Chapter 5
No-Collapse Interpretations of Quantum Theory

Oliver Passon

In Sect. 2.3.1, the measurement problem was formulated in the form of a trilemma. In this view, either (i) the wavefunction is not a complete description; or (ii) the time evolution is not a continuous unitary process; or (iii) measurements do not lead to well-defined results. The GRW theory described in Sect. 2.3.1 chooses alternative (ii); it adds a nonlinear term to the Schrödinger equation, which models a physical mechanism for the "actual" collapse of the wavefunction. The Copenhagen interpretation also denies a continuous time evolution which follows the Schrödinger equation; in contrast to the GRW theory, this process is however not given a *realistic* interpretation.

In this chapter, we treat the most prominent advocates of those strategies which either deny the completeness of the wavefunction (de Broglie–Bohm theory), or question the uniqueness of the measurement results (Everett's or the many-worlds interpretation). In these theories, the state vector is thus subject to a continuous unitary time evolution. Their common feature is dispensing with the "collapse" of the wavefunction; only the *appearance* of this non-unitary change of state needs to be justified in these interpretations. Thus, the name *no-collapse interpretations* has become common as a generic label for these theories.

5.1 The de Broglie–Bohm Theory

Within the debates over the interpretation of the quantum theory—especially in view of the measurement problem—the question of whether or not quantum mechanics in its present form is simply *incomplete* is immediately raised. The statistical interpretation of quantum mechanics suggests that it must be based on an additional structure, whose elucidation would give the interpretation of the theory a completely new direction. Since this additional structure is unknown in the present version of

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quantum mechanics, this research programme was originally called "the search for 'hidden' variables".

In 1952, David Bohm published his article *"A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables"* (Bohm 1952). At the time, he was unaware that Louis de Broglie had introduced a mathematically equivalent formulation of this theory already in 1927 at the 5th Solvay Conference (de Broglie 1927). For this reason, we refer to this interpretation as the "de Broglie–Bohm theory" (DBB theory).[1] de Broglie himself referred to the interpretation as the "theory of pilot waves" (*l'onde pilote*). The conference proceedings of the 5th Solvay Conference have been accessible in English only since 2009 (Bacciagaluppi and Valentini 2009). Antony Valentini and Guido Bacciagaluppi not only undertook the translation, but also, in their knowledgeable commentary, they discuss the role of this conference for the interpretation of the quantum theory in general. According to their analysis, it is misleading to reduce the significance of the 5th Solvay Conference to the (unquestionably important) debates between Bohr and Einstein. Bacciagaluppi and Valentini argue in favour of a re-evaluation of the role of de Broglie within the early interpretation debates, and in that connection, they state:

> Today, pilot-wave theory is often characterized as simply adding particle trajectories to the Schrödinger equation. An understanding of de Broglie's thought from 1923 to 1927, and of the role it played in Schrödinger's work, shows the gross inaccuracy of this characterization: after all, it was actually Schrödinger who removed the trajectories from de Broglie's theory (Bacciagaluppi and Valentini 2009, p. 78).

A discussion of the priorities in the early development of wave mechanics can and should not be carried out here. We have cited this thought-provoking passage mainly because it expresses the basic idea of the de Broglie–Bohm theory in such a simple and clear-cut manner. This is a theory which alleges the incompleteness of the usual quantum mechanics and adds "particles" in the literal sense to the wavefunction. As we have already indicated above, the term "hidden variables" has been adopted for these additional determining quantities. This term is, to be sure, somewhat misleading, since even the harshest critics cannot deny that particles, and their locations in particular, are directly observable (and thus in this sense not at all hidden). Instead, it is simply the *wavefunction* which is not susceptible to direct observation.[2]

For reasons which of course must be explained in more detail in the following, the de Broglie–Bohm theory succeeds in this way in describing the measurement process as a normal interaction which leads to a uniquely defined final state. At the same time, it is (in the technical sense) a *deterministic* theory—while in addition, it can also reproduce all the predictions of the quantum theory. However, this theory

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[1]Bohm's lack of knowledge of the earlier work is understandable if one is aware that de Broglie himself did not develop his theory further, but instead became a supporter of the "conventional" quantum theory. Only after reading Bohm's publication of 1952 was his interest in these questions again aroused.

[2]We shall see that the lack of knowledge (and control) of the *precise* initial conditions plays an important role in the DBB theory. This aspect of the additional variables can indeed be considered to be "hidden". Furthermore, the concept of "hidden variables" also refers to the fact that they do not occur in the standard interpretation.

makes *no* new predictions which deviate from those of the quantum theory, so that experimentally, there is no way to decide between the two.[3]

In Bohm's formulation of 1952, we are dealing with an extension of non-relativistic quantum theory. We will take up the question of a relativistic generalization in Sect. 5.1.7. The following description of the theory makes use at various points of a comparison with the "standard interpretation" or the "usual textbook version" of quantum mechanics. These concepts are naturally not strictly defined, and the reader can think here of the Copenhagen interpretation or a textbook description of quantum mechanics, which do not deal with the problems treated in this book.

5.1.1 Mathematical Description

The de Broglie–Bohm theory is an extension of the standard quantum theory. Among the relations which define the theory mathematically, we thus find the usual Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + V(r)\psi\,. \tag{5.1}$$

Here, V refers to the potential which characterizes the corresponding system (see also Eq. (1.39) in Sect. 1.2.4; there, the Schrödinger equation was introduced for only one spatial dimension). We have chosen the positional representation not by chance, since it is, as we shall see, in fact distinguished within the de Broglie–Bohm theory. In the standard interpretation, ψ is presumed to contain the complete description of the system, and from its absolute square $|\psi|^2$, the probability of observing a particle by a measurement within a particular spatial region can be obtained. In the standard interpretation, one however cannot speak of a particle's trajectory or orbit, i.e. that which brought it to the position where it was observed.

In the de Broglie–Bohm theory, the concept of "particle" is taken so seriously that at each moment in time (i.e. even *without* measurements), it is associated with a well-defined position. A quantum-mechanical N-particle system is thus no longer described by the wavefunction alone, but rather by the *pair* consisting of the wavefunction and the position coordinates of the particles: $(\psi, Q(t))$. Here, $Q(t) = (Q_1(t), \cdots, Q_N(t))$, where $Q_i : t \to \mathbb{R}^3$ denotes the trajectory of the ith particle. $Q(t) \in \mathbb{R}^{3N}$ is called the configuration of the system, and $\mathbb{R}^{3N}$ is its so-called configuration space.[4]

[3]This statement holds strictly under the so-called quantum equilibrium hypothesis (see Sect. 5.1.2). Without that assumption, predictions which differ from the ordinary quantum theory may result (cf. Cushing 1995 and Valentini 2004).

[4]Configuration space is of central importance even in conventional quantum theory, because the wavefunction is likewise defined on this space.

For the particle positions $Q(t)$, one must specify an equation of motion, i.e. a (differential) equation which describes the temporal and spatial evolution of the particle positions under the influence of the given external conditions. This prescription must reproduce—on average—the statistical predictions of quantum theory. There have been various suggestions for the motivation of this equation of motion (cf. Passon 2010, pp. 32–36). In the following, we will make use of the analogy between quantum theory and hydrodynamics, which was pointed out as early as 1926 by Erwin Madelung (cf. Madelung 1926). Let us therefore briefly consider a liquid (or a gas) with a mass density of ρ_m. Under the assumption that the mass is a conserved quantity, the mass density within a certain region in space can then change its magnitude only if fluid flows out of or into that region. In order to describe the flow of the fluid, we define the "current-density vector" or, for short, the "current density", as the product of the mass density and the flow velocity of the fluid: $j_m = \rho_m v$. The x component of j_m denotes the amount of fluid which flows per unit time through a unit surface element (perpendicular to the x-axis) and correspondingly for the y and z components. Then the conservation of mass is represented by the following mathematical expression:

$$\underbrace{\frac{\partial \rho_m}{\partial t}}_{\text{time rate of change}} = \underbrace{-\nabla \cdot j_m}_{\text{spatial rate of change}} . \tag{5.2}$$

Here, the symbol "∇" denotes the *divergence*, i.e. the sum of the spatial changes over all three directions. This *equation of continuity* from hydrodynamics expresses—as explained—the conservation of the fluid mass.

We now turn back to quantum theory, in which likewise an equation of continuity holds—but now for the "probability density" $\rho = |\psi|^2$. This equation is formally identical[5] to the hydrodynamic equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0 . \tag{5.3}$$

At this point, naturally, the mathematical details should not be so much the subject of our considerations as the structural relations. The decisive point is that this equation can be *derived* from the Schrödinger equation, and for the probability current density, we find the following (somewhat complicated) expression:

$$j = \frac{\hbar}{2mi}\left[\psi^*(\nabla\psi) - (\nabla\psi)\psi^*\right] . \tag{5.4}$$

[5] There is, however, a decisive difference with respect to the hydrodynamic equation of continuity: While the mass density ρ_m is defined on real position space, the probability density $\rho = |\psi|^2$ is a function on configuration space. A naive identification of $|\psi|^2$ with a matter density thus appears to be impossible.

In the usual textbook descriptions of quantum theory, the equation of continuity (5.3) is interpreted as an expression of the "conservation of probability". Probability (like mass within hydrodynamics) can be neither "created" nor "destroyed".

In the de Broglie–Bohm theory, one takes a step further, since the goal is finally to arrive at an equation of motion for the "Bohmian particles". The expression ρ in quantum theory is interpreted as the probability density of the real particle configuration, and we recall that in hydrodynamics, the relation $j = \rho v$ holds. If we put in the corresponding quantum-mechanical expressions for ρ and j (and use the "polar representation" $\psi = Re^{\frac{i}{\hbar}S}$ for the wavefunction), we find, after a simple computation, the equation of motion for the particle positions $Q(t)$ that we were seeking (for its velocity, we have of course $v = \frac{dQ}{dt}$):

$$\begin{aligned} v &= \frac{j}{\rho} \\ \frac{dQ}{dt} &= \frac{\nabla S}{m} \,. \end{aligned} \tag{5.5}$$

This Eq. (5.5) is called the *guidance equation* of the de Broglie–Bohm theory. Pictorially speaking, the particle trajectories are thus guided by the wavefunction (or rather by its phase S). Treating a physical problem with the help of the DBB theory thus means first of all solving the Schrödinger equation (as in the usual quantum mechanics). In Sect. 5.1.4, we will discuss concrete applications.[6]

The validity of the equation of continuity (5.3) has still another important consequence for the de Broglie–Bohm theory. It follows from this equation namely that a configuration once distributed according to $|\psi|^2$ retains this property under Bohmian dynamics. This observation is the key to the fact that the de Broglie–Bohm theory reproduces all the predictions of the usual quantum theory, since naturally a differential equation fixes the motion only through its boundary and initial conditions. If one now chooses the initial configuration $Q(t_0)$ at random according to the probability distribution $|\psi_{t_0}|^2$ for a system that is described by the wavefunction ψ, then the configuration $Q(t)$ will *remain* distributed according to $|\psi_t|^2$ at each later moment in time, t. In other words, according to Born's rule, all the predictions of the usual quantum theory will be reproduced.[7] This condition is called the "quantum equilibrium hypothesis", and we will take a closer look at it in Sect. 5.1.2.

The three relations which define the de Broglie–Bohm theory mathematically are thus

1. The **Schrödinger equation**: $i\hbar\frac{\partial\psi}{\partial t} = -\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + V(\mathbf{r})\psi$
2. The **guidance equation**: $\frac{dQ}{dt} = \frac{\nabla S}{m}$

[6]In fact, the condition of being able to reproduce the statistical predictions of quantum mechanics does not fix the dynamics *uniquely*. In this sense, there are indeed infinitely many "de Broglie–Bohm-like" theories. In these theories, the individual trajectories do *not* follow Eq. (5.5), but they however reproduce the same statistics (Deotto and Ghirardi 1998).

[7]The equivalence to quantum mechanics presumes that all predictions can be uniquely described in terms of position coordinates—e.g. by "pointer positions" of a measurement apparatus.

3. The **quantum equilibrium hypothesis**: The position distribution ρ of states with the wavefunction ψ is given by the probability density $\rho = |\psi|^2$.

The second and the third relations deserve a more careful consideration, since they signal the differences relative to conventional quantum theory.

5.1.2 The Quantum Equilibrium Hypothesis

According to the quantum equilibrium hypothesis, the positions of the particles of a state which is described by the wavefunction ψ are distributed in accord with the probability density $|\psi|^2$. The occupation probability within a spatial region V is calculated by integration, $\int_V |\psi|^2 dV'$.

If this initial condition is fulfilled at one time, it follows from the equation of continuity (5.3) that Born's rule will remain valid at all later times. Furthermore, the quantum equilibrium hypothesis guarantees that the particle positions cannot be more precisely controlled. Bell writes on this topic:

> Note that the only use of probability here is, as in classical statistical mechanics, to take account of uncertainty in initial conditions (Bell 1980, p. 156).

If thus follows that the Heisenberg uncertainty relations can also not be violated within the de Broglie–Bohm theory! At the same time, one might be tempted to call the "determinism" of the de Broglie–Bohm theory "fictitious". In its descriptive content, the de Broglie–Bohm theory does not differ from the standard interpretation of quantum mechanics, and it likewise can make only statistical predictions. The quote from Bell however indicates a conceptual difference. Within the de Broglie–Bohm theory, the statistical character of the predictions is attributable to our lack of knowledge and is thus epistemic in nature. Within the standard interpretation of quantum mechanics, the ignorance interpretation of the probability is not possible; it is thus an ontic probability.

Let us now turn to the question of how this equilibrium distribution can be justified. The first attempt dates back to Bohm (cf. Bohm 1953), who gave a *dynamical* explanation of the $|\psi|^2$ distribution. His approach was developed further by Valentini (1991). In Valentini and Westman (2005), one finds for example numerical simulations of systems which, under the dynamics of the guidance equation, lead from a non-equilibrium distribution to the quantum equilibrium distribution. In the framework of this approach, it would seem reasonable to consider systems in "quantum *non*-equilibrium"—together with all possible deviations of the predictions between conventional quantum mechanics and the de Broglie–Bohm theory (cf. Valentini 2004). Another strategy—for which Bell seems to express support at various times—consists in simply *postulating* the quantum equilibrium hypothesis. This would give it the status of a fundamental law.

In contrast, Dürr *et al.* (1992) argue that neither postulating the quantum equilibrium condition, nor its dynamic justification is reasonable or convincing. At the core, the question is namely how—within a deterministic theory—probability statements can occur at all. This problem is naturally much older than the de Broglie–Bohm theory, and it has dominated the discussion on the relation between (Newtonian) statistical mechanics and classical thermodynamics since the nineteenth century. In their justification of the quantum equilibrium distribution, Dürr *et al.* therefore hark back to a concept introduced by Ludwig Boltzmann (1844–1906), namely that of "being typical" for a physical event. "Being typical" has a terminological meaning here, namely the appropriateness for the "overwhelming majority" (as defined by measure theory) of initial configurations (Dürr 2001, pp. 49ff).

The application of this concept to de Broglie–Bohm theory is now carried out in two steps. First, the authors clarify the question of under which conditions subsystems can be associated with a wavefunction at all. This can naturally not be expected of arbitrary subsystems, owing to interactions with their environment. In principle, the de Broglie–Bohm theory thus holds for the wavefunction of the universe, Ψ. The concept of the "wavefunction of the universe" sounds presumptuous. In fact, it does *not* mean that the de Broglie–Bohm theory claims universal validity. Rather, it is the wavefunction of a system in which probability statements can no longer be explained in terms of an "external influence", i.e. by the existence of a still larger system in which the system considered is embedded. For the *fundamental* justification of probability statements, this standpoint thus *must* be adopted.

For the wavefunction of the universe, however, the assertion that its position coordinates are distributed according to $\rho = |\Psi|^2$ appears problematic. After all, there is only *one* universe,[8] and a test of this probability statement by measurements of relative frequencies of occurrence is impossible. For the wavefunction of the universe, one cannot ascribe the meaning of a probability density to the expression $|\Psi|^2$, at least not in an operational sense. Instead, Dürr *et al.* suggest that we see in it a measure of what a "typical" initial condition (in Boltzmann's sense) for the universe would be like. They justify their choice with the "equivariance" of the distribution, i.e. with the fact already mentioned that a configuration which at one moment is distributed according to $|\psi|^2$ will retain this property. The choice of any other (non-equivariant) distribution as the measure of "typical" initial configurations would have to distinguish a particular moment in time, and the moment at which precisely that distribution was present in an unnatural way.

In addition, there is a class of subsystems which can be described by using "effective wavefunctions". This means that the particle dynamics of these subsystems are almost completely determined by that effective wavefunction.[9]

[8]Speculations about "multiverses" change nothing in this situation—for there, also, as a rule any contact to the other "universes" is forbidden.

[9]The effective wavefunction $\psi(x)$ of a subsystem with the variables x on configuration space, which belongs to the overall system $\Psi(x, y)$, is defined as a part of the following decomposition: $\Psi(x, y) = \psi(x)\Phi(y) + \Psi^{\perp}(x, y)$. Here, Φ and $\Psi^{\perp}$ have disjunct carriers, and the configuration of the environment (Y) lies in the carrier of Φ. For the overall system, one could think for example of subsystem + environment or, concretely, subsystem + measurement apparatus. The above decomposition occurs namely during a measurement interaction: If the configuration of the measurement setup corresponds to Y (this could be a particular "pointer position" of the measurement apparatus),

Finally, Dürr *et al.* can *prove* that subsystems with an effective wavefunction ψ within a "typical" universe fulfil the quantum equilibrium hypothesis. In this sense, the deterministic de Broglie–Bohm theory obtains the appearance of randomness, and the empirical distributions correspond to the quantum-mechanical predictions. If one accepts this "Boltzmann argument", then the quantum equilibrium condition becomes even a *theorem* of the de Broglie–Bohm theory.[10]

5.1.3 The Guidance Equation

Thus far, we have considered only the single-particle case. The general form of the guidance equation for an N-particle system is given by[11]:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \Im \frac{\nabla_i \psi}{\psi} = \frac{\nabla_i S}{m_i} . \tag{5.6}$$

Here, m_i denotes the mass of the ith particle, $\Im$ the imaginary part of the following expression and ∇_i is the gradient with respect to the spatial coordinates of the ith particle. In case the wavefunction is a spinor, i.e. $\psi : \mathbb{R}^{3N} \rightarrow \mathbb{C}^{2N}$, the probability current is changed, so that one obtains the following guidance equation:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \Im \frac{\psi^* \nabla_i \psi}{\psi^* \psi} , \tag{5.7}$$

where $\psi^* \psi$ is the scalar product on $\mathbb{C}^2$. The latter equation is mentioned here not only for completeness, but also because it will be used in the treatment of the measurement problem in Sect. 5.1.5.

The existence and uniqueness of the solutions of the guidance equation for all the relevant types of potentials have been demonstrated (see Teufel and Tumulka 2005). Two points should be emphasized: First, the order of the guidance equation (as well as the resulting general properties of its solutions); second, its so-called non-locality. The next two subsections are devoted to these two issues.

the x system is guided by the wavefunction $\psi(x)$. The remaining parts of Ψ are then irrelevant for the particle dynamics, and in this way, an "effective collapse" is described (cf. Sect. 5.1.5).

[10]Our treatment here could of course only roughly sketch the train of reasoning, and it suppresses many mathematical details. Thus, an impression of circularity may have (falsely) arisen: One postulates the $|\Psi|^2$ distribution of the universe and obtains the $|\psi|^2$ distribution of subsystems. See Dürr (2001, p. 201) for more on this topic.

[11]The following is naturally difficult to understand for those readers who are not well versed in mathematics. The decisive point is that the position (and velocity) of the Bohmian particles are mathematically determined by the *wavefunction*.

General Properties of the Trajectories

Since the guidance equation is a differential equation of first order, *one* initial condition $Q(t_0)$ already determines the trajectories uniquely. In configuration space, the paths are thus not overlapping. It follows for the single-particle case, in which position space and configuration space are identical, that the trajectories within the DBB theory do not intersect each other. If they are in fact identical at one point, then they must be identical at all points. In many cases, this information alone allows us to visualize a qualitative picture of the trajectories.

Non-locality

The guidance equation determines the trajectory of the ith particle essentially by taking the derivative of the wavefunction (more precisely: by taking its gradient). The wavefunction is however defined on configuration space and is evaluated at the position $Q(t)$. In other words, the change of position of each particle at the time t depends on the positions of all the other particles at the *same moment in time*. Since these influences do not propagate through space in the sense of a short-range interaction, one speaks of a non-local influence, or of the non-locality of the de Broglie–Bohm theory. However, it is precisely this non-locality which permits the de Broglie–Bohm theory to violate the Bell inequalities (in agreement with experiments; see Chap. 4). At the same time, the quantum equilibrium hypothesis guarantees that this non-locality cannot be used for the transmission of signals, since it is in the end a question of stochastic events. The evident problem of the relativistic generalization of this theory will be addressed in Sect. 5.1.7.

5.1.4 Applications of the de Broglie–Bohm Theory

We now turn our attention to the obvious question of which form the particle trajectories take, whose existence distinguishes the de Broglie–Bohm theory from the usual quantum theory. The guidance equation has in fact been solved numerically for various problems. For those who favour this theory, the *existence* of these trajectories is notably more important than their concrete characteristics or their numerical simulation. Dürr writes on the question of whether Bohmian trajectories should be calculated at all:

> Roughly speaking, no! Sometimes, however, the asymptotic behaviour of the trajectory – essentially that of free particles – can be quite useful. [...] All that we learn from the trajectories is indeed only that at every time t, particles are present whose positions are distributed according to the quantum equilibrium hypothesis as $|\psi(q,t)|^2$ (Dürr 2001, p. 142).

In the following, we nevertheless consider explicit Bohmian trajectories for the tunnel effect, for interference from gratings (and from the double slit), as well as for the hydrogen atom; thus for several examples of quantum phenomena which, in the usual understanding, can not possibly be explained in terms of continuous trajectories.

The Tunnel Effect

A spectacular prediction of quantum theory is the "tunnel effect". It consists in the fact that quantum-mechanically described particles can overcome a potential barrier, although the energy height of the barrier is *greater* than the energy of the particles. Radioactive α decay, as well as nuclear fusion in the interior of the Sun, is understandable only in terms of the tunnel effect.[12] Pictorially speaking, the particles pass *below* the barrier—they thus "tunnel" beneath it.[13] In an orthodox manner of speaking, there is a finite probability that the particles will be detected on the other side of the barrier. Within the de Broglie–Bohm theory, a continuous particle trajectory must naturally lead from inside the potential barrier to a position outside the barrier.

Figure 5.1 shows the paths taken by some of these trajectories. The y-axis corresponds to the position coordinate (in arbitrary position units) and the x-axis to the time coordinate. A Gaussian wave packet ψ was assumed as initial condition, and it approaches the barrier from below in the figure. This potential barrier is located at $0.72 \leq y \leq 0.78$, and it is twice as high as the average energy of the wave packet.[14] Then the Schrödinger equation is solved numerically and input into the guidance equation. In this way, the course of the trajectories can be computed. One can first recognize how all the particles are braked within the barrier (the slopes of the trajectories in Fig. 5.1 correspond to the particles' velocities). The tunnel effect occurs for those particles which reach the barrier first, while those arriving later are reflected earlier and earlier. If this were not the case, the particles' trajectories could intersect. Thus, the property of being *intersection-free* determines the shape of the trajectories already qualitatively.

This description of the tunnel effect notably opens up the possibility of calculating the "tunnelling time". The obvious question of the time required by a particle in order to overcome the tunnel barrier cannot even be reasonably asked of conventional quantum theory, since time is not an observable. Cushing (1995) discusses the possibility of an experimental test of the de Broglie–Bohm theory on this basis.

Two-Slit Interference

Diffraction and interference of an electron beam by a double slit and the pattern of the typical interference fringes (see Fig. 5.2, left) were successfully observed by Claus Jönsson in 1959 (see Möllenstedt and Jönsson 1959). Particularly impressive

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[12] In the case of α decay, helium nuclei overcome the potential barrier at the surface of the decaying nucleus, although their energies, considered classically, are too small to permit this. In the case of nuclear fusion in the interior of the sun, hydrogen atoms combine to form helium. Here again—considered classically—the pressure and temperature are too low to overcome the repulsion of the positively charged hydrogen nuclei.

[13] This manner of speaking, "tunnelling" or "passing beneath", is naturally to be understood as metaphorical, since the "height" of the potential barrier is not a spatial quantity, but rather an energy.

[14] The wavefunction is Gaussian, with its centre initially at 0.5 and a width (variance) of 0.05. The density of the trajectories between 0.66 and 0.68 was increased in order to be able to study the oscillatory behaviour within the barrier more precisely (see Dewdney and Hiley 1982).

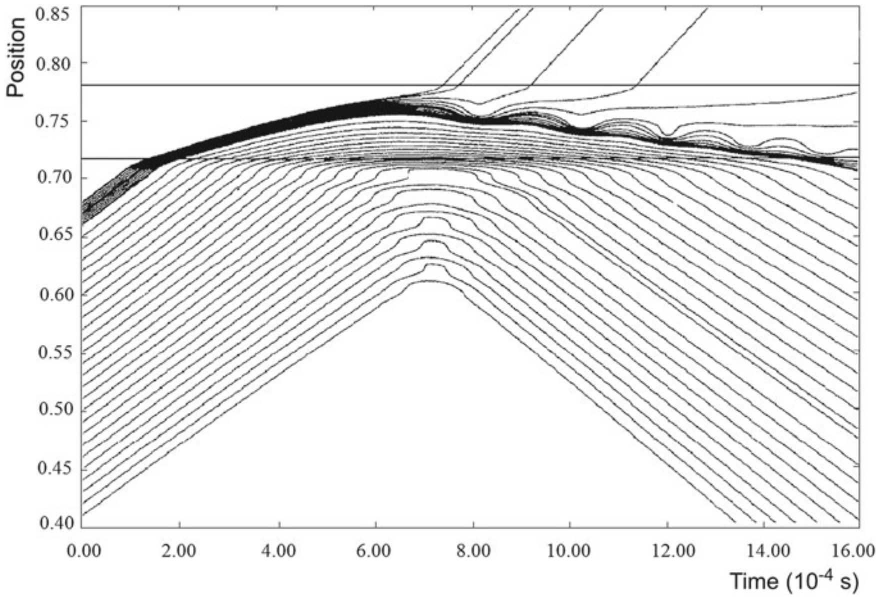

Fig. 5.1 A numerical simulation of some trajectories in the 1-dimensional tunnel effect (from Dewdney and Hiley 1982, reprinted with kind permission of the *Springer Verlag*). The x-axis corresponds to the time coordinate and the y-axis to the position coordinate

are the experiments in which the particle beam has such a low intensity that the formation of the interference pattern can be observed over a longer period of time. Then, *point-like* detected particles on the detection screen are seen to gradually build up the interference pattern.

This experiment would appear to illustrate with unusual clarity that the concept of particle trajectories is not applicable in quantum theory. If—thus runs the usual argument—the particle trajectories pass through the upper *or* the lower slit, it should be irrelevant whether at that moment the *other* slit were opened or closed. The result should be that the distribution, after passing through a double slit, should correspond to the sum of those from each of two single slits.

However, the observed pattern is evidently quite different. Popularizations occasionally claim that the particle has passed "through both slits". This paradoxical formulation is apparently intended to suggest that particle trajectories in the classical sense can no longer be considered to exist.

Within the de Broglie Bohm theory, this problem is resolved in a simple manner. The particle trajectories naturally each pass through only one of the openings of the double slit (or of the grating). The trajectories are however led by the wavefunction, according to the guidance equation. The wavefunction encodes the information on the slit geometry and steers the trajectories correspondingly towards the interference maxima. Here, it becomes clear how within the de Broglie–Bohm theory, the

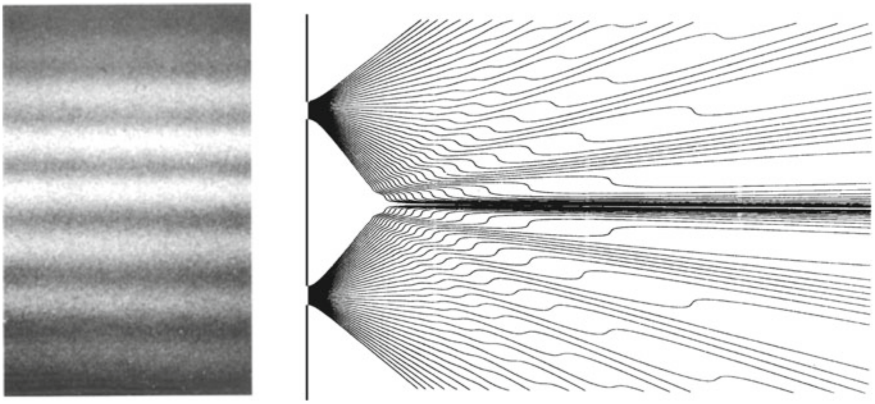

Fig. 5.2 Left: Measurement of the interference fringes of electrons from a multiple-slit arrangement (from Möllenstedt and Jönsson 1959). Right: A numerical simulation of some of the trajectories through a double slit (from Philippidis *et al.* 1979). Both of these images are reprinted with the kind permission of the *Springer Verlag*

wavefunction no longer represents a "probability wave", but instead a real physical effect.[15] Every reference to wave–particle dualism thus becomes superfluous.

If the initial values of the particle are distributed according to the quantum equilibrium hypothesis, the DBB theory exactly reproduces the occurrence frequency distribution of quantum theory. A numerical simulation of some of the corresponding trajectories can be seen on the right in Fig. 5.2. Again, it can be clearly discerned that the trajectories do not intersect. At the same time, they exhibit a completely "non-classical" behaviour, in that they show abrupt changes of direction (in—classically—"field-free" regions). Here, one can already see that momentum or energy conservation do not hold on the level of individual particles, since they obey Bohmian mechanics and not Newtonian mechanics. In Sect. 5.1.5 (see also Footnote 19), this aspect is explained in more detail.

The Hydrogen Atom

The discrete and characteristic spectra of excited atoms provided important impulses to the early development of the quantum theory. The successful description of the discrete energy levels of the hydrogen atom belongs among its early triumphs.

The solution of the Schrödinger equation for this problem (i.e. for the potential $V = -\frac{e^2}{r}$) is mathematically rather involved and will not be repeated here. The decisive point is that one is led to eigenstates of the energy for which the wavefunction is a product of a real function and the expression $e^{i(m\phi - Et/\hbar)}$. In the ground state, m (the "magnetic" or "directional" quantum number) is zero, so that the phase is given by $S = -Et$. Inserting this expression into the guidance equation (5.5), we obtain for the velocity field of course everywhere zero; we have computed the spatial

[15] On the status of the wavefunction; see however Sect. 5.1.7, and also Dürr *et al.* (1996).

derivative of an expression which has no spatial dependence. In other words, the particle in the ground state is at rest, at positions which are distributed according to the quantum equilibrium condition for the associated wavefunction. One might call this result counter-intuitive—but it must be admitted that no one has an "intuition" of the processes within an atom.[16]

5.1.5 *The Solution of the Measurement Problem*

At its core, the measurement problem consists in the fact that following a measurement, the measurement apparatus indeed *shows a result*. After the measurement, the apparatus should (considered quantum-theoretically) thus be in an eigenstate of the corresponding operator.

In general, the microscopic state (on which the measurement is carried out) is described as a superposition of various components, which each correspond to a different "pointer position" of the measurement apparatus. Under the dynamics of the linear Schrödinger equation, the measurement apparatus should also take on a superposition state and not an eigenstate. In reality, however, a superposition of macroscopic states is neither readily imaginable, nor has one ever been observed.[17]

The solution of the measurement problem by the de Broglie–Bohm theory can be illustrated in a completely non-technical and nevertheless appropriate way. It is based on the idea that it is only the *pair* consisting of the wavefunction *and* the configuration which makes up the complete description of a system and not just the wavefunction alone. Due to the definite particle positions, *every* system is in a well-defined state at *every time*. This therefore holds also for the measurement apparatus after a measurement has taken place. The different pointer positions of the measurement apparatus are in the end none other than different configurations, $Q(t)$. In other words, in the de Broglie–Bohm theory, the "wavefunction of the measurement apparatus" will in general be in a superposition state. The configuration however indicates the result of the measurement which is actually realized. That part of the wavefunction which "guides" the particle(s) can be reasonably termed the *effective* wavefunction. All the remaining parts can be ignored, since they are irrelevant for the particle dynamics. As a result of decoherence effects (see Sect. 5.2.4), the probability that they will produce interference effects with the effective wavefunction is vanishingly small. In

[16] In the excited states, where $m \neq 0$, the azimuthal-angle ϕ is time-dependent, and the Bohmian particle orbits around an axis (see Passon 2010, pp. 87f). Note that also this motion does not correspond to Bohr's atomic model, which is well known (and rebutted) in school physics.

[17] Formally speaking, we are considering here the superposition of several states of the overall system, consisting of a measurement object ($\psi = \sum c_i \psi_i$) and a measurement apparatus (Φ_i). In the case that the measurement apparatus is initially at the position Φ_0, during the measurement interaction it undergoes a time evolution $\psi \otimes \Phi_0 \to \sum c_i \psi_i \otimes \Phi_i$. Here, Φ_i denotes the state of the apparatus after the measurement, on measuring the property which is associated with the state ψ_i.

this sense, the de Broglie–Bohm theory describes an "effective collapse" (see also Footnote 9). In the words of Dürr:

> This 'collapse' is not a physical process, but rather an act of convenience. It takes place only through the choice of description [...] because the price of forgetting about the other, non-effective parts of the wavefunction is extremely low, since future interferences are practically excluded (Dürr 2001, p. 160).

This solution of the measurement problem makes an additional tacit assumption: All the results of measurements must be uniquely characterizable in terms of position coordinates. Think for example of the "pointer positions", or of the positions of inked pixels on a sheet of paper.[18] This however does not mean that only the measurement of particle *positions* can be described by the de Broglie–Bohm theory. Naturally, this solution of the measurement problem applies also to spin, momentum or the measurement of any other "observable". Their status is however drastically re-interpreted in this theory, as is described by the keyword "contextuality".

Contextuality

Already in Sect. 1.1.1, we dealt with the Stern–Gerlach experiment for the measurement of the spin component of an electron. A beam of silver atoms is passed through an inhomogeneous magnetic field, so that the spins of their outer electrons lead to a splitting of the beam.

Here, also, we are dealing with a measurement whose definite result is described by the de Broglie–Bohm theory. The discussion is made more complicated by the fact that the Schrödinger equation cannot describe particles with spin. Instead, one has to resort to the so-called Pauli equation. This modification of the Schrödinger equation describes spin-$\frac{1}{2}$ particles using a 2-component wavefunction. A guidance equation for the particles is found analogously to the case of the Schrödinger equation (this procedure was already described in Sect. 5.1.3, Eq. 5.7). This however yields no conceptional differences relative to the above discussion. Figure 5.3 gives a naive representation of how the results of the measurements are determined in the de Broglie–Bohm theory. If the particle coordinate is *above* the plane of symmetry (like the small black dot under the magnifying glass in the figure), a deflection into the upper branch of the wavefunction occurs ("spin up"), and *vice versa*. It is thus the particle's *location* which determines the result of a spin measurement! The property "spin" is not attributed to the particle itself, but instead, it is a property of the wavefunction.[19]

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[18] At this point, it again becomes clear that the term "hidden variables" for the particle positions is misleading. It is just their *un-hiddenness* which qualifies them to describe the observable results of a measurement!

[19] The same is true of all other physical quantities. The particles in the de Broglie–Bohm theory have no properties besides their positions and their velocities. Even mass, momentum or charge cannot reasonably be attributed to the particles; think for example of quantum-mechanical interference experiments in which the influence of gravity or an electromagnetic interaction can (in principle) modify the wavefunction. Therefore, we have thus far avoided referring to the "Bohmian particles" as "electrons", "atoms", etc. However, in Holland (1993) as well as in Bohm and Hiley (1993), a possible spin variable is discussed. Our treatment here follows Bell (2001, pp. 5ff) and Dürr (2001).

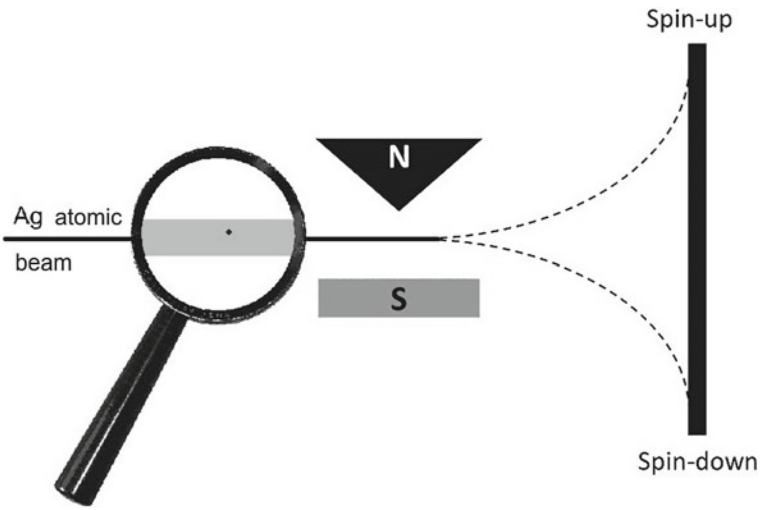

Fig. 5.3 Deflection of silver atoms in an atomic beam by an inhomogeneous magnetic field (Stern–Gerlach experiment). The initial position (indicated by the small dot under the magnifying glass) is decisive for the result of the spin (-component) measurement in the de Broglie–Bohm theory

We could argue in a similar way about the measurement of energy, momentum or other observables. For all of these quantities, the de Broglie–Bohm theory thus introduces *no* additional "hidden variables" which would describe their actual values. Instead, their values are determined by the wavefunction, the particle position *and* the particular implementation of the measurement. Taking the example of the Stern–Gerlach experiment, we can illustrate the influence of the particular measurement setup in an intuitively clear manner: If the orientation of the magnetic field in Fig. 5.3 were reversed, we would measure the *opposite* spin for the *same* particles! The de Broglie–Bohm theory thereby composes a picture in which only position measurements yield a value that was already present within the system *before* the measurement and is thus a *property* of the particle in the narrow sense. All other measurements owe their outcomes to the "context" of the implementation of the measurement. The terms "measurement" and "observable" are rather misleading here. This property of the de Broglie–Bohm theory is called "contextuality". Indeed, this concept has a somewhat more extended meaning in the discussions and includes those mutual influences which occur in *combined* measurements of different quantities.

The relations treated here can be formulated rather concisely by making use of the terminology which has been developed in philosophy for the description of various types of properties. The spin, or all other properties aside from the position, are not categorial properties within the de Broglie–Bohm theory, but rather they are dispositions.[20] The contextuality of dispositions is however not remarkable; it is simply a part of their definition (cf. Pagonis and Clifton 1995).

Proofs of the Impossibility of Hidden Variables

This contextuality of the de Broglie–Bohm theory explains why the numerous *"no-go" theorems* or "proofs of impossibility" relating to theories with hidden variables do not apply to it. The best known of these theorems is due to von Neumann. A generalization was formulated by Kochen and Specker; see Mermin (1990) and the references there. These theorems are based on the intuition that hidden variables fully encode the (only apparently) statistical outcome of the measurements. The proofs demonstrate the impossibility of a mapping which ascribes to every state a unique value in regard to *every* possible measurement—indeed, without taking the context into account. The de Broglie–Bohm theory does not claim even the *existence* of actual values in regard to every physical quantity which can be measured; for only the position is a categorial property of this theory. Think again of the example given above of the measurement of the spin component: The particle is not associated with any fixed orientation of its spin, independently of a concrete "measurement"; when the direction of the magnetic field is changed, the spin can even take on the reversed value. According to Daumer *et al.* (1996), the understanding of measurements which is based for example on such *no-go* theorems reveals a "naive realism" in relation to the role of operators. These authors understand this as the usual identification between operators and observables, bound up with the widespread manner of speaking that "operators can be measured". This expression is however highly misleading, since the influence of the experimental context on a measurement as described above is not taken into account.

5.1.6 The Schools of the de Broglie–Bohm Theory

The *Compendium of Quantum Physics* (Greenberger *et al.* 2009) contains two entries on the subject of this chapter. One of them is entitled the "*Bohm Interpretation of Quantum Mechanics*", while the other is called simply "*Bohmian Mechanics*". One is left with the suspicion that "Bohmian mechanics" is not identical to "Bohm's interpretation of quantum theory". This impression is correct and deserves a closer look—even if only to facilitate the orientation of the reader in studying the relevant literature.

The article on "*Bohmian Mechanics*" was written by Detlef Dürr, Sheldon Goldstein, Roderich Tumulka and Nino Zanghì. This group supports a version of the theory which was formulated by John Bell, beginning in the 1960s. Our own treatment is closely related to this version. At its centre stand the guidance equation and the re-interpretation of the concept of *observables* (keyword: "contextuality").

[20]While categorial properties are associated with an object *without* any reference to its environment (e.g. "being round"), dispositions describe those properties which manifest themselves only in certain special contexts (e.g. "being fragile").

The author of the article "Bohm Interpretation of Quantum Mechanics" is Basil Hiley. He was a colleague and close coworker of David Bohm at Birkbeck College; and together with Chris Dewdney, Chris Philippidis and others, this "English group" strongly supports Bohm's formulation of the theory from 1952. Bohm and his coworkers referred (or refer) to this theory notably as an "ontological" or a "causal" interpretation of the quantum theory. In this variant of the theory, the concept of the "quantum potential" plays a central role. Let us consider the derivation of the guidance equation in this respect, as it was given by David Bohm in 1952. He chose a path for the derivation which invokes an analogy to the Hamilton–Jacobi equation of classical mechanics. In the classical case, the Hamilton–Jacobi theory contains the relation $p = \nabla S$ (with the momentum $p = mv$ and the action S). Bohm could show in his work that a similar relation holds in quantum theory, if the action is replaced by the phase S of the wavefunction. This then led him to the well-known guidance equation, $v = \frac{dQ}{dt} = \frac{\nabla S}{m}$. Indeed, this theory can then be represented in such a way that it appears to be a modification of Newtonian mechanics:

$$m \frac{\mathrm{d}^2 Q(t)}{\mathrm{d}t^2} = -\nabla (V + U_{\mathrm{quant}}) \tag{5.8}$$

with the classical potential V and the additional quantum potential

$$U_{\mathrm{quant}} = -\frac{\hbar^2 \nabla^2 |\psi|}{2m|\psi|} \, . \tag{5.9}$$

Note, however, that in contrast to Newtonian physics, the velocity is already fixed via the guidance equation. The representation in terms of a second-order differential equation is thus misleading, since it suggests that position and momentum may be chosen independently.

In fact, the quantum potential has completely non-classical properties, which allow the adherents of this "causal viewpoint" to justify the uniqueness of quantum phenomena. They find for example that wavefunctions which differ only through a complex factor lead to the same quantum potential, since in U_{quant}, the wavefunction enters both into the denominator and the numerator. Here, Bohm and Hiley (1993, p. 31) introduce the concept of "active information", and they find in it the justification for a new kind of "holism" (see also Hiley 1999).

Although these two readings of the de Broglie–Bohm theory are *mathematically* equivalent, and the real contradiction between the usual quantum theory and these variants of the DBB theory holds for both of them, the rivalry of these schools is considerable. Hiley writes:

> It should be noted that the views expressed in our book (Bohm and Hiley 1993) differ very substantially from those of Dürr *et al.* (1992), who have developed an alternative theory. It was very unfortunately that they chose the term 'Bohmian mechanics' to describe their work. When Bohm first saw the term he remarked, 'Why do they call it 'Bohmian mechanics'? Have they not understood a thing that I have written'? He was referring [...] to a footnote in his book *Quantum Theory*, in which he writes, 'This means that the term 'quantum mechanics' is a misnomer. It should, perhaps, be better called quantum nonmechanics.' It would have

> been far better if Dürr *et al.* had chosen the term 'Bell mechanics'. That would have reflected the actual situation far more accurately. (Hiley 1999, p. 119)

The acrimony in this dispute is essentially due to the fact that the "ontological interpretation" associates far-reaching natural-philosophical speculations to its concept of the quantum potential, while supporters of "Bohmian mechanics" see the strength of the theory in its being able to *eliminate* philosophical speculations from the formulation of the theory. Characteristically, the title of an article by Dürr and Lazarovici (2012) is "Quantum physics without quantum philosophy".

5.1.7 Criticism of the de Broglie–Bohm Theory

John Bell, who, beginning in the 1960s contributed to the popularization of the de Broglie–Bohm theory with a number of articles, writes concerning the topic of this section:

> It is easy to find good reasons for disliking the de Broglie–Bohm picture. Neither de Broglie nor Bohm liked it very much; for both of them, it was only a point of departure. Einstein also did not like it very much. He found it 'too cheap', although, as Born remarked, 'it was quite in line with his own ideas'. But like it or lump it, it is perfectly conclusive as a counter example to the idea that vagueness, subjectivity, or indeterminism are forced on us by the experimental facts covered by nonrelativistic quantum mechanics (Bell 2001, p. 152).

According to Bell, all the counter arguments cannot reduce the importance of the theory in principle. Nevertheless, in the following we will take a closer look at some of those arguments. Heisenberg expresses the opinion that the identical descriptive content of the theory (relative to standard quantum mechanics) disqualifies it as an independent theory. He writes (Heisenberg 1959, p. 106):

> From a strictly positivistic point of view, one could even say that we are dealing here not with an alternative suggestion to the Copenhagen interpretation, but instead with an exact repetition of it, only with different terminology.

In the face of the conceptional differences between the de Broglie–Bohm theory and the usual quantum theory, this statement would seem to be overly strong. Furthermore, Heisenberg naturally presumes here that the Copenhagen interpretation offers a convincing solution to the measurement problem. Closely related are references to "Ockham's razor". According to the prevailing opinion, when two theories are equivalent, the one which requires the lesser number of premises should be preferred. Does Ockham's razor thus "cut off" the guidance equation as superfluous ballast from a theory which offers no additional predictions? This demand fails to notice that the additional equation in the de Broglie–Bohm theory renders unnecessary all of the postulates concerning the outcome of a measurement and the interpretation of the wavefunction.

While the previous arguments take the considerable *similarity* of the DBB theory to the quantum theory as an object of criticism, others see the reason to repudiate the

DBB theory in their radical *dissimilarity*. They find fault with the explicit distinction of the position, and the lack of symmetry between, e.g., the position and the momentum spaces (see the objection of Pauli in Myrvold 2003). In the DBB theory, with its first-order equation of motion, momentum and energy at the level of individual particles are however no longer conserved quantities. The justification of the demand for symmetry between position and momentum can thus be reasonably questioned.

Still other critics are bothered by the (double) role of the wavefunction: It fixes the particle dynamics and is at the same time (i.e. its absolute square) a measure for the equilibrium distribution. In addition, it acts upon the particle's motion *without* any back-reaction effects. Another point of criticism refers to the fact that according to the de Broglie–Bohm theory, the world is populated by innumerable "empty" wavefunctions. This is indeed somewhat inelegant.

The role or the status of the wavefunction is also the object of a discussion among those scientists who work with the DBB theory. Originally, the wavefunction was taken to represent a real, physical field. Dürr *et al.* (1996) suggest, in contrast, that it should play a "nomological" role (i.e. with a law-like character). The wavefunction would then more closely correspond to, e.g., the Hamilton function in classical mechanics than to the usual type of physical field. This could reduce the weight of the criticisms of the lack of reaction effects and of the "empty" wavefunctions. While the interested reader can find a more detailed discussion of the criticisms of the DBB theory in Passon (Passon 2010, pp. 117ff), we will now turn to the principal objection against it: The question of the possibility of a *relativistic generalization* of the theory.

The particle dynamics of the de Broglie–Bohm theory connects positions at arbitrary distances. This non-locality would appear to violate Einstein's postulate of the speed of light as an upper limiting velocity. However, the DBB theory discussed so far is an extension of non-relativistic quantum mechanics. The allusion to the fact that it is not compatible with the requirements of the special theory of relativity is thus not really a criticism, but rather simply a statement of fact. This rejoinder is however too superficial, since it is indeed just the non-local dynamics which allows the de Broglie–Bohm theory to explain the violation of the Bell inequalities (cf. Chap. 4, and there in particular Sect. 4.4).

As a rule, the criticism of the non-locality of the DBB theory is primarily associated with doubts as to whether or not the theory can be relativistically *generalized*. At the same time, there is an orthodox relativistic quantum theory (the Dirac theory) and a relativistic quantum field theory (see Chap. 6), so that the final (and negative) judgment about the DBB theory seems to be passed. However, this argument would be significantly more convincing if those (orthodox) relativistic theories had *no* measurement problem. But there, also, e.g. the question of definite measurement outcomes is controversially debated.

Thus, the development of a "Bohm-like" relativistic quantum theory (and quantum field theory) without the foundational problems of the conventional formulation is part of the current research program of scientists who work in this field. Some of the approaches discussed there apply notably not *particle ontologies*, but instead *field* *ontologies*. Furthermore, some of the relativistic generalizations dispense even with a deterministic description.[21]

It turns out that not only the dynamics of a generalized guidance equation pose a problem for a relativistic generalization, but also the (generalized) quantum equilibrium distribution. This requirement distinguishes a frame of reference, namely that in which the distribution is determined. The equivalence of all inertial systems is, however, at the very core of special relativity, according to the usual understanding. There are some approaches in which the "distinguished" frame of reference has no experimentally accessible influence and which can reproduce all the predictions of relativistic quantum theory. A new evaluation of the relationship between quantum theory and relativity is however certainly bound up in such approaches. The supporters of the DBB theory recall in this connection quite rightly that (as mentioned above) it is precisely this non-locality, as expressed in the violation of the Bell inequalities, which is also a part of conventional quantum mechanics and quantum field theory. Therefore, from the viewpoint of many adherents of the DBB theory, the conventional interpretations of quantum mechanics and quantum field theory likewise have this same problem, but they mask it by their vague formulations (cf. also Bricmont 2016, pp. 169–173) for more on this topic).

We shall now leave the de Broglie–Bohm theory for a while and turn to Everett's work, another controversially discussed interpretation of quantum theory. In Sect. 5.3, we will then come back to the DBB theory within the framework of a comparison between various interpretation approaches.

5.2 Everett's Interpretation

In 1957, the American physicist Hugh Everett III (1930–1982) published his "*relative state*" formulation of quantum mechanics (see Everett 1957). It contains the results of his doctoral thesis, mentored by John A. Wheeler at Princeton University. Its goal was a re-formulation of the theory, in which the discontinuous change of state ("collapse") would be superfluous, and instead, a unitary time evolution of the wavefunction would hold throughout. In contrast to the de Broglie–Bohm theory, however, the *completeness* of the quantum-mechanical description is asserted, and thus the third statement of Maudlin's trilemma (Sect. 2.3.1) is denied: Measurements *appear* to have only one definite outcome in Everett's approach, although in fact the wavefunction (with its superposition states) contains a complete description.

Everett's guiding concept was to derive the interpretation from the mathematical formalism.[22] He was motivated explicitly by the measurement problem, or by the distinction of an *external* observer in the usual formulation:

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[21]The existing approaches and attempts to find a Lorentz-invariant generalization of the DBB theory are discussed in Passon (2006) and in Tumulka (2007).

[22]Everett himself writes of his methodology: "The wavefunction is taken as the basic physical entity with no *a priori* interpretation. Interpretation comes only *after* an investigation of the logical

> No way is evident to apply the conventional formulation of quantum mechanics to a system that is not subject to *external* observation. The whole interpretive scheme of that formalism rests upon the notion of external observation (Everett 1957, p. 455).

But at the latest when considering cosmological problems, the standpoint of an *external* observer can no longer be reasonably assumed, and the applicability of the quantum theory would appear to be frustrated by that fact.

Along with the justification for how—in view of the superposition states—the *appearance* of definite measurement outcomes is produced, Everett must furthermore explain how and why the statistics of those measurement results follows Born's rule (i.e. $|c_i|^2$ corresponds to the probability of occurrence of the given outcome). Now, Everett's work has itself become the object of an interpretation debate. Jeffrey Barrett writes on this subject:

> The fact that most no-collapse theories have at one time or another been attributed to Everett shows how much the no-collapse tradition owes to him, but it also shows how hard it is to say what he actually had in mind (Barrett 1999, pp. 90f).

In the following, we will trace roughly how the open technical and conceptional questions relating to the 1957 article have led to the development of variants and modifications. We begin however with a description of Everett's basic idea.

5.2.1 The Basic Idea

Everett's re-interpretation of the measurement problem is just as surprising as it is brilliant. This problem results, as is well known, from the application of the quantum theory to the measurement process. In general, a superposition state results from this process, consisting of, e.g., *different* "pointer states", while our experience tells us that measurements lead to *unique* results. Superposition states appear under these circumstances not to give an appropriate description of the physical situation. The drastic consequences which result ("either the Schrödinger equation is false or it is not complete" (Bell 2001, p. 173)) are avoided by Everett with the aid of the following consideration: Under the premise that the quantum theory is applicable also to the observation process, the observer must therefore also enter into a "superposition state"—and *this* superposition undermines the reliability of the judgement that caused us to doubt the appropriateness of superposition states as a description in the first place! Instead, Everett suggests that we identify *every* term of the superposition with an (equally weighted) state of the observer.[23] The evolution of measurements (or observations) can then be described as follows:

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structure of the theory. Here as always the theory itself sets the framework for its interpretation" (Everett 1957, p. 455).

[23] He models the "observer" by a physical system, in the concrete case a machine which has access to sensors and storage media.

> We thus arrive at the following picture: Throughout all of a sequence of observation processes there is only one physical system representing the observer, yet there is no single unique *state* of the observer (...). Nevertheless, there is a representation in terms of a *superposition*, each element of which contains a definite observer state and a corresponding system state. Thus with each succeeding observation (or interaction), the observer state 'branches' into a number of different states. (...) All branches exist simultaneously in the superposition after any given sequence of observations (Everett 1957, p. 459).

In which sense Everett can still consider just *one* observer ("one physical system representing the observer"), who is simultaneously in the *multiplicity* of states as described, is initially unclear. The various different answers to this question lead essentially to the different variants of the Everett interpretation which were mentioned in the above quote from Barrett.

5.2.2 The Many-Worlds Interpretation

Bryce DeWitt and Neil Graham (1973) popularized the Everett theory through their anthology "*The Many-Worlds Interpretation of Quantum Mechanics*" and coined the catchy name with their choice for its title. They interpret the branching of the wavefunction mentioned in the Everett quote above in a completely realistic manner, as a real splitting into different "worlds", and write[24]:

> The universe is constantly splitting into a stupendous number of branches, all resulting from the measurement-like interactions between its myriads of components. Moreover, any quantum transition taking place on every star, in every galaxy, in every remote corner of the universe is splitting our local world on earth into myriads of copies of itself (DeWitt and Graham 1973, p. 161).

"World" means here the totality of all the (macroscopic) objects, and the human observer likewise is subject to this splitting into a manifold of "copies".

David Wallace (Wallace 2010, p. 4) illustrates this astounding idea by means of an analogy with classical electrodynamics. Imagine an electromagnetic configuration $F_1(r,t)$ which describes a pulse of light that is propagating from the Earth to the Moon. A second configuration $F_2(r,t)$ could describe a light pulse underway from Venus to Mars. How, asks Wallace, should one now interpret the configuration

[24]It is very questionable as to what extent this suggestion corresponds to Everett's own understanding of the theory. Since Everett worked in the strategic planning department of the Pentagon after finishing his doctorate, and published no more work on quantum theory, this question can be answered only by consulting his sporadic correspondence and papers from his estate. These sources give the impression that Everett *did not* have a splitting up into different "worlds" in mind, whose definition would seem to make a connection with classical concepts. In some respects, the current version of the many-worlds interpretation, which we will discuss more detail in the following sections, appears to be more similar to Everett's original conception. However, he did not categorically reject the language of DeWitt—especially since he was very grateful to the latter for the popularization of his ideas. See Barrett (2011), and the essay by Peter Byrne in Saunders *et al.* (2010), for more on this subject.

$$F(r,t) = \frac{1}{2} \cdot F_1(r,t) + \frac{1}{2} \cdot F_2(r,t)\ ? \tag{5.10}$$

Does it describe a light pulse which is moving *simultaneously* between the Earth and the Moon as well as between Venus and Mars, since it occurs as a superposition? This is of course nonsense; instead, Eq. (5.10) does not describe a "strange" light pulse in a superposition state, but rather two "ordinary" light pulses at different locations. Wallace continues:

> And this, in a nutshell, is what the Everett interpretation claims about macroscopic quantum superpositions: they are just states of the world in which more than one macroscopically definite thing is happening at once. Macroscopic superpositions do not describe indefiniteness, they describe multiplicity (Wallace 2010, p. 5).

Here, however, we are not dealing with a spatial separation (as in the example from electrodynamics), but instead—as Wallace expresses it—with a *dynamic* separation. This means that the parallel worlds have no mutual interactions, i.e. described pictorially, they are "transparent" to one another. The innumerable "worlds" are located unperturbed in the same, single spacetime region.[25]

The interpretation of Everett's construction given by DeWitt and Graham has entered into the popular-scientific literature and has since ignited the fantasy of (not only) laypersons interested in physics and science fiction authors. In an obvious sense, the measurement problem is resolved by this construction, since in each "world", an eigenstate of the measurement apparatus in fact exists. Whether this condition suffices for a complete resolution of the measurement problem is however questioned by Maudlin (2010). In Sect. 5.2.6, we will discuss this criticism of Everett's interpretation. The situation regarding the question of non-locality is similar: While Bacciagaluppi (2002) supports the view that the violation of Bell's inequality (see Chap. 4) can be explained here *without* action at-a-distance, Allori *et al.* (2011) argue that the many-worlds interpretation produces this appearance of locality only because of its imprecise formulation. In Allori *et al.* (2011), a modification of the many-worlds interpretation is suggested, which likewise contains action at-a-distance (cf. Sect. 5.2.6).

In the version that we have thus far sketched, the theory however does not appear to be complete. Leslie Ballentine has pointed out that the meaning of probability statements within the Everett interpretation is unclear. Finally, all possible events do actually occur (see Ballentine 1971, pp. 233–235). Furthermore, the branching is subject to an ambiguity with respect to the choice of basis. This problem of the "preferred" basis will be discussed first, in the following section.

[25] This spacetime is subject to splitting only when the many-worlds idea is applied to theories of quantum gravitation.

5.2.3 *The Problem of the Preferred Basis*

Let us consider a typical example of the superposition of various spin states (e.g. those of a silver atom): $|\Psi\rangle = \frac{1}{\sqrt{2}}\,(|\uparrow_x\rangle + |\downarrow_x\rangle)$. If one wishes to determine the orientation of the spin along the x direction, one would investigate this state using a correspondingly oriented Stern–Gerlach magnet. At the end of the measurement, the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_x\rangle|M_{\uparrow_x}\rangle + |\downarrow_x\rangle|M_{\downarrow_x}\rangle\right) \tag{5.11}$$

is present. This state thus describes—according to the many-worlds interpretation—two "worlds", in which the x component of the spin is either $\uparrow_x$ or $\downarrow_x$. The decomposition into basis vectors is however in general not unique and could be just as well carried out with eigenvectors with respect to some other measurable quantity. For example, the following linear combination could be considered[26]:

$$\begin{aligned} |\uparrow_z\rangle &= \frac{1}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle) \qquad & |\downarrow_z\rangle &= \frac{1}{\sqrt{2}}(|\uparrow_x\rangle - |\downarrow_x\rangle) \\ |M_{\uparrow_z}\rangle &= \frac{1}{\sqrt{2}}(|M_{\uparrow_x}\rangle + |M_{\downarrow_x}\rangle) \qquad & |M_{\downarrow_z}\rangle &= \frac{1}{\sqrt{2}}(|M_{\uparrow_x}\rangle - |M_{\downarrow_x}\rangle)\,. \end{aligned}$$

With respect to this basis, the state (5.11) now has the following representation:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_z\rangle|M_{\uparrow_z}\rangle + |\downarrow_z\rangle|M_{\downarrow_z}\rangle\right)\,. \tag{5.12}$$

If the two "worlds" branch in terms of *these* basis vectors, the spin along the x direction would not have a well-defined value, and instead, its z component[27] would be well defined. The choice of a basis within the quantum theory is to be sure purely conventional and should have no physical relevance. A factual difference between the representations in (5.11) and (5.12) must therefore be separately justified. In other words, the choice of a "preferred basis" is necessary. One might object at this point that the choice of a specific measurement setup leads to precisely such a distinction of the pointer basis (5.11). In the other basis (5.12), in contrast, in every term there is a superposition of the various states of the x measurement apparatus. The non-occurrence (or rather the non-observability) of superpositions of macroscopically different states was however just what we were trying to explain with the Everett interpretation—it should thus not be a *precondition* of the investigation. Furthermore, such a distinction of a particular basis for the measurement process contradicts the

[26] The ambiguity of the representation is the subject of the "biorthogonal decomposition theorem" (cf. Bub 1997, p. 151). The decomposition is unique if and only if all the components have different and nonzero coefficients.

[27] Note that there are no *common* eigenvectors of σ_x and σ_z.

spirit of an interpretation which merely wishes to let the formalism remain valid and in which the observation plays no fundamental role.[28]

In today's view, the suggestions for solving this problem fall into two classes: The older ones, which do not refer to decoherence, and those which make use of the mechanism of decoherence. A brief treatment of those Everett variants which are currently considered to be obsolete since the advent of the approaches based on decoherence is desirable with a view to the discussion of the concept of probability (Sect. 5.2.5). We will therefore first cast a brief glance at these older approaches before considering the role of decoherence theory in Sect. 5.2.4.

David Deutsch's Variant of Everett's Interpretation

David Deutsch, in his early works, suggested a mechanism for distinguishing a basis (cf. Deutsch 1985).[29] He extends the quantum-theoretical formalism in terms of an algorithm which produces the corresponding basis. This depends (without going into the details here) only on the corresponding physical state and its dynamics. The choice is limited by the requirement that in the case of "measurements", the relevant basis in fact corresponds to the "pointer basis". This guarantees that after a measurement, a unique result is in fact obtained (Deutsch 1985, pp. 22f).

Wallace (2010, p. 7) calls this variant of the Everett interpretation the "many-exact-worlds" interpretation. In Sect. 5.2.4, we will see that in the meantime, there are more promising candidates for the solution of the problem of the preferred basis, and David Deutsch himself has also rejected this interpretation since the end of the 1990s. First, however, we will consider yet another variant.

The Many-Minds Interpretation

The many-worlds interpretation includes the act of observation within the physical description. This apparently presumes that mental states are also a part of the physical world and are subject to the laws of quantum theory.[30]

In this sense, a many-worlds interpretation *appears* to always imply a theory of branching consciousness states (the exception will be discussed below). This evident significance is however *not* meant when one speaks of the *many-minds* interpretation.

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[28]The problem treated here thus occurs in other interpretations of quantum mechanics as well, and it shows that the measurement problem actually consists of two sub-problems: (i) The problem of the preferred basis and (ii) the problem of the definite outcome of a measurement. Within, e.g., the Copenhagen interpretation, however, (i) can be resolved by specifying the measurement setup (choice of direction).

[29]Deutsch mentions here (on p. 2) that he has taken up an idea of Everett's, based on private conversations with him.

[30]This position is called "physicalism". Physicalism (expressed in simplified form) asserts the metaphysical hypothesis that everything which exists is *physical*. It can be understood as a further development of materialism. In particular, it rejects any kind of dualism between physical and mental ("mind") states. The relation between physical and mental states is not necessarily an identity, however. In the philosophy of the mind, the viewpoint is widespread that these two property areas are connected through a "supervenience relation". The supervenience of A over B is understood to mean that (in "slogan" form) "no change in A is possible without a change in B". This also permits speculations on a possibly non-reductionistic physicalism.

A prominent suggestion of this variant is due to Albert and Loewer (1988). They were motivated by the problem of the preferred basis, as well as the difficulty of understanding the significance of probability statements within the many-worlds interpretation (this problem will be treated in more detail in Sect. 5.2.5).

The point of departure of the *many-minds* interpretation is the statement that mental states can never be in superpositions, according to our introspective experience. Loewer and Albert reason from this that mental states (i.e. beliefs, intentions, memories, etc.) are *not* physical.[31]

They then postulate that every observer is outfitted with an infinite number of "*minds*". While in the case of a measurement or an interaction, the physical brain states take on a superposition state, a probabilistic time evolution leads to a state in which a certain portion of these *minds* corresponds in each case to the perception of *one single* outcome for the experiment. This process takes place within *one* world.

Now, how does this interpretation deal with the problem of the "preferred basis"? In an evident sense, the choice of basis vectors for the evolution of a state has no physical significance, since in the *many-minds* interpretation, there is only *one* world. An ambiguity with respect to the splitting into "many worlds" thus cannot arise here. However, Barrett (1999, p. 195) has pointed out that the "basis" of the consciousness states plays a comparable role.[32]

Both in the *many-minds* interpretation and also in the interpretation of Deutsch (1985), a preferred basis must thus be postulated. This common strategy is accompanied by a common difficulty: All attempts to introduce a preferred basis *ad hoc* must *postulate* properties which should in fact be *explained* in a fundamental theory (cf. Wallace 2010, p. 8). In the next section, we will treat the theory of decoherence. With it, one associates the hope that a convincing solution of the problem of the preferred basis can be found, since it does without such *ad hoc* assumptions.

5.2.4 The Role of Decoherence Theory

As a rule, physics investigates "isolated systems", i.e. it considers the influence of the "environment" to be a negligible perturbation and, above all, an unnecessary complication. We now find that within the quantum theory, precisely the *inclusion* of the interactions with the environment can lead to conceptional progress in describing measurements, as well as the classical limits of the theory. The research which has been accomplished since the early 1970s in this field has not been associated with any particular interpretation of quantum theory and makes use simply of the mathematical properties of the standard formalism. Pioneers in this field of

[31]One may consider the astute self-observation on which this conclusion is based not to be a particularly powerful tool for philosophical reflection. But for questions involving our conscious minds, it is however our *only* tool!

[32]Just how Barrett means this for a non-physicalistic conception of the mind remains unclear.

"decoherence"[33] were Zeh (1970) and Zurek (1981). Already in Sect. 2.3, we have discussed the decoherence programme. We make use here of the concepts introduced there, amplify them, and codify the results within the context of the Everett interpretation.

In Sect. 5.2.3, we have already explained how the decomposition of a state into its basis vectors can be ambiguous. The decompositions (5.11) and (5.12) are mathematically equivalent—their physical differences must therefore be justified.

The first step towards the resolution of this problem can now be accomplished through a purely mathematical consideration: If we look at the entanglement with a *third* system E (the *environment*, in our example likewise represented by a two-dimensional Hilbert space with states $|e_i\rangle$), we will be led to a state of the form

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\uparrow_x\rangle|M_{\uparrow_x}\rangle|e_{\uparrow_x}\rangle + \frac{1}{\sqrt{2}}|\downarrow_x\rangle|M_{\downarrow_x}\rangle|e_{\downarrow_x}\rangle\,. \tag{5.13}$$

Andrew Elby and Jeffrey Bub (1994) were able to show that this decomposition into orthogonal states on a *triple* product space is *unique*.[34] It thus eliminates the ambiguity in the choice of a basis, in a formal sense (and also that of the associated physical measurand). Naturally, this purely mathematical argument as yet yields no indication of *which* basis is to be distinguished—especially since the extremely detailed states of the environment are unobservable. In this situation, physical criteria for the identification of this unique ("preferred") basis must still be developed, as Schlosshauer mentions:

> The decoherence programme has attempted to define such a criterion based on the interaction with the environment and the idea of robustness and preservation of correlations. The environment thus plays a double role in suggesting a solution to the preferred-basis problem: it selects a preferred pointer basis, and it guarantees its uniqueness via the tridecompositional uniqueness theorem (Schlosshauer 2005, p. 1279).

These criteria were thus not *postulated*, but rather they follow from the quantum-theoretical investigation of the dynamic influence of the environment. For this purpose, one treats complicated models of the environment. The interactions between it and the measurement apparatus take place as a rule via force laws which contain powers of the spatial distance (e.g. the Coulomb force $\propto r^{-2}$). It follows that the unique decomposition as a rule distinguishes the basis of *positional space*, and in the case of a measurement, the "pointer basis" is the relevant basis. Schlosshauer summarizes this approach, called *environment-induced superselection*, as follows:

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[33]The adjective "coherent" in the general vocabulary means "connected". The physical-terminological expression "coherent" is usually applied to optics, and it describes, roughly speaking, the precondition that must be fulfilled by different wavetrains in order that they may be able to interfere with each other. Expressed non-technically, the decoherence program thus attempts to clarify the conditions and preconditions under which quantum states lose this "non-classical" property.

[34]This *tridecompositional uniqueness theorem* is valid under rather general conditions. The existence of the decomposition is by the way not guaranteed. The proof of this theorem can also be found in (Bub 1997, Sect. 5.5).

> The clear merit of the approach of environment-induced superselection lies in the fact that the preferred basis is not chosen in an *ad hoc* manner simply to make our measurement records determinate or to match our experience of which physical quantities are usually perceived as determinate (for example, position). Instead the selection is motivated on physical, observer-free grounds, that is, through the system-environment interaction Hamiltonian. The vast space of possible quantum-mechanical superpositions is reduced so much because the laws governing physical interactions depend only on a few physical quantities (position, momentum, charge, and the like), and the fact that precisely these are the properties that appear determinate to us is explained by the dependence of the preferred basis on the form of the interaction. The appearance of *classicality* is therefore grounded in the structure of the physical laws – certainly a highly satisfying and reasonable approach (Schlosshauer 2005, pp. 14f).

This quote once again emphasizes that the results of decoherence are not tied to any particular interpretation of the quantum theory, i.e. that they can be applied within every interpretation.[35]

Since the interaction with the environment is described quantum-mechanically (i.e. via a unitary time evolution), the combination of the [object + measurement apparatus + environment] remains in a so-called pure state. This overall state will in general contain both a superposition of various "pointer positions" and also interference terms. The exact state of the environment is not only not susceptible to influences, but as a rule also not to observation. If one computes the predictions for the real observables in the subsystem [object + measurement apparatus], one obtains a result in which there are *practically* no more interference terms.[36] This part of the programme is termed the *environment-induced decoherence* and consists—in summary—in the fact that from a *coherent* superposition (a "pure state"), an *incoherent* (or "decoherent"—thus the name) superposition with respect to a uniquely defined basis emerges. Due to an apparent reason, this process *alone* does not constitute a solution to the measurement problem, for it still cannot explain *which* branch of this now decoherent superposition corresponds to the outcome of the measurement. In Footnote 28, the measurement problem was divided up into two sub-problems: (i) "Preferred basis" and (ii) "definite outcome". The decoherence theory thus merely solves the first sub-problem.

For the Everett interpretation, this question is naturally not relevant: In its context, the basis which is preferred in this manner *defines* the splitting into independent "worlds". These are however not "exact" (as, e.g., in the suggestion of Deutsch 1985), but are rather merely approximations. In the end, the preferred basis is approximately distinguished by a dynamic process.

According to David Wallace (2010, p. 11), since the mid-1990s there has been a broad consensus among physicists that the problem of the preferred basis has been solved by environment-induced decoherence. Only in some areas of the philosophy of science is there still criticism of the fact that the approximate dynamic process of

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[35]For the adherents of the de Broglie–Bohm theory, the results of decoherence, for example, permit a more exact justification of the so-called effective collapse of the wavefunction (cf. Sect. 5.1.5).

[36]Expressed technically, one takes the trace of the density matrix over the degrees of freedom of the environment. This makes it (in the preferred basis) *approximately* diagonal. The off-diagonal elements are however just what give rise to the interference effects.

decoherence is used to define objects which one then "takes seriously in the ontological sense". In Wallace's opinion, the quasi-classical branches of the wavefunction are "emergent structures", whose ontological status corresponds, for example, to that of the temperature in statistical mechanics (see the essay by Wallace in Saunders *et al.* 2010, p. 53).

The Everett interpretation has experienced a considerable revival through these results, since the decoherence-based modification is ontologically certainly less extravagant than the versions of DeWitt and Graham (1973), Deutsch (1985) or Albert and Loewer (1988).[37] The definition of the"worlds" is based here on a dynamic process which can be analysed using the methods of the standard formalism. Furthermore, this approach can be relativistically generalized in a manifest way. The significant open question which remains is that of the status of probability statements, to which we will now turn our attention.

5.2.5 Probability in Everett's Interpretation

Within the Copenhagen interpretation, if we consider a state $|\Psi\rangle = \sum_i c_i |\psi_i\rangle$, the square of the amplitude $|c_i|^2$ denotes the probability of obtaining the state $|\psi_i\rangle$ as the result of a measurement of the corresponding observable on the system $|\Psi\rangle$. In the de Broglie–Bohm theory, the same is true – but there, on the grounds that the configuration of the particle selects out this part of the wavefunction. In the GRW theory, finally, this is the probability that the dynamic collapse of the wavefunction of the measurement apparatus will lead to this state. In all of these cases, there are two preconditions for the practicable application of the probability concept: Various *possible* outcomes and the *lack of knowledge* of the actual result. Within the many-worlds interpretation, however, *all* of the results will occur *with certainty*. It thus initially appears unclear just what the probability statements could *refer to* in this connection (the "incoherence problem")—not to mention why *these* probabilities should correspond to $|c_i|^2$ (the "quantitative problem"). Precisely these two aspects (which are however closely related) are singled out in the discussion of the probability problem.

The status of probability statements within the Everett interpretation has led to a technically and conceptionally highly complex debate. Some of the important contributions to this discussion will be treated in the following. Here, again, it is seen that the advent of the decoherence theory marked a division point within the overall debate.

The Incoherence Problem

Naturally, the square of the amplitude $|c_i|^2$ still retains the mathematical properties within the Everett interpretation that qualified it to be a measure of probability (over

[37] Now and then at scientific conferences, surveys are conducted (not always with complete seriousness) about which interpretation of quantum theory is favoured by the conference attendees. Tegmark (1998) reports the result of such a survey at a workshop on quantum theory; it found that the Everett interpretation was the preferred alternative to the Copenhagen interpretation.

the set of all branchings). However, the c_i are just "branching amplitudes", and *every* branch claims the same reality in this interpretation. Both Everett, as well as later DeWitt and Graham, appear to have appreciated this difference insufficiently, since they claimed that they could even derive Born's rule:

> The conventional probability interpretation of quantum mechanics thus emerges from the formalism itself (DeWitt and Graham 1973, p. 163).

This claim is supported by DeWitt and Graham on the basis of the following mathematical result[38]: If one considers a series of N measurements of a superposition state with coefficients c_i, then in the limit $N \to \infty$, the state of the overall system (= N measurement apparatus + N systems) converges towards an eigenstate of the so-called relative frequency operator for the measured value i. This operator measures—as its name implies—just the relative frequency with which the experiment yields the outcome i. The associated eigenvalue is then indeed given by $|c_i|^2$. However, to see a proof of Born's rule in this fact is a failure to recognize that in real experiments, the value of N must always remain finite, and therefore, branches occur with *statistical deviations*. Now, one can justifiably expect that their squared amplitudes remain "small". The assertion that these events thus also occur with small *probabilities* is however correct only if the squares of the branching amplitudes are indeed identifiable with probabilities. This however renders the argument circular, for just this identification is what was supposed to be *justified* (cf. Barrett 1999, p. 163; Deutsch 1985, p. 20; or Ballentine 1971, p. 234).

A genuine solution to the incoherence problem was suggested by David Deutsch in the same article in which he also treated the question of the preferred basis. It is based on the intuition that the *most probable* outcome should also be the *most frequent*. While with DeWitt, *individual* worlds branch off, Deutsch postulates an (uncountable) infinity of identical copies of the *same* world (see Axiom 8 in Deutsch 1985, p. 20). In the case of a measurement (with i possible measured values), a relative fraction p_i branches off into worlds with the corresponding experimental outcome. This fraction then corresponds to the probability of occurrence of the event i (in "my" world). Deutsch thus solves the incoherence problem by means of an extension of the "ontology" of the theory.

The *many-minds* interpretation of Albert and Loewer (1988) proceeds identically in a structural sense. As we have seen, there also, each observer state is associated with infinitely many *minds*. In the case of a measurement (with i possible measured values), these are supposed to assume the "consciousness content" that "the event has occurred", likewise with the fractional weight p_i.

If we now set this fraction p_i of the *minds* or the *worlds* (Deutsch), respectively, equal to the squared amplitude $|c_i|^2$, we also obtain an (*ad hoc*) solution to the quantitative problem.[39]

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[38]This theorem was discovered by Neil Graham in 1970, during his doctoral work which was mentored by DeWitt. Already in 1968, James Hartle had proved an equivalent result (Hartle 1968).

[39]The *many-minds* interpretation buys the solution to the probability problem at the price of a substance dualism, which is accepted in modern philosophy of the mind by only a small minority

These two suggestions are of course based on special solutions of the problem of the preferred basis (cf. Sect. 5.2.3), which at latest since the advent of decoherence-based approaches is regarded as obsolete. We thus find here the curious situation that precisely the most convincing solution (in the eyes of many physicists) to the problem of the preferred basis leads to the result that the concept of probability once again appears to be a "foreign body" within the Everett interpretation.

Now, there exist various approaches which—as escape routes out of this dilemma—attempt to justify a concept of "uncertainty" or "indeterminacy" within the Everett interpretation. This concept appears to many authors to be a necessary condition for making it possible that probability statements can reasonably be made at all.

Vaidman (1998) undertook such an attempt. He considers a measurement whose possible outcomes are denoted as A and B. It is true, maintains Vaidman, that in the world A, the probability for the occurrence of the outcome could be trivially $A = 1$; however, it could also be that an experimenter *in* the world A might have no knowledge of this circumstance—for example as long as that observer in world A had not yet read off the result from the measurement apparatus. See also Vaidman *et al.* (2008)

Whether this type of "lack of knowledge" suffices to give the concepts of "probability" and "chance" a reasonable meaning however remains unclear. David Albert (see Albert 2010, pp. 367f) objects that this uncertainty is on the one-hand avoidable, and on the other, it occurs only *after* the experiment has been carried out.

Simon Saunders has developed a stronger version of this "subjective indeterminacy", which according to its claims can also be applied to situations *before* a measurement has been carried out. He argues that the branching into different worlds occurs in a way that is subjectively indeterministic. On the basis of a specific definition of "personal identity", Saunders sees in every "copy" of the observer a future self of the original observer. In this sense, the person should experience uncertainty *before* a measurement as to which person he or she will become *after* the measurement (cf. Saunders 1998). Another justification of subjective indeterminacy in the Everett interpretation is due to David Wallace, who makes the semantics of probability statements his starting point (cf. Wallace 2005). These results however remain the objects of a controversial debate (see, e.g., Greaves 2004 for a criticism of these positions). At the end of the next subsection, we will meet up with one more suggestion for treating the incoherence problem.

The Quantitative Problem

Let us postpone for the moment the incoherence problem and turn to the question of why probability statements within the Everett interpretation should obey Born's rule in particular. The thought offers itself that when a splitting into N worlds has occurred, each branch should be associated with the *same* probability, $\frac{1}{N}$. After all,

of philosophers. This problem motivated Lockwood (1996) to suggest a variant of the *many-minds* interpretation, which dispenses with dualism and a probabilistic dynamics. Ironically, it is however controversial as to whether or not Lockwood's theory permits a plausible probability interpretation at all (see Barrett 1999, pp. 206–211).

their "reality" is alleged to be equivalent. This strategy is however not allowed in the decoherence-based Everett interpretation, since no counting arguments can be utilized for the dynamically and only approximately defined worlds.

Some authors however challenge the justification of demanding a positive substantiation of Born's rule in the Everett interpretation (see Saunders 1998, p. 384, as well as the contribution by Papineau in Saunders *et al.* 2010). Born's rule should be able to be postulated here just as in conventional quantum theory (and like analogous propositions in other theories)—the status of probability statements would then be just as secure (or insecure) as in other areas of physics.

A completely new twist was given to this discussion by the publication of Deutsch (1999) (later, his approach was rendered more precise by Wallace (2003)). In this article, David Deutsch transferred the methods and results of decision theory to a quantum-theoretical context, and even claimed that he could derive Born's rule.

The (classical) decision theory models decision processes which are carried out by "rational agents" in uncertain situations. Probabilities are thus construed here as functional, namely as factors which guide behaviour. The fundamental concepts of this theory are "states of the world" ($s_i \in \mathcal{S}$), "actions" ($A, B, \ldots$), their "consequences" ($\mathcal{C}$), as well as "preferences", which an agent ascribes to possible actions. These preferences define an ordering within the set of actions: $A \geq B \geq C \cdots$ (in words: "action A is preferred relative to B; both are preferred over C, etc.").

Formally, actions are mappings between the states of the world and the consequences ($A(s) \in \mathcal{C}$). The agent considered has only incomplete knowledge of the actual state of the world—and therefore of the consequences of his or her actions. Decision theory can now prove the so-called representation theorem: If the preferences for actions are subject to so-called *rationality conditions*,[40] those preferences can be expressed in terms of a unique utility function U for the consequences, as well as a probability measure p for the states:

$$EU(A) = \sum_{s_i \in \mathcal{S}} p(s_i) \cdot U(A(s_i))\,. \tag{5.14}$$

In this expression, $EU(A)$ stands for the *expected utility* of the action, and the preference of action A over action B as chosen by the agent is translated into the condition $EU(A) > EU(B)$. Greaves summarizes this relation as follows:

> This result guarantees an operational role for subjective probability: any rational agent will (at least) act as if she is maximizing expected utility with respect to some probability measure (Greaves 2007, p. 113).

These relations are often illustrated in an economic context, for example, as the rational behaviour for choosing how to bet a sum of money in a wager.

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[40] The concept of "rationality" is used here in a very narrow or weak sense. Decision theory investigates logical limitations of the preferences and makes no claim to determine them with regard to content. A typical rationality requirement is the transitivity of preferences: If I prefer action A over action B, and B over C, then A must be preferred over C.

David Deutsch and David Wallace were able to prove an analogous result for the Everett interpretation by utilizing the following correspondences: "states of the world" correspond to the number of branchings after carrying out a particular measurement; "actions" correspond to the wagers on particular measurement outcomes (in a "quantum game"); and "consequences" correspond to the winnings (or losses) in the case that a certain *single* event occurs. Making use of analogous "rationality conditions", it was then possible to prove a representation theorem like that in Eq. (5.14). For the probability measure, one finds just the squares of the amplitudes, $p_i = |c_i|^2$. The rational agent will thus behave in such a way *as if* the multiple branchings represented alternatives whose occurrence frequency is given by Born's probability rule.[41]

In the eyes of the supporters of this position, the probability concept is even better accommodated within the Everett interpretation than in all other physical theories. Instead of posing a special problem, the role of probability would now even represent a strong argument *in favour* of the many-worlds interpretation.

This result however by no means ended the discussion, since there is no unanimity over the question of how conclusively the *premises* for the proof can be justified. Some authors doubt that in fact only *non-probabilistic* parts of decision and quantum theories enter into the proof. That would of course invalidate the alleged *proof* of the probability rule (cf. Hemmo and Pitowsky 2007).

Likewise problematic is the fact that decision theory investigates actions "in uncertain situations". Its applicability thus depends again on the question of whether "uncertainty" is present or not in the Everett interpretation (or whether its subjective appearance can be conclusively justified). This is at its core of course simply once again the incoherence problem of the previous section. Here, Hilary Greaves now takes a radical position: She admits freely that genuine probability and subjective uncertainty indeed have no place within the Everett interpretation. She adopts the position (Greaves 2004) that in the framework of the decision-theoretical programme, this is not at all necessary, and she argues that the rationality conditions can also be justified in the context of (deterministically) branching worlds. The associated measure $p(s_i)$ cannot however be reasonably called "probability" here. Greaves suggests instead the term *caring measure* and describes its meaning as follows:

> We might instead call it the agent's 'caring measure', since the measure quantifies the extent to which (for decision-making purposes) the agent cares about what happens on any given branch (Greaves 2007, p. 118).

The rational agent thus behaves in such a way that the expected utility is maximized over all the branches of the wavefunction with respect to the $|c_i|^2$ measure, *because* he or she knows that all the results will in fact occur.

A further objection to the decision-theoretical justification of probability relates to the fact that this programme *presupposes* that the rational agent accepts the validity

[41] Therefore, the Everett variant of the representation theorem makes an even stronger statement than its counterpart in classical decision theory. The latter determines the probability measure only relative to the corresponding preferences of the agent. There are however several such preferences which fulfil the rationality conditions!

of the Everett interpretation. Which arguments speak for it? This question refers to the so-called *evidence problem* of the Everett interpretation, that is, the question of how a verification of this theory could be obtained from measurements. Within the Everett interpretation, a series of measurements leads to a splitting into branches of the wavefunction which correspond to every *arbitrary* statistical distribution of the measured values. The occurrence of a distribution which deviates strongly from the Born prediction would thus not represent a reason for doubting the quantum theory; instead, it would be rather expected. A suggestion for solving this problem was made by Greaves and Myrvold (see Saunders *et al.* 2010, pp. 264ff). According to those authors, the decision-theoretically justified "branching weights"[42] would likewise play a confirmation-theoretical role.

5.2.6 Criticism of Everett's Interpretation

The problematic status of probability statements within the Everett interpretation was already discussed in the previous section. Let us now turn directly to another obvious objection to the interpretation, namely its extravagance. David Wallace notes at the end of his essay on the Everett interpretation:

> I have left undiscussed the often-unspoken, often-felt objection to the Everett interpretation: that it is simply unbelievable. This is because there is little to discuss: that a scientific theory is wildly unintuitive is no argument at all against it, as twentieth century physics proved time and again (Wallace 2010, p. 23).

Against this succinct remark, we could answer that the Everett interpretation carries its application of "scientific realism" further than other theories in modern physics. The scientific realist supports the view that the success of a scientific theory can best be explained by assuming that the objects and properties that it *postulates* do in fact *exist* (cf. Bartels 2007). This hypothesis thus refers expressly to not-directly-observable objects such as quarks or black holes.[43] The adherents of the Everett interpretation reason in precisely the sense that the branching of the wavefunction implies the existence of parallel worlds. This circumstance is described with notable accuracy by Ballentine:

> Rather than deny that a state vector can be a complete model of the real world, Everett and DeWitt choose to redefine 'the real world' so that a state vector [...] can be a model of it (Ballentine 1971, p. 232).

The modern (decoherence-based) approaches would seem however to have markedly improved the ontological status of the many-worlds interpretation. The almost arbitrary and unlimited multiplication of universes (or *minds*) within the earlier variant

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[42]The concept of "probability" is thus again avoided here.

[43]In fact, variants of scientific realism are also possible which attribute a valid claim of truthfulness to certain *theories*, while the *entities* in question are not considered to be realistic (see Russell's position in Hacking 1983, p. 27).

of David Deutsch, or the *many-minds* interpretation of Albert and Loewer, become superfluous in these newer approaches.

The solution of the measurement problem in the many-worlds interpretation is based on an additional strong metaphysical assumption: In order to eliminate an "external observer", the measurements and observations are referred to the worlds which are continually branching off. This presumes that the mental states of the observer can likewise be described quantum-theoretically.[44] This physicalism is indeed a widespread position, but it is controversial within the philosophy of the mind as to whether it provides a solid basis for explaining the qualia problem or the typical intentionality of mental states. Tying the solution of the measurement problem to this precondition would seem to be maladroit, at the very least.

A still much more fundamental criticism was expressed by Tim Maudlin (cf. Maudlin 2010). He doubts that the Everett interpretation in fact offers a solution to the measurement problem. According to the usual view (e.g. according to Maudlin 1995!), the measurement problem consists essentially in interpreting the superposition of macroscopically different states (i.e. different pointer positions, living and dead cats, etc). From this reading, a measurement on an *eigenstate* would be unproblematic. Let $|M_0\rangle$ be the state of a measurement apparatus before the measurement, and $|\psi_1\rangle$ the eigenstate of a system relative to the quantity which is to be measured by M. Then after its measurement, the overall state $|M_1\rangle|\psi_1\rangle$ is found. Maudlin now expresses doubt as to the alleged simplicity of this special case and poses the question of in which sense a state (e.g. $|M_1\rangle$) in a high-dimensional vector space can *at all* represent the well-defined *spatial* state of a measurement apparatus, e.g. "pointer at the position 1". He criticizes the usual manner of speaking, according to which the wavefunction is defined on the *configuration space*, since the "spatial configuration" of all the parts that are represented by a point within this configuration space is not at all a component of all the interpretations of the quantum theory. While the spatial configuration of all the parts on $\mathbb{R}^3$ is an explicit component of the description in the de Broglie–Bohm theory, in a "wavefunction-monistic" theory, in contrast, this concept can not even be referred to (see Maudlin 2010, pp. 126f). The adherents of the Everett interpretation (and the same applies to some variants of the GRW theory) thus, according to Maudlin, lack the resources that would be required to establish a connection to the localized objects of our four-dimensional spacetime:

> For if the result of a measurement consists in, say, a pointer pointing a certain way, and if a pointer is made of particles, then if there are no particles there is no pointer and hence no outcome. All of this talk of a wavepacket 'representing' an outcome is unfortunate: what the wavefunction monist has to defend is that the outcome just *is* the wavefunction taking a certain form (in some high-dimensional space) (Maudlin 2010, p. 130).

According to Maudlin, the technical discussions about the concept of probability within the Everett interpretation thus obscure a decisive point: The probabilities sought after must not only be probabilities for the occurrence of physical events, but also of the *right* physical events. "Right" refers here of course to the ability

[44] Albert and Loewer (1988) formulate, in contrast, a dualistic position in their *many-minds* interpretation.

to establish a connection to elements of our everyday physical world in the sense described above.

In fact, the work of Allori *et al.* (2011) provides a variant of the many-worlds interpretation which takes Maudlin's objection into account (although, curiously, Maudlin's work has not been cited). As we shall however see, this modification dispenses with the fundamental assumption of previous many-worlds interpretations that a physical system is to be described by the wavefunction *alone*. But since it is this *formal* simplicity which is emphasized by the supporters of the many-worlds interpretation as its principal distinctive characteristic, the Allori suggestion is naturally not considered from *their* perspective to be an alternative which can be taken seriously.[45]

Let us take a brief look at this work, which also makes use of an interesting scientific-historical allusion. At the beginning of the work of Allori *et al.* (2011), the original suggestion of Schrödinger (1926) of interpreting the wavefunction "realistically" is namely analysed. In that view, in the single-electron case, the charge density is given by the expression $e \cdot |\psi|^2$. For the many-electron case, Schrödinger formulates a prescription which involves integration over the additional coordinates in configuration space. Then, as is well known, the dynamics of the Schrödinger equation leads in general to a spreading of this charge density over a large region of space within a short time. This is also the reason why Schrödinger rapidly discarded this interpretation of the wavefunction.[46] Allori *et al.* however apparently find this step to have been premature, for while Schrödinger's idea indeed stands in contradiction to point-like charges within a "one-world" theory, it can be re-interpreted in an evident way in terms of a *many-worlds theory*. Instead of the charge density, Allori *et al.* notably make use of the mass density $m(x, z)$ for technical reasons (see Footnote 1 on p. 4 in their article):

$$m(x,t) = \sum_{i=1}^{N} m_i \int d^3x_1 \cdots d^3x_N \delta(x - x_i) |\psi(x_1, \cdots, x_N)|^2 . \qquad (5.15)$$

The mass density at a point x is thus obtained by integrating the probability density $|\psi|^2$ over all of the rest of the configuration space (this is quite analogous to the prescription of Schrödinger 1926). The many-worlds character of this theory is now obvious: If, for example, the wavefunction of Schrödinger's cat branches into the disjunctive parts ψ_{alive} and ψ_{dead}, then (5.15) will lead to *interaction-free* mass densities m_{alive} and m_{dead}. The objects described by these mass densities can, pictorially speaking, be considered to be "reciprocally transparent" (Allori *et al.* 2011, p. 7).

Allori *et al.* refer to the additionally introduced mass density $m(x, t)$ (in comparison to the usual many-worlds interpretation) as the *primitive ontology* (PO) of their

[45]Thus, we have here a conceptional similarity to the de Broglie–Bohm theory, which is not surprising if one casts a glance at the list of authors: with Valia Allori, Sheldon Goldstein, Roderich Tumulka and Nino Zanghì, we find here several prominent supporters of Bohmian mechanics.

[46]This difficulty was pointed out to Schrödinger by Hendrik Antoon Lorentz in a letter from March, 1926 (Jammer 1974, p. 31).

theory. They point out the necessity of such a structure, in order (as in Maudlin's argument) to describe material objects in real space via a physical theory.[47] As mentioned at the outset, the guiding idea of Everett's, of working with the wavefunction *alone*, is intentionally disregarded in this theory. While thus Maudlin's criticism of the many-worlds interpretation is formally invalidated by this variant, it aims in its content at the converse, for there is now no reason to prefer this interpretation over the de Broglie–Bohm theory.

5.3 The Relation Between the Various Interpretations

We conclude this chapter with a brief summary, which in particular establishes some relations among Bohm, Everett, and the interpretations introduced in Chap. 2 (the ensemble and the Copenhagen interpretations).

Both the de Broglie–Bohm theory and the Everett interpretation of quantum mechanics dispense with a discontinuous change of state ("collapse") of the wavefunction. Both interpretations thus in fact contain all the branches of the wavefunction which accumulate through splitting off as a result of every interaction. The non-observability of superpositions of macroscopically different states (e.g. during the act of measurement—but a measurement is of course a typical example of an interaction with a macroscopic object) must be explained in both interpretations. They however choose different strategies for solving this problem.

The "Bohmian solution" of the measurement problem consists in the fact that the additional spatial configuration of the "Bohmian particle" distinguishes precisely that part of the wavefunction which corresponds to the output of the measurement apparatus.[48] There can thus be no ambiguity in the "pointer position", since each state of a measurement apparatus is characterized by a unique configuration of these Bohmian particles. Applying suitable initial conditions, this allows the theory to reproduce all of the statistical predictions of quantum mechanics. In this sense, the de Broglie–Bohm theory complements the ensemble interpretation of quantum mechanics by a mechanism which describes the behaviour of the members of the ensemble.

With the exception of position measurements, however, one finds here no properties of the quantum objects which were already present *before* the measurement. In a way, this form of contextuality could be seen as the detailed development of a remark of Bohr's, which can be found for example within the following quote: "The procedure of measurement has an essential influence on the conditions on which the

[47]In Sect. 5.2.2, we have already mentioned that this variant of the many-worlds interpretation is non-local. The problem of the preferred basis and the role of probability statements can likewise be treated differently in this theory.

[48]The description of the "effective collapse" of the wavefunction in addition profits from the results of the work on decoherence.

very definition of the physical quantities in question rests" (Bohr 1935, p. 1025).[49] The "production" or "establishment" of the result through and in the act of measuring is likewise part of the Copenhagen interpretation. In contrast to the Copenhagen interpretation, the de Broglie–Bohm theory however offers a *physical mechanism* which explains this process realistically. That naturally says nothing yet about the plausibility of this mechanism.

On a quite different level, we can establish a parallel between the de Broglie–Bohm theory and the Copenhagen interpretation: A characteristic of the de Broglie–Bohm theory is its description of physical reality in terms of the *pair* consisting of the wavefunction and the configuration (formally: (ψ, Q)). As we have mentioned in Sect. 2.2.2, the Copenhagen interpretation claims that there is an "indissoluble connection" between the microscopic system and the measurement apparatus (i.e. the macroworld). In this sense, the Copenhagen interpretation thus also describes the physical world in terms of a pair—expressed formally, for example, as (ψ, 'macroworld').[50] In the de Broglie–Bohm theory, the second element of this pair is therefore replaced by the objects which according to this theory represent the constituents of the macroscopic world.

In the case of the Everett interpretation, all the possible outcomes of a measurement are realized in fact. This however is not subject to observations, since each observer is likewise subject to the splitting up of the worlds. The integration of a plausible concept of probability and the justification of Born's rule (i.e. the observable relative frequencies of occurrence) remain problematic, as we have discussed in Sect. 5.2.5. However, the results of decoherence theory have made it plausible how the pointer basis of a measurement apparatus is in fact distinguished. This "decoherence-based" version of the many-worlds interpretation thus dispenses with a good deal of ontological ballast which had been held against its earlier formulations.

The splitting into infinitely many worlds of course still appears radical and eccentric. With this background, one may tend to prefer the de Broglie–Bohm theory, at least with reference to the solution of the measurement problem. Numerous authors have however pointed out that the latter likewise contains all of the branches of the wavefunction which split off with every interaction. A more plausible resolution of the measurement problem is thus only then possible within the de Broglie–Bohm theory if it defines the ontological status of the wavefunction in a corresponding manner. In Brown and Wallace (2005), the question is discussed as to which difficulties are faced by this strategy. The suggestion of Dürr *et al.* (1996), already mentioned in Sect. 5.1.7, that the wavefunction should be seen as nomological, is criticized

[49] Bohr however saw no sort of causal relationship here; instead, he compared the influence of a measurement on its outcome with the connection between the frame of reference and the observations within the special theory of relativity.

[50] This "pairing" is intended to illustrate that even within the Copenhagen interpretation, a complete description of the physical world with reference to the wavefunction alone is not possible. The classic textbook of Landau and Lifschitz formulates this relation in a particularly pointed way: "Quantum mechanics thus occupies a rather remarkable position among physical theories: It contains classical mechanics as a limiting case, and at the same time, it requires this limiting case for its own justification" (Landau and Lifschitz 2011, p. 3).

for depending in a speculative manner on cosmological considerations. According to Brown and Wallace, the "empty wavefunctions" in the de Broglie–Bohm theory likewise correspond to real worlds—the solution of the measurement problem in the de Broglie–Bohm theory therefore must (and can) make no decisive reference at all to the particle and is congruent with that in the many-worlds interpretation. Brown and Wallace thereby emphasize the dictum of David Deutsch, who characterizes the guidance-field theories as "parallel-universe theories in a state of chronical denial" (Deutsch 1996, p. 225). A reply to this accusation is to be found, for example, in Maudlin (2010). In the section on criticisms of the Everett interpretation (Sect. 5.2.6), we have already cited this work, which casts doubt on the possibility of finding a solution to the measurement problem at all, as long as the spatial configuration is not taken into account. This points out an important and still open problem for the de Broglie–Bohm theory: The status of the wavefunction is not completely clarified in that theory either, and this signals a further line of separation between various schools within the de Broglie–Bohm theory (cf. Sect. 5.1.6).

Exercises

1. The de Broglie–Bohm theory is frequently called a theory of "hidden variables". This term implies the criticism that the theory introduces in principle unobservable quantities into its description. Write a brief dialogue between an advocate of the de Broglie–Bohm theory and a supporter of the Copenhagen interpretation, in which the former defends the theory against this criticism and accuses the "Copenhagen" advocate of actually making this error herself. In the course of this debate, additional arguments *pro* and *contra* could be introduced!
2. Explain why within the de Broglie–Bohm theory, the uncertainty relation $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$ is not violated!
3. Compare the solutions to the measurement problem in the de Broglie–Bohm and the Everett interpretations. Give examples of structural similarities and differences between them.

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