Chapter 5
Stories Neglected About Children's Mathematics Learning in Play

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Abstract In this paper we describe stories of mathematics learning in play that are often neglected in this era of schoolification and discussions of what counts as learning in early childhood. Drawing on theories of early childhood teaching and learning that emphasize the importance of teachers' (a) content knowledge, pedagogical content knowledge, and knowledge of children's development, (b) action competencies, and (c) attitudes and beliefs, we explore three stories of child-teacher interactions in play. We found that, despite different political and public perceptions of what counts as learning in three different countries, preschool teachers evidenced competencies in similar ways – each illustrating a neglected story of children's mathematics learning.

Keywords Mathematics learning · Play · Early childhood · Kindergarten · Preschool · Schoolification · Global neglected stories · Narratives · Counter-narrative · Narrative repair

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Introduction

Schoolification, what the Organisation for Economic Co-operation and Development (OECD, 2006) and the United Nations Educational, Scientific and Cultural Organization (UNESCO, 2010) use to describe the increasing pressure to make preschools more academic, is the subject of ongoing global debates. The debates taking place in the media and in academia raise philosophical, political, and ethical questions about schooling for young children. In the USA, this is particularly true for children whose skin color, language, cultural practices, and economic background differ from the white middle and upper class families who have access to well-resourced play-based preschool and to a lesser extent in Sweden and Norway. The abundance of international comparative data on education has had an adverse effect that is "fueling a competitive 'global race' where governments become increasingly concerned with national rating" (Ang, 2014, p. 188). The debates about schoolification are taking place in public and academic circles as evidenced by the proliferation of articles in popular press and in academic journals where it is argued that formal curriculum for young children devalues the early years' experience (Faulkner & Coates, 2013; Pugh, 2010; Rose & Rogers, 2012). The debates about academic push down can be particularly evident in mathematics. It is not our intent to engage in a debate on "false dichotomies" about play versus didactic instruction (Fuson, Clements, & Sarama, 2015) but to explore stories of children's learning of mathematics in play that are often neglected in public and policy arenas.

In this paper, we address this issue by examining ways in which mathematics in play or in play-like activities is getting taken up in different contexts. In particular, we are interested in preschool teachers' practices across contexts in an environment that is increasingly academic for our youngest children. We provide stories of how teachers engage with children in three preschools in Sweden, the USA, and Norway. These examples are situated in varying political and public narratives about mathematics in play and learning. We have found that despite varying political or policy climates in each country, preschool teachers share similar knowledge, competencies, and attitudes, and this is reflected in their practice. This has emerged in several ways; we have put forward three stories to demonstrate the neglected narratives from practice that do not find their way into public perception and political action.

Theoretical Perspective

Our study is at its' core grounded in a perspective that considers play not only a human right of childhood (OECD, 2006) but a critical space for children to learn and grow (cite). We use the definition of play recently espoused in the EECERA (2017) position statement where "children actively participate in constructing their play world based on their own interests and needs" rather than the interests and needs of adults (p. 2). With respect to mathematics, we are in particular interested in

mathematics that is embedded in (Ginsburg, 2006) or emerges through (Wager & Parks, 2016) play, not activities that are planned by the teacher to engage children in mathematics play. In order to support and reflect on the stories of teacher-child interactions in play that are often neglected, we need a framework that gives us an understanding of the competencies needed for preschool teachers. The stories and the neglected stories that we as researchers and the teachers can see in particular in play situations is a way of highlighting the teachers' competences and also showing where it is important to support the teachers. To do this we draw on three ways of examining teachers work with children: Benz's (2016) professional competencies; Graue, Delaney, and White's (2014) improvisation; and shared thinking (Doverborg & Samuelsson, 2011; Siraj-Blatchford & Sylva, 2004).

Benz (2016) synthesized the professional competences needed for supporting children's early mathematical thinking. She found three categories of competences: (a) content knowledge, pedagogical content knowledge, and knowledge of children's development, (b) action competencies, and (c) attitudes and beliefs. According to Benz (2016) content knowledge, pedagogical content knowledge, and knowledge of children's development are the knowledge that supports the teachers in noticing children's mathematical competencies in their activities.

With respect to the first competency, teachers need to know the mathematics content which is the teacher's knowledge about understanding of what and why (Shulman, 1986, p. 9). Early childhood educators "have to see the relations between mathematics in the early years and later on to guarantee coherent mathematical learning" (Gasteiger, 2014, p. 278). Based on Shulman (1986) and Gasteiger (2014), teachers also need to have pedagogical content knowledge, in this case how mathematics content might be evident in play and how to support it. Further, they must understand and support children's development and the kinds of interactions, content, and questions that are appropriate for young children.

For the second competency, the teacher needs to not only know how to get the children to reflect on their own thinking but also "how to ask questions and communicate in order to strengthen children's understanding" (Doverborg & Samuelsson, 2011, p. 60). Benz (2016) bases the idea of action competencies on Ginsburg, Lee, and Boyd (2008) where the focus is on identifying "teachable moments." According to Ginsburg et al. (2008), this is quite challenging for teachers and, as seen in Wager and Parks (2014), this is especially challenging in children's play. One way of supporting this is to think about teachers' improvisational acts (Graue et al., 2014) wherein they respond in the moment to the play that children lead.

The third point about attitudes and beliefs is also important when it comes to noticing mathematics in children's play. Teachers need a broad view of what counts as mathematics in order to actually notice it. If the teachers do not notice the mathematics, they will not be able to tell the stories about all the mathematics that children engage in. In order for the teachers to notice or tell the stories of developing children's mathematical thinking, they need the abovementioned competencies and knowledge.

The Data Resources and the Methodology

This is a case study of three teachers, one each in Sweden, Norway, and the USA. To provide some context for these settings, a brief overview of the political and public perceptions with regard to mathematics in play and the terms used for early childhood settings is described in Table 5.1.

As is the case in many countries around the world, in Swedish preschools, play is considered the foundation for children's learning experiences (Skolverket, 2011). This is reflected in the curriculum. "Play is important for the child's development and learning. Conscious use of play to promote the development and learning of each individual child should always be present in preschool activities. Play and enjoyment in learning in all its various forms stimulate the imagination, insight, communication and the ability to think symbolically, as well as the ability to cooperate and solve problems" (Skolverket, 2011, p. 6). Connecting play with enjoyment assumes that learning will produce more easily "imagination, insight, communication and the ability to think symbolically, as well as the ability to cooperate and solve problems."

In the USA, the very notion of play in early childhood has become contested. In the not so distant past, early childhood classrooms were child (not content)-centered spaces (Elkind, 2009), but schoolification has taken hold and kindergarten classrooms have become "glorified first graders" that are increasingly standardized with limited time for play and driven by assessment (Graue, 2009). The US early childhood education system is becoming more aligned with practices in older grades that are heavily influenced by state and federal standards-based accountability movements (Brown, 2015).

The Norwegian framework plan for the content and tasks of kindergartens (Ministry of Education and Research, 2011) has play-oriented guidelines with focus on children's participation and interest. Still, there is a strong push among politicians

Table 5.1 Mathematics play and learning across contexts

Country Grades/age	Policy	Media
Sweden: Preschool (1–5) Preschool class (6) First grade (7)	Play-based goals for the preschools	Mixed message based on play but talks about school results (TIMSS and PISA)
USA: Preschool (0–5) Prekindergarten (4–5) Kindergarten (5–6) First grade (6–7)	Universal preschool Schoolification	Mixed messages about play
Norway: Preschool/ kindergarten (1–6) First grade (6)	Play-based goals for the preschools	Mixed message based on play but talks about school results (TIMSS and PISA)

and in media that children should engage in mathematics to get better school results. For instance, the Norwegian Minister of Education, Torbjörn Röe Isaksen, stated that: "Jeg mener at en enda sterkere vektlegging av matematikk kan være et godt tiltak for å snu trenden med dårlige matteresultater i skolen" (I believe that an even stronger emphasis of mathematics [in preschool] could be a good step to reverse the trend of poor math performance in school.) (Isaksen, 2014).

We draw on the idea of the counter-narrative used in critical race methodology (Solorzano & Yosso, 2002) and narrative repair (Nelson, 2001) to provide stories that counter and repair the notion that children do not have opportunities to learn and preschool teachers do not teach mathematics in play. Much as scholars use critical race methodology to study those at the margins of society (Solorzano and Yosso, p. 23), we are studying the youngest most vulnerable children who experience a different form of marginalization; who, because of their age, are not able to tell their own stories; and who have instruction done "to" them rather than "for" them (Wood, 2010). We use narrative repair (Nelson) to repair, write, and rewrite the stories that get told about preschool teachers who support children's learning of mathematics in play. We knew anecdotally there were numerous opportunities for mathematics in play learning and that these could provide evidence countering the schoolification discourse. As such, we approached our work by examining data from our studies to identify stories that provided evidence of opportunities children have to learn and teachers have to support mathematics learning in play. But beyond identifying the stories, we also aim to explore the themes evident in the stories.

To construct new narratives that counter and repair existing stories in each of our countries – what we are referring to as stories neglected – we first met to unearth themes we found common in studies of professional development that we have been involved in. Each of the authors has participated in research studies of professional development programs to support early childhood teachers to notice mathematics learning and teaching opportunities in play. We explored a subset of the data from these studies that included classroom video, teacher reflection on course work, teacher reflection on children's play, and researcher reflection. Through our analysis of the data across the projects in each country, we identified four "tropes" that emerged in all of our work with teachers: (a) the conflicting teacher-researcher narrative, (b) the congruent child-teacher narrative, (c) the conflicting child-teacher narrative, and (d) the shifting teacher narrative. We then selected a representative story of mathematics learning in play from the larger studies and examined those through the lens of Benz's (2016) categories of competencies. These stories are told through the experiences of the teacher, the child, and the researcher.

The Tropes

In all three setting, we found similar ways in which teachers involved in professional development engaged with children in mathematics during play. The four themes, or tropes, are summarized here and then exemplified in the stories below.

Conflicting teacher-researcher narratives Not surprisingly, we found that as researchers analyzing data after the fact, and even in the moment, there were instances when we saw evidence of and opportunities for teachers to engage children in mathematical thinking in their play or activities. We refer to these as conflicting teacher-researcher narratives to make evident the difference in what we see as researchers and what teachers notice in the moment. We do not intend this to be an opportunity to highlight what teachers miss but rather what is possible as we continue to work with teachers in professional development to recognize mathematics learning opportunities. These conflicting narratives tended to include two areas of conflict: what "counts" as mathematics and "where" we see mathematics in play and in children's everyday activities.

Child-teacher congruent narratives We found that those situations in which the child(ren) and teacher were in agreement met the following criteria: the activity was play oriented wherein the mathematics emerged in the play initiated by the child; both were engaged in the mathematics in the activity; and both were involved in the play. Further, there was a shifting back and forth between who was leading the activity – in other words neither the teacher nor the child was solely responsible for the direction of the play or the mathematics. And finally, communication acts were necessary to provide evidence of the interaction (but they were not always verbal). For example, there were times when there was evidence of mathematical thinking, such as a child nodding their head as they counted, that did not include a verbal exchange between the teacher and child.

Child-teacher conflicting narratives In some situations, we found that the goals of the child and teacher differed. In these cases, the teacher may have been trying to infuse the mathematics into an activity initiated by the child, and the child resisted the change to their play/activity. Teachers handled these situations by either walking away from the child's play, dropping the mathematics, or continuing to try to engage in mathematics.

Teachers' shifting narratives We think about shifting narratives as the ways teachers' stories change over time in response to curriculum, policy, public (media, parents, and community), other teachers, children, and professional development. In this manuscript, we provide examples of how teachers' narratives shift as a result of professional development and engagement with children. In all of our research doing professional development, we found that teachers' experience shifts toward recognizing the role of play in teaching and learning mathematics. And, as teachers respond to children's mathematics as they engage in play, the teachers' narratives about what counts as mathematics also shifts.

The Stories

Walking Along the Bench: Conflicting Teacher-Researcher Narrative

Looking at what counts as mathematics depends on the researchers' or the preschool teacher's attitudes and beliefs about what counts as mathematics. This view will also affect the teacher's possibility of supporting or promoting children's learning in mathematics. The following example can be found in Helenius et al. (2015) where it is analyzed using didactic space to see how the foci for the child and for the teacher changes during the course of a very short event occurring in a free play situation.

This situation takes place outside; a toddler is walking back and forth along a bench. The child gets to the end of the bench and looks down, and then the child turns and walks to the other side of the bench where the teacher is standing looking at the child. The girl's exploration could be seen as exploring space, locating in space, and learning about spatial relations such as being up on the bench above the ground, walking along the bench, back and forth, and looking down to the ground and hence is seen by the researchers as being mathematical (Helenius et al., 2015). The situation then continues with the toddler raising her arms toward the teachers; this could be interpreted as the toddler wanting assistance to get down. The teacher in this case puts her arms in a gesture that could be interpreted as the teacher wanting the child to find a way of getting down herself. Here the teacher has a goal of actually wanting the child to find a way of getting down. This could be interpreted as the teacher having action competence, she changes the situation by not acting on the child's intention but rather challenge the child in the learning situation.

The teacher might not see this activity as a mathematical situation but rather a situation where the teacher is encouraging the child to explore her motoric skills of climbing down the bench herself, so the teacher's pedagogical content knowledge might affect her actions. One reason for this could be the lack of mathematical language connected to other parts of the mathematics not only numbers and shapes. On the other hand, the researchers as seen in Helenius et al. (2015) see this as a mathematical situation, and it is categorized as such using Bishop's (1988) categories of mathematical activities. Seen from the teacher's perspective, it is not clear that she sees the situation as mathematical or that her actions actually have the goal of challenging the child's mathematical learning.

This video has also been used in different professional development courses, and in the discussions of the video, most of the preschool teachers attending this course do not at first see this as a mathematical situation; instead, it is framed as being about the child learning to get down from the bench or to develop her motor skills in climbing. So the preschool teacher's pedagogical content knowledge and their attitudes and beliefs will affect the situation, and this is why we

see this story as a neglected story where the child is in fact challenged in the play situation but not necessarily in way of challenging the child's mathematical competences.

Water Table Fill and Spill: Both a Congruent and Conflicting Teacher-Child Narrative

In the following, we see within one brief exchange an example of both a child-teacher congruent story and a conflicting story. This play-oriented interaction (in other words, the mathematics emerges in play) is congruent as both the teacher and the children engage in the mathematics and the play in the activity; and there is shift back and forth between who is leading the activity. But it is also conflicting as there are times when the goals of the teacher differ from those of the children.

Nick and Ramone were playing at the water table. They had plastic buckets, cups, and funnels of varying sizes, cylinders, and one bucket with a bottom that popped off when it was too full. Nick was filling the big bucket so that the bottom would fall off and water would spill. He asked Ms. D to hold the big bucket so he could use a cup to fill it with water. Ramone joined in and held the big bucket; after the bucket was about half full, the bottom fell off and water spilled into the table. The boys found this hilarious and they put the bottom back on to try again. Ms. D asked the boys "so I'm wondering how many cups of water it takes before it spills."

Nick pours cups of water into it the big bucket Ms. D is holding. She counts each cup as he pours it in. Ramone is watching and nodding slightly each time Ms. D counts and eventually she stopped counting to have Ramone take over.

Ms. D:	*[counts each cup as Nick pours, getting louder and more enthusiastic with each cup poured in]* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, *[whispers and nods to Ramon]* 13
Ramone:	14, 15, 16, 17, 18, 19 *[the boys are clearly getting excited that the water is probably about to spill, at 19 cups the bottom comes out and water pour out and all three jump back and laugh]*
Ramone:	19!
Nick:	19!
Ms. D:	19, oh my goodness *[holding up a cylinder]* What if you tried one of these?
Nick:	No *[now Ramone is holding the big bucket and Nick is filling it using two cups at a time, one in each hand]*
Ms. D:	Are you counting?
Ramone:	No, I don't want to count because I'm holding *[the bucket]*
Ms. D:	oh, okay *[Ms. D walks to another part of the classroom and the boys continue but start to disagree about what they are doing. Ms. D returns to the water table to moderate the disagreement and the boys happily continue]*
Ms. D:	Now you have two cups and are trying to fill it.
Ramone:	It will go faster.

Ms. D: Faster, you are right. I'm still wondering about… it took 19 cups, of these cups to fill it *[holding up the original cup]*, I wonder how many of these cups it would take *[holds up a cylinder]*. Do you think it will be 19 or do you think it will be a different number?
Ramone: I think, I think, I think it will be 16.
Ms. D: You think it will be 16. After Nick is done here maybe, you can try that out Ramone because it is hard to count when he is going by twos. Oh… *[The bucket spills again and now Ms. D encourages the boys to use the cylinder. Nick starts out by using two cylinders]* Let's see, let's count them and see if we get 16.
Ms. D: It will be hard to count using two, one at a time.
Ramone: *[to Nick]* One at a time! *[Ms. D starts the counting again and Ramone is mouthing the words for the count, Ms. D counts silently until they get to 16.]*
Ms. D: We are getting close, 16, did it pop off?
Ramone: Nope
Ms. D: 17, 18, 19, 20, 21 *[the bucket is full at this point and Nick pops the bottom off so the water will spill]*
Ms. D: So I think you had 21 before it flowed over the top.
Nick: I did it. I did it. I beat the record.

It is difficult in text to explain the excitement and enthusiasm the children expressed throughout this activity. In this example, we see evidence of the content knowledge, pedagogical content knowledge, and knowledge (Benz, 2016; Shulman, 1986) of children's development Ms. D possessed as she engaged the boys in counting during their play activity. And, importantly, she understood where each boy was in his mathematical understanding and build on that knowledge (Carpenter, Franke, Johnson, Torrou, & Wager, 2016). Ms. D was aware of Ramone's silent counting as she said the number sequence aloud. She also moved gently in and out of the play introducing possible ways to use mathematics – counting the number of cups and comparing different-sized cups – but did not push too far and interrupt the boys' ideas about the play. She took advantage of several teachable moments in this interaction demonstrating her action competencies or what Graue, Delaney, and Whyte (Graue et al., 2014) refer to as improvisation. With respect to Benz's (2016) third category, Ms. D clearly sees opportunities for mathematics in multiple ways in this brief interaction at the water table. There are certainly other things Ms. D could have done, such as count by two when Nick was using two cups at a time, but we can't know why she did not make that choice. Perhaps she knew she was pushing the boys as far as she could already, and perhaps she knew they were not yet counting by twos and wanted to reinforce the counting sequence. But we do know that Ms. D made some mathematically sound decisions as she asked questions during the fill and spill activity.

This story shifts between teacher-child congruent and teacher-child conflicting narratives as the teacher negotiates her place (and mathematics place) in the play. It starts with shared (congruent) engagement in the activity between Ramone and Ms. D, as they are both interested in counting the cups to determine how many it will take to spill

out. They are also interested in knowing if the number of cups would differ if they use a different size cup. The story shifts to a conflicting narrative as Ms. D asks Ramone if he is counting and he replies, "No, I don't want to count because I'm holding." At this point Ramone has moved from an interest in the mathematics of the activity to the fun of holding the bucket. The narrative shifts back toward congruence when Ms. D returns to the table and again engages Ramone in thinking about the mathematics.

The Stone Story: The Shifting Teacher Narrative

In the following, we present a play-oriented activity that is in opposition to the schoolification process. Because it shows how children engage from an adult initiated activity to a child initiated activity. The spontaneous conversation shifts from an adult-guided counting activity to a child-oriented measuring activity.

Below are excerpts from a Norwegian preschool teacher and her reflection over her ability to support children's learning. She tells about an incident where she had planned an outdoor activity with counting and sets with use of one die.

Excerpt 1
[The die showed four dots and all the children ran around finding four objects. Then two children started arguing.]

Child 1: My stone is bigger than yours.
Child 2: Is it?
Teacher: *[What now, what with all my plans!]*
Teacher: How could we work it out?
Child 1: We must measure. We can hold the stones next to each other.
Teacher: Yes that was smart.
Child 2: They are the same length.
Child 3: But, how long in numbers are they?
Teacher: How can we work it out then?
Child 3: We must find something to measure them.

[The children measure the length of the stones using a folding ruler]

In the reflections after, the teacher said: "I thought, what now, what with all my plans? It was so difficult not to interrupt the children, but I managed to follow the children's interests." Her goal and plan was to look at the children's competencies in counting and sets, but she was able to support children's activity when it shifted to measuring discussed as improvisation by Graue, Delaney, and Whyte (Graue et al., 2014).

Shulman (1986) defined "pedagogical content knowledge" as knowledge about teaching and not just knowledge about content. In this excerpt the preschool teacher managed to support children's curiosity and the children's desire to explore mathematical connections. Her ability to modify her plans to encourage the children's participation is included in the concept pedagogical content knowledge. Such flexibility is part of being a preschool teacher.

The incident continues:

Excerpt 2

Child 1: My stone is thicker than your stone.
Child 2: We have to measure.
Child 1: Yes, but it is impossible for the folding rule to bend.
Teacher: But, how can we measure the thickness of the stones?

[The children struggle to measure the circumference using a ruler.]

Child 3: We can take two blades of grass and put them around the stones and then we can see which one are the longest.
[The children measure the circumference of the stones with blades of grass.]
Teacher: That was smart.
Child 1: But we cannot exactly see numbers on the blade of grass.
Teacher: No, you're absolutely right. Can you look in the bag if there is anything we can use to measure the thickness of the stones?

[Having thrown everything on the ground and examined several of the objects, at least for 10 min, I thought the children were distracted and the measurement activity forgotten.]

Child 5: Here is something with numbers that are soft and we can bend
[looking at a measuring tape.]

By following the children's initiative, the preschool teacher's goal for the activity shifts from hers to the children's. For example, the preschool teacher is asking them questions like "How could we work it out?" and "But, how can we measure the thickness of the stones?" These questions are supportive to the children's initiative. Instead of trying to guide them back toward her own goal, she let them take charge in the new activity. She is taking the role as a supporter for the children. In the dialog, we find the preschool teacher's utterances like "That was smart" and "No, you're absolutely right. Can you look in the bag if there are anything we can use to measure the thickness of the stones?" Not only is she supporting but she is also guiding them toward their goal. The preschool teacher shows us that she has pedagogical content knowledge and content knowledge (Benz, 2016; Shulman, 1986) about mathematics. She shows content knowledge when she supports children related to the subject numbers and measuring in this incident.

Doverborg and Samuelsson (2011, p. 60) emphasize both to know what early mathematics can be and know how to communicate and challenge children as important aspects of teacher knowledge. In this incident, the preschool teacher invites to "shared thinking." Siraj-Blatchford and Sylva (2004) define sustained shared thinking as "an interaction where two or more individuals 'work together' in an intellectual way to solve a problem, clarify a concept, evaluate activities, or

extend a narrative" (p. 718). The communication between the preschool teacher and the children show us that the preschool teacher presents the children with problems like "But, how can we measure the thickness of the stones?" and the children responds.

Further:

Excerpt 3

Child 2:	My stone is so thick, *[the child holds its finger at digit 9 on the tape measure].*
Teacher:	Yes 9 centimeter thick.
Child 4:	How thick is your stone?
Child 1:	Mine is so thick, *[the child explains and keeps both hands around the stone].*
Child 4:	Yes but how many meters is it?
Child 1:	*[Put the tape measure around the stone and asked]* How thick is this?
Teacher:	It is 12 centimeter. Which stone is than thickest?
Children 1–5:	This *[everyone is pointing to the stone that is 12 centimeter].*
Teacher:	Which stone is the heaviest, do you think?
Child 1:	The one that is thickest is the heaviest because that how it is with humans.

In this excerpt, the teacher gives oral response to the child's experience when the teacher answers, "Yes 9 centimeter thick." Both the digit "9" and the unit "centimeter" are new for the child. This experience may contribute to the child's interests for numbers and measurements. By reading the numbers for the children, the preschool teacher identifies a teachable moment (Benz, 2016; Ginsburg et al., 2008, p. 7), a situation that might promote learning.

The preschool teacher helps the children to be on track by referring to the first problem: "Which stone is the heaviest, do you think?" She connects the children's previous and current experience in order to solve the original problem from the child 1: "My stone is bigger than yours."

Preschool teachers reflect on her experience:

> "Earlier I thought of the goal for the activity, but it was my thought about the goal. In my head, I focused on my written plans for activities and my aim with the activities. Now I realize that there is so much learning for the children if I listen and pay attention to the mathematical ideas that they express through play and conversation. I think there might be even more learning for the children if I pay more attention to them and their interest even when it is in conflict with my goal for the activity."

Here the preschool teacher reflects on her ability to facilitate learning opportunities and her knowledge for teaching. When she tells about her reflection, she develops her content knowledge about mathematics but also her pedagogical content knowledge (Benz, 2016; Shulman, 1986). "Noticing children's mathematics can be a way of respecting children and engaging with them to promote greater and deeper understandings" (Dockett & Goff, 2013, p. 774).

According to Benz (2016) the preschool teacher's attitude to children and their learning is an important part of being a preschool teacher. In the preschool teacher's self-reflection, there is a positive attitude toward the learning possibilities in the children's initiated activity. In her reflection, she acknowledges children's own exploration in play activities.

Discussion

The push for schoolification in the media and academia may affect the kinds of interactions, support, and engagement preschool teachers have in children's play and everyday activities. Schoolification, often characterized by teaching for the future, is in opposition to our examples that focus on teaching in here and now situations. The three findings across these stories are about teacher knowledge, teachable moments, and teachers' attitudes with respect to engaging children's learning of mathematics in play. These stories and findings are global – they come from different countries but each could have happened in any of these countries.

Preschool teachers' knowledge about how to engage children in mathematics in play is sometimes evident and sometimes it is not. For example, in the bench story, we as researchers see the math in the situation but it might be that the teachers does not see it or notice it because they may not have the language to describe what they are doing or we do not ask them about it. In the water table example, we are making assumptions about what the teacher knows based on what she does, and in the stone example the teacher reflects about how to engage children when she is telling about it. In our three stories, we have highlighted the value of flexible preschool teachers that have knowledge about what is mathematics for children discussed as content knowledge. In the bench story, the teacher challenged the child in the learning situation but not necessarily in the mathematical situation, maybe because of lack in pedagogical content knowledge. Also in other studies (Svensson, 2016) using Bishops 6 mathematical activities, the most common activity is counting and measuring. In the water table story, the preschool teacher understood that to encourage Ramone to count, she had to start first – she had knowledge of Ramone's counting skills and also his hesitancy to demonstrate those skills. In the stone story, the preschool teacher listens to the children and supports their exploration of measurement even if her original idea was to facilitate their learning in numbers and sets. This supports the argument that she has content knowledge about different topics in mathematics and is able to switch between these topics. The preschool teacher shows pedagogical content knowledge by listening to the children and challenging them with questions.

Teachable moments or actions are evidenced across the three stories in the ways that teachers respond in the moment to children's play. In the bench story we can see that the teacher notices the child's action and interacts with the child in a learning situation, even though it is not clear from the example that the teacher sees this as mathematics but still handles the situation as a learning situation. In some cases, the teachers support the child's thoughts by responding through oral response and in

some cases with actions. In the bench story, the teacher challenged the child in changing the way of acting, i.e., not following the child initiative. So the teacher is initiating a new direction for the situation where the child is challenged. In the water table story, the teacher seizes on an opportunity to engage children in counting in an activity they have started. In the stone story, the teacher gives oral response "Yes, 9 centimeter thick" when the child holds her finger at digit 9 on the tape measure. Perhaps the child did not know how to read the symbol "9," and the teacher grabs the opportunity to extend the child's understanding of the concept of "9."

The three stories provide evidence of teachers' attitudes about what counts as mathematics in play and when children should be left to play or encouraged to engage in mathematics. In the bench story the teacher's attitude about what counts as mathematics will affect her actions and the way she challenges the child. In the water table story, the teacher wants the children to count the cups of water. That is not their initial intention but they willingly engage in the counting when she makes it about a challenge – how many cups will it take? But she also stops encouraging the counting during the times the children don't take it up. In the stone story, the preschool teacher supports the children's curiosity when they wonder who has the biggest stone. She pays attention to children's interests and their motivation for learning.

Conclusion

The goal in this article is to give language to neglected stories of mathematics teaching and learning in play to counter and repair those stories told by parents, politicians, and the media. These parties are telling early childhood teachers what to do, and the best stories are probably the ones they already do. These stories of repair empower the preschool teacher and the child by "reclaiming" their agency as teachers and learning of mathematics (Nelson, 2001).

Our aim is to highlight a different aspect of mathematics in early childhood and to develop language among preschool teachers to talk about mathematics in play activities. We describe these as neglected stories in the larger narrative that children do not have enough mathematics learning opportunities in play, which would suggest a need for schoolification. The teacher in the *bench story* tells a story of a play situation, which could be seen as a mathematical situation by the researcher, but the child is still challenged in her learning even though the teachers might not have seen this as a mathematical situation. If the teachers do not notice the mathematics, they will not be able to tell these kinds of stories. In the *water table story*, the preschool teacher sees mathematical learning opportunities in multiple spaces in her play-based classroom. She does engage in more didactic activities that she plans but also supports children in rich mathematical discussions during play. Stories such as this where the teacher spontaneously follows and leads the children's play to support their mathematics learning are not often shared in public spaces. The preschool teacher in the *stone story* tells us a narrative about a planned play activity that changed after children's engagements. This is a neglected story because stories of

mathematical activities are usually about those that are well planned, organized, and instruction based. The measurement activity deviates from the original plan. Those stories we tend to tell are stories that are well planned, and the aim of the activities are fulfilled.

The stories, coming from three different countries, indicate that preschool teachers share similar knowledge, competencies, and attitudes despite varying political or policy climates in each country. Furthermore this state of affairs rubs off on teachers' practices regardless of different preschool situations occurring in different countries.

The stories we have shared provide an important counter-narrative and repair to narratives encouraging schoolification by demonstrating the possibilities of mathematics learning that can happen in play. In each example we show how neglected narratives of children's learning need to be shared not only among scholars but also the public and policy makers that drive what "counts" as mathematics learning. In this way, we may empower the preschool teachers' knowledge and children's learning of mathematics. Maybe if these stories of what happens in preschool were not neglected, the various public and policy approaches to schoolification may happen to a lower degree. Then policy and public approaches could change to match what teachers see as best for children related to learning of mathematics in play.

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