is a sequence of *observation matrices*; $\boldsymbol{G} \triangleq (G_i)_{i=1}^N \in {}_\mathrm{N}\mathbb{F}^{n_\mathrm{y},n_\mathrm{w}}$ is a sequence of *observation noise matrices*; and $\boldsymbol{H} \triangleq (H_i)_{i=1}^N \in {}_\mathrm{N}\mathbb{F}^{n_\mathrm{x},n_\mathrm{v}}$ is a sequence of *process noise matrices*.

The transitions, or jumps, between operational modes of an MJLS are governed by a discrete-time Markov chain θ, which is a collection of random variables θ_t all taking values in the same state space, i.e., $\{\theta_t : t \in \mathbb{T}\}$, and satisfying the Markov property. The initial probability distribution of the Markov chain is defined $\forall i \in \mathbb{M}$ by $p_i(0) \triangleq \Pr(\theta_0 = i)$, and the initial probability distribution of all the operational modes is defined as the vector $\mathrm{p}_0 \triangleq [p_1(0) \ldots p_N(0)]' \in \mathbb{R}^{N,1}$. The transition probability between the operational modes $i, j \in \mathbb{M}$ of a Markov jump linear system is formally defined as

$$p_{ij}(k) \triangleq \Pr(\theta_{k+1} = j \mid \theta_k = i)\,, \tag{13.2}$$

where $\forall i \in \mathbb{M}$ and $\forall k \in \mathbb{T}$, $\sum_{j=1}^N p_{ij}(k) = 1$. The corresponding transition probability matrix is defined as

$$P(k) \triangleq \left[p_{ij}(k)\right] \in \mathbb{R}^{N,N}. \tag{13.3}$$

The initial conditions for a Markov jump linear system consist of the initial state of the dynamical system $\mathrm{x}_0 = \boldsymbol{x}_0 \in \mathbb{F}^{n_\mathrm{x}}$, the initial state of the Markov chain $\theta_0 = \vartheta_0 \in \mathbb{M}$ and the initial probability distribution of the states of the Markov Chain denoted by $\mathrm{p}_0 = \boldsymbol{p}_0 \in \mathbb{R}_0^N$ s.t. $\left\|\boldsymbol{p}_0\right\|_1 = 1$, with $\|\cdot\|_1$ the standard $1-$norm of a vector and $\mathbb{R}_0^N$ the N-dimensional linear space with entries in the set of nonnegative real numbers $\mathbb{R}_0$.

Although in engineering problems the operation modes are not often available, there are enough cases where the knowledge of random changes in system structure is directly available to make these applications of great interest [9, 17]. The typical examples include a ship steering autopilot, control of pH in a chemical reactor, combustion control of a solar-powered boiler, fuel–air control in a car engine, and flight control systems [17]. In this chapter, we focus on the fact that MJLSs' model is well suited also for the WNCS scenario, when a channel estimation is performed (see e.g., [57] and the references therein), so the channel state information is known at each time step. In fact, the knowledge of θ_k at each time instant k is a standard assumption in the setting of MJLSs, and in this chapter we do the following assumption.

Assumption 13.1 At every time step $k \in \mathbb{T}$, the jump variable θ_k is measurable and available to a controller.

Depending on the considered problem, (some of) the system's vector variables x_k, u_k, y_k, and z_k may also be viewed as measurable.

We have previously discussed that in most real cases the transition probability matrix $P(k)$ introduced in (13.3) cannot be computed exactly and is time-varying, and that there exists a considerable number of works on discrete-time Markov jump systems (both linear and nonlinear) with polytopic uncertainties, which can be either time-varying or time-invariant. From now on, we assume that $P(k)$ is varying over time, with variations that are arbitrary within a polytopic set of stochastic matrices. In order to express this statement formally, let $V \in \mathbb{Z}_+$ be a number of vertices of a convex polytope, and $\mathbb{V}$ be an index set of vertices of a convex polytope, i.e., $\mathbb{V} \triangleq \{i \in \mathbb{Z}_+ : i \leq V\}$. Then, the set of vertices of a convex polytope of transition probability matrices is formally defined as

$$_{\mathbb{V}}\mathbb{P} \triangleq \left\{P_l \in \mathbb{R}^{N,N} : l \in \mathbb{V}\right\}. \tag{13.4}$$

Clearly, being a transition probability matrix, each vertex P_l satisfies (13.2) and (13.3). These vertices are obtained from measurement on the real system or via numerical reasoning, taking into account accuracy and precision of the measuring instruments and/or numerical algorithms. They bound the possible values each transition probability can assume. Then, the polytopic time-inhomogeneous assumption is stated as follows.

Assumption 13.2 The time-varying transition probability matrix $P(k)$ is **polytopic**, that is, for all $k \in \mathbb{T}$, one has that

$$P(k) = \sum\nolimits_{l=1}^{V} \lambda_l(k) P_l, \quad \lambda_l(k) \geq 0, \quad \sum\nolimits_{l=1}^{V} \lambda_l(k) = 1, \tag{13.5}$$

where for each $l \in \mathbb{V}$, $P_l \in {}_{\mathbb{V}}\mathbb{P} \subset {}_{\mathbb{V}}\mathbb{R}^{N,N}$, i.e., P_l are elements of a given finite set of transition probability matrices, which are the vertices of a convex polytope; moreover, $\lambda_l(k)$ are unmeasurable.

Assumption 13.2 plays an important role also in our model of Markov jump switched linear system, which is a dynamical system having the same form as (13.1), with the only difference being that the operational modes of the system are determined by the stochastic variable s_k that represents a standard Markov Decision Process. A Markov Decision Process is a quintuple ($\mathbb{M}$, $\mathbb{A}$, Pr, g, γ), where

- $\mathbb{M}$ is a finite set of states of a process, with $|\mathbb{M}| = N$.
- $\mathbb{A}$ is a finite (index) set of actions among which a decision maker (a.k.a. a discrete controller or a supervisor) is able to chose, i.e., $\mathbb{A} \triangleq \{i \in \mathbb{Z}_+ : i \leq M\}$. Typically, only a subset of $\mathbb{A}$ is available in any given state of an MDP.[1] We take this into account by defining for each state $i \in \mathbb{M}$ the related set $\mathbb{A}_i$ of actions α available in that state. We write this statement symbolically as $\mathbb{A}_i \subseteq \mathbb{A}$, $\alpha \in \mathbb{A}_i$.
- Pr is state- and action-dependent transition probability distribution. For any $k \in \mathbb{T}$, $i, j \in \mathbb{M}$, $\alpha \in \mathbb{A}_i$, the future transition probability distribution, conditioned on the present state s_k of the MDP and the action α_k to be taken from that state, is denoted by

$$p_{ij}^{\alpha}(k) \triangleq \Pr\{s_{k+1} = j \mid s_k = i,\ \alpha_k = \alpha\}. \tag{13.6}$$

  Being a probability distribution, $p_{ij}^{\alpha}(k) \in \mathbb{R}_0$ and satisfies $\forall k \in \mathbb{T}$, $i, j \in \mathbb{M}$, and $\alpha \in \mathbb{A}_i$

$$\sum\nolimits_{j=1}^{N} p_{ij}^{\alpha}(k) = 1. \tag{13.7}$$

  For any $\alpha \notin \mathbb{A}_i$, the action is not available in a given state of the MDP. Hence, $\forall j \in \mathbb{M}$

$$p_{ij}^{\alpha}(k) \triangleq 0. \tag{13.8}$$

  In the following, we assume that the transition probability matrices constituting Pr are varying over time, with variations that are arbitrary within a polytopic set of stochastic matrices according to Assumption 13.2.
- Selecting an (available) action in any given state of a Markov decision process entails a (nonnegative) cost, which is seen as a function $g : \mathbb{M} \times \mathbb{A} \to \mathbb{G}$, where $\mathbb{G} \subseteq \mathbb{R}_0$ is a set of immediate costs.
- γ is a discounting factor, which represents the difference in importance between future costs and present costs; $\gamma \in \mathbb{R}_0$, $\gamma \leq 1$. Since taking the discount factor into

[1] For instance, in a decision problem of the optimal transmission power management in a wireless communication, the possible actions available to a controller may be those of increasing or decreasing of a transmission power: in a finite set of transmission power levels, it is impossible to increase a power from a maximum level or decrease it from a minimum level.

account does not affect any theoretical results or algorithms in the finite-horizon case (but might affect the decision maker's preference for policies) [71, p. 79], we do not consider a discounting factor here.

In the same way as done before for a Markov jump variable, we make the following assumption.

Assumption 13.3 The state s_k of the Markov decision process is measurable and available for the discrete controller at each time step $k \in \mathbb{T}$.

We are now ready to define a MJSLS as the following system of recursive equations, where the system's variables and matrices are the same as above:

$$\begin{cases} \mathrm{x}_{k+1} &= A_{s_k}\mathrm{x}_k + B_{s_k}\mathrm{u}_k \\ \mathrm{z}_k &= C_{s_k}\mathrm{x}_k + D_{s_k}\mathrm{u}_k, \\ \mathrm{x}_0 &= \boldsymbol{x}_0,\ s_0 = \boldsymbol{s}_0,\ \mathrm{p}_0 = \boldsymbol{p}_0. \end{cases} \tag{13.9}$$

13.3.2 Co-design of Controller and Routing Redundancy

As discussed in Sect. 13.2, routing and scheduling schemes in a WNCS where the network is based on a WirelessHART-like protocol have a direct and relevant impact on closed-loop performance and the aforementioned standards leave the possibility, for the designers, to implement complex and time-varying routing and scheduling ad hoc optimal policies. Indeed, to make a WirelessHART WNCS robust to transmission nonidealities routing redundancy can be exploited by relaying data via multiple paths and then appropriately recombining them, which is reminiscent of network coding. Basically, if a communication path fails, another one can be available to maintain the communication flow. As paths are characterized by different communication properties, such as delays and packet losses, the routing design must take into account the effect on the control performance of the closed-loop system.

In this section, we illustrate a motivating example from [21], where we model a WNCS implementing WirelessHART as a MJSLS and address the problem of optimally co-designing controller and routing with respect to a control performance index, e.g., the classical quadratic cost used in LQR.

Consider a state-feedback WNCS as in Fig. 13.2, where the communication between the controller and the actuator can be performed via a set of r routing paths $\{\rho_i\}_{i=1}^r$ in a wireless multi-hop communication network. Each path ρ_i is characterized by a delay $d_i \in \mathbb{N}^+$ and a packet loss probability $p_i \in [0, 1]$ that represents the probability that the packet transmitted on that path will not reach the actuator due to communication failure. Therefore let us define, for each path ρ_i, the stochastic process $\sigma_i(k) \in \{0, 1\}$, with $\sigma_i(k) = 0$ if the packet expected to arrive via the routing path ρ_i at time k suffered a packet drop and $\sigma_i(k) = 1$ if the packet is successfully received at time k. For simplicity we assume that $\sigma_i(k)$ is a sequence of

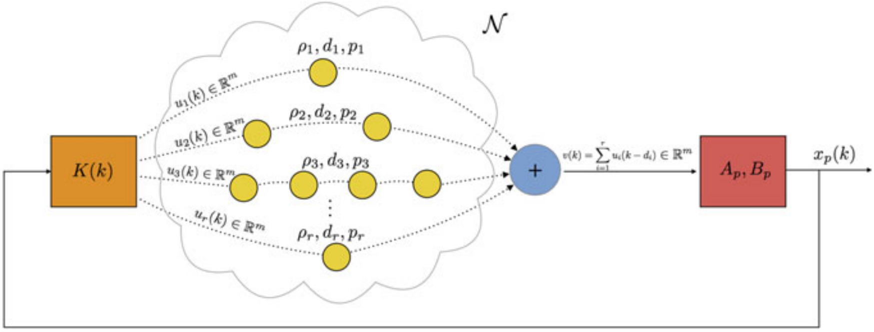

Fig. 13.2 State-feedback control scheme

i.i.d. random variables, each characterized by a Bernoulli distribution with probability measure $\mathbb{P}[\sigma_i(k) = 0] = p_i$. It is also assumed here that the events of occurrence of packet losses in the different paths are i.i.d.: as a consequence the stochastic process $\sigma(k) \doteq [\sigma_1(k), \ldots, \sigma_r(k)]'$ is a vector of i.i.d. random variables, where $\sigma(k)$ can assume 2^r values. The controller cannot measure the signal $\sigma(k)$, i.e., it is not possible to measure the occurrence of packet losses. It is assumed that, in general, the controller can decide for each time instant k the set of paths where data will be sent: i.e., the controller can decide to send data at time k on all paths, on a subset of paths, on one path, or even not to send any data. To this aim let us define for each path i the discrete control signal $a_i(k) \in \{0, 1\}$, with $a_i(k) = 1$ if the controller decides to send a packet via the routing path i at time k, and $a_i(k) = 0$ if no packet is sent via path i at time k. Consequently, the discrete control signal $a(k) \doteq [a_1(k), \ldots, a_r(k)]'$, where $a(k)$ can be chosen among 2^r different values.

Let the plant be a discrete-time LTI system described by the matrices $A_P \in \mathbb{R}^{\ell \times \ell}$, $B_P \in \mathbb{R}^{\ell \times m}$ and assume that we can measure the full system's state, then the dynamics of the networked system are as follows:

$$\begin{cases} x(k+1) = A_{\sigma(k)} x(k) + B_{a(k)} u(k) \\ y(k) = x(k) \end{cases} \tag{13.10}$$

with

$$A_{\sigma(k)} = \begin{bmatrix} A_P & \Lambda_1(\sigma(k)) & \Lambda_2(\sigma(k)) & \cdots & \Lambda_r(\sigma(k)) \\ 0 & \Gamma_1 & 0 & \cdots & 0 \\ 0 & 0 & \Gamma_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \Gamma_r \end{bmatrix} \in \mathbb{R}^{(\ell+\nu(r) \times \ell+\nu(r))},$$

$$B_{a(k)} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ a_1(k)I_m \otimes \mathbf{e}_{d_1} & 0 & \cdots & 0 \\ 0 & a_2(k)I_m \otimes \mathbf{e}_{d_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_r(k)I_m \otimes \mathbf{e}_{d_r} \end{bmatrix} \in \mathbb{R}^{(\ell+\nu(r)\times mr)},$$

with

$$\Lambda_i(\sigma(k)) \doteq \sigma_i(k)\left[B_P \ 0 \cdots 0\right] \in \mathbb{R}^{\ell\times md_i},$$

$$\Gamma_i \doteq \begin{bmatrix} 0 & I_m & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_m & 0 \\ 0 & 0 & \cdots & 0 & I_m \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{md_i\times md_i},$$

and with $\nu(i) \doteq m \sum_{j=1}^{i} d_j$, I_m the m-dimensional identity matrix, $\mathbf{e}_i$ a column vector of appropriate dimension with all zero entries except the $i-th$ entry equal to 1, and $\otimes$ the Kronecker product. Note that in the feedback scheme presented, it is assumed that the controller can measure the whole state $x(k) = [x_P(k)' x_N(k)']'$ of (13.10), where $x_P(k) \in \mathbb{R}^\ell$ is the state of the plant and $x_N(k) \in \mathbb{R}^{\nu(r)}$ are state variables modeling the delay induced by each path. It is assumed that the controller can measure the state $x_P(k)$ of the plant via sensors. Also, the controller is aware of the current and past actuation signals $u(k)$ that have been sent to the actuator, as well as of the current and past signals $a(k)$: as a consequence the controller has direct access to the state of $x_N(k)$, which models the actuation commands that are expected to arrive at the actuator, but is not aware of their actual arrival to the actuator since $\sigma(k)$, which models packet drops, is not measurable.

We consider in this example the simpler case when routing is designed a priori, i.e., $\forall k \geq 0, a(k) = a_k$: note that with this assumption system (13.10) is a MJLS as defined in (13.2). Let us now consider an instance of system (13.10) characterized by a four-dimensional unstable randomly generated plant

$$A_P = \begin{bmatrix} 1.1062 & -1.0535 & 0.7944 & -0.4543 \\ 0.0202 & -0.0654 & 0.9697 & -0.6888 \\ 0.1131 & -0.5755 & 1.7434 & -0.7174 \\ 0.0745 & -0.2565 & 0.2999 & 0.7252 \end{bmatrix}, B_P = \begin{bmatrix} -0.1880 \\ 0.0182 \\ 0.1223 \\ 0.2066 \end{bmatrix},$$

and by a wireless network characterized by two paths: ρ_1 with packet loss probability $p_1 = 0.25$ and delay $d_1 = 1$ and ρ_2 with packet loss probability $p_2 = 0$ and delay $d_2 = 5$. We setup the following standard LQR optimization problem:

Problem 13.1 Given System (13.10) and a routing sequence a_k, $k = 0, 1, \ldots, N-1$, design for any $k \in \{0, \ldots, N-1\}$ an optimal state-feedback control policy $u^*(k) = K^*(\theta(k), k)x(k)$ minimizing the following objective function:

$$\mathscr{J}(\theta_0, \mathrm{x}_0) \triangleq \min_{\mathrm{u}} \sum\nolimits_{k=0}^{T-1} \mathbb{E}\big(\|\mathrm{z}_k\|_2^2\big) + \mathbb{E}\big(\mathrm{x}_T^* Z_{\theta_T} \mathrm{x}_T\big) \tag{13.11}$$

with θ and z_k as in (13.1) and $\mathbf{Z} \triangleq (Z_i)_{i=1}^N \in {}_{\mathrm{N}}\mathbb{F}_0^{n_\mathrm{x}, n_\mathrm{x}}$ a sequence of the terminal cost weighting matrices.

We solve Problem 13.1 using the optimal LQR solution for MJLS for comparing the performance of three simple routing strategies: (1) using for all time instants only path ρ_1; (2) using for all time instants only path ρ_2; using for all time instants both paths simultaneously. Solution is computed for a time horizon $T = 300$. For a detailed description of the weight matrices of the cost function (13.11) and the initial conditions we refer the reader to [21]. For each routing strategy, $5K$ MC simulations of the state trajectories are performed. Figure 13.3 shows the trajectories of the first component of the extended state vector when only path ρ_1 is used. The system can be stabilized, but clearly the variance of the trajectories is large. This routing policy is clearly a bad choice. Figure 13.4 shows the trajectories when only path ρ_2 is used (red) and when both paths ρ_1 and ρ_2 are used (blue and green). Routing data only to path ρ_2 clearly generates always the same trajectory since $p_2 = 0$. The system trajectories are stable but the associated cost is quite large because of the delay, as evidenced by the overshoot and the settling time performances. Figure 13.4 evidences that routing data via both paths ρ_1 and ρ_2 the control performance strongly

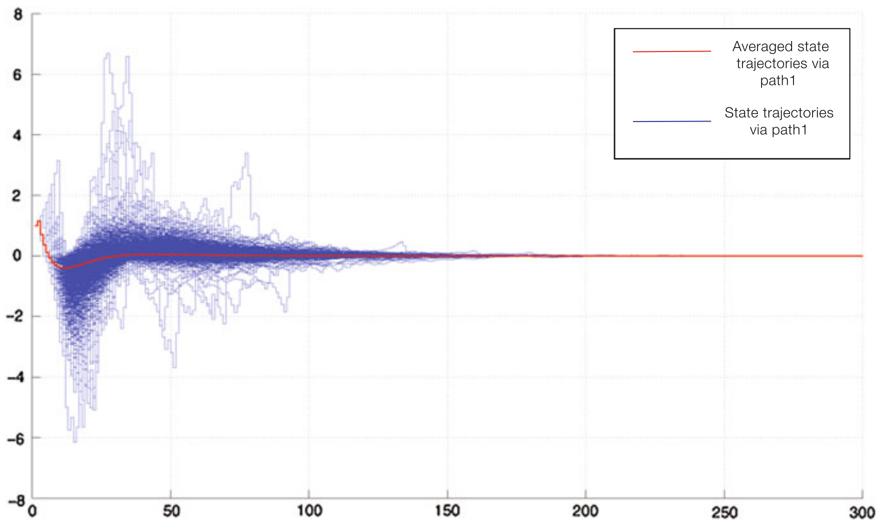

Fig. 13.3 State trajectories routing only via path ρ_1 (blue) and their average (red)

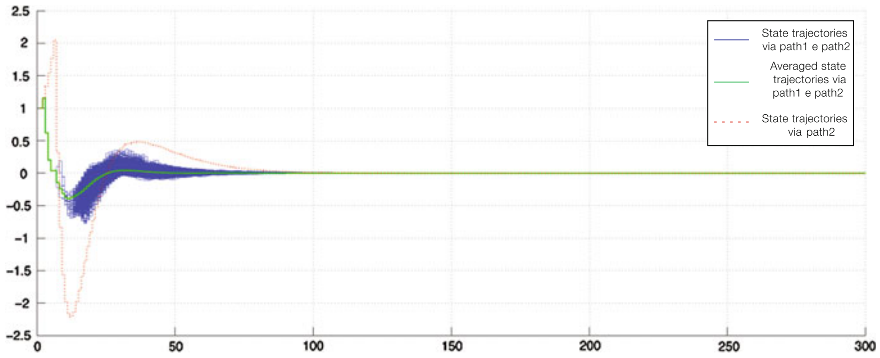

Fig. 13.4 State trajectory routing only via path ρ_2 (red dashed); state trajectories routing via both paths ρ_1 and ρ_2 (blue) and their average (green)

Table 13.1 Cost averaged over 5K MC simulations

	Averaged cost
Via path ρ_1	~ 900
Via path ρ_2	~ 250
Via paths ρ_1, ρ_2	~ 100

improves: in particular, the trajectory of the system computed by averaging over all MC simulations is characterized by much smaller overshoot and faster settling time. The single trajectories generated routing data via both paths ρ_1 and ρ_2 clearly have some variance due to the high packet loss probability p_1: however, in the $5K$ MC simulations, the performance of any of the single trajectories is much better than the case when only path ρ_2 is used. Table 13.1 shows the tremendous improvement of the controller performance obtained by exploiting both paths and co-designing optimal control and routing redundancy.

13.4 Main Results

In the previous section, we have illustrated that time-inhomogeneous MJSLSs represent a mathematical model to jointly take into account the dynamics of a physical plant and nonidealities of wireless communication such as packet losses, and that their exploitation for optimal design of routing redundancy can strongly improve the closed-loop control performance. In this section, we illustrate recent advances related to the co-design of controller and communication protocol configuration subject to stochastic packet drops.

13.4.1 Stability Analysis of Linear Systems with Switching Delays

In our research line, as a first approach to address the above problem, we considered in [50] the case when routing is purely nondeterministic and packet losses are not present. We will exploit the mathematical framework of switching linear systems, which can be considered a special case of a time-inhomogeneous MJLS, where the polytopic uncertainty set contains all stochastic transition probability matrices. Because of the TDMA scheduling each path is characterized by a fixed delay in forwarding the data (see [24] for details), as a consequence each actuation data is delayed of a finite number of time steps according to the chosen routing and our system is characterized by switching time-varying delays of the input signal.

Systems with time-varying delays have attracted increasing attention in recent years (see e.g., [41, 44, 75] and references therein). In [46], it is assumed that the time-varying delay is approximatively known and numerical methods are proposed to exploit this partial information for adapting the control law in real time. The LMI-based design procedures that have been developed for switching systems with time-varying delays (see e.g., [45, 86]) do not take into account the specific structure of the systems induced by the fact that the switching is restricted on the delay-part of the dynamics. Our goal is to leverage this particular structure in order to improve our theoretical understanding of the dynamics at stake in these systems. This enables us to design tailored controllers, whose performance or guarantees are better than for classical switching systems. Our modeling choice is close to the framework in [44]. However, our setting is more general and realistic in that, differently from [44], it allows for several critical phenomena to happen: in our model, control commands generated at different times can reach the actuator simultaneously, their arrival time can be inverted, and it is even possible that at certain times no control commands arrive at the actuator. An investigation similar to ours, applied to the different setting of Lyapunov exponents of randomly switching systems, has been pursued in [76].

In our setting assume that, at each time t, the controller is aware of the propagation delays of the actuation signals sent at times $t, t+1, \ldots, t+N-1$. We assume that N can be larger than 1, i.e., that the controller is aware of the current and $N-1$ next future routing path choices and keeps memory of the past delays: we define N the *look-ahead* parameter and we call this situation the *delay-dependent case*.

The practically admissible values for N depend on the protocol used to route data: indeed, note that in several practical situations, the networking protocol can be designed to choose at any time t the future routing paths up to $t+N-1$.

As a first contribution, we show that our particular networked systems can be modeled by pure switching systems, where the switching matrices assume a particular form. As a direct consequence, the well-known LMI stability conditions for switching systems (see e.g., [70]) can be directly used to compute the worst rate of growth with fixed and arbitrarily small conservativeness. Also, while it is well known that the stability analysis problem is NP-hard for general switching systems [14], we prove that it is NP-hard even in our particular case of switching delays.

As a second contribution, we address the controller design problem. We first consider the case when one can design the communication system such that we have an arbitrarily large but finite look-ahead N. Of course, we are interested in requiring the smallest N: to this aim, we first provide an algorithm for efficiently constructing this controller, or deciding it does not exist. In case it exists, we prove a general upper bound N^* on the needed look-ahead, depending only on the dimension of the plant and the set of delays. This result has strong practical implications, since it implies (if a system is controllable) that it is never necessary to have infinite look-ahead and moreover a look-ahead equal to N^* is always sufficient. If $N \geq N^*$, stabilizability is equivalent to controllability of a projection of the initial system (as is customary for linear time-invariant systems). This implies that our techniques are also valid for the stabilizability problem.

The results described above, presented in [50], provide necessary stability and controllability conditions when extending our modeling framework to the stochastic setting in order to address packet loss models, which is the main topic of the next section.

13.4.2 Analysis and Design of Time-Inhomogeneous Discrete-Time MJSLS

In this section, we first provide necessary and sufficient stability conditions for time-inhomogeneous discrete-time MJLS ([60, 62]). Then we illustrate optimal solutions for the LQR problem for time-inhomogeneous discrete-time MJSLS ([61]). In [59] we recently addressed the optimal filtering problem and proved a separation principle.

The robust stability problem: Let us consider an autonomous discrete-time Markov jump linear system described by the following state-space model:

$$\begin{cases} \mathrm{x}_{k+1} = A_{\theta_k}\mathrm{x}_k + H_{\theta_k}\mathrm{v}_k, \\ \mathrm{x}_0 = \boldsymbol{x}_0, \ \theta_0 = \vartheta_0. \end{cases} \tag{13.12}$$

Let us denote by $\mathbb{E}(\cdot)$ the expected value of a random variable, and by $\|\cdot\|$ either any vector norm or any matrix norm. Then, the *mean square stability* of a system (13.12) is defined as follows.

Definition 13.1 [17, p. 36–37] A Markov jump linear system (13.12) is **mean square stable** if for any initial condition $\mathrm{x}_0 \in \mathbb{F}^{n_\mathrm{x}}$ and $\theta_0 \in \Theta_0$ there exist $\boldsymbol{x}_e \in \mathbb{F}^{n_\mathrm{x}}$ and $Q_e \in \mathbb{F}_+^{n_\mathrm{x},n_\mathrm{x}}$ (independent from initial conditions x_0 and θ_0), such that

$$\lim_{k\to\infty} \|\mathbb{E}(\mathrm{x}_k) - \boldsymbol{x}_e\| = 0, \tag{13.13a}$$

$$\lim_{k\to\infty} \left\|\mathbb{E}\left(\mathrm{x}_k\mathrm{x}_k^*\right) - Q_e\right\| = 0. \tag{13.13b}$$

Remark 13.1 It is worth mentioning [17, p. 37, Remark 3.10] that in noiseless case, i.e., when $\mathrm{v}_k = 0$ in (13.12), the conditions (13.13) defining mean square stability become

$$\lim_{k\to\infty} \mathbb{E}(\mathrm{x}_k) = 0, \qquad \lim_{k\to\infty} \mathbb{E}\big(\mathrm{x}_k \mathrm{x}_k^*\big) = 0. \tag{13.14}$$

There exist also other forms of stability for Markov jump linear systems without process noise, notably *exponential mean square stability* (EMSS) and *stochastic stability* (SS), that we define as follows.

Definition 13.2 [17] An MJLS (13.12) is **exponentially mean square stable** if for some reals $\beta \geq 1, 0 < \zeta < 1$, we have for all initial conditions $\mathrm{x}_0 \in \mathbb{F}^{n_\mathrm{x}}$ and $\theta_0 \in \Theta_0$ that, for every $k \in \mathbb{T}$, if $\mathrm{v}_k = 0$, then

$$\mathbb{E}\big(\|\mathrm{x}_k\|^2\big) \leq \beta \zeta^k \, \|\mathrm{x}_0\|_2^2 \tag{13.15}$$

We observe that $\|\cdot\|_2$ denotes the Euclidean norm, also known as $\mathbb{L}^2$-norm or simply 2-norm.

Definition 13.3 [17] A Markov jump linear system (13.12) is **stochastically stable** if for all initial conditions $\mathrm{x}_0 \in \mathbb{F}^{n_\mathrm{x}}$ and $\theta_0 \in \Theta_0$, we have that, if $\mathrm{v}_k = 0$ for every $k \in \mathbb{T}$, then

$$\sum_{k=0}^{\infty} \mathbb{E}\big(\|\mathrm{x}_k\|^2\big) \leq \infty. \tag{13.16}$$

In the *time-homogeneous* case, i.e., when the transition probability matrix defined by (13.2) and (13.3), is such that $P(k) = P$ for all $k \in \mathbb{T}$, there is a condition based on a value of a spectral radius of a matrix associated to the second moment of x_k that is necessary and sufficient for the mean square stability of system (13.12); furthermore, in the noiseless setting, MSS, EMSS, and SS are equivalent [17, pp. 36–44]. Specifically, the matrix related to the second moment of x_k that we have mentioned above is

$$\Lambda \triangleq \big(P^T \otimes I_{n_\mathrm{x}^2}\big) \Big(\bigoplus_{i=1}^{N} \big(\bar{A}_i \otimes A_i\big)\,,\Big) \tag{13.17}$$

where $\otimes$ denotes the Kronecker product, $I_{n_\mathrm{x}^2}$ is the identity matrix of size n_x^2, and the direct sum $\oplus$ of the manipulated elements of a sequence of state matrices $\boldsymbol{A}$ produces a block diagonal matrix, having the matrices $\big(\bar{A}_i \otimes A_i\big)$ on the main diagonal blocks.

The necessary and sufficient condition for the mean square stability of time-homogeneous Markov jump linear systems we have hinted at before is

$$\rho(\Lambda) < 1, \tag{13.18}$$

where $\rho(\cdot)$ denotes the spectral radius of a matrix. This condition for mean square stability does not hold in time-inhomogeneous case. The results of this section are based on a noiseless version of (13.12), i.e., when $\mathrm{v}_k = 0$ for every $k \in \mathbb{T}$. They are based on our first work on MJLSs [60].

Let us consider a noiseless autonomous discrete-time Markov jump linear system described by the following system of difference equations

$$\begin{cases} \mathrm{x}_{k+1} = A_{\theta_k}\mathrm{x}_k, \\ \mathrm{x}_0 = \boldsymbol{x}_0,\ \theta_0 = \vartheta_0 \end{cases} \tag{13.19}$$

where, as before, $\mathrm{x}_k \in \mathbb{F}^{n_\mathrm{x}}$ is a system's state vector, $\boldsymbol{A} \triangleq (A_i)_{i=1}^N \in {}_\mathrm{N}\mathbb{F}^{n_\mathrm{x}, n_\mathrm{x}}$ is a sequence of *state matrices*, each of which is associated to an operational mode; while $\mathrm{x}_0 \in \mathbb{F}^{n_\mathrm{x}}$ and $\theta_0 \in \Theta_0$ are initial conditions. Let the transition probability matrix $P(k) = [p_{ij}(k)]$ of the system (13.19) be polytopic time-inhomogeneous, i.e., satisfying Assumption 13.2.

Theorem 13.4 [60] *The discrete-time Markov jump linear system (13.19) with unknown and time-varying transition probability matrix* $P(k) \in \mathrm{conv}_\mathbb{V}\mathbb{P}$ *is mean square stable if and only if* $\hat{\rho}({}_\mathbb{V}\boldsymbol{\Lambda}) < 1$.

In Theorem 13.4 $\mathrm{conv}_\mathbb{V}\mathbb{P}$ denotes the convex hull of the set of transition probability matrix vertices as defined in (13.4) and $\hat{\rho}(\cdot)$ denotes the joint spectral radius (JSR)[2] of the set of matrix vertices ${}_\mathbb{V}\boldsymbol{\Lambda}$ obtained by replacing in (13.17) the transition probability matrix P with the vertices ${}_\mathbb{V}\mathbb{P}$. While it is well known that the stability analysis problem for general switching systems (that is, deciding whether the joint spectral radius is smaller than 1) is NP-hard [14], we proved that it is NP-hard even in our particular model.

Theorem 13.5 [60] *Given a discrete-time Markov jump linear system (13.19) with unknown time-varying transition probability matrix* $P(k) \in \mathrm{conv}_\mathbb{V}\mathbb{P}$, *unless* $P = NP$, *there is no polynomial-time algorithm that decides whether it is mean square stable.*

Our last but not least important result on stability of autonomous noiseless Markov jump linear systems as in (13.19) having polytopic time-inhomogeneous transition probabilities has been presented in the following theorem.

Theorem 13.6 [60] *The following assertions are equivalent.*

1. *The system* (13.19) *is mean square stable (MSS);*
2. *The system* (13.19) *is exponentially mean square stable (EMSS);*
3. *The system* (13.19) *is stochastically stable (SS).*

We developed an extension of the above results in presence of bounded-energy disturbance in [62].

The switched LQR problem. Using the approach in Sect. 13.3.2 it is possible to compute, for a finite set of predefined routing policies, the associated expected quadratic cost and choose the optimal policy. To further improve the performance one can

[2] It is well known that the maximal rate of growth among all products of matrices from a bounded set is given by its JSR $\hat{\rho}(\cdot)$, which is the generalization of the notion of spectral radius to sets of matrices. See [49] and references therein for a detailed treatment of the JSR theory.

dynamically choose, for each time step and according to the plant state measurement, the routing choice: we address this problem by considering the mathematical framework of time-inhomogeneous MJSLS. In particular, we consider the problem of joint cost minimization of continuous and discrete control inputs for the worst possible disturbance of the transition probabilities. The provided solution has been derived in [61] and consists of a finite set of recursive-coupled Riccati difference equations. This result is an extension of state of the art which is nontrivial from the technical point of view, since in the proof we needed to show that due to the time-varying nature of perturbations, at generic time step k the vertex that attains the maximum is unknown and state dependent. With respect to previous works on MJLSs having exactly known transition probabilities, we also needed to define and address the issue of explosion of the number of coupled Riccati difference equations.

Let us consider the discrete-time Markov jump switched linear system (13.9) with the switching between operational modes of the system being governed by a Markov decision process $(\mathbb{M}, \mathbb{A}, \mathrm{Pr}, g, \gamma)$. Its transition probabilities associated to each action available in an operational mode are polytopic time-inhomogeneous, as by Assumption 13.2. Also, all the operational modes of the system are considered to be measurable (Assumptions 13.3). We recall that the state-space representation of the system (13.9) under consideration is

$$\begin{cases} \mathrm{x}_{k+1} = A_{s_k}\mathrm{x}_k + B_{s_k}\mathrm{u}_k \\ \mathrm{z}_k = C_{s_k}\mathrm{x}_k + D_{s_k}\mathrm{u}_k, \\ \mathrm{x}_0 = \boldsymbol{x}_0,\ s_0 = \boldsymbol{s}_0,\ \mathrm{p}_0 = \boldsymbol{p}_0 \end{cases}$$

where the system variables and matrices are those of Sect. 13.3.1. Without loss of generality [17, Remark 4.1, p. 74] we assume that for each $i \in \mathbb{M}$ $C_i^* D_i = 0$ and $D_i^* D_i \succ 0$.

For each $k \in \mathbb{T}$, we denote by π_k the *hybrid control pair* (α_k, u_k), where $\alpha_k \in \mathbb{A}_i$ and u_k are respectively a discrete and a continuous action at time instant k. The sequence π of hybrid control pairs $(\pi_k)_{k=0}^{T-1}$ is called *hybrid control sequence*. At each time step (or decision epoch, in MDP terminology) k, a *particular choice* $\boldsymbol{u}_k$ of u_k is called the continuous control law; similarly, $\boldsymbol{\alpha}_k$ is denominated discrete switching control law. The pair $(\boldsymbol{\alpha}_k, \boldsymbol{u}_k)$ forms the hybrid control *law* $\boldsymbol{\pi}_k$, and the sequence of hybrid control laws over the horizon T constitutes a finite horizon *feedback policy*, $\boldsymbol{\pi} \triangleq (\boldsymbol{\pi}_k)_{k=0}^{T-1} \triangleq (\boldsymbol{\alpha}_k, \boldsymbol{u}_k)_{k=0}^{T-1}$. We also indicate by $\mathrm{p}_{s\bullet}^{\alpha} \triangleq \left(\mathrm{p}_{s_k\bullet}^{\alpha}(k)\right)_{k=0}^{T-1}$ the sequence of length $T \in \mathbb{T}$ of the transition probability row vectors $\mathrm{p}_{i\bullet}^{\alpha}(k)$, with $k \in \mathbb{T}_{T-1}$. Note that the transition probability row vectors $\mathrm{p}_{i\bullet}^{\alpha}(k)$ belong to a polytopic set of transition probability row vectors induced by the tranisition probability matrix vertices $_{\mathbb{V}_\alpha}\mathbb{P}$ similarly to Assumption 13.2. For more details the reader is referred to [61].

We cast an optimal linear quadratic state-feedback control problem for Markov jump switched linear systems with bounded perturbations of the transition probabilities as a min-max *problem of optimizing robust performance*, i.e., *finding the minimum over the finite-horizon feedback policy of the maximum over the transition probability disturbance obtained in correspondence of the chosen feedback policy*.

This problem can be cast from the game-theoretic point of view, where at each time step $k \in \mathbb{T}$ the perturbation-player (environment and/or malicious adversary) tries to maximize the cost while the controller tries to minimize the cost. The game-theoretic formulation of the optimal robust control problem requires to make explicit the following assumption on the information structure for the controller and the adversary.

Assumption 13.7 The perturbation-player has no information on the choice of the controller and vice versa.

The problem of designing the optimal mode-dependent state-feedback Markov jump controller, which is robust to all possible polytopic perturbations in transition probabilities, is formally defined as follows.

Problem 13.2 Given a discrete-time Markov jump switched linear system (13.9) with unknown and time-varying transition probability row vectors $\mathrm{p}^{\alpha}_{i\bullet}(k) \in {}_{\mathbb{V}_\alpha}\mathbb{P}$ and satisfying Assumption 13.2, find the mode-dependent state feedback policy $\boldsymbol{\pi}$ that achieves the following optimal cost of robust control.

$$\mathscr{J}(\mathrm{s}_0, \mathrm{x}_0) \triangleq \min_{\pi} \max_{\mathrm{p}^{\alpha}_{s\bullet}} \sum\nolimits_{k=0}^{T-1} \mathbb{E}\big(\|\mathrm{z}_k\|_2^2 + g(s_k, \alpha_k)\big) + \mathbb{E}\big(\mathrm{x}_T^* Z_{s_T} \mathrm{x}_T\big) \tag{13.20}$$

with $\boldsymbol{Z} \triangleq (Z_i)_{i=1}^{N} \in {}_{\mathrm{N}}\mathbb{F}_0^{n_\mathrm{x}, n_\mathrm{x}}$ being a sequence of the terminal cost weighting matrices.

Our solution to Problem 13.2 has been derived in [61] based on the *dynamic programming* approach in *Bellman's optimization formulation* [12], by backward induction. Note that even if the cost $g(s_k, \alpha_k)$ of performing a discrete action α_k in an operational mode s_k here is treated as time-invariant, the result will obviously remain the same in the case of the time-varying cost $g(s_k, \alpha_k, k)$, as long as the current value of the cost is known by the decision maker.

Exploiting the optimal solution defined in Problem 13.2 in the example of Sect. 13.3.2 the dynamic routing choice results in an event-driven policy that depends at each time step on the current state measurement. Note that the controller may also decide not to send control data over the network. This approach is closely related to the Event-Triggered control paradigm (see ([7, 39, 65] and references therein), where a triggering condition based on current state measurements is continuously monitored and control actuations are generated and applied when the plant state deviates more than a certain threshold from a desired value.

13.5 Conclusions and Future Work

This chapter presents an overview of some recent results on co-design of controller and network parameters of WNCS implementing communication protocols similar to the WirelessHART standard.

We leverage the class of discrete-time Markov jump linear systems, putting a specific focus on dealing with abrupt and unpredictable dynamic perturbations of transition probabilities between the operational modes of such systems and adding to the model the possibility to make discrete decisions, i.e., defining the class of time-inhomogeneous discrete-time Markov jump switching linear systems. In order to account for uncertainties and time-variance inherent to real world scenarios, we use the time-inhomogeneous polytopic model of transition probabilities, which is very general and widely used. We illustrate that time-inhomogeneous MJSLS represent a mathematical model to jointly take into account the dynamics of a physical plant and non-idealities of wireless communication such as packet losses, and that their exploitation for optimal design of routing redundancy can strongly improve the closed-loop control performance. We provide novel results in this setting addressing the robust stability and the switched LQR problems.

Our interest in this particular class of systems is inspired by their application as possible models for WNCS implementing communication protocols specifically developed for automation applications: we believe that this topic is timely, especially in view of the ongoing efforts made by academia and industry in developing a fifth generation of mobile technology (5G), which also uses models based on Markov chains and is expected to meet the requirements of ultra-reliable, low-latency communications for factory automation and safety-critical internet of things. Based on the research illustrated in this chapter we will attempt to improve our models of the communication protocols and wireless communication non-idealities and our analysis and design algorithms, with the aim of bringing substantial improvements in wireless closed-loop automation systems of the next generation by optimally co-designing the controller as well as the different layers of the communication protocol stack.

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