Chapter 3
Using Children's Patterning Tasks During Professional Development for Preschool Teachers

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Abstract Patterning activities in preschool are considered one way for enhancing young children's appreciation for structure. Preschool teachers, however, are not always aware of the mathematics behind these activities. This paper describes one part of a professional development program that employs the use of tasks for children to promote preschool teachers' knowledge for teaching patterns. Segments of the program reflect how the refined Cognitive Affective Mathematics Teacher Education framework helped to ensure that while engaging in pattern tasks for children, teachers enhanced their mathematics knowledge, knowledge of students, and knowledge of tasks.

Keywords Repeating patterns · Preschool teachers · Unit of repeat · Professional development · Pattern tasks · The CAMTE framework

Introduction

In Israel, the preschool curriculum encourages teachers to engage children with pattern activities with the aims of having children identify, draw, and continue repeating patterns as well as use mathematical language to describe these patterns (Israel National Mathematics Preschool Curriculum [INMPC], 2008). Yet prospective preschool teachers receive little, if any, preparation for teaching patterning in preschool. This paper describes a professional development program aimed at increasing preschool teachers' knowledge for teaching patterning. Preschool, in this paper, will relate to children ages 4–6, 1 and 2 years prior to first grade. In the next section, we offer some background on research related to children and

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patterning activities. Following that, we introduce the framework we used to investigate preschool teachers' knowledge for teaching and how that framework may be used when planning professional development for teachers.

Research Related to Patterning and Young Children

Why Engage with Patterning Tasks?

Several national curricula have recognized the potential of pattern activities in promoting early algebraic thinking among young children. For example, the National Council of Teachers of Mathematics' (2000) Algebra Standard for Pre-K-2 states that "algebraic concepts can evolve and… develop… through work with classifications, patterns, and relations…" (p. 91). Exploring patterns during the elementary years may enhance the meaning of algebra during the secondary years. Algebraic thinking relates to finding and using generalizations. "Every pattern is a type of generalization in that it involves a relationship that is 'everywhere the same'" (Papic, Mulligan, & Mitchelmore, 2011, p. 240). Thus, working with patterns can promote this aspect of algebraic thinking. At the preschool level, educators have specifically noted that exploring repeating patterns may promote children's appreciation of underlying structures (Starkey, Klein, & Wakeley, 2004).

Repeating patterns are patterns with a cyclical repetition of an identifiable "unit of repeat" (Zazkis & Liljedahl, 2006). For example, the pattern ABBAB… may have a minimal unit of repeat of length three (ABB) and ends with an incomplete unit of repeat. However, without specifically stating what the minimal unit of repeat is, one may claim that the minimal unit of repeat in the above sequence is ABBAB. In general, sequences may be generated in an infinite number of ways. For example, the sequence 1, 2, 4, 7 may continue with 11, with 12, or with 13, depending on the respective rules: $x_n = n(n - 1)/2 + 1$, $x_n = x_{n-1} + x_{n-2} + 1$, or $x_n = x_{n-1} + x_{n-2} + x_{n-3}$. According to the Israel National Mathematics Preschool Curriculum (2008), "patterning activities provide the basis for high-order thinking, requiring the child to generalize, to proceed from a given 'unit', to a pattern in which the unit is repeated in a precise way" (p. 23).

Children's Engagement with Various Patterning Tasks

Young children naturally engage in pattern activities such as building block towers with an ABAB pattern (Seo & Ginsburg, 2004). However, while most children by the end of kindergarten will be able to copy a repeating color pattern, few will be able to extend or explain it (Clarke & Clarke, 2004). Being able to copy a pattern may not necessarily indicate that the child recognizes the structure of the pattern. Papic et al. (2011) found that some preschool children may be able to draw an

ABABAB pattern from memory by recalling the pattern as single alternating colors of red, blue, red, blue, basically recalling that after red came blue and after blue came red. This strategy is sometimes called the "matching one item at a time" strategy, or the "alternation strategy," especially successful with simple AB patterns, and less so in patterns such as ABCD that have more elements. When shown a more complicated pattern such as ABBC, they could not replicate the pattern. Rittle-Johnson, Fyfe, McLean, and McEldoon (2013) found that when young children were asked to duplicate or extend an ABB pattern, some children could not produce more than one unit of repeat correctly, while some reverted to producing an ABAB pattern.

Recently, Tsamir, Tirosh, Barkai, Levenson, and Tabach (2015) found that when children were requested to choose possible ways to continue repeating patterns, more children were able to continue a pattern which ended with a complete unit of repeat than a pattern which ended with a partial unit. When deciding whether or not to choose some continuation, some children merely seemed to guess, while others exhibited some strategy. One strategy was to physically move each continuation to the end, trying it out before deciding whether or not it was appropriate. Another strategy was aligning up each continuation with the beginning of the pattern to see if it matched. One child chose continuations based on the last element of the pattern, claiming that the next element cannot be the same as the last element of the given patterns. They suggested that in addition to promoting children's recognition of the unit of repeat, we should encourage children to recognize the sequencing aspect of the pattern and how to continue a pattern from any point.

In addition to duplication and extension tasks, there are other patterning tasks which focus more on the pattern structure. For example, one could request the child to directly identify the smallest unit of the pattern by either circling it or placing a string around the unit (Papic et al., 2011). Similarly, one could build a tower with a repeating pattern and request the child to build the smallest tower that still keeps the same pattern as the one already built (Rittle-Johnson et al., 2013). An activity which calls for more abstraction on the part of the child is to request the child to construct (or draw) the "same kind of pattern" as a given pattern but with different materials (Rittle-Johnson et al., 2013). For example, if an AABB pattern is constructed from red and blue cubes, then the child is given triangles and circles to construct a similar pattern. Sarama and Clements (2009), in their description of children's developmental progression for patterns and structure, state that being able to translate patterns into new media is a more advanced stage than being able to duplicate, extend, or fix a pattern. Rittle-Johnson et al. (2013) also found that abstraction tasks are more difficult than duplication and extension tasks and children often turn to building random sequences when solving abstraction tasks.

While in the above activities children are requested to act, other tasks focus on verbalization. According to the NCTM (2000), describing how two patterns, such as "red, red, blue, red, red, blue" and "step, step, clap, step, step, clap," are the same and how they are different encourages children to focus on underlying structures and sets the foundation for recognizing that seemingly different mathematical expressions, such as $2x + y$ and $2a + b$, have the same algebraic structure, $ax + b$.

While this specific activity was not implemented in any of the studies reviewed here, Papic et al. (2011) did note that comparing two patterns may occur spontaneously among children. They describe an incident where a child claimed that a blocks pattern he created was similar to a flower pattern because one is “blue, yellow, yellow, blue, yellow, yellow” and the other is “curved, spiky, spiky, curved, spiky, spiky.” When asked to elaborate on their similarity, the child responded that “There is one curved and one blue, and then there’s two spiky and two yellow, that’s the same pattern” (p. 255). Papic et al. took this claim as evidence of the child’s emergent recognition of an ABB pattern and the child’s readiness to consider structure.

In the above studies, children were observed without adult intervention. However, when given proper assistance, young children are capable of recognizing the unit of repeat in a repeating pattern and come to comprehend the underlying structure of the pattern (Papic et al., 2011). In other words, for children to achieve the benefit of engaging in pattern activities, adult guidance is advisable. Yet, teachers may not always provide worthwhile patterning opportunities for children, and when children engage spontaneously in patterning, teachers sometimes fail to capitalize on the child’s interest, missing out on opportunities to extend children’s interest and knowledge in patterning (Fox, 2005). One possible reason for these missed opportunities might be teachers’ lack of focus or partial knowledge regarding some structural aspects of repeating patterns. Elements of structure include the minimal unit of repeat, the length of the unit of repeat and the number of times it is repeated, and whether or not the pattern ends in a complete unit.

The Cognitive Affective Mathematics Teacher Education Framework

Describing the Framework

It is widely accepted that the knowledge necessary for teaching a subject goes beyond knowing the subject matter and that knowledge of subject matter may also have various elements (e.g., Ball, Thames, & Phelps, 2008). It is also recognized that teachers’ self-efficacy beliefs may have an impact on their instruction (Allinder, 1994). Bandura defined self-efficacy as “people’s judgments of their capabilities to organize and execute a course of action required to attain designated types of performances” (1986, p. 391). The Cognitive Affective Mathematics Teacher Education (CAMTE) framework takes into consideration teachers’ knowledge for teaching mathematics as well as their self-efficacy for teaching mathematics in preschool. Like our previous studies concerning professional development for preschool teachers (e.g., Tsamir, Tirosh, Levenson, Tabach, & Barkai, 2014), the program described in this study was planned using this framework.

This paper focuses on the knowledge elements of the framework. These elements draw on the works of Ball and her colleagues (Ball et al., 2008) who differentiated between two aspects of pedagogical content knowledge (PCK) (Shulman, 1986): knowledge of content and students and knowledge of content and teaching. As before, we differentiated teachers' subject-matter knowledge (SMK) into knowledge for producing solutions and knowledge of evaluating given solutions. In this study, however, we refined our previous framework by dividing teachers' knowledge of students to include teachers' knowledge of ways in which students produce solutions and teachers' knowledge of students' abilities to evaluate others' solutions. As was shown in the previous section, most studies regarding children's patterning activities describe "production activities" (i.e., tasks where children have to produce something, such as building, copying, or extending a repeating pattern). However, it is also valuable for students to be given opportunities to engage in evaluation tasks (NCTM, 2000), tasks which require the learner to evaluate a given situation or solution. Likewise, teachers' knowledge of tasks was refined to include teachers' knowledge of designing and evaluating different tasks, specifically tasks that require students to produce solutions and tasks that require children to evaluate given solutions. Table 3.1 presents the framework and offers examples of knowledge elements with respect to each cell within the context of patterning.

Using the CAMTE Framework

Recently, we began investigating elements of preschool teachers' knowledge for teaching patterns (Tirosh, Tsamir, Levenson, Barkai, & Tabach, 2015). Related to Cell 1, we studied teachers' definitions for repeating patterns and their ways of drawing and continuing repeating patterns. Results indicated that participants found it difficult to write a definition for the notion of a repeating pattern yet were able to draw and extend a repeating pattern. In addition, although teachers correctly extended repeating patterns, there was a strong tendency on the part of the teachers to end patterns with a complete unit of repeat. That is, if the structure of the pattern is ABC, and they are shown the beginning of a pattern, for example, ABCABCABCA…, teachers tend to add BC or BCABC, and not just add a B, or BCA. Yet, repeating patterns, such as repeating decimals, do not always present themselves by ending in a complete unit. When dividing one by seven on a calculator, students might receive a solution of 0.142857142857142. Students need to recognize the pattern and surmise that after the two comes an eight, etc. Thus, it was suggested that the issue of ending or not ending a pattern in a complete cycle might be an aspect of pattern knowledge in need of more attention. Regarding cells 4a and 4b of the framework (see Table 3.1), another study found that most preschool teachers prefer production pattern tasks (e.g., extend the pattern) rather than evaluation tasks (e.g., is this a pattern?) (Tirosh, Tsamir, Levenson, Barkai, & Tabach, 2016). Taking into consideration the

Table 3.1 The refined CAMTE framework

	Subject matter		Pedagogical content			
			Students		Tasks	
	Producing	Evaluating	Producing	Evaluating	Producing	Evaluating
Knowledge	Cell 1 Identifying, describing, and creating repeated patterns, continuing a repeating pattern, identifying the unit of repeat	Cell 2 Evaluating correct and incorrect solutions to pattern tasks	Cell 3a Knowing children's strategies for solving patterning tasks, knowing correct and incorrect ways in which children will continue repeating patterns	Cell 3b Knowing examples and non-examples of patterns that children will easily identify as patterns or non-patterns	Cell 4a Knowing to design "producing" tasks	Cell 4b Knowing to design "evaluating" tasks
Self-efficacy	Cells 5 and 6 Mathematics self-efficacy related to cells 1 and 2		Cells 7a, 7b, 8a, and 8b Pedagogical-mathematics self-efficacy related to cells 3a, 3b, 4a, and 4b, respectively			

benefits of promoting preschool teachers' knowledge for teaching repeating patterns, including their knowledge of patterns as well as their knowledge of patterning tasks, this paper describes a professional development program that takes into consideration the necessity to promote preschool teachers' SMK and PCK for teaching repeating patterns.

Although this chapter does not focus on the affective side of the framework, the professional development program was designed to promote teachers' knowledge in a non-threatening way. Instead of explicitly stressing mathematics knowledge, the program was designed to take into account what Watson and Sullivan (2008) called teachers' obvious interest in planning and teaching lessons or, in the case of preschool teachers, their interest in activities that can be realistically implemented in classrooms with young children. As such, we designed patterning tasks that teachers could implement with children, but at the same time, we used those tasks to engage the teachers with the mathematics involved in patterning and to promote their knowledge of patterning tasks and children's ways of solving patterning tasks. The aims of this chapter are (1) to illustrate some elements of a professional development course for preschool teachers focusing on repeating patterns and (2) to investigate the affordances and constraints of using various pattern activities to promote preschool teachers' SMK and PCK related to teaching patterns.

Setting

The Program

Twenty-three preschool teachers participated in the program described in this study. All had a first degree in education and between 1 and 38 years of teaching experience in preschools. Many prospective preschool teachers in Israel attend only two mathematics education courses during their 4-year education degree. These courses sometimes include one semester for learning about the development of number concepts and one semester for the development of geometrical concepts. Thus, providing ongoing professional development focused on mathematics preschool education is imperative. Yet, while professional development is strongly recommended, and teachers are given credit for courses taken, the choice between programs is varied, and teachers are not necessarily mandated to specifically enroll in mathematics programs.

The program described in this study was planned for 21 h. The teachers met seven times over a period of about 4 months in the local professional development center in their area. Approximately five of the seven sessions were devoted to patterning with the other two focusing on number concepts. The main themes of each of the five sessions were as follows: (1) identifying repeating patterns, mathematical language, focusing on unit of repeat; (2) analyzing repeating pattern tasks, action and verbalization, using concrete materials, pictures, etc.; (3) choosing tasks for children, how to implement them, and how to use video as a tool; and (4 and 5) watching videos of the teachers in the program engaging children with repeating pattern tasks and analyzing the videos together. All lessons and tasks were planned by the four authors of this paper. The third author did the actual teaching and will be called in this paper the teacher educator (TE). All sessions were videotaped and transcribed.

The Tasks

Four main patterning tasks were used throughout the program (see Figs. 3.1, 3.2, 3.3, and 3.4). The first two were pictorial extension tasks (i.e., the patterns were presented as pictures on cards). However, the first task (see Fig. 3.1) was a production task, where one had to choose an element from a bank of elements and extend the given pattern. The second task (see Fig. 3.2) was an evaluation task where one had to evaluate different ways of extending various patterns and choose which ways were correct.

Note that for Task 1 the patterns presented have essentially three different pattern structures: AB, ABC, and ABB. In each case, the minimal unit of repeat is repeated at least three times. Taking this view, the first three patterns end with a complete unit of repeat; the last three do not.

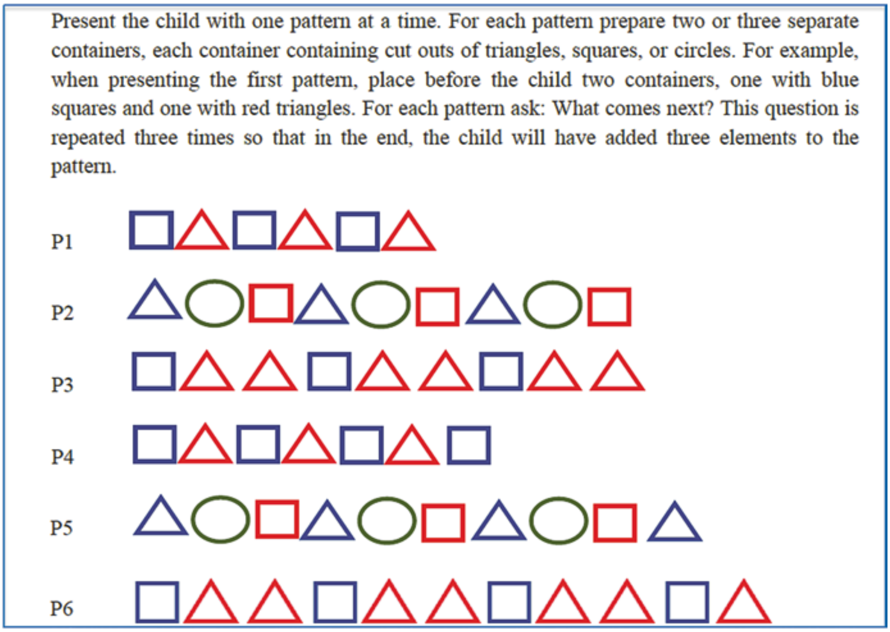

Fig. 3.1 Task 1 – what comes next?

For Task 2 (see Fig. 3.2) two patterns are used, both with an ABB structure. However, the first pattern ends with a complete unit of repeat, while the second does not. Furthermore, some of the possible correct extensions will end the pattern with a complete unit of repeat, and some will not. A main difference between Task 1 (see Fig. 3.1) and Task 2 (see Fig. 3.2) is that in the first task, extending the pattern is done one element at a time, while in the second task, the child has to look ahead and extend the pattern by two, three, or four elements at a time.

The third and fourth tasks involve concrete tangible items (colored beads) and are both production tasks in the sense that children are required to produce solutions as opposed to evaluate possible solutions. For Task 3, children are presented with two different pattern pairs and are requested to say how each two patterns are similar and how they are different (see Figs. 3.3 and 3.4). The first pattern pair consisted of two actual strands of colored beads. The first strand, S1, had three repetitions of the unit of repeat AAB, making a total of nine beads. The second strand, S2, had the same structure, but the colors of the beads in S2 were different from the colors of the beads in S1. The second pair of strands, S3 and S2, contained strands of the same colored beads and the same total amount of beads as S1, but the unit of repeat in each pair was different.

The fourth task consisted of having children construct a strand of beads with the same structure as one presented to them but with different colored beads (see Fig. 3.5). A key difference between Tasks 3 and 4 is that Task 3 calls for verbalization while in Task 4, children are requested to act.

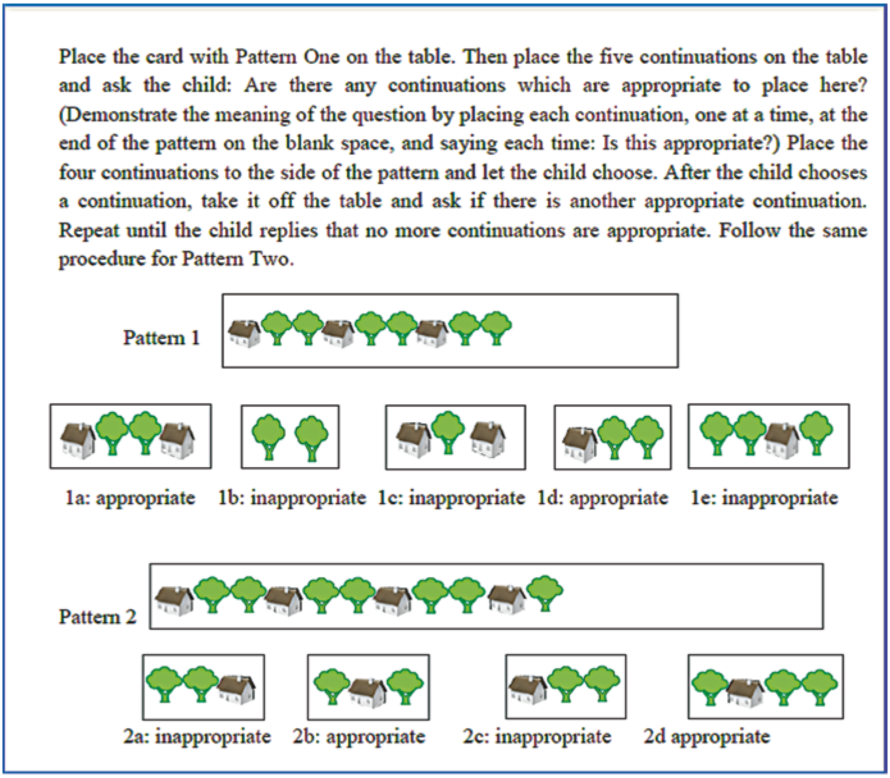

Fig. 3.2 Task 2 – which continuation is appropriate?

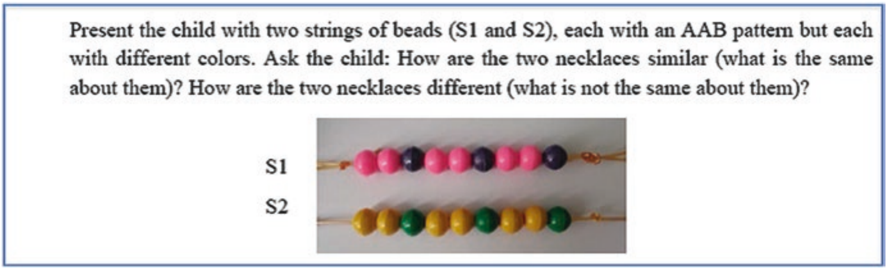

Fig. 3.3 Task 3, first pattern pair – what is similar and what is different?

A summary of the task characteristics, apart from the difference between the minimal units of repeat of the patterns, is offered in Table 3.2. These characteristics were inherent to the task. However, as will be seen in the next section, teachers were encouraged to discuss what may be varied within this framework. An essential element of learning about tasks (Cell 4 of the CAMTE framework) is understanding

Present the child with two strings of beads of the same colors, but one with an AAB pattern and one with an AB pattern. Ask the same questions as stated previously.

S3

S2

Fig. 3.4 Task 3, second pattern pair – what is similar and what is different?

Present the child with a string of beads with an AB structure (see S3 in Figure 4b above). Place on the table a string and two different colored beads (different colors from those on the presented strand). Ask the child to make a strand with the same pattern as the one you showed him. Repeat this activity but now present the child with a string of beads with an AAB structure (see S2 in Figures 4a and 4b above).

Fig. 3.5 Task 4 – construct a strand of beads

Table 3.2 A summary of the four tasks presented to preschool teachers

	Task 1	Task 2	Task 3	Task 4
Instructions	Extend the pattern one element at a time	Extend the pattern a few elements at a time	Compare two patterns	Build a pattern with a similar structure as a given pattern
Pictorial/ tangible	Pictorial	Pictorial	Tangible	Tangible
Production/ evaluation	Production	Evaluation	Production	Production
Verbal/action	Action	Action	Verbal	Action
Given pattern elements	3 full repeats of the minimal unit	(a) 3 full repeats of the minimal unit (b) 3 full repeats of the minimal unit and 2 additional elements	(a) 3 full repeats of the minimal unit (b) 4 1/2 repeats of the minimal unit	3 full repeats of the minimal unit

what may be varied in the task without changing the aim of the task. For example, if Task 4 used tangible items, and the TE presented this task with green and yellow beads, the teacher in the classroom might use red and blue beads, or yellow and green blocks. However, changing a task that was meant to use tangible items to one that would be pictorial was considered an inherent change to the task.

The variations in the tasks present different kinds of challenges to children. First, we regard the instructions. Extending a pattern, whether by one element or a few elements, is considered easier than tasks which require abstracting the pattern structure, such as Tasks 3 and 4 (Sarama & Clements, 2009). Regarding the use of pictorial representations versus tangible items, in general, one might think that the use of tangible items is more appropriate for young children. However, when it comes to repeating patterns, tangible items might be a distraction because they can be moved. For the first two tasks, it was important that children not be distracted by different spacing between the shapes. In the last two tasks, the string on which the beads were placed acted as an anchor for the elements. Thus, in this case presenting patterns using static pictures did not necessarily challenge the children more than presenting patterns with tangible items. Regarding production versus evaluation tasks, on the one hand, it might be easier to evaluate a solution that is presented rather than come up with a solution of one's own. However, in Task 2, which was an evaluation task, not only did children have to look ahead beyond the very next element, they had to consider that there might be more than one possible solution to the problem. In other words, production and evaluation tasks each present their own set of challenges. Regarding the issue of verbalization, one study reported an episode with a young girl who created a necklace out of game materials and described her necklace as "diamond, funny shape, diamond, funny shape" (Waters, 2004, p. 326). The challenge in this case is verbalizing not only what one sees but finding a way to express the abstractness of structure. Finally, as stated previously, children find it easier to extend a pattern when it ends with a complete unit (Tsamir et al., 2015). When a pattern is presented with an incomplete unit of repeat, the minimal unit of repeat might be more difficult to identify. For example, if the pattern ABAAB is a repeating pattern, is the minimal unit ABA or ABAAB? In either case, the next element would be A. But after that? In an attempt to make this minimal unit stand out a bit more, in our tasks, we always presented at least three repeats of what we considered to be the minimal unit of repeat.

Results: Program Segments

The following program segments were taken from the second and third sessions of the program because it was during these sessions that the above pattern tasks were introduced to the teachers. The teachers were told that these activities could be used with children in preschool and that later on in the program they would be asked to implement these activities with children, video the activity, and analyze together, in the course, the children's solutions. We analyze the segments according to the cells of the framework, pointing out how the tasks served to stimulate discussion revolving around the different knowledge cells of the refined CAMTE framework (see Fig. 3.1). The segments also illustrate how promoting different aspects of knowledge is intertwined.

Segment 1: To end or not to end the pattern with a complete unit of repeat
The participants, along with the TE, discuss the first three patterns of the first task and the differences between AB, ABC, and ABB patterns. They then examine the fourth pattern (P4).

1	TE:	What is the difference between P1 and P4?
2	Maya:	It (P4) has another cycle.
3	TE:	What do you mean by another cycle? I still have three repetitions of the minimal unit of repeat.
4	Sophie:	ABA.
5	TE:	Ok. The first ended with a complete unit and this one doesn't. It has the first element of the next cycle. When you video your children, we will see that some children choose which element comes next in the pattern by going back to the first element [of the given pattern] and adding that one. If the pattern ends in a complete unit, then that strategy works. But if they use that strategy here, what will they place?
6	Sherry:	A square.
7	TE:	But what really should come next is a triangle. That happens because most of the pattern tasks presented to children have patterns that end with a complete unit of repeat.

The minimal unit of repeat for both P1 and P4 is AB or, more specifically, . P1 has three cycles of the minimal unit of repeat, while P4 has 3 1/2 cycles. In the above segment, the TE begins by promoting teachers' mathematical knowledge of patterns. This knowledge includes using precise mathematical terminology such as minimal unit of repeat and pointing out that the word cycle is not appropriate if only one element of the unit of repeat is given. In Line 5, the TE goes on to describe one strategy children use when asked to find the next element of a pattern. This promotes teachers' knowledge of students' ways of solving problems (Cell 3a of the CAMTE framework, see Fig. 3.1). The TE ends (in Line 7) by explaining to the teachers that many children do not realize that this strategy does not always work because nearly all of the patterns they engage with are patterns which end in a complete unit of repeat. In that case, the strategy of extending a pattern by adding the first element of that pattern works. In other words, sometimes a task can be successfully completed without a child fully understanding the underlying concept. Knowing this about tasks, how to analyze a task by taking into consideration children's ways of thinking, is necessary for choosing tasks. The TE is promoting teachers' knowledge of tasks which call for children to solve mathematical problems (Cell 4a of the framework) and the need for engaging children with various patterns as well as various tasks.

Segment 2: Focusing on the minimal unit of repeat
In the following segment, there is some disagreement as to the minimal unit of P1.

8 Shena: I have a question. You said that the unit of repeat (in P1) is square triangle, but I see it as square triangle square and then a triangle and then square triangle square, and then a triangle. Could it be that a child will see it my way?

9 TE: Let's see. What Shena is saying is that the unit of repeat could be square triangle square and then a triangle.

10 Shena: The triangle is between the square triangle square.

11 TE: And what would you put here (pointing to the end of P1)?

12 Shena: A square and then a triangle. It comes out the same, but I see it differently.

13 TE: Ok. But what's the big difference between the ways we each see it. The way I see it, P1 ends in a complete unit of repeat, but in your eyes, the pattern ends with a partial unit. And that is why it is very important to ask the child to explain how and why he chooses to continue the pattern in a certain way. We need to be able to evaluate the child's solution. The next element will be the same either way, but the child may see it differently. He may have in his mind a different minimal unit of repeat than we do.

The above segment focuses on the intertwining of Cell 2 (being able to evaluate mathematical solutions) and Cell 3b (knowledge of students' ways of evaluating solutions) of the CAMTE framework. First, teachers must themselves be able to evaluate the correctness of their students' mathematical solutions. Can one say that the minimal unit of P1 is ABAB and not just AB? In addition, it is important to recognize that children see things in their own way and that their way of evaluating patterns may be different than ours, but their way of thinking is not necessarily apparent from their solutions. In this case, a child may complete the task successfully by adding the correct elements, but still may not recognize the minimal unit as AB.

Segment 3: Focusing on the task instructions
In the following segment, the TE presents to the teachers the second task and has them say what is similar and what is different about the two strings of beads (S1 and S2). After they discuss different ways in which the patterns are similar and different, the following interaction occurs:

14 TE: Maybe it would have been better to first ask the children what is different, and then ask what is the same.

15 Osher: It's hard to say what is more difficult, saying what is similar or saying what is different.

16 TE: And after the children answer, you should ask if there is anything else similar, anything else different. Keep on asking till the children have nothing to add…. Now look at what would happen if I turn this one (S2) around (the TE flips S2 over so the left most bead is now green).
17 Osher: It's confusing.

When promoting knowledge of tasks, there are several elements to consider. First, there are the instructions, what the child is asked to do. But, there is also the sequence of instructions. What do we ask the child to do first and what do we ask the child to do second? The sequencing of the steps in the task may have an impact on the child's ability to complete the task. Thus, knowledge of children's patterning abilities (Cell 3) may impact on how the task is set up (Cell 4). Furthermore, if we request the child to complete a task only once, the extant of that child's knowledge may not be evident. Thus, the TE suggests asking the child over and over again to say what is the same and what is different. In other words, how we implement a task may impact on the knowledge we, as teachers, gain of our children's patterning conceptions.

Segment 4: Discussing task materials and characteristics
During the third session, the TE reviewed all four tasks which had been presented previously, this time drawing the teachers' attention not to the patterns but to the task features. Although during the professional development program we had supplied the task materials, it was understood that preschool teachers would use materials and supplies found in their own classes. Thus, discussing the actual material and how they might impact on the students' engagement with the task was important.

18 TE: Now, the materials (used in the tasks) are all different. There is what is called pictorial, drawings that I show them (the children), like the stickers that you use (stickers with pictures) on paper because it's hard for children to draw. There is also a tangible pattern, where I place a blue bottle cap on the table, then a red, and so on. Within those types there is still a wide variety – geometric shapes as opposed to abstract symbols. In addition, there are movement patterns (the TE demonstrates by patting her shoulders, raising her arms, and repeating three times) and sound patterns.
19 Rachel: What are pictorial patterns?
20 TE: Like these that I showed you (pointing to the patterns used in Task 1). The necklace (used in Task 4) uses real beads so that's tangible. The child actually strings the beads.
21 Rachel: But what is a sound pattern?

22 TE: Like a rhythm you hear that repeats. Or if I say Shena, Rachel, Osher, Shena, Rachel, Osher, Shena, Rachel, Osher. Now with a sound and movement pattern, you only see or hear the last element in the pattern. Right? If I do this (makes a pattern with hand movements), as soon as I do the second movement, the first is gone. And when I make the third movement the second is gone. With a drawing or with tangible items, I see the whole pattern.

In the above segment, the TE notes four types of pattern presentations. The first is a pictorial presentation, which (in Line 18) the TE says may be stickers. The stickers in this case are not used in the pattern as tangible items such as the beads are used, but are stickers with pictures on them. In addition, stickers are not mentioned merely because they are fun and available. Previously, the participants had discussed a pattern which only had triangles, but triangles of different sizes. The teachers and the TE discussed how difficult it was for young children to accurately draw these different size triangles, and so stickers with pictures of triangles could be used instead.

The various types of pattern representations allow the teachers to encourage children to use their sight, hearing, and touch senses. While this is especially important for young children, the different representations have different impacts on children's ability to extend a pattern. This is pointed out by the TE in Line 22.

In addition to the modes of representations, one of the teachers brings up the use of color in the beads when discussing Task 4.

23 TE: For the fourth task, you bring beads. Here is my strand (the TE holds up a strand of yellow and green beads with an AB structure). See what a nice pattern I have here. And then you show the boy or girl different color beads (the TE holds up a bowl with purple beads and another bowl with pink beads). It doesn't have to be these colors. Then ask the child to make a strand of beads with the same pattern as the one you have but with different colors.

24 Shena: Why a strand with different colors?

25 TE: Because otherwise, (if the child is given the exact same color beads as the one presented to him) it will just be a simple duplication task. One to one, he (the child) can say, first there is a yellow bead, so I will take a yellow bead. In our case, we can see if the child understands the concept of a pattern.

In the above segment, we see how a discussion of materials and a seemingly innocent question of color led to a more in-depth analysis of the task. Sometimes, the issue of color is unimportant. In Line 23, the TE says that the teachers can use any colors they wish. However, at times, the issue of color is important and can lead to very different types of tasks. Duplicating a pattern is simpler than abstracting a pattern from one medium and creating it in another medium. In this case, the hard material (beads) remains the same, but using different colors adds complexity.

The above segments specifically dealt with promoting teachers' knowledge of production tasks (Cell 4a). However, in the refined CAMTE framework (Fig. 3.1) we differentiated between two types of tasks – solving and evaluating tasks. This is pointed out in the following interaction:

26	TE:	The first task and the second task are different types of tasks. The first task is a producing task. The child has to extend the pattern. The second task is different. I show the child a pattern, and I say this is a pattern. Then I take this strip of paper with trees and houses on it, and I ask the child if this can be the continuation of the pattern. So, what kind of task is this – a production task or an evaluation task?
27	Many teachers:	An evaluation task.
28	Gale:	I don't understand why it's an evaluation task?
29	TE:	Because the child doesn't have to choose an element to continue the pattern. Instead, when I give him the strip of paper (with the drawings), he has to decide if this is a correct way to continue the pattern.

To summarize, several elements of patterning tasks, such as materials, are important to analyze for all patterning tasks, and thus all three vignettes above relate to Cell 4 of the CAMTE framework, promoting teachers' knowledge of patterning tasks. Specifically, the last few lines illustrate the difference between tasks which require the child to produce solutions and tasks which require the child to evaluate others' solutions.

Summary

The above segments represent only a sample of what took place during the entire program. Yet, several different aspects of teachers' knowledge for patterning were mentioned. Regarding teachers' SMK, we saw the emphasis on using precise mathematical language, recognizing the minimal unit of repeat and evaluating solutions. (In other studies, we focused to a greater extent on teachers' knowledge of solving patterning tasks and their knowledge of evaluating patterning tasks solutions (cf., Tirosh et al., 2015).) Regarding teachers' knowledge of students, the main issues discussed with participants while working on the tasks were different strategies children may use when extending a pattern and which tasks are simpler or more difficult for children. These and other aspects of knowledge of children's patterning abilities were again brought up later in the program when teachers viewed the videos of children engaging with the pattern tasks. Most of all, however, the above segments illustrate the promotion of teachers' knowledge for teaching patterning. Among the various points discussed were task instructions, modes of pattern representations, materials, and types of tasks. Recognizing that attention to detail is important, the TE and participants also related to the sequencing of patterns and the sequencing of instructions, colors, and how many times to repeat a question.

Epilogue

It is beyond the scope of this paper to convey all the different elements of the program. Instead, we jump right to the end. During the program, each teacher was required to video themselves implementing one pattern production task and one pattern evaluation task with at least one child in their preschool class. As a final project, the teacher was required to analyze and summarize what she learned from implementing the activity in terms of the mathematics involved, children's patterning concepts, and patterning tasks and summarize the experience. Here are excerpts from what three teachers wrote:

> T1: I sat with a five-year-old child, but in my opinion, the activity is relevant for all ages. The materials used were simple and appropriate. The patterns were not too difficult. Also, the way the activity is implemented is important, the way the task is presented, the instructions, the activity.
>
> T2: I greatly enjoyed watching the video with the other program participants. I felt proud of Shelly (the girl with whom she implemented the tasks) and the way she so nicely cooperated.
>
> T3: I was pleasantly surprised by the activity because I sat with a child who is 3 years and 10 months old and he knew how to identify patterns, continue a pattern which ended in a complete unit and one which did not end with a complete unit. Beforehand, I never worked with preschool children on patterns because I thought they were too little…. This experience caused me to understand that I can begin to work on patterning even with young children.

T1 stresses the activity, mentioning the patterns, materials, and the way it is implemented. T2 and T3 focus on the participants, the child they sat with, as well as themselves. T2 notes the child's cooperation, and T3 mentions the child's correct performance. Both write about their own enjoyment as teachers. T3 also points at that she has gained new knowledge about children's patterning abilities. Although we only bring three short excerpts, taken together, they reflect an overall positive experience with the professional development program, which can raise their self-efficacy for teaching patterning in preschool.

Discussion

Watson and Sullivan (2008) suggested that tasks for teachers have multiple purposes: to inform teachers about the variety and purpose of classroom tasks, to provide opportunities to learn more mathematics, to provide insight into the nature of mathematical activity, and to stimulate teachers' theorizing about students' learning. In this paper, we described a program that used classroom tasks to do just that.

At the start of the program, most teachers claimed that in their classroom, children engage in patterning tasks when they draw boarders or frames for pictures. It could be that the teachers were not aware of various patterning activities that can develop children's appreciation for pattern structure. This is in line with Zazkis and Liljedahl (2006) who found that although the topic of patterns may be found in curriculum objectives, pattern activities are often relegated to enrichment activities and not dealt with as seriously as intended. Reflecting on the program segments described above, it might be said that the instructor took a leading role during the sessions, introducing mathematical terms and posing questions, while the preschool teachers responded and reacted. One reason for this stance was the necessity of introducing mathematical language which would allow the teachers to engage with patterns on a mathematical level. In addition, as can be seen in several instances, the instructor's question often led one of the teachers to ask a question, leading to a deeper analysis of the pattern or of the task.

Our professional development program introduced teachers to various mathematically engaging patterning activities. The tasks also provided opportunities for the teachers to learn mathematics. In a previous study, it was found that when writing a definition for a repeating pattern, some preschool teachers wrote statements that mentioned the content of the pattern and that there is repetition, but did not mention structure (Tirosh et al., 2015). Building on that study, during the program described here, we discussed with the teachers critical attributes of a pattern as we analyzed tasks (e.g., that there is a specific core unit made up of a string of elements, the string of elements are not randomly laid out but have a fixed structure, and the unit is repeated.) The TE also used the first preschool patterning task to discuss with the teachers the broader issue of sequences, asking them, for example, what element of the pattern would appear in the 18th place. Affording preschool teachers a glimpse of how patterning in preschool will be developed in later school years may also increase their motivation to engage children with patterning activities during the early years.

In addition to promoting teachers' specific content knowledge, focusing on the unit of repeat and structure may enhance teachers' appreciation for the nature of mathematics. According to Schoenfeld, mathematics is "a living subject which seeks to understand patterns that permeate both the world around us and the mind within us" (Schoenfeld, 1992, p. 334). Thus, by engaging in patterning activities and, specifically, the comparison task, the preschool teachers were able to gain insight into the nature of mathematical activity and to see that mathematics is more than number concepts. This is especially significant in light of studies which found that most preschool teachers believe preschool mathematics consists of mastering arithmetic (e.g., one-to-one correspondence, counting, writing numerals) (Lee & Ginsburg, 2007). When asked to describe mathematical activities that take place in their preschool, the activity most often mentioned was counting (Benz, 2010). Finally, as the teachers engaged with the activities, they began to theorize about their young students' learning. This was evident in the above segments as teachers began to contemplate how the setup of an activity might affect children's performance.

Beyond listing the affordances of using classroom tasks during professional development, we also note a few constraints. While engaging teachers with classroom tasks (meant for children) might increase teachers' engagement, it may sometimes deflect their attention from other aspects. For example, some teachers were so enthusiastic about receiving from the TE ready-made activities that they tried out some of the activities in their class before it was fully analyzed in the program. A few teachers did not use the materials as were presented to them. For example, regarding Task 1 (see Fig. 3.2), teachers were told to prepare five separate containers for each possible cutout, and only present to the child those containers which contained cutouts for that pattern. Some teachers kept all of the containers on the table, no matter which pattern was being extended, while other teachers separated the elements into only three containers according to shape (e.g., putting blue and red squares in one container) and then placed all three containers on the table, no matter which pattern was being extended. A few teachers did not give the instructions as they were meant to be given. For example, some teachers asked the children to say out loud each element from the beginning of the pattern to the end, before asking them continue the pattern. Reflecting back on the instructional stance of the course, one concern might be for teachers' autonomy. These results demonstrate that despite the instructive stance of the instructor, the preschool teachers retained their independence and flexibility and varied aspects of the task that had been "instructed" to them. While a certain amount of variety is welcome, and teachers are certainly autonomous to enact activities in the way they see fit for their class, studies have shown that the way a task is implemented will affect the level of cognitive challenge felt by the student (Henningsen & Stein, 1997). Thus, it is important to understand how to implement a task in order to understand how each element of the task affects another element.

In addition, like T2 in the epilogue, many teachers were satisfied with the children's cooperation and apparent enjoyment of the activity. While this is commendable, we would also like to see teachers enthusiastic about the mathematical learning that is going on. Perhaps the focus on activities that were specifically designed for children, as opposed for teachers, took away from the fact that the activities were designed to promote mathematical learning. Yet, is this truly a constraint? There is a folk saying which says, "That which is done by rote will eventually come to be done with meaning." In our case, we believe that teachers, who are enthusiastic about implementing mathematical activities in their early childhood classrooms, even if it is "just" because they are fun, will come to see, with the guidance of professional development, the mathematical value of such activities. Furthermore, the refined CAMTE framework can help TEs, as well as teachers, pay attention to the different aspects of mathematical knowledge needed for teaching while enjoying the mathematical activities.

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